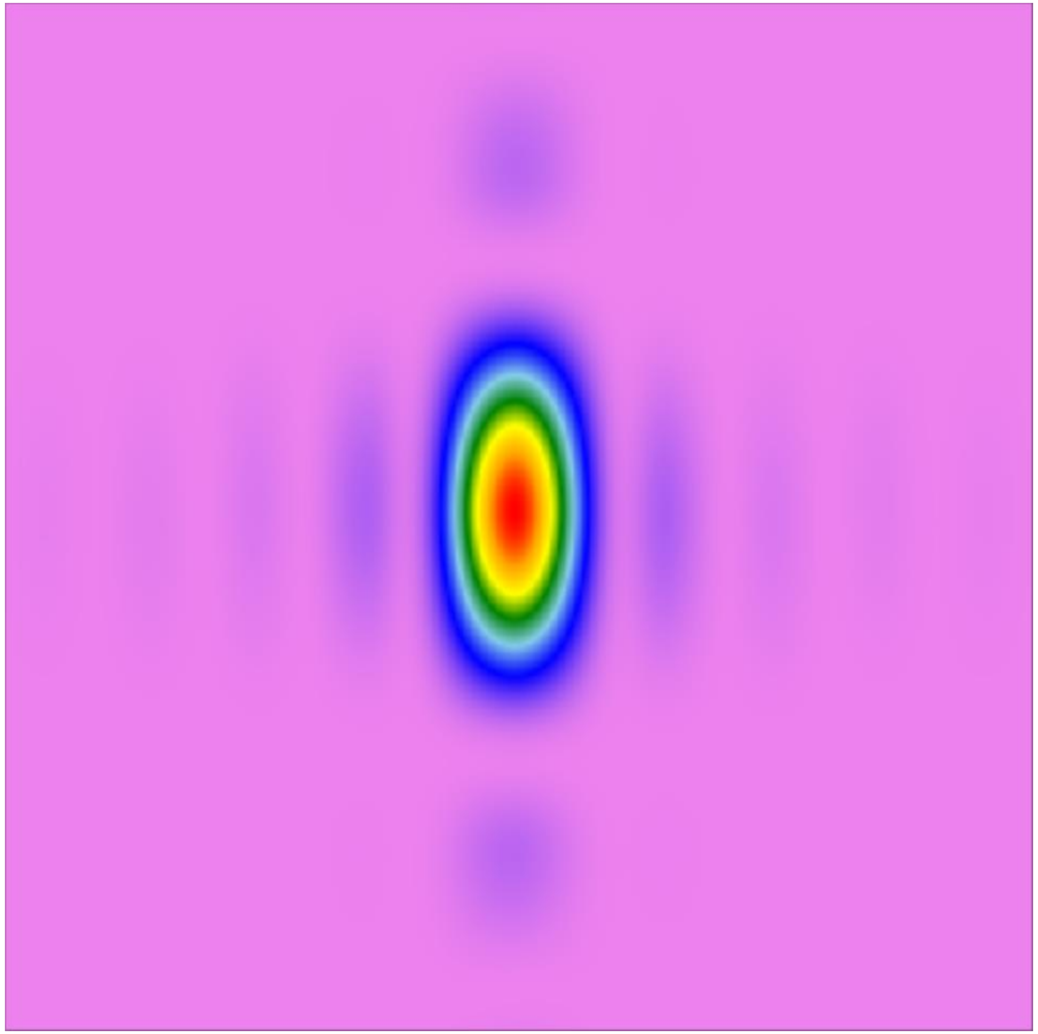
Grazing-Incidence Focusing Mirrors for X-Ray Beams

