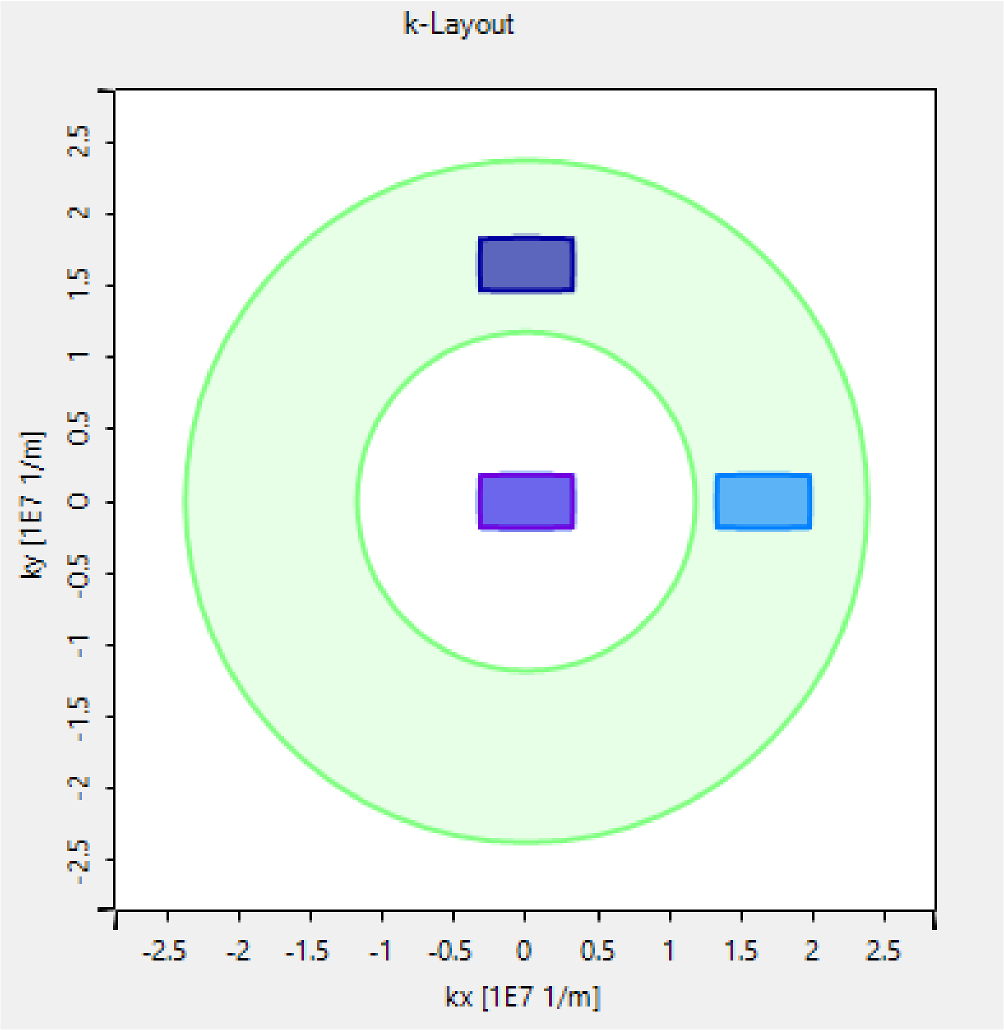
k-Domain Layout Visualization ("The Magic Circle")

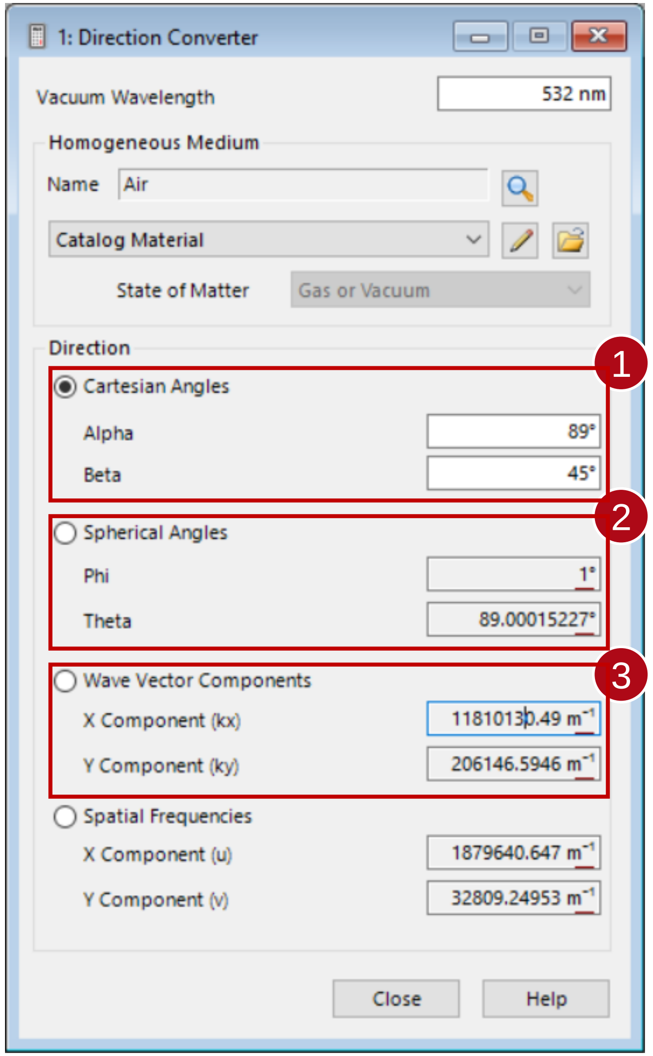
Abstract



Lightguide devices for augmented- and mixed-reality applications must fulfill two simultaneous functions: they must allow the light scattered by the environment to pass through (i.e. be transparent) and at the same time transport the artificial image to be superimposed from the emitter to the eye of the user. This is most often realized with transparent lightguides in which light can be guided under TIR in combination with different types of grating structures on the surfaces to couple light in and out. In VirtualLab Fusion, the k-Layout Visualization tool provides a powerful illustration method in the k-domain to analyze the guiding and coupling conditions of a specific lightguide layout.

The Concept

Direction Converter Calculator



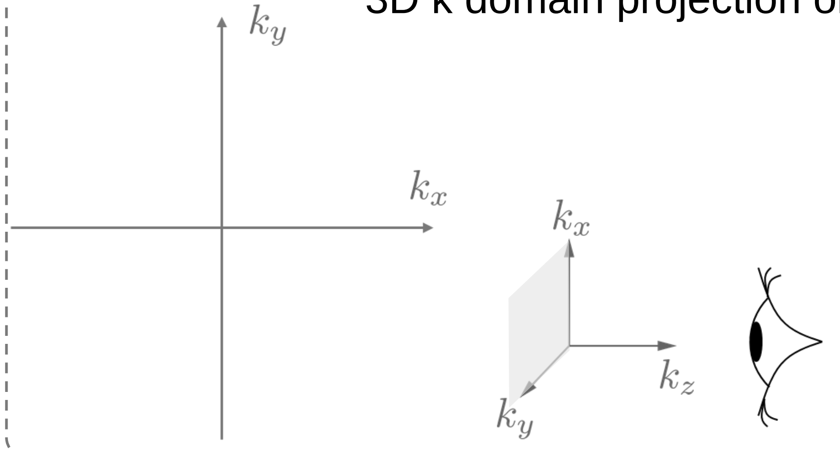
- A plane wave (FOV* mode) is charactered by its direction ($\hat{\mathbf{s}} \stackrel{\text{def}}{=}$ unitary direction vector). This direction can be expressed using different conventions:
  1. For lightguides, it is common to use Cartesian angles, θ_x and θ_y,
  $$\hat{\mathbf{s}} = \frac{(\tan\theta_x, \tan\theta_y, 1)}{\sqrt{1 + \tan^2\theta_x + \tan^2\theta_y}}$$
  2. Using spherical angles (θ and ϕ), the equivalent is
  $$\hat{\mathbf{s}} = (\sin\theta\cos\phi\,, \sin\theta\sin\phi\,, \cos\theta)$$
  3. The direction of the plane wave is related to its wave-vector $\boldsymbol{k}$, via
  $$\boldsymbol{k} = k_0 n \hat{\mathbf{s}}$$

  Where $k_0 = 2\pi/\lambda$ with λ the wavelength in vacuum, and n the refractive index of the medium in which we consider the wave.
- The *Direction Converter Calculator* can be found via *Start > Calculators*, which helps to demonstrate different ways how angles can be specified.

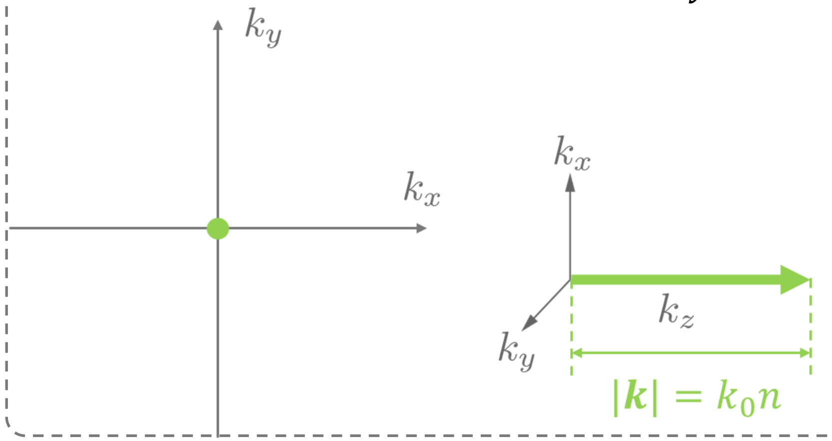
* field of view

k-Domain Visualization

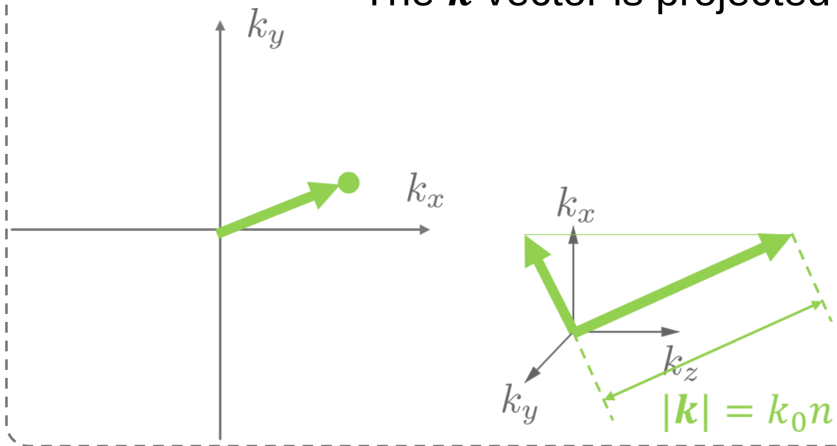
3D k domain projection on (k_x, k_y) plane.



