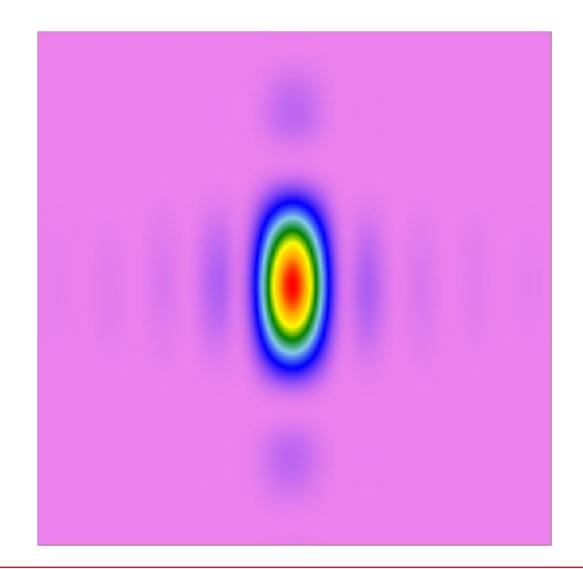


Grazing-Incidence Focusing Mirrors for X-Ray Beams

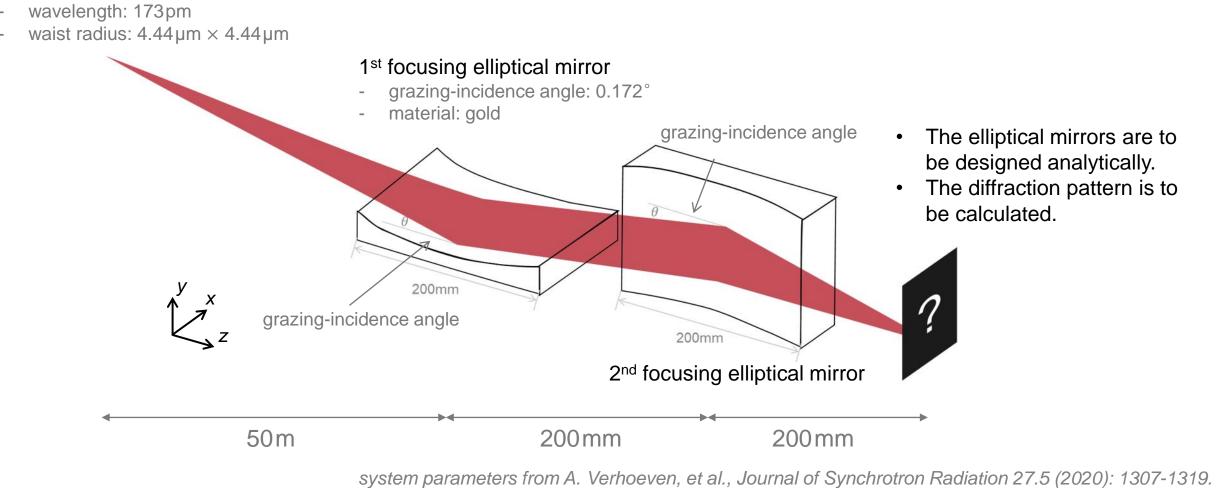
Abstract



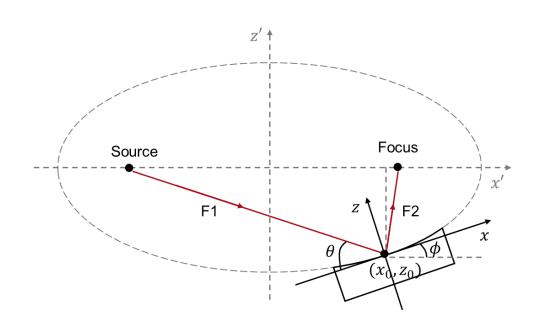
Grazing-incidence reflective optics are widely used in X-ray beamlines, in particular Kirkpatrick-Baez (KB) elliptical mirror systems [A. Verhoeven, et al., Journal of Synchrotron Radiation 27.5 (2020): 1307-1319]. Focusing is accomplished by using two physically separated elliptical mirrors to focus the beam in two dimensions. The incoming Xrays can be focused by the system down to nanometer-scale spot size. Such system is modeled and simulated in VirtualLab Fusion and the focal field is calculated.