Abstract



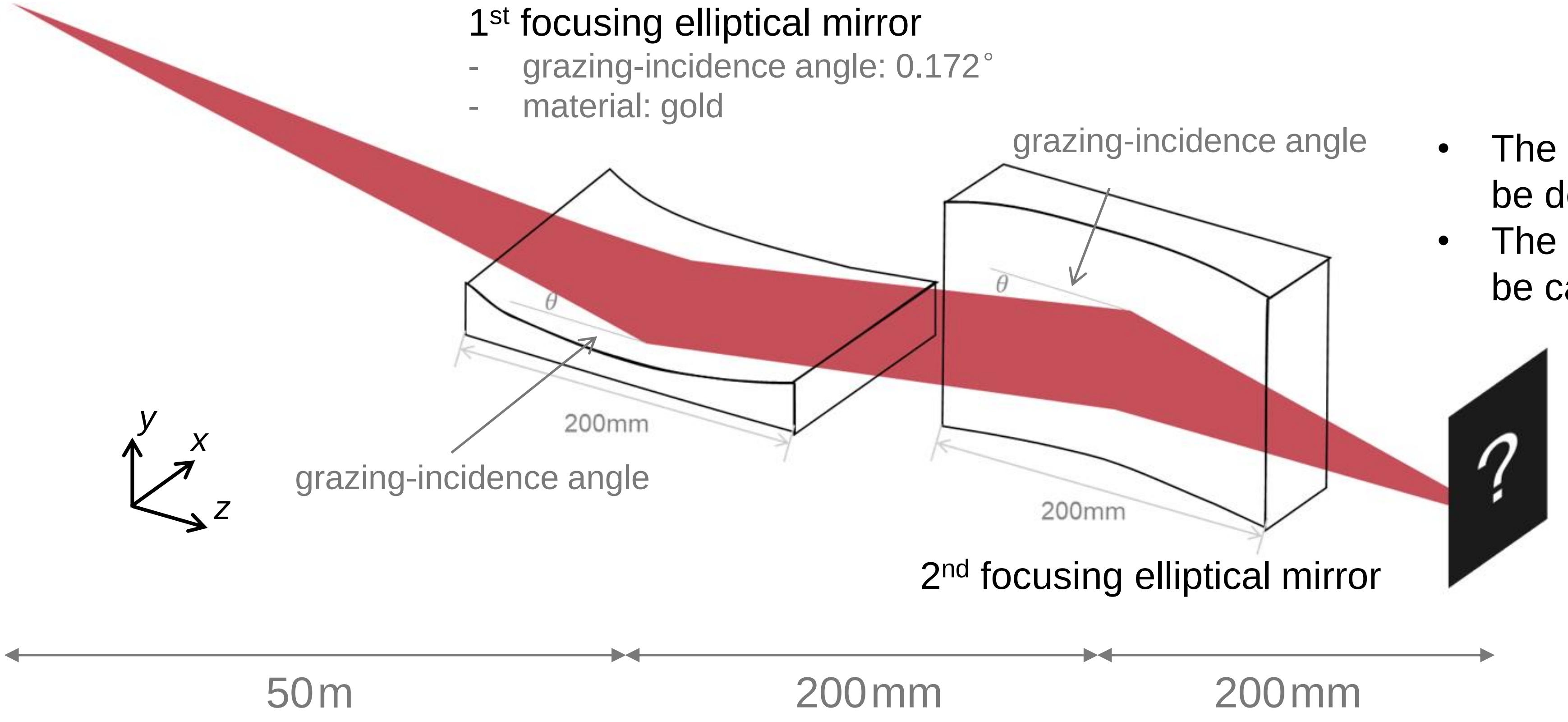
Grazing-incidence reflective optics are widely used in X-ray beamlines, in particular Kirkpatrick-Baez (KB) elliptical mirror systems [A. Verhoeven, *et al.*, *Journal of Synchrotron Radiation* 27.5 (2020): 1307-1319]. Focusing is accomplished by using two physically separated elliptical mirrors to focus the beam in two dimensions. The incoming X-rays can be focused by the system down to nanometer-scale spot size. Such system is modeled and simulated in VirtualLab Fusion and the focal field is calculated.

Modeling Task

fundamental Gaussian
- wavelength: 173 pm
- waist radius: 4.44 µm × 4.44 µm

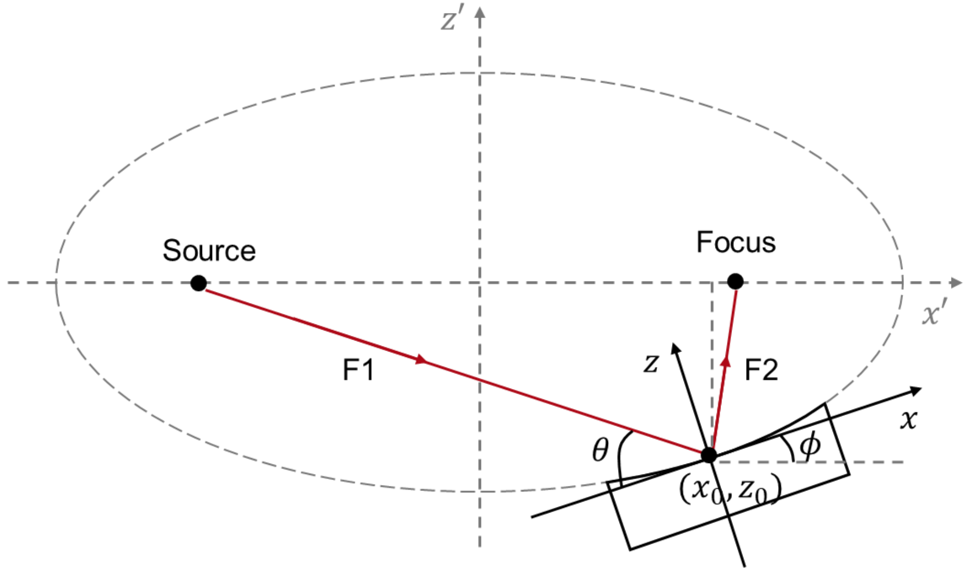
1st focusing elliptical mirror
- grazing-incidence angle: 0.172°
- material: gold