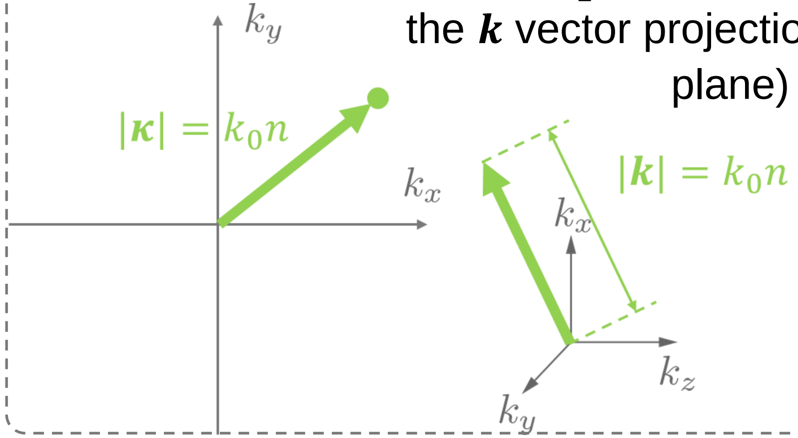
When $k_x = 0, k_y = 0$, the $\boldsymbol{k}$ vector lies along k_z axis.



The $\boldsymbol{k}$ vector is projected on the (k_x, k_y) plane.



When $k_z = 0$, the $\boldsymbol{\kappa}$* value (length of the $\boldsymbol{k}$ vector projection on the (k_x, k_y) plane) is the maximum.

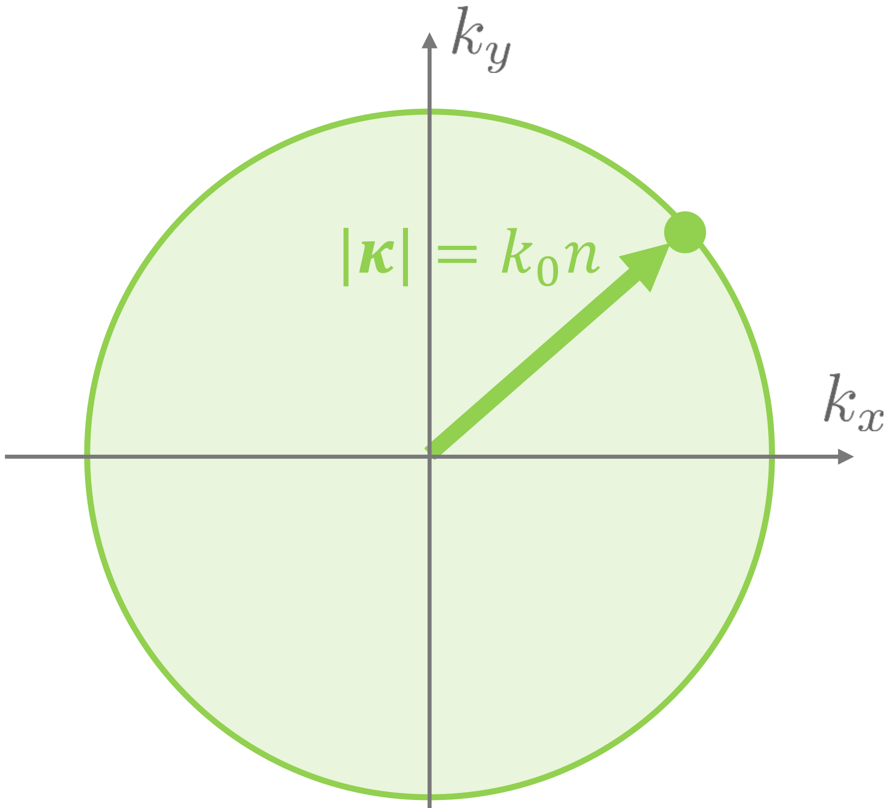


* $\boldsymbol{\kappa} \stackrel{\text{def}}{=} \boldsymbol{k}_\perp$, $\boldsymbol{\kappa} = (k_x, k_y)$

k-Domain Visualization: Propagation of the Plane wave

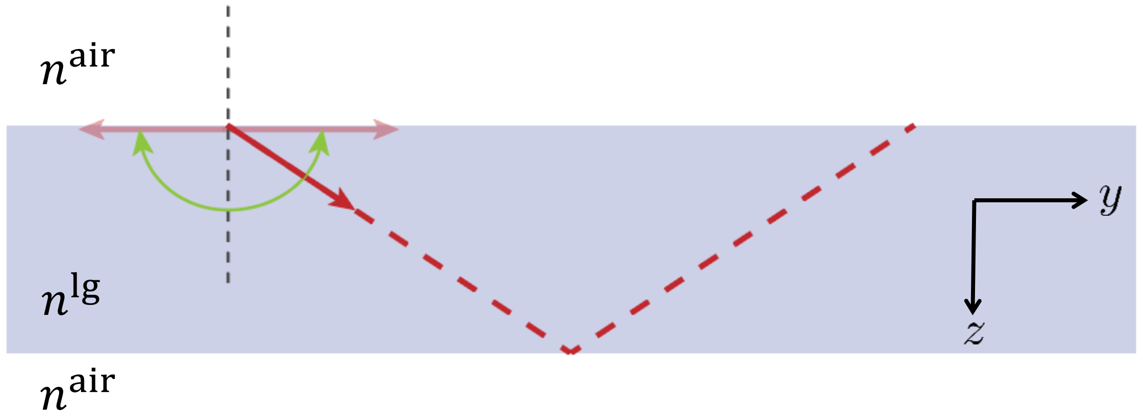
- For a plane wave to propagate in a given medium, the coordinates of the wave-vector are corresponded to one of the possible directions in space. As shown above, the propagation directions must be below 90°.

- Consider this propagation direction range in the k-domain, the maximum $\boldsymbol{\kappa}$ value is corresponding to the maximum angle of 90°, i.e., $k_z = 0,$ therefore, the projections of all the $\boldsymbol{k}$ vector that can propagate in the material are inside the circle of maximum $\boldsymbol{\kappa}$.



All points contained inside this circle correspond to $\boldsymbol{k}$ vectors allowed to propagate in a medium with refractive index n, for radiation of a given wavelength λ.

k-Domain Visualization: Total Internal Reflection

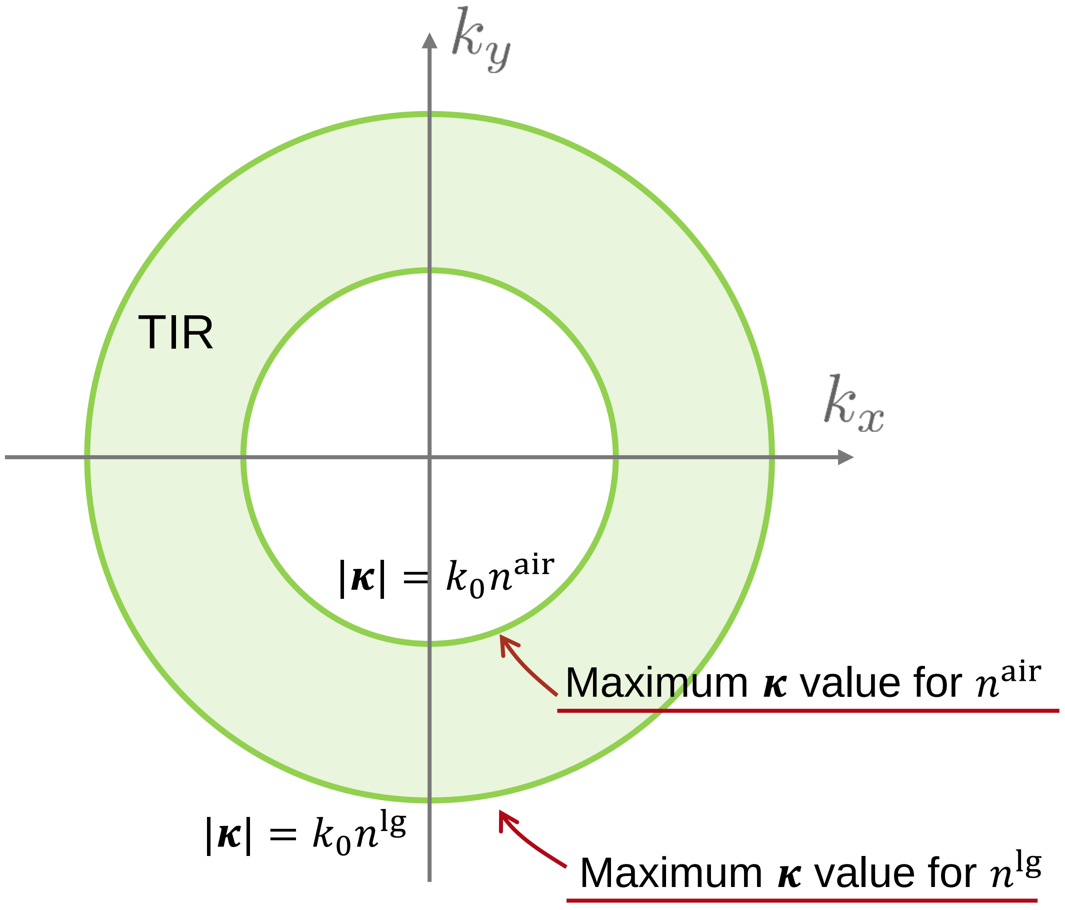


- In order to "trap" the light inside the lightguide, the range of $\boldsymbol{\kappa}$ values should inside of the circle of lightguide material but outside the circle of air.

