VirtualLab Fusion Technology – Solvers and Functions

Runge-Kutta Beam Propagation Method (RK-BPM) for GRIN Medium

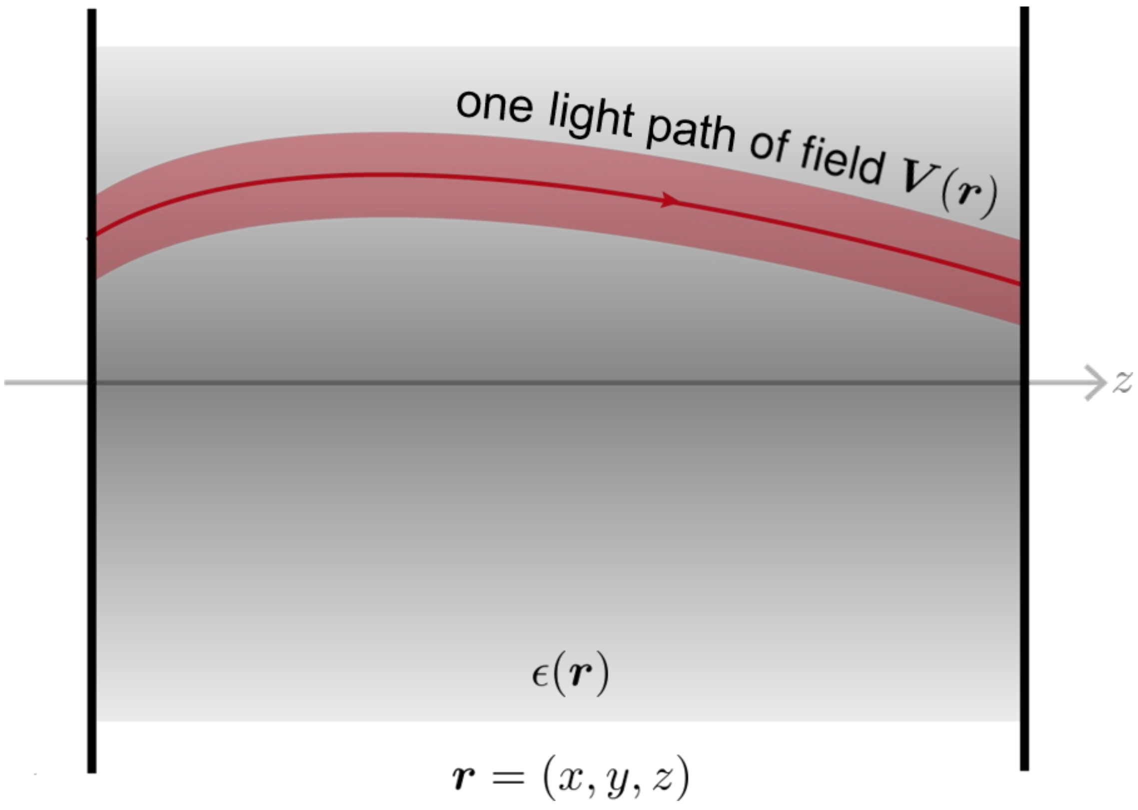
For the **GRIN Lens Component, Inhomogeneous Medium Component**

Abstract

The RK-BPM solver works in the spatial domain (**x domain**), in a pointwise manner. Mathematically, it solves, simultaneously,

1. one ordinary differential equation (ODE) for the light path, and
2. another ODE for the field polarization vector.

The solution of the ODEs is based on the standard Runge-Kutta (RK) forth-order method. In comparison to the ray tracing for GRIN medium – that solves the light path – we extend it to embrace the field quantities i.e. the complex amplitude and polarization.



Solver Algorithm

- The Runge-Kutta beam propagation method (RK-BPM) starts with an input field given in the **GRIN medium** $\epsilon(\boldsymbol{r})$, and it is in the same medium that the output field is calculated.