- The elliptical mirrors are to be designed analytically.
- The diffraction pattern is to be calculated.



system parameters from A. Verhoeven, et al., Journal of Synchrotron Radiation 27.5 (2020): 1307-1319.

Analytical Design of the Elliptical Mirror (1)



For the calculation of the elliptical surface height profile $z(x)$ the following parameters are required:

- distance between source and the mirror center F_1
- distance between image/focus and the mirror center F_2
- grazing-incidence angle θ

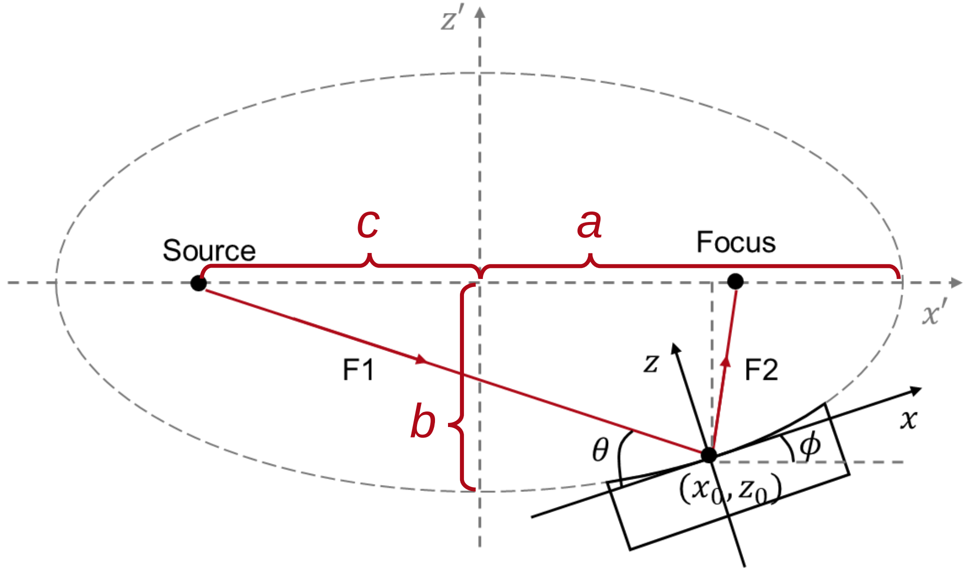
1st elliptical mirror

- $F_1 = 50\text{m}$
- $F_2 = 400\text{mm}$
- $\theta = 0.172°$

2nd elliptical mirror

- $F_1 = 50.2\text{m}$
- $F_2 = 200\text{mm}$
- $\theta = 0.172°$

Analytical Design of the Elliptical Mirror (2)



- To calculate the height function $z(x)$, two equations need to be considered.
  - elliptical equation

$$\frac{x'^2}{a^2} + \frac{z'^2}{b^2} = 1$$

  - coordinate transform

$$\begin{pmatrix} x' \\ z' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} x \\ z \end{pmatrix} + \begin{pmatrix} x_0 \\ z_0 \end{pmatrix}$$