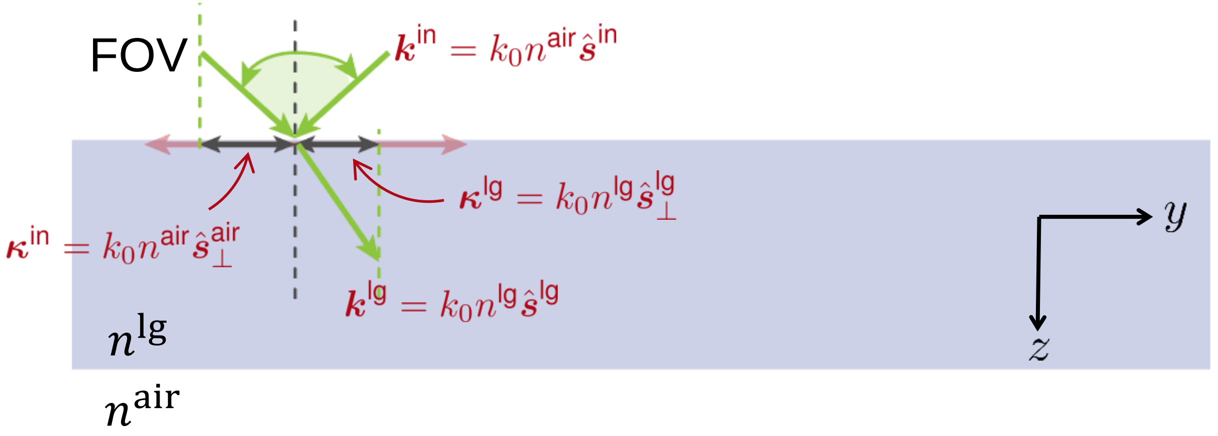
- Therefore, the total internal reflection (TIR) occurs for those $\boldsymbol{\kappa}$ values which can propagates in the denser material of the lightguide, but not outside.

- This corresponds to the ring enclosed by the two circumferences:

$$k_0 n^{\mathrm{air}} \leq k^{\mathrm{TIR}} \leq k_0 n^{\mathrm{lg}}$$



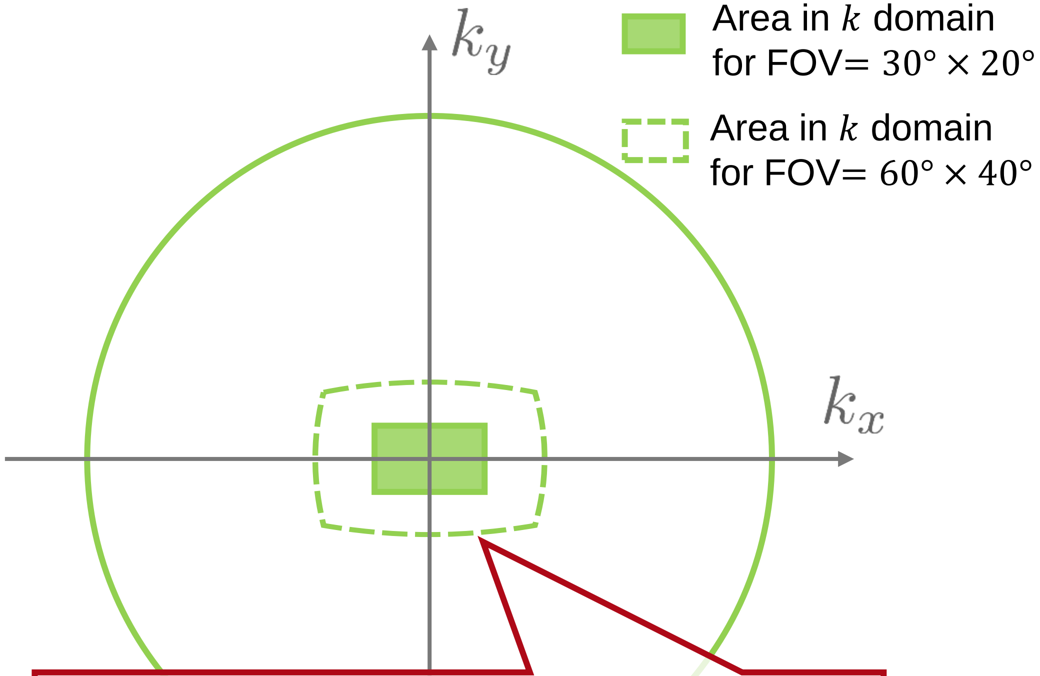
k-Domain Visualization: the FOV “Box”



- The FOV angles in air and the $\boldsymbol{k}$ vector in air are related via

$$\boldsymbol{k}_{\perp}^{\text{in}} = \boldsymbol{\kappa}^{\text{in}} = k_0 n^{\text{air}} \hat{\boldsymbol{s}}_{\perp}^{\text{in}} \propto k_0 n^{\text{air}} (\tan\theta_x^{\text{in}}, \tan\theta_y^{\text{in}})$$

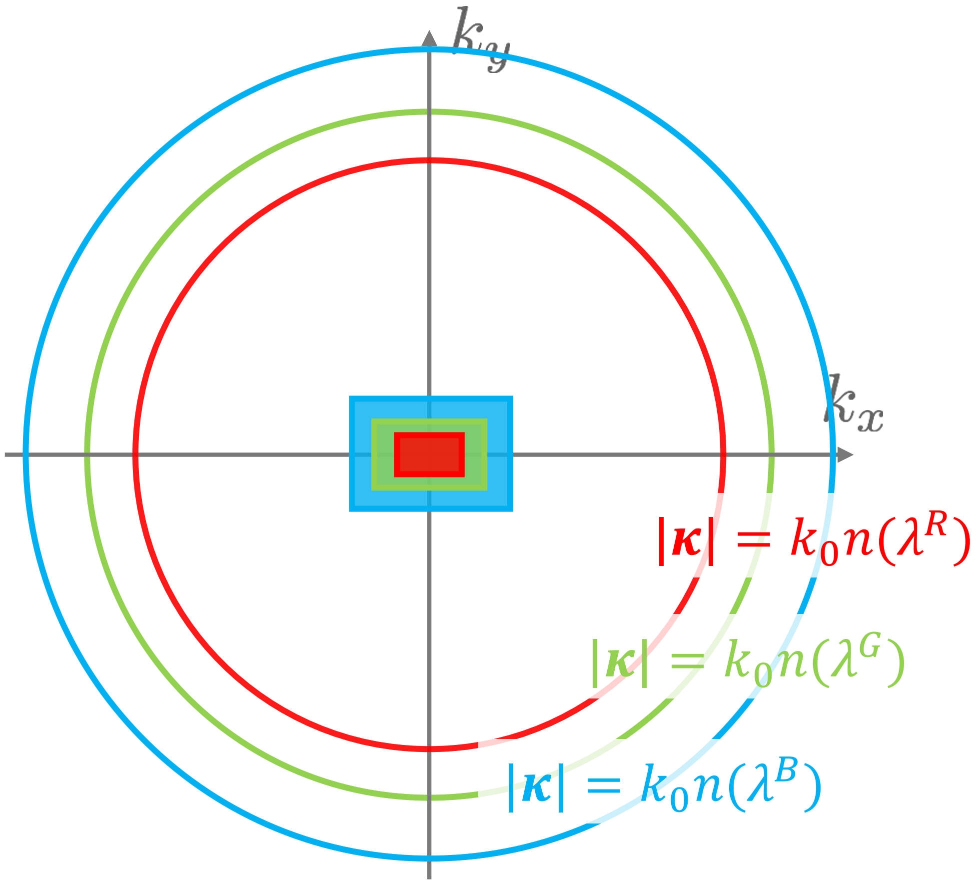
- The $\boldsymbol{\kappa}$ values for a certain range of FOV form a k-domain area on the (k_x, k_y) plane, that is the FOV “box”. The wider the FOV is, the larger the “box”.
- According to the boundary condition, $\boldsymbol{\kappa}^{\text{in}} = \boldsymbol{\kappa}^{\text{lg}}$, i.e., in the k-domain, the FOV region will be the same inside the lightguide as for the input field.



