

VirtualLab Fusion Technology – Solvers and Functions

# Runge-Kutta Beam Propagation Method (RK-BPM) for GRIN Medium

For the **GRIN Lens Component**, **Inhomogeneous Medium Component** 

The RK-BPM solver works in the spatial domain (**x domain**), in a pointwise manner. Mathematically, it solves, simultaneously,

- 1. one ordinary differential equation (ODE) for the light path, and
- 2. another ODE for the field polarization vector.

The solution of the ODEs is based on the standard Runge-Kutta (RK) fourth-order method. In comparison to the ray tracing for GRIN medium – that solves the light path – we extend it to embrace the field quantities i.e. the complex amplitude and polarization.



#### **Solver Algorithm**

- The Runge-Kutta beam propagation method (RK-BPM) starts with an input field given in the **GRIN medium**  $\epsilon(r)$ , and it is in the same medium that the output field is calculated.
- · Let us consider both input and output fields on planes, as

 $\boldsymbol{V}^{\mathsf{in}}(\boldsymbol{\rho}) = \boldsymbol{V}(\boldsymbol{\rho}, z = 0) = \boldsymbol{U}^{\mathsf{in}}(\boldsymbol{\rho}) \exp\left(\mathrm{i}\psi^{\mathsf{in}}(\boldsymbol{\rho})\right),$  $\boldsymbol{V}^{\mathsf{out}}(\boldsymbol{\rho}) = \boldsymbol{V}(\boldsymbol{\rho}, z) = \boldsymbol{U}^{\mathsf{out}}(\boldsymbol{\rho}) \exp\left(\mathrm{i}\psi^{\mathsf{out}}(\boldsymbol{\rho})\right),$ 

where  $\psi$  is the wavefront phase part, U is the residual fields, and  $\rho = (x, y)$  as the transverse coordinates.

• The output field is to be calculated pointwisely

$$\boldsymbol{V}^{\mathsf{out}}(\boldsymbol{\rho}) = \int \mathbf{B}(\boldsymbol{\rho}, \boldsymbol{\rho}') \delta(\boldsymbol{\rho} - f(\boldsymbol{\rho}')) \boldsymbol{V}^{\mathsf{in}}(\boldsymbol{\rho}') \, \mathrm{d}\boldsymbol{\rho}' \,,$$

with  $\rho = f(\rho')$  represents a one-to-one mapping for the spatial coordinates, which is to be calculated by the RK method.



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In the RK method, we are dealing with the 3D electric field vectors instead of the transverse components.

 $oldsymbol{r} = (x,y,z)$ Therefore, a 3x3 B-matrix

 $\epsilon(\boldsymbol{r})$ 

ρ

 $\rightarrow z$ 

is used to connect the input and output fields.

### **Solver Algorithm**

 Applying the property of the Dirac delta function, the expression of the output field can be simplied to

$$\boldsymbol{V}^{\mathsf{out}}(\boldsymbol{\rho}) = \mathbf{B}(\boldsymbol{\rho}) \boldsymbol{V}^{\mathsf{in}}(f^{-1}(\boldsymbol{\rho}))$$
 .

- Next, the algorithm can be explicitly written, with respect to  $\psi$  and  ${\pmb U}$  separately, as
  - the redisual field:  $U^{\text{out}}(\rho) = \mathbf{b}(\rho)U^{\text{in}}(f^{-1}(\rho))$ , and
  - the wavefront phase part:  $\psi^{\text{out}}(\rho) = \psi^{\text{in}}(f^{-1}(\rho)) + \Delta \psi(\rho)$ .
- Following [1, 2], we introduce
  - unit direction vector  $\hat{\boldsymbol{s}}(\boldsymbol{r}),$  arc length  $\Delta s$  [3], and
  - unit polarization vector  $\hat{\boldsymbol{u}}(\boldsymbol{r}) = \boldsymbol{U}(\boldsymbol{r})/||\boldsymbol{U}(\boldsymbol{r})||,$

as auxiliary variables in the algorithm.



# **Solver Algorithm**

• With the two auxiliary variables, two ordinary differential equations can be formulated [1, 2] as

$$\frac{\mathrm{d}}{\mathrm{d}s}\sqrt{\epsilon(\boldsymbol{r})}\hat{\boldsymbol{s}}(\boldsymbol{r}) = \nabla\sqrt{\epsilon(\boldsymbol{r})}, \qquad \begin{array}{c} \text{Ray tracing for GRIN} \\ \text{medium [4] deals with the} \\ \text{first equation only.} \end{array}$$

$$\sqrt{\epsilon(\boldsymbol{r})}\frac{\mathrm{d}}{\mathrm{d}s}\hat{\boldsymbol{u}}(\boldsymbol{r}) = -\left(\hat{\boldsymbol{u}}(\boldsymbol{r})\cdot\nabla\sqrt{\epsilon(\boldsymbol{r})}\right)\hat{\boldsymbol{s}}(\boldsymbol{r}).$$

- Both ordinary differential equations are solved simultaneously:
  - the first equation determines the change in path i.e. the ray, and thus  $\Delta \psi(\rho)$ ;
  - the second equation determines change in polarization, and together with energy conservation law, it determines  $\mathbf{b}(\boldsymbol{\rho})$ .
- To solve the differential equations, a standard RK 4th-order numerical routine is employed.

# **Usage in VirtualLab Fusion**

- Take the GRIN Lens Component as an example:
  - The GRIN medium can be loaded from the catalog, and its parameter can be further modified.
  - Two plane interfaces are used to define the medium boundaries, as the default for most cases in practice.
  - The medium may have non-planar boundaries, and that can be specified by loading the corresponding interfaces.



## **List of References**

- [1] Huiying Zhong, Site Zhang, Rui Shi, Christian Hellmann, and Frank Wyrowski, "<u>Fast propagation of</u> <u>electromagnetic fields through graded-index media</u>," J. Opt. Soc. Am. A 35, 661-668 (2018)
- [2] Max Born and Emil Wolf, *Principles of optics* (Cambridge University Press, 1999)
- [3] Gerald Farin, *Curves and Surfaces for CAGD: a Practical Guide* (Morgan Kaufmann Publishers Inc.,2001)
- [4] Anurag Sharma, D. Vizia Kumar, and A. K. Ghatak, "<u>Tracing rays through graded-index media: a new</u> method", Appl. Opt. 21, 984-987 (1982)

### **Document Information**

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