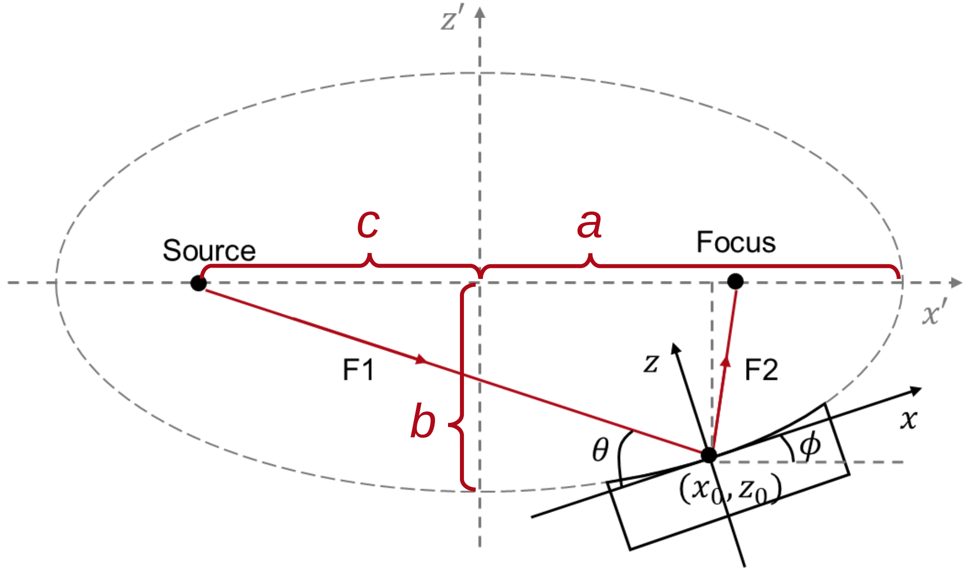
- The final height function $z(x)$ is

$$z(x) = (z' - z_0)\cos\phi - (x' - x_0)\sin\phi$$

with $\phi = \arctan\left(-\frac{b^2}{a^2}\frac{x_0}{z_0}\right)$, $x_0 = \frac{F_1^2 - F_2^2}{4c}$, and $z_0 = -b * \sqrt{1 - x_0^2/a^2}$

unknowns a, b, x' and z' in next slide

Analytical Design of the Elliptical Mirror (3)



- a and b can be calculated from F_1, F_2 and θ

$$a = \frac{F_1 + F_2}{2}$$
$$c = \frac{1}{2}\sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos(\pi - 2\theta)}$$
$$b = \sqrt{a^2 - c^2}$$

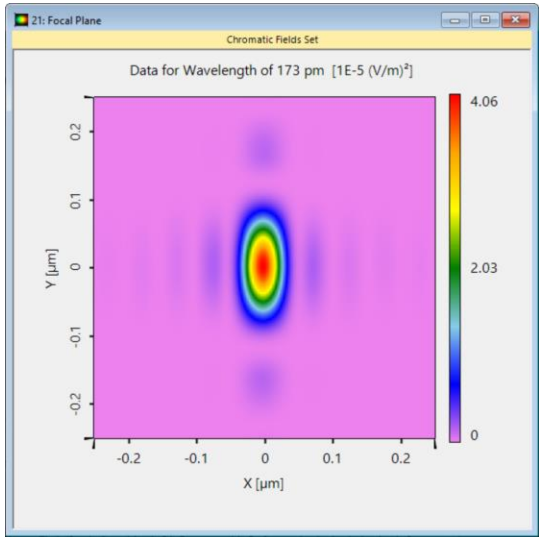
- parameter z' and x' are

$$z'(x') = -b\sqrt{1 - \frac{x'^2}{a^2}}, \qquad x'(x) = \frac{-n + \sqrt{n^2 - 4mt}}{2m}$$

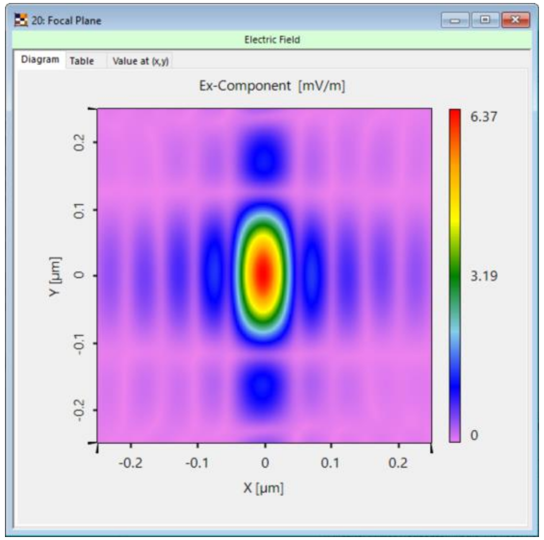
with

$$m = \cos\phi^2 + \frac{b^2}{a^2}\sin\phi$$
$$n = -2\cos\phi\,(x + x_0\cos\phi + z_0\sin\phi)$$
$$t = (x + x_0\cos\phi + z_0\sin\phi)^2 - b^2\sin^2\phi$$