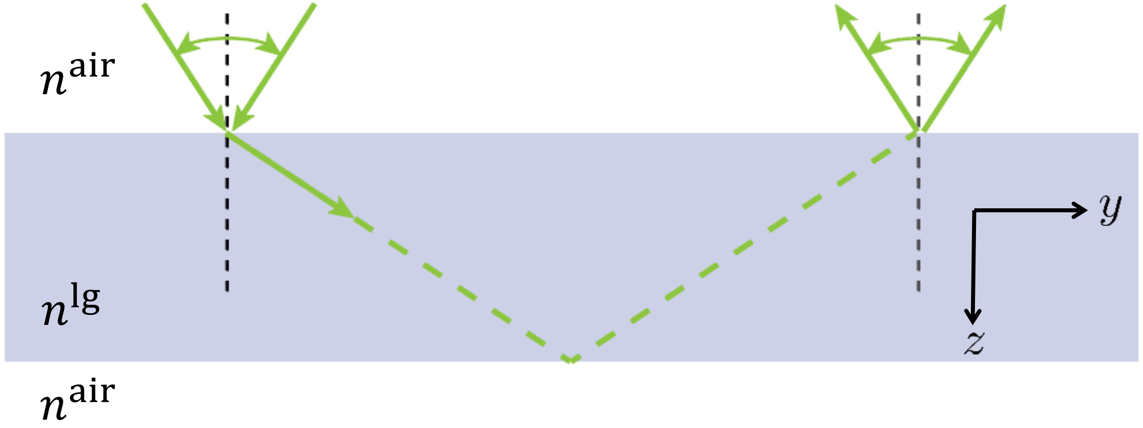
Due to the non-linear (trigonometric) relationship between the angles and their $\boldsymbol{\kappa}$ projections, the FOV “box” is not really a rectangle. The distortion becomes more noticeable the larger the angles involved.

k-Domain Visualization: RGB and FOV in k Domain

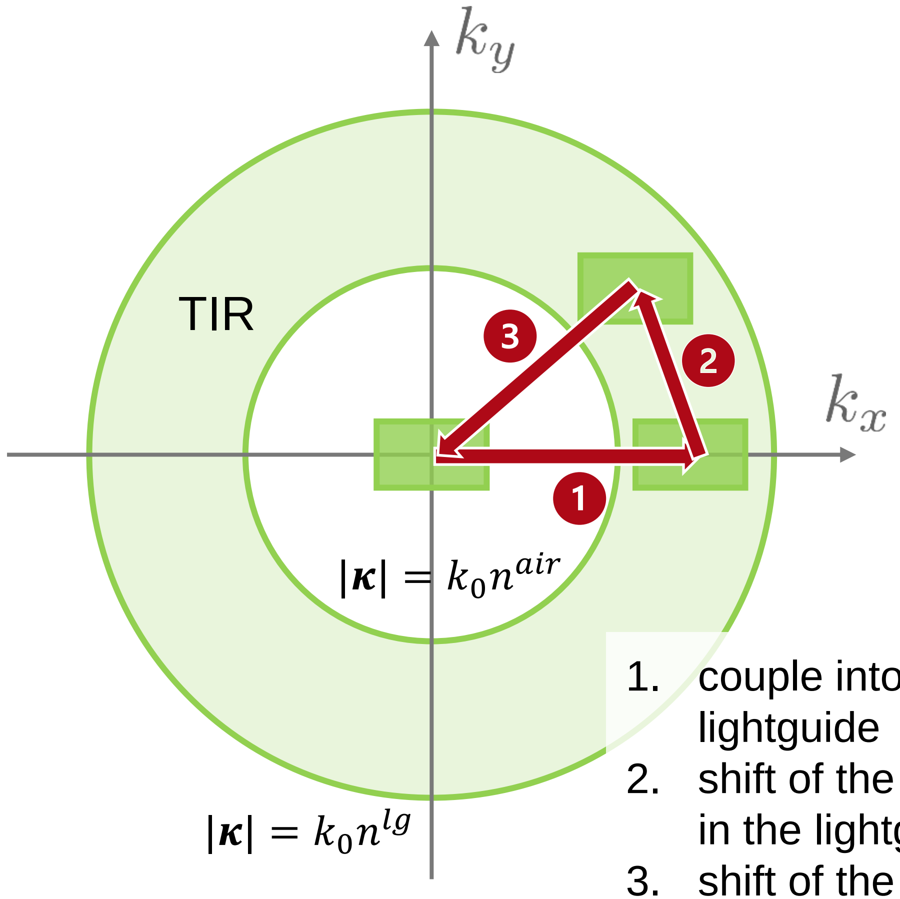
- In all calculations in the k-domain, the wavelength affects the quantities via
$$k_0 = 2\pi/\lambda.$$
- In addition, the material dispersion $n(\lambda)$ should be included in accurate modelling.
- The main effect of λ on the k-domain quantities is a scaling with the factors 1, 1.6, 0.82 for green, blue and red, respectively.



k-Domain Visualization : Transportation of the FOV “Box”



- The initial FOV is defined in the air, therefore the initial FOV “box” must fit in the inner circle.
- To be able to transport the whole FOV range to the eye via TIR, the entire FOV “box” must fit inside the ring enclosed by the two circumferences.
- At the end of the propagation, the FOV “box” should be coupled back out of the lightguide at the position of the eye box, to meet the eye of the wearer.



1. couple into the lightguide
2. shift of the FOV box in the lightguide
3. shift of the FOV box in the lightguide

k-Domain Visualization: Coupling with Gratings

- In a system consisting exclusively of the slab of glass, the $\boldsymbol{\kappa}$ vector, $\boldsymbol{\kappa} = (k_x, k_y)$, does not change ($\boldsymbol{\kappa}^{\mathrm{air}} = \boldsymbol{\kappa}^{\mathrm{lg}}$) and the beam just passes through the plate, without changing direction (it only experiences a small lateral shift).
- Therefore, in order to "trap" the beam inside the lightguide, it is expedient that the $\boldsymbol{\kappa}$ vector be modified upon entrance to the slab of glass.
- Gratings offer a very elegant way of realizing this change of $\boldsymbol{\kappa}^{\mathrm{lg}}$. According to the grating equation for linear gratings in the k-domain :

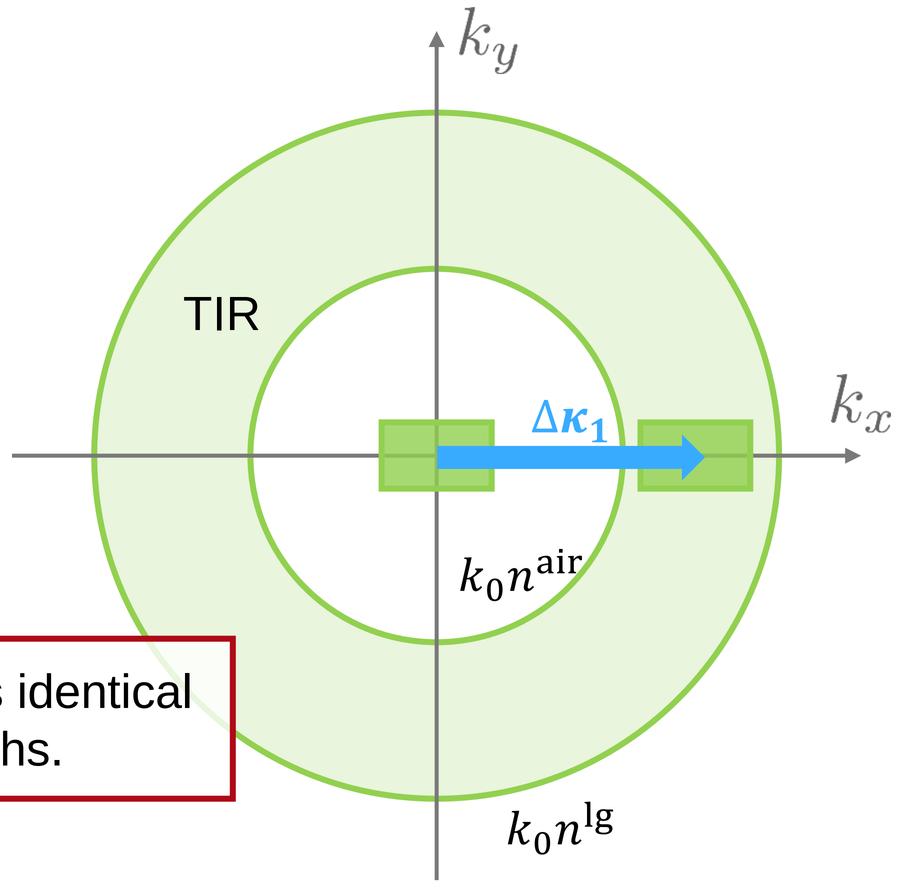
$$\boldsymbol{\kappa}^{\mathrm{lg}} = \boldsymbol{\kappa}^{\mathrm{in}} + m\Delta\boldsymbol{\kappa}$$

where $m \in \mathbb{Z}$ indicates the grating order under consideration and with the grating vector