- Let us consider both input and output fields on planes, as

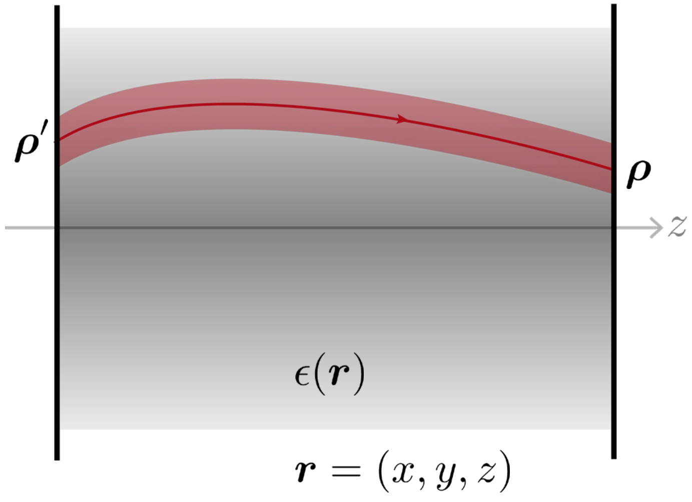
$$\boldsymbol{V}^{\text{in}}(\boldsymbol{\rho}) = \boldsymbol{V}(\boldsymbol{\rho}, z=0) = \boldsymbol{U}^{\text{in}}(\boldsymbol{\rho}) \exp\left(\mathrm{i}\psi^{\text{in}}(\boldsymbol{\rho})\right),$$
$$\boldsymbol{V}^{\text{out}}(\boldsymbol{\rho}) = \boldsymbol{V}(\boldsymbol{\rho}, z) = \boldsymbol{U}^{\text{out}}(\boldsymbol{\rho}) \exp\left(\mathrm{i}\psi^{\text{out}}(\boldsymbol{\rho})\right),$$

  where ψ is the wavefront phase part, $\boldsymbol{U}$ is the residual fields, and $\boldsymbol{\rho} = (x, y)$ as the transverse coordinates.

- The output field is to be calculated pointwisely

$$\boldsymbol{V}^{\text{out}}(\boldsymbol{\rho}) = \int \mathbf{B}(\boldsymbol{\rho}, \boldsymbol{\rho}')\delta(\boldsymbol{\rho} - f(\boldsymbol{\rho}'))\boldsymbol{V}^{\text{in}}(\boldsymbol{\rho}')\,\mathrm{d}\boldsymbol{\rho}',$$

  with $\boldsymbol{\rho} = f(\boldsymbol{\rho}')$ represents a one-to-one mapping for the spatial coordinates, which is to be calculated by the RK method.



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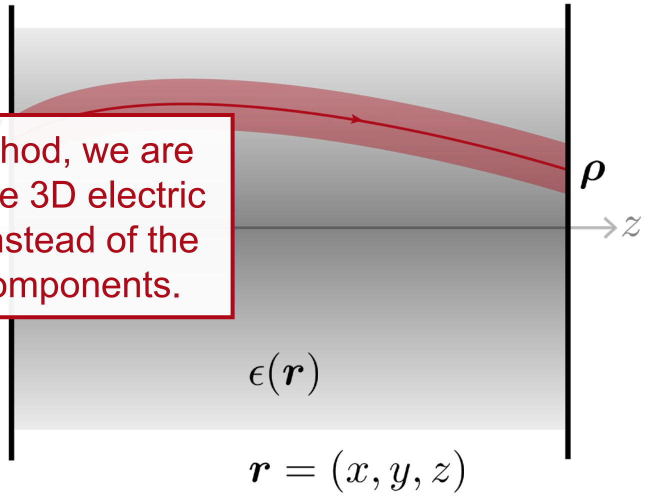
In the RK method, we are dealing with the 3D electric field vectors instead of the transverse components.



- The output field is to be calculated pointwisely

$$\boldsymbol{V}^{\text{out}}(\boldsymbol{\rho}) = \int \mathbf{B}(\boldsymbol{\rho}, \boldsymbol{\rho}')\delta\left(\boldsymbol{\rho} - f(\boldsymbol{\rho}')\right)\boldsymbol{V}^{\text{in}}(\boldsymbol{\rho}')\,\mathrm{d}\boldsymbol{\rho}',$$

  with $\boldsymbol{\rho} = f(\boldsymbol{\rho}')$ represents a one-to-one mapping for the spatial coordinates, which is to be calculated by the RK method.