Modeling Task

fundamental Gaussian



Analytical Design of the Elliptical Mirror (1)



For the calculation of the elliptical surface height profile z(x) the following parameters are required:

- distance between source and the mirror center F_1
- distance between image/focus and the mirror center F_2
- grazing-incidence angle θ

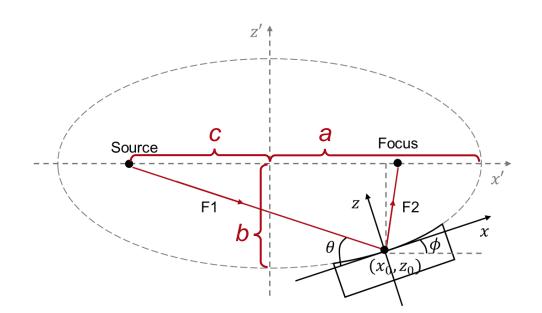
1st elliptical mirror

- $F_1 = 50 \text{m}$
- $F_2 = 400 \text{ mm}$
- $\theta = 0.172^{\circ}$

2nd elliptical mirror

- $F_1 = 50.2 \text{m}$
- $F_2 = 200 \text{ mm}$
- $\theta = 0.172^{\circ}$

Analytical Design of the Elliptical Mirror (2)



- To calculate the height function z(x), two equations need to be considered.
 - elliptical equation

$$\frac{x^{\prime 2}}{a^2} + \frac{z^{\prime 2}}{b^2} = 1$$

coordinate transform

$$\begin{pmatrix} x'\\z' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi\\\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} x\\z \end{pmatrix} + \begin{pmatrix} x_0\\z_0 \end{pmatrix}$$

• The final height function z(x) is

$$z(x) = (z' - z_0) \cos \phi - (x' - x_0) \sin \phi$$

with
$$\phi = \arctan\left(-\frac{b^2}{a^2}\frac{x_0}{z_0}\right)$$
, $x_0 = \frac{F_1^2 - F_2^2}{4c}$, and $z_0 = -b * \sqrt{1 - x_0^2/a^2}$

unknowns a, b, x' and z' in next slide

Analytical Design of the Elliptical Mirror (3)

• *a* and *b* can be calculated from F_1 , F_2 and θ

$$a = \frac{F_1 + F_2}{2}$$

$$c = \frac{1}{2}\sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos(\pi - 2\theta)}$$

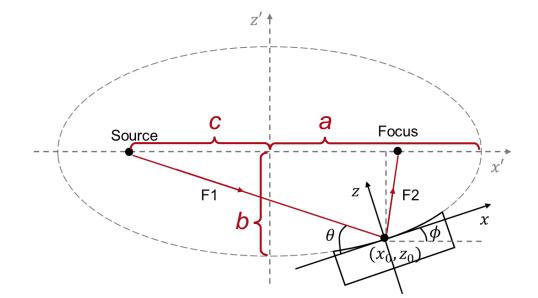
$$b = \sqrt{a^2 - c^2}$$

• parameter z' and x' are

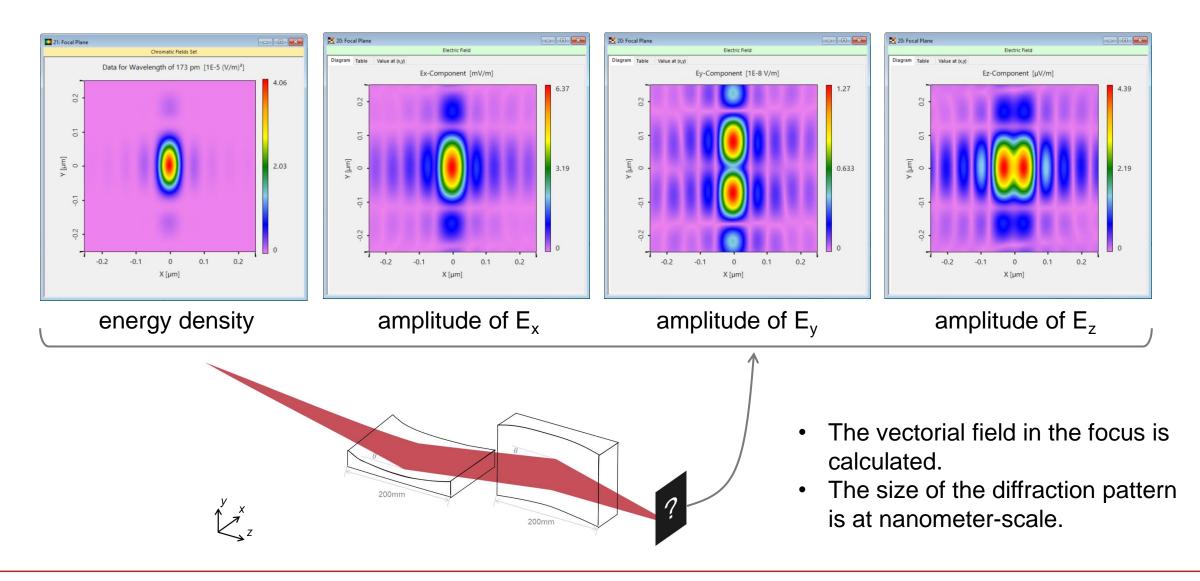
$$z'(x') = -b\sqrt{1 - \frac{x'^2}{a^2}}, \qquad x'(x) = \frac{-n + \sqrt{n^2 - 4mt}}{2m}$$

with

$$m = \cos \phi^2 + \frac{b^2}{a^2} \sin \phi$$
$$n = -2\cos \phi \left(x + x_0 \cos \phi + z_0 \sin \phi\right)$$
$$t = (x + x_0 \cos \phi + z_0 \sin \phi)^2 - b^2 \sin^2 \phi$$