$$\Delta\boldsymbol{\kappa} = 2\pi\left(\frac{1}{d_x}, \frac{1}{d_y}\right)$$

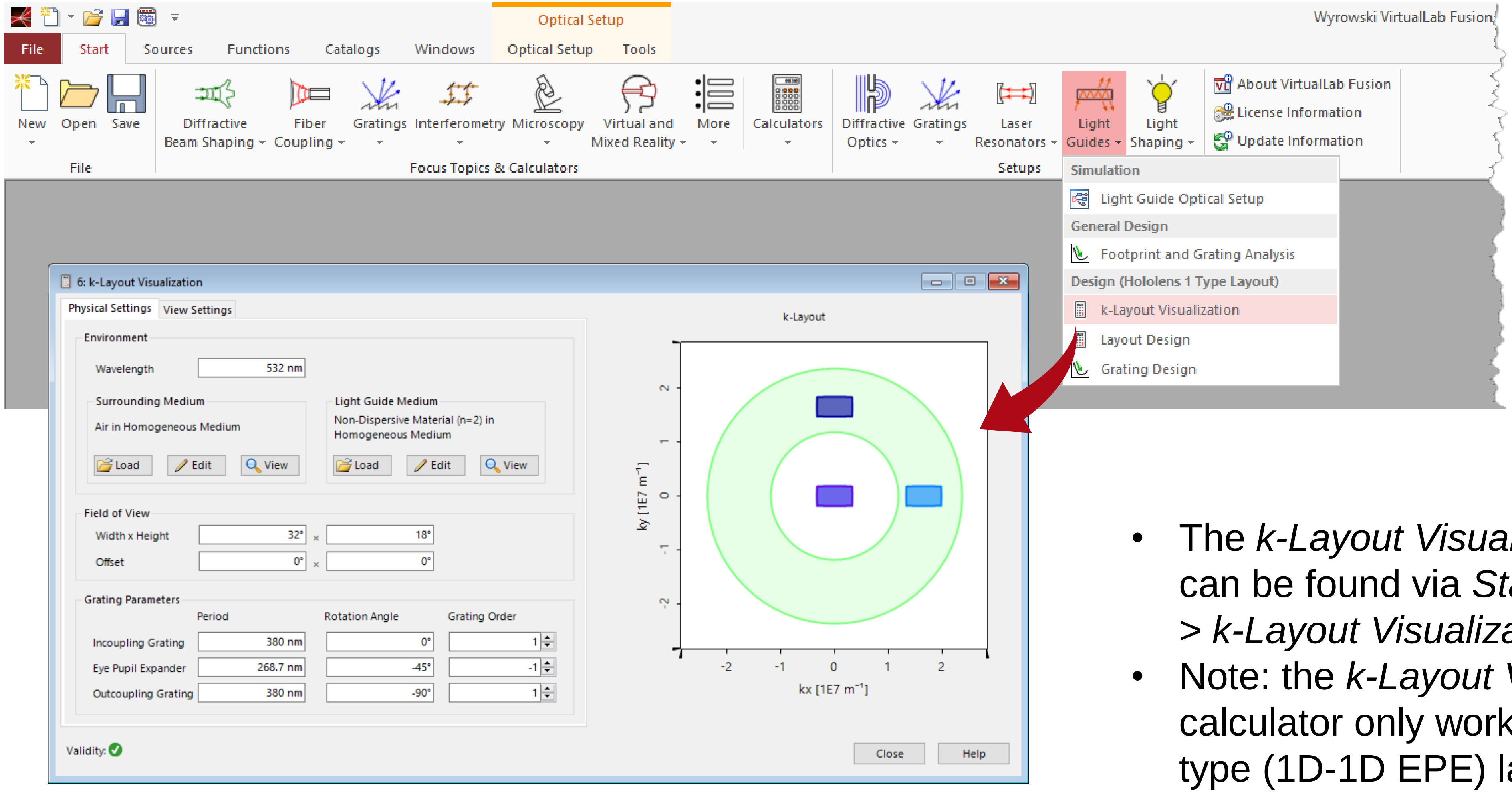
with (d_x, d_y) the period of the grating along the indicated dimensions.

Grating vector is identical for all wavelengths.



Using the k-Layout Visualization Calculator

k-Layout Visualization Calculator In VirtualLab Fusion



- The *k-Layout Visualization* calculator can be found via *Start > Light Guides > k-Layout Visualization*
- Note: the *k-Layout Visualization* calculator only works for HoloLens 1-type (1D-1D EPE) layouts

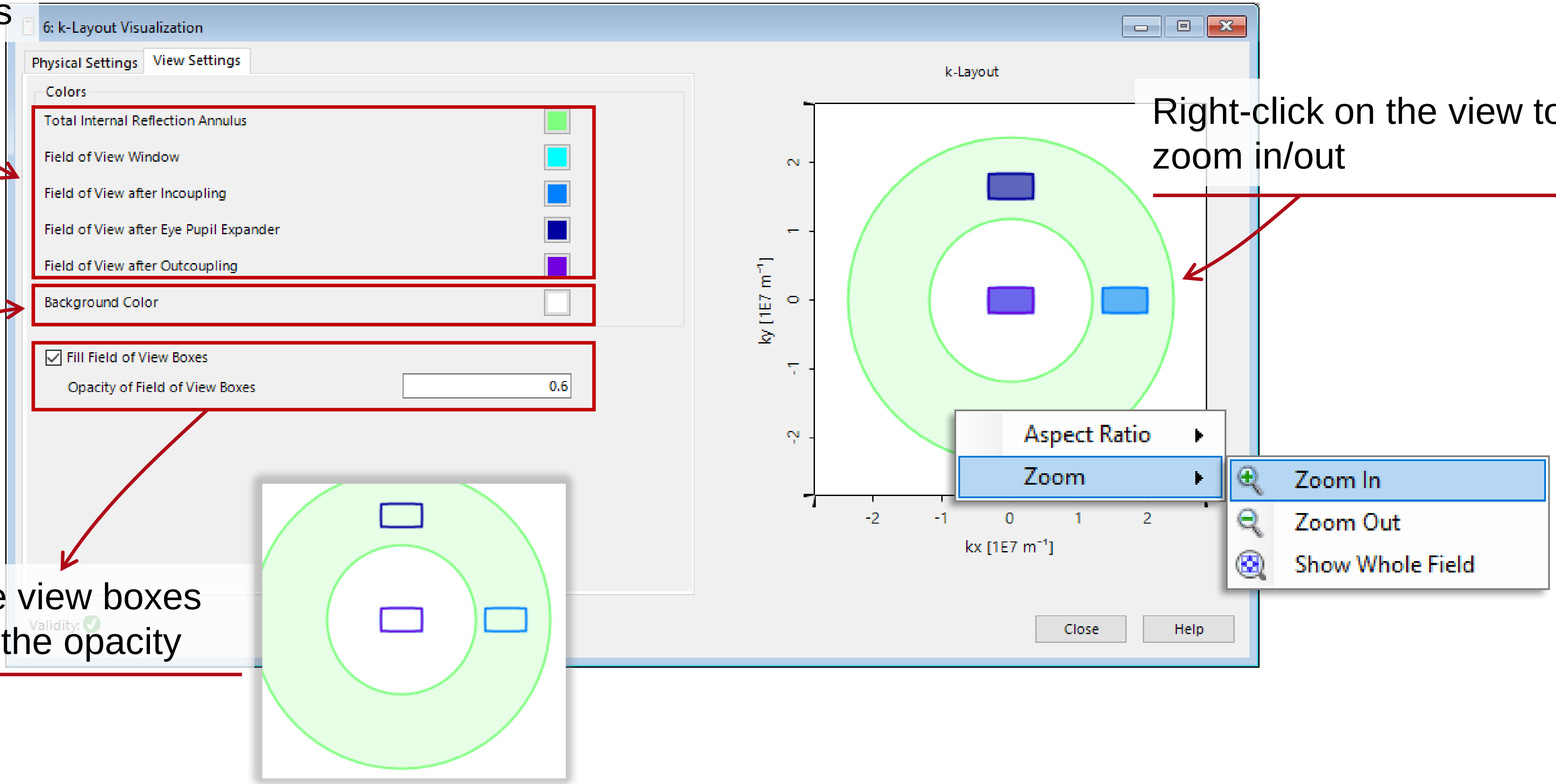
View Settings

define colors of the TIR annulus and the FOV boxes

define background color

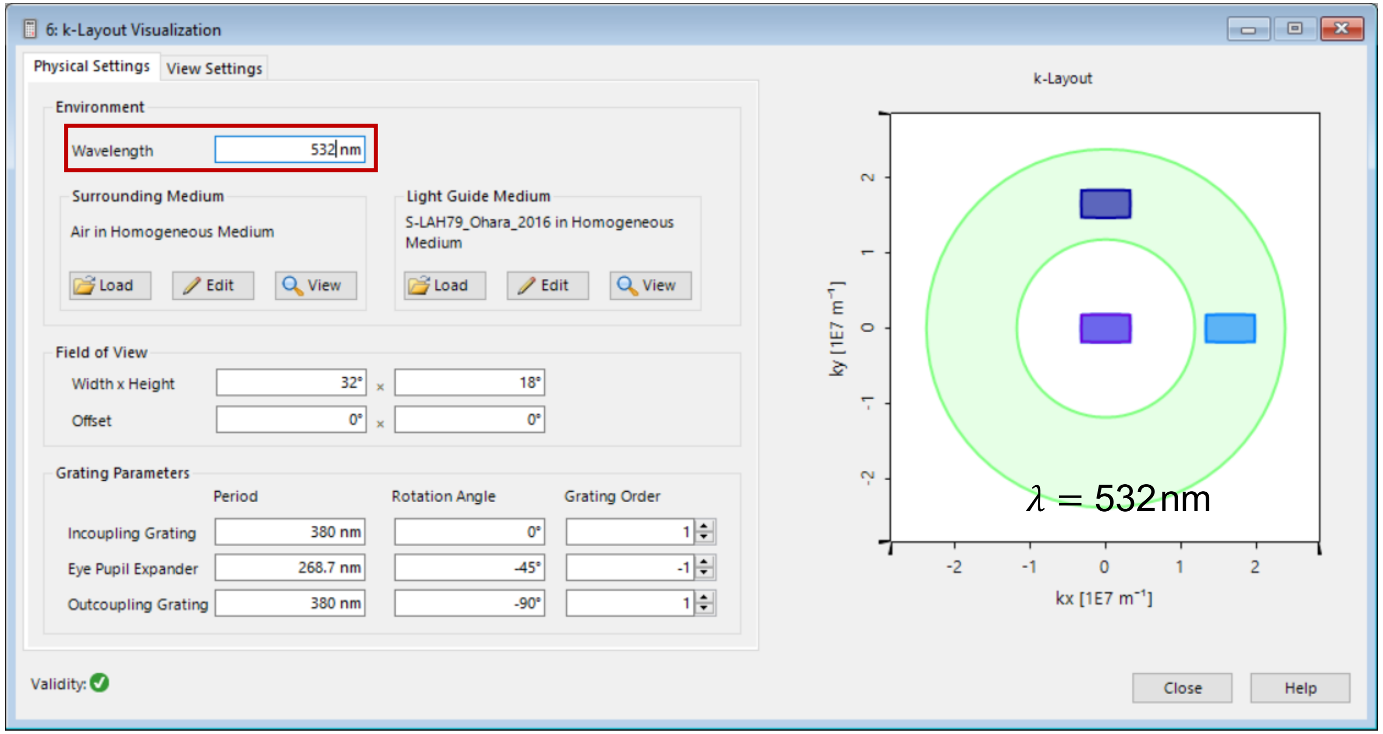
fill/clear the view boxes and adjust the opacity