Therefore, a 3x3 B-matrix is used to connect the input and output fields.

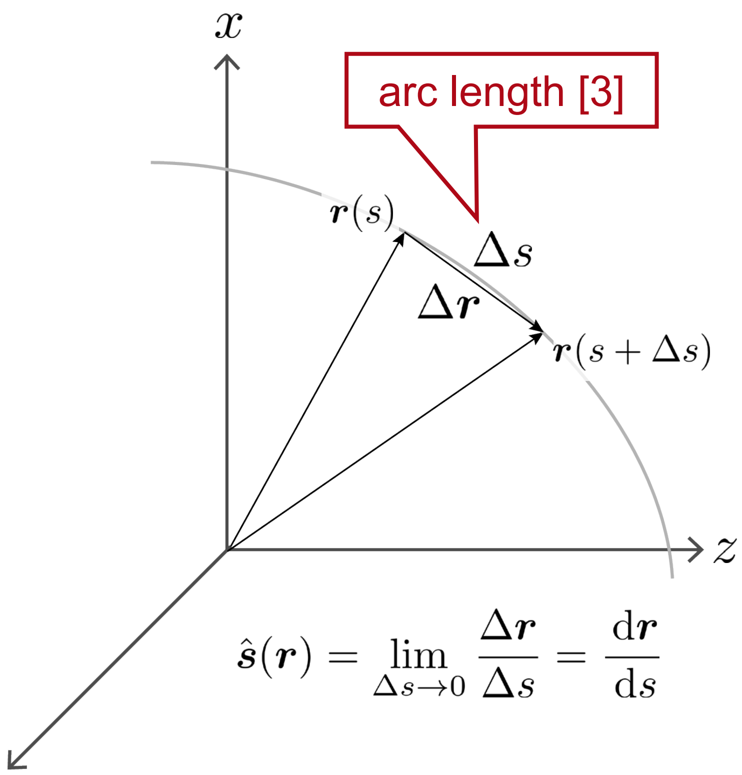
Solver Algorithm

- Applying the property of the Dirac delta function, the expression of the output field can be simplied to

$$\boldsymbol{V}^{\text{out}}(\boldsymbol{\rho}) = \mathbf{B}(\boldsymbol{\rho})\boldsymbol{V}^{\text{in}}(f^{-1}(\boldsymbol{\rho}))\,.$$

- Next, the algorithm can be explicitly written, with respect to ψ and $\boldsymbol{U}$ separately, as
  - the redisual field: $\boldsymbol{U}^{\text{out}}(\boldsymbol{\rho}) = \mathbf{b}(\boldsymbol{\rho})\boldsymbol{U}^{\text{in}}(f^{-1}(\boldsymbol{\rho}))$, and
  - the wavefront phase part: $\psi^{\text{out}}(\boldsymbol{\rho}) = \psi^{\text{in}}(f^{-1}(\boldsymbol{\rho})) + \Delta\psi(\boldsymbol{\rho})$.
- Following [1, 2], we introduce
  - unit direction vector $\hat{\boldsymbol{s}}(\boldsymbol{r})$, arc length Δs [3], and
  - unit polarization vector $\hat{\boldsymbol{u}}(\boldsymbol{r}) = \boldsymbol{U}(\boldsymbol{r})/||\boldsymbol{U}(\boldsymbol{r})||$,

  as auxiliary variables in the algorithm.



$$\hat{\boldsymbol{s}}(\boldsymbol{r}) = \lim_{\Delta s\to 0}\frac{\Delta\boldsymbol{r}}{\Delta s} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}s}$$

Solver Algorithm

- With the two auxiliary variables, two ordinary differential equations can be formulated [1, 2] as

$$\frac{\mathrm{d}}{\mathrm{d}s}\sqrt{\epsilon(\boldsymbol{r})}\hat{\boldsymbol{s}}(\boldsymbol{r}) = \nabla\sqrt{\epsilon(\boldsymbol{r})}\,,$$