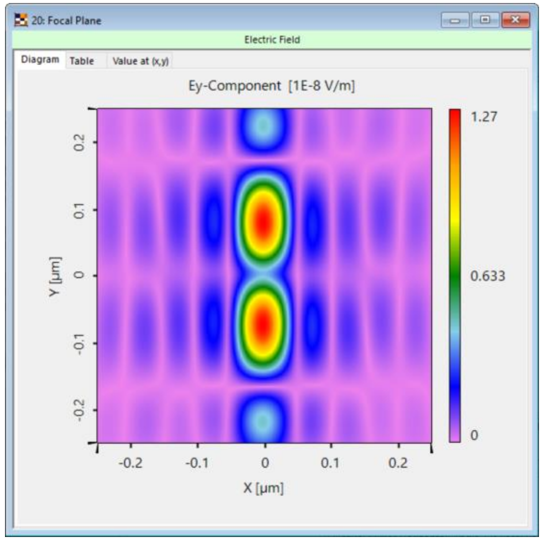
Energy Density & E-Field at Focal Plane



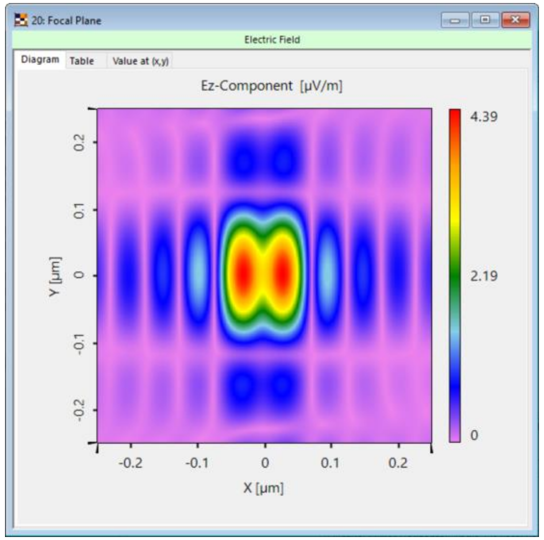
energy density



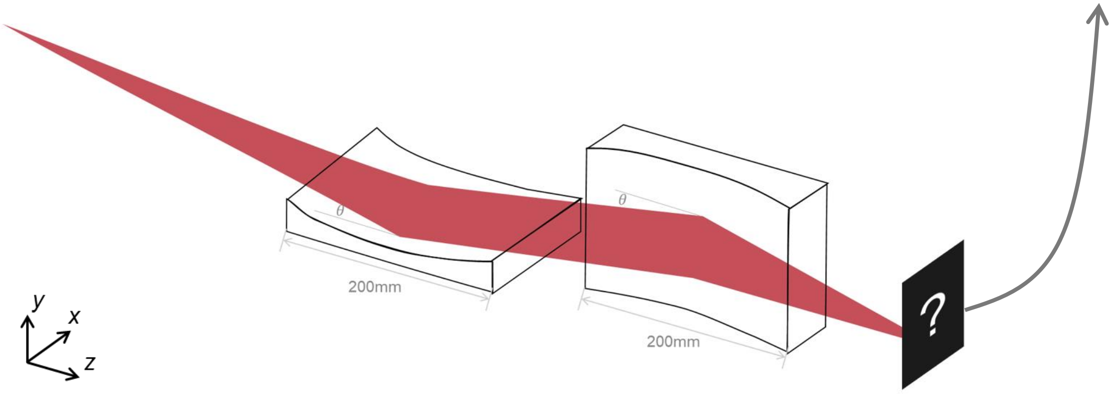