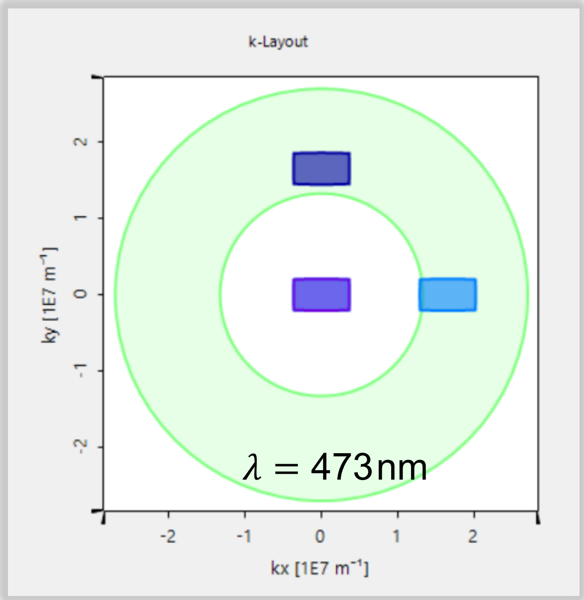
Right-click on the view to zoom in/out



k-Layout Visualization Calculator: Wavelength

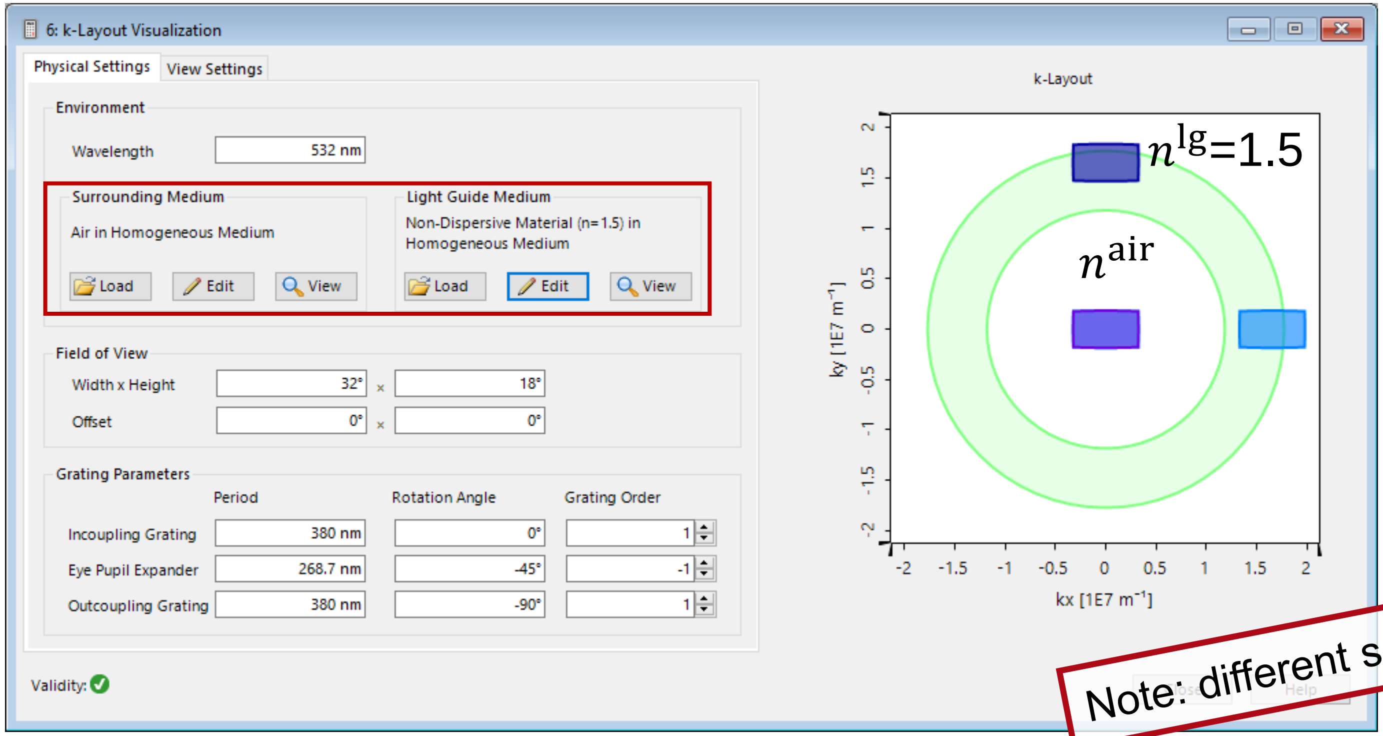
- In all calculations in the k-domain the wavelength affects the quantities via
$$k_0 = 2\pi/\lambda.$$
- In addition, the material dispersion $n(\lambda)$ should be included in accurate modelling.
- Therefore, the wavelength affects the size of the FOV box and the size of the guiding condition rings.



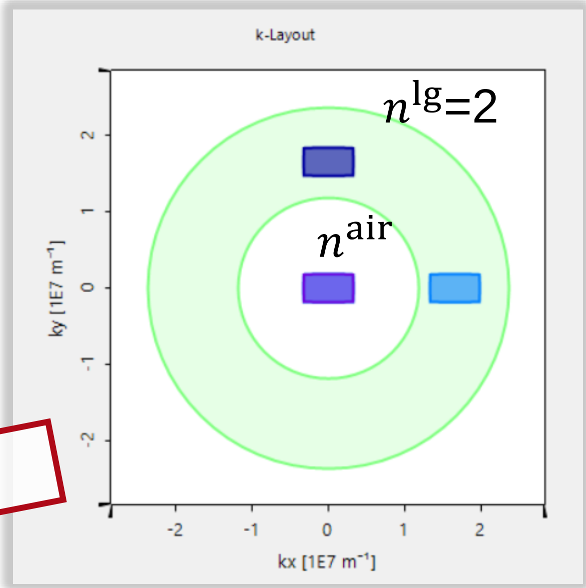


k-Layout Visualization Calculator: Media

- Via the refractive index, the surrounding medium affects the inner TIR bound, and the lightguide medium affects the outer TIR bound.
- The dependence of dispersion (the dependence of n^{lg} on λ, mostly) must also be taken into account.