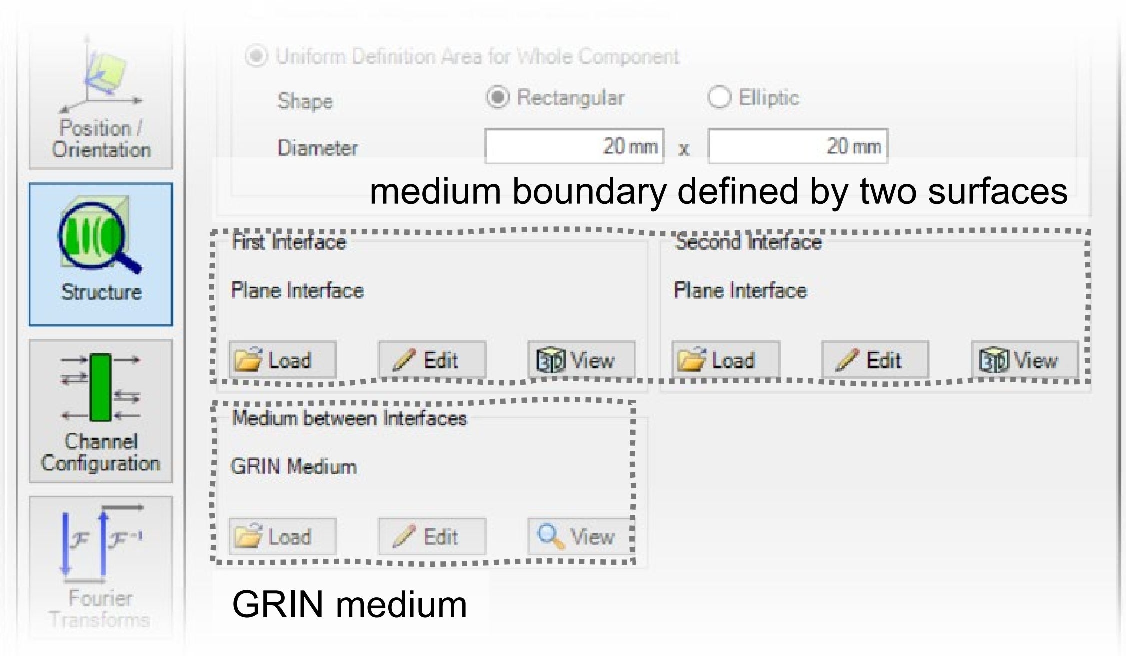
$$\sqrt{\epsilon(\boldsymbol{r})}\frac{\mathrm{d}}{\mathrm{d}s}\hat{\boldsymbol{u}}(\boldsymbol{r}) = -\left(\hat{\boldsymbol{u}}(\boldsymbol{r})\cdot\nabla\sqrt{\epsilon(\boldsymbol{r})}\right)\hat{\boldsymbol{s}}(\boldsymbol{r})\,.$$

Ray tracing for GRIN medium [4] deals with the first equation only.

- Both ordinary differential equations are solved simultaneously:
  - the first equation determines the change in path i.e. the ray, and thus $\Delta\psi(\boldsymbol{\rho})$;
  - the second equation determines change in polarization, and together with energy conservation law, it determines $\mathbf{b}(\boldsymbol{\rho})$.
- To solve the differential equations, a standard RK 4th-order numerical routine is employed.

Usage in VirtualLab Fusion

- Take the GRIN Lens Component as an example:
  - The **GRIN medium** can be loaded from the catalog, and its parameter can be further modified.
  - Two plane interfaces are used to define the medium boundaries, as the default for most cases in practice.
  - The medium may have non-planar boundaries, and that can be specified by loading the corresponding interfaces.



List of References

[1] Huiying Zhong, Site Zhang, Rui Shi, Christian Hellmann, and Frank Wyrowski, "Fast propagation of electromagnetic fields through graded-index media," J. Opt. Soc. Am. A 35, 661-668 (2018)

[2] Max Born and Emil Wolf, *Principles of optics* (Cambridge University Press, 1999)

[3] Gerald Farin, *Curves and Surfaces for CAGD: a Practical Guide* (Morgan Kaufmann Publishers Inc.,2001)

[4] Anurag Sharma, D. Vizia Kumar, and A. K. Ghatak, "Tracing rays through graded-index media: a new method", Appl. Opt. 21, 984-987 (1982)

Document Information

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