amplitude of E_x



amplitude of E_y

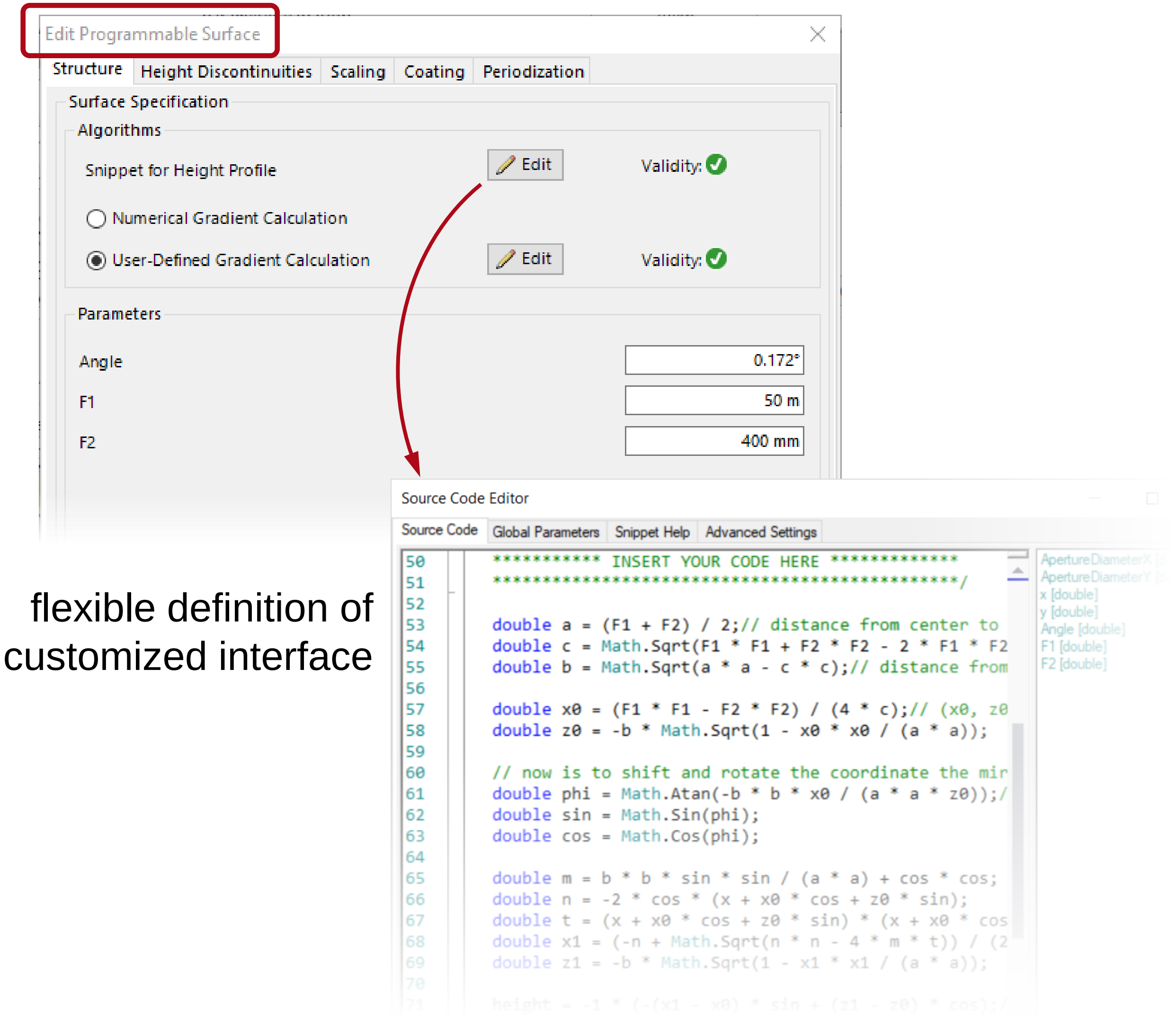


amplitude of E_z

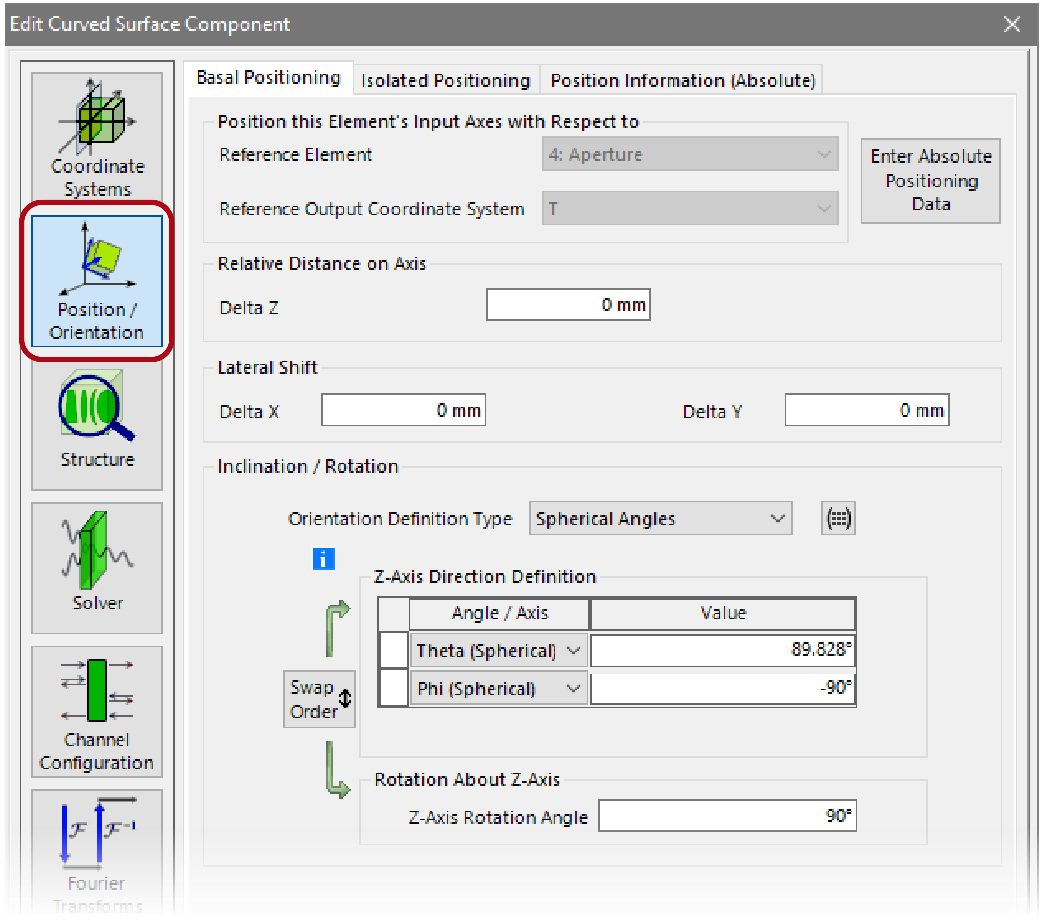


- The vectorial field in the focus is calculated.