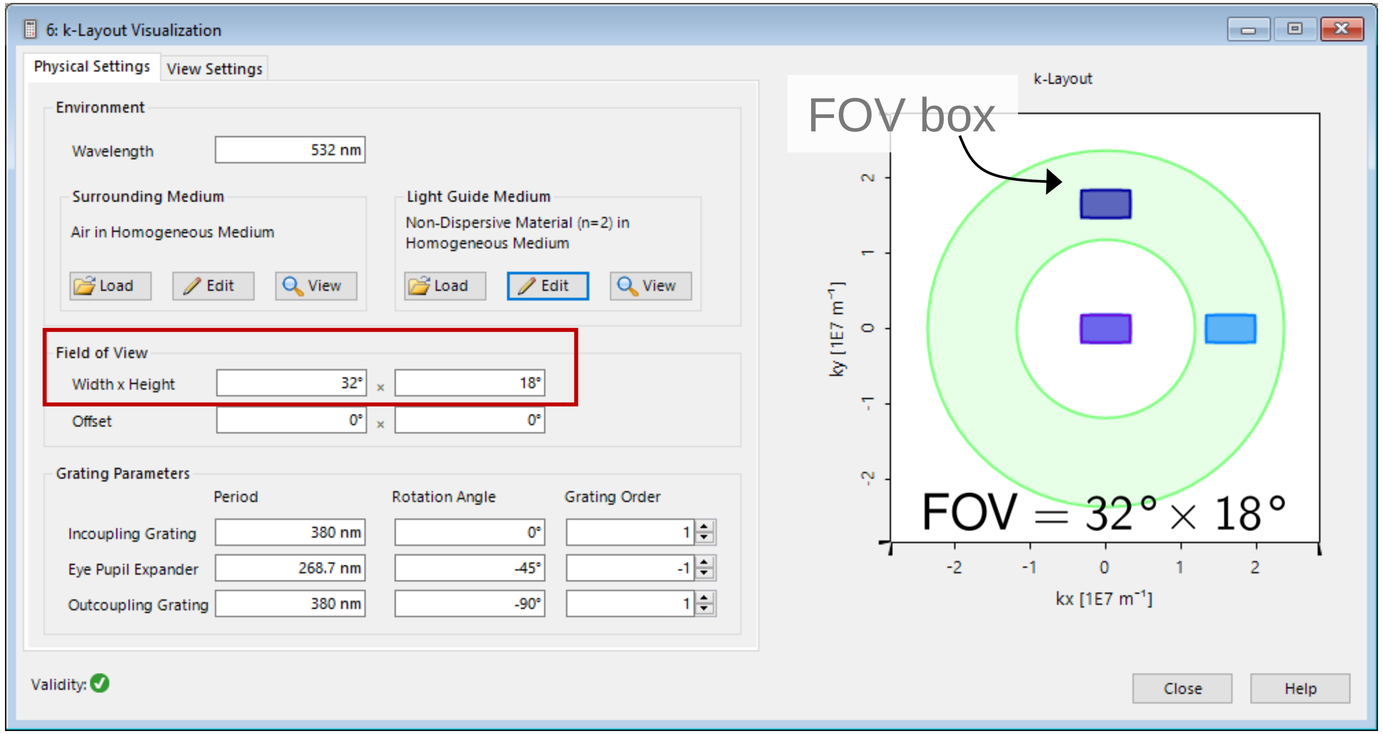


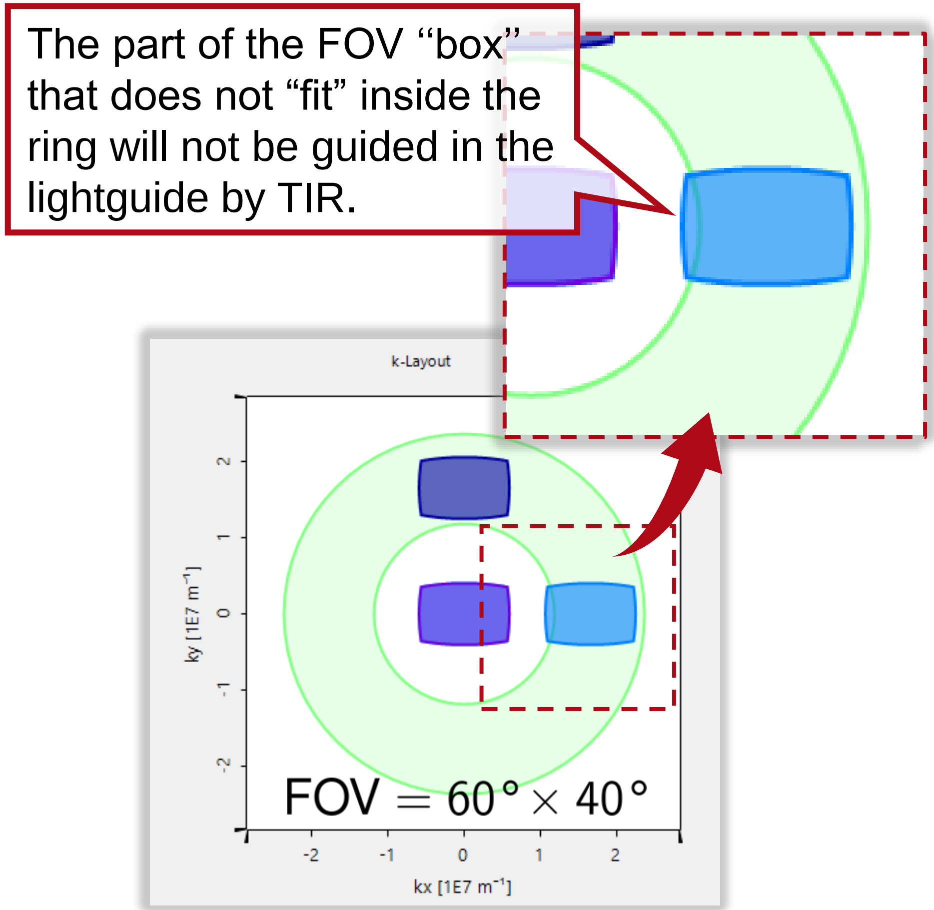
Note: different scaling



k-Layout Visualization Calculator: FOV Range

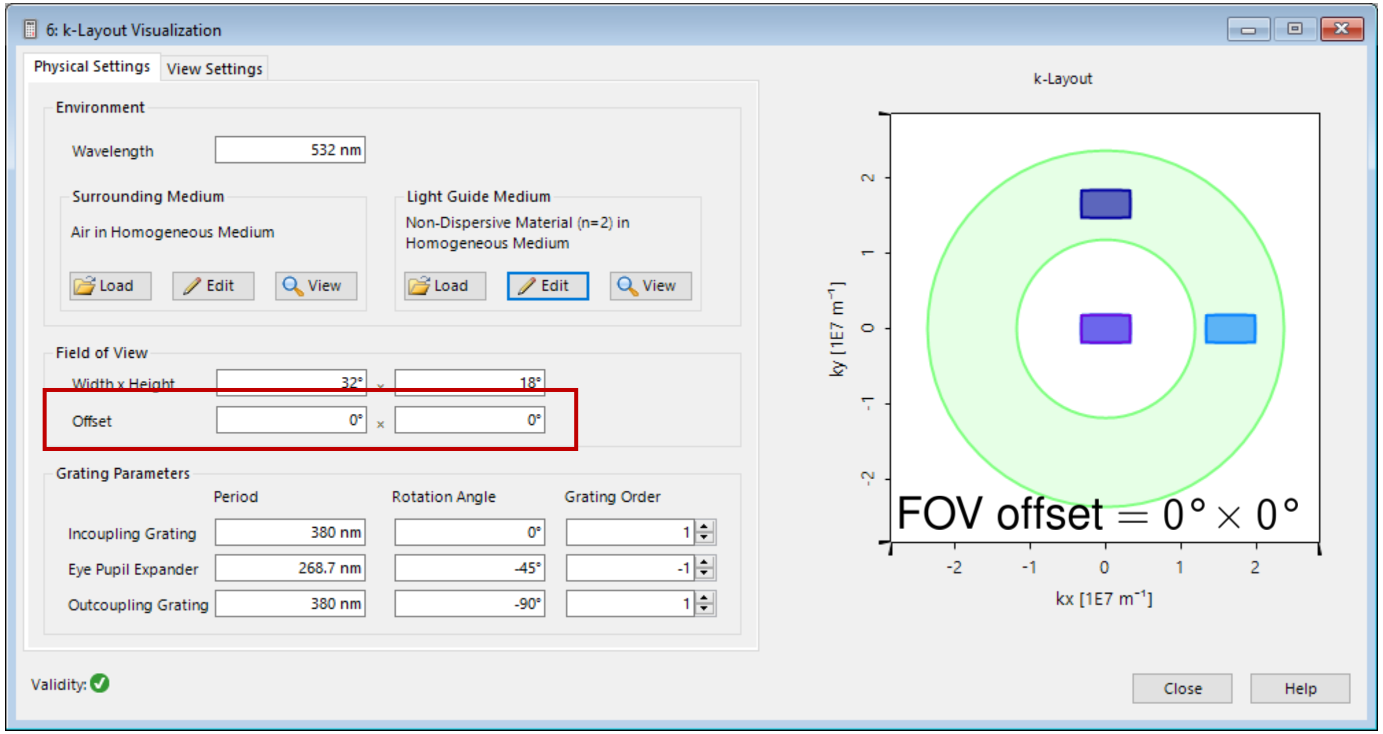
- The field of view range changes the size of the FOV "box" .
- Note the non-linear relationship between θ_x, θ_y and k_x, k_y (courtesy of the tangent in $k_x = k_0 n \cdot \tan\theta_x$).

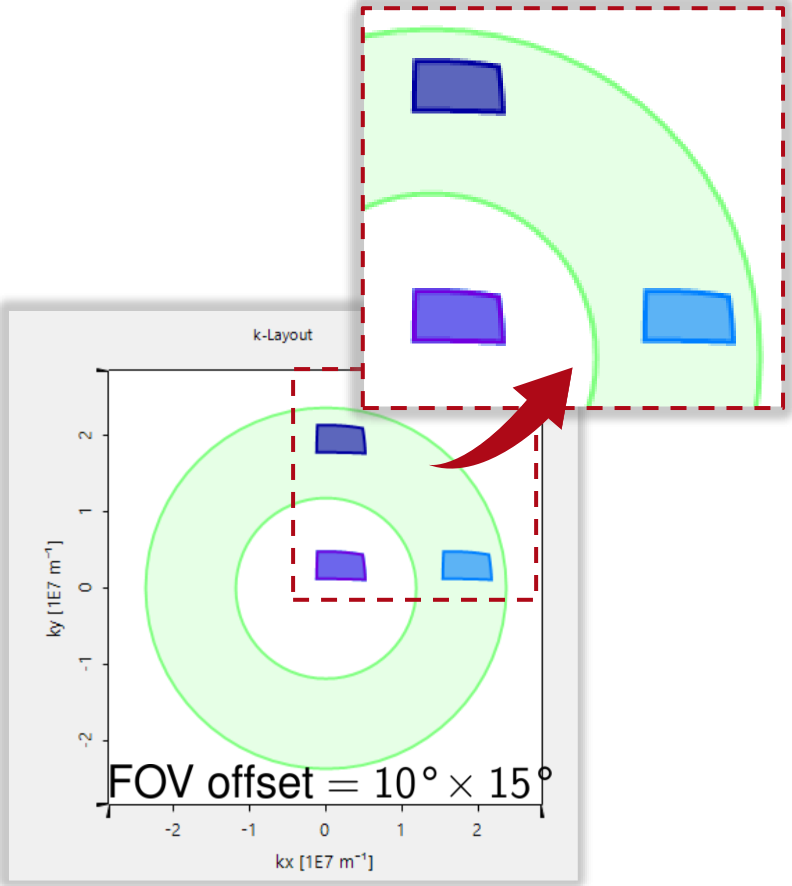




k-Layout Visualization Calculator: FOV Offset

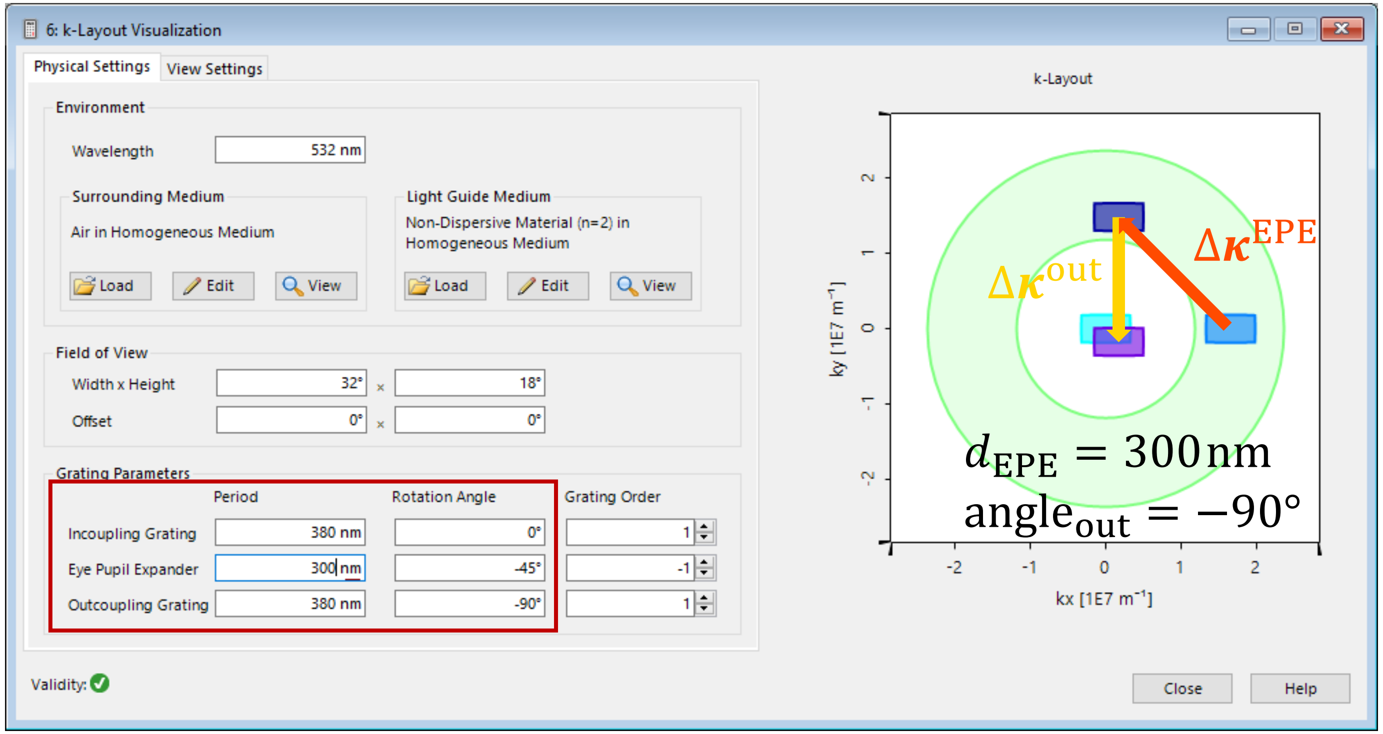
- The FOV offset shifts the FOV “box” .
- Due to the effect of the non-linear relationship, the shape of the FOV “box” is distorted as a result.

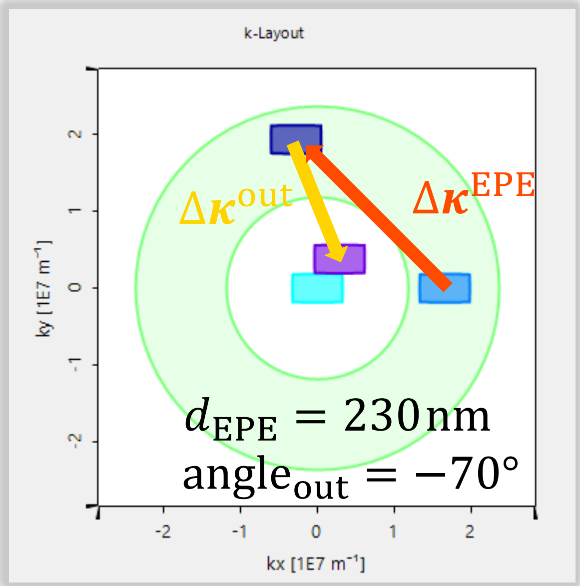




k-Layout Visualization Calculator: Gratings

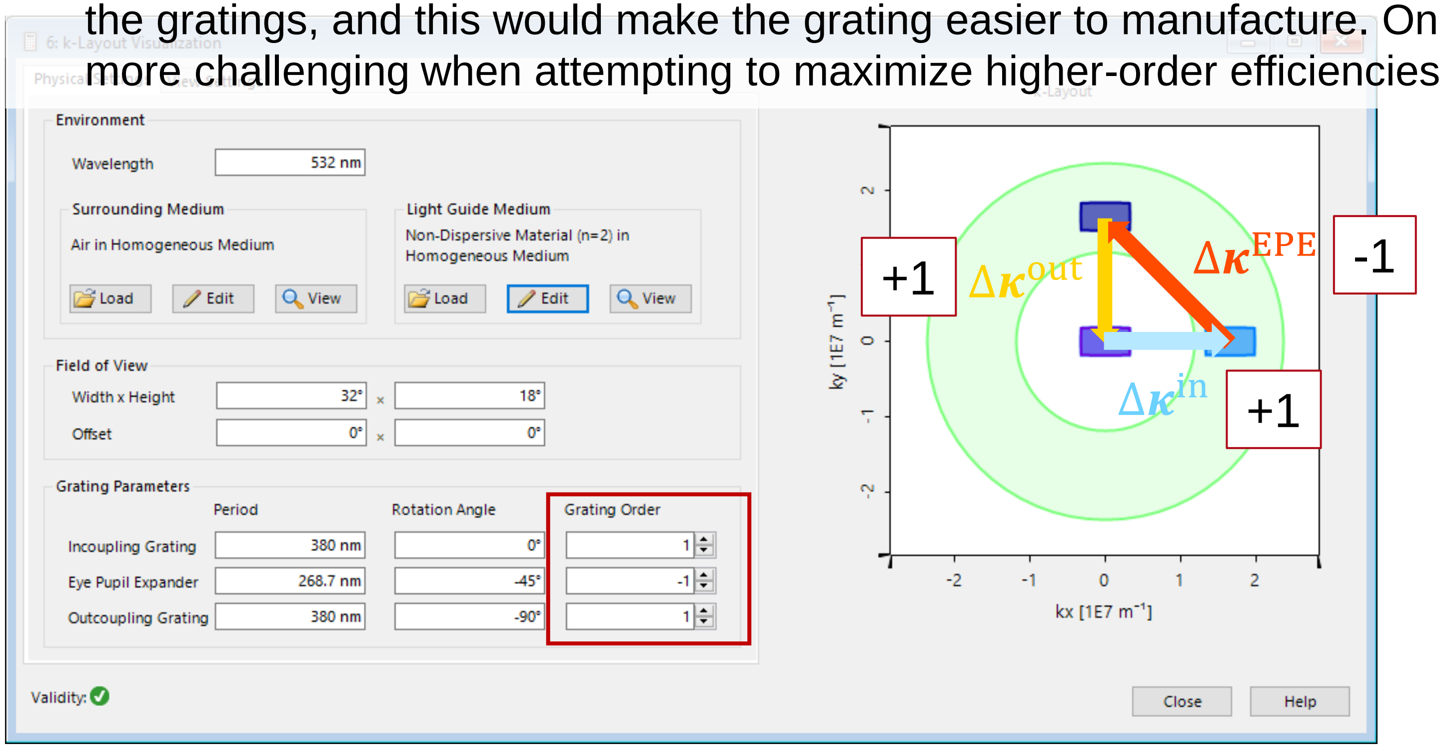
- The grating period affects the length of the grating vector $\Delta\boldsymbol{\kappa} = 2\pi(\frac{1}{d_x}, \frac{1}{d_y})$.
- The rotation angle affects the orientation of the grating vector in the diagram.

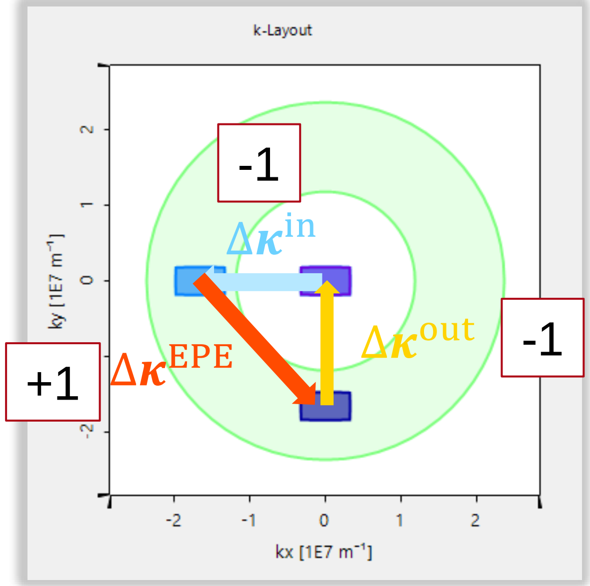




k-Layout Visualization Calculator: Grating Order

- The sign of the grating order the moving direction of the FOV “box”, due to the resulting summation or subtraction of the grating vector $\Delta\boldsymbol{\kappa}$.
- The intended working orders and their signs must be selected in conjunction with the orientation of the grating. For instance, a rotation angle of -45° with the -1st order as selected order is physically equivalent to a rotation of 45° with the +1st as selected order.
- It is also technically possible to select higher orders, especially since that would allow us to employ larger period values in the gratings, and this would make the grating easier to manufacture. On the downside, the design process becomes more challenging when attempting to maximize higher-order efficiencies.





Document Information

title	k-Domain Layout Visualization ("The Magic Circle")
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software edition	• VirtualLab Fusion Advanced • VirtualLab Fusion Basic with Light Guide Toolbox Gold
software version	2021.1 (Build 1.180)
category	Feature Use Case
further reading	- Construction of a Light Guide - Modeling of a “HoloLens 1”-Type Layout with Light Guide Component - Flexible Region Definition - Specification of Diffraction Orders and Efficiencies for Grating Regions - Light Guide Layout Design Tool