- The size of the diffraction pattern is at nanometer-scale.

Peek into VirtualLab Fusion



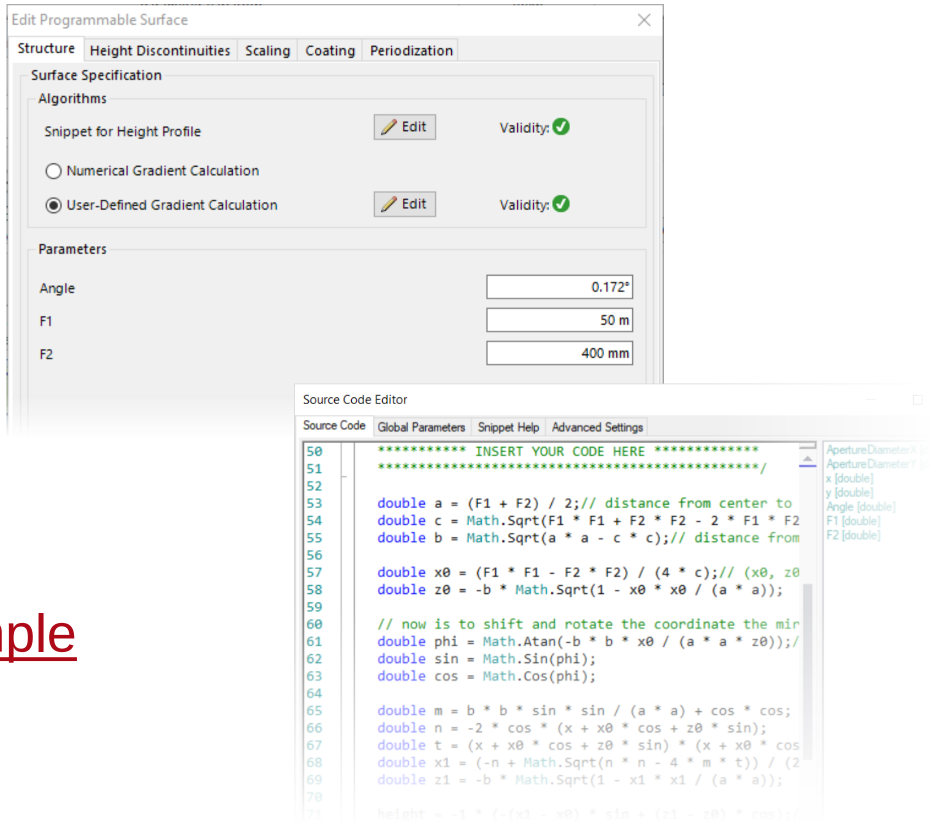
flexible definition of customized interface



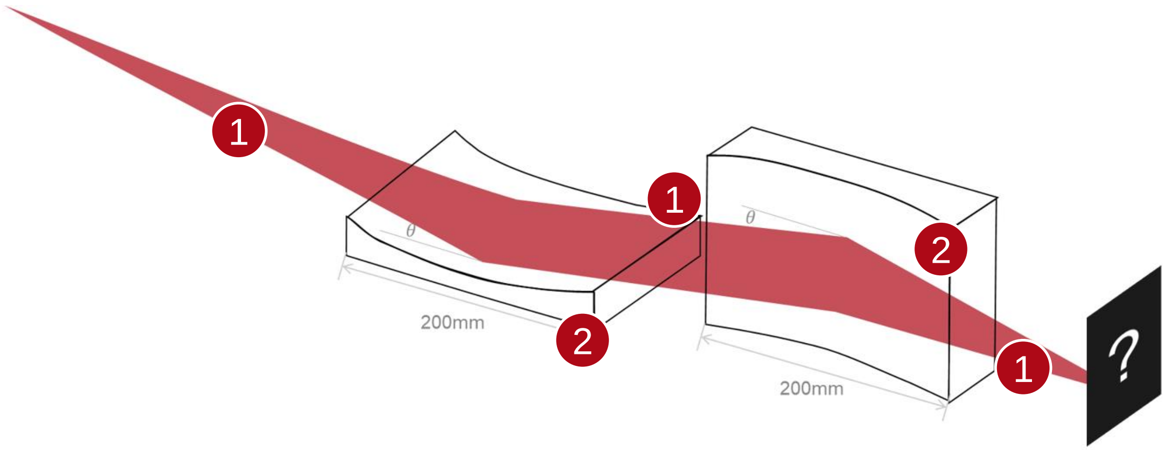
convinient definition of position and orientation

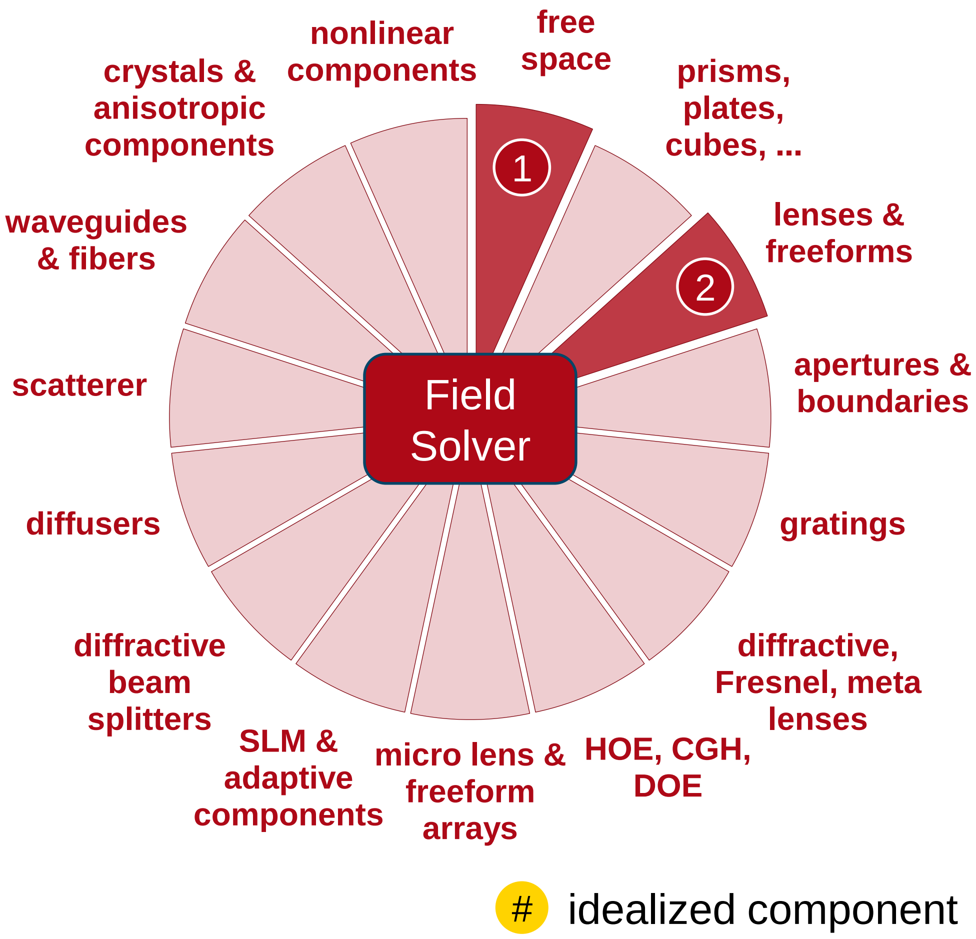
Workflow in VirtualLab Fusion

- Set up input Gaussian field
  - Basic Source Models [Tutorial Video]
- Set the position and orientation of components
  - LPD II: Position and Orientation [Tutorial Video]
- Programmable the elliptical interface
  - How to Work with the Programmable Interface & Example (Spherical Surface) [Use Case]



VirtualLab Fusion Technologies





Document Information

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