Energy Density & E-Field at Focal Plane



Peek into VirtualLab Fusion

Structure He	eight Discontinuities	Scaling	Coating	Periodization		
- Surface Spe	cification					
Algorithm	;					
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			62 63		<pre>= Math.Sin(phi); = Math.Cos(phi);</pre>	
			64	double cos -	= Mach.cos(phi);	
			65	double m = b	o * b * sin * sin / (a * a) + cos * cos;
			66		-2 * cos * (x + x0 *	
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					<pre>(-n + Math.Sqrt(n * -b * Math.Sqrt(1 - x</pre>	
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lit Curved Surface	: Component	<
21	Basal Positioning Isolated Positioning Position Information (Absolute)	
Coordinate Systems	Position this Element's Input Axes with Respect to Reference Element 4: Aperture Reference Output Coordinate System T Relative Distance on Axis Delta Z 0 mm	
E Structure	Lateral Shift Delta X 0 mm Delta Y 0 mm Inclination / Rotation 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	
Solver	Orientation Definition Type Spherical Angles ✓ (iii) Z-Axis Direction Definition Angle / Axis Value Theta (Spherical) ✓ 89.828° Phi (Spherical) ✓ -90°	
Channel Configuration	Rotation About Z-Axis Z-Axis Rotation Angle 90°	

convinient definition of position and orientation

Workflow in VirtualLab Fusion

- Set up input Gaussian field
 - Basic Source Models [Tutorial Video]
- Set the position and orientation of components
 - LPD II: Position and Orientation [Tutorial Video]
- Programmable the elliptical interface
 - How to Work with the Programmable Interface & Example (Spherical Surface) [Use Case]

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	53 54		F1 + F2) / 2;// dist		Angle [double]
	54 55 56			2 * F2 - 2 * F1 * F2 c);// distance from	F1 [double] F2 [double]
	57	double $x\theta =$	(F1 * F1 - F2 * F2)	/ (4 * c);// (x0, z0	
	58		-b * Math.Sqrt(1 - x		
	60	// now is to	shift and rotate th	e coordinate the mir	
	61	double phi =	Math.Atan(-b * b *	x0 / (a * a * z0));/	
	62	double sin =	Math.Sin(phi);		
	63	double cos =	Math.Cos(phi);		
	64				
	65		* b * sin * sin / (
	66		2 * cos * (x + x0 *		
	67			sin) * (x + x0 * cos	
		double x1 =	(-n + Math.Sqrt(n *	n - 4 * m * t)) / (2	
		double z1 =	-b * Math.Sqrt(1 - x	1 * x1 / (a * a));	

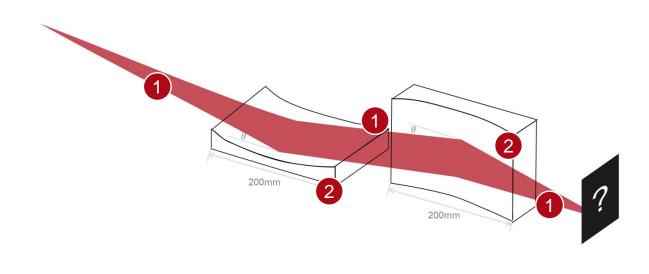
Edit Program

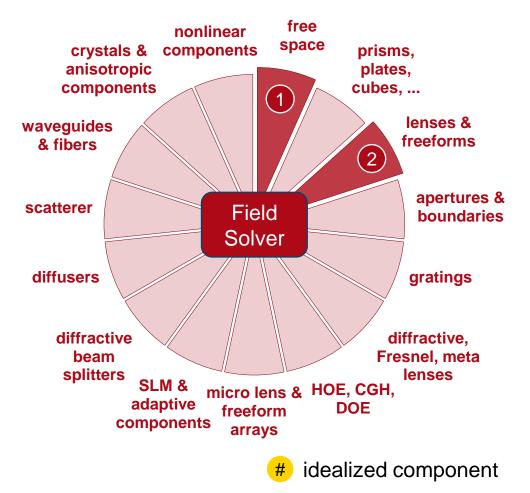
Surface Sp Algorithr Snippet

Num
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F1 F2

VirtualLab Fusion Technologies





title	Grazing-Incidence Focusing Mirrors for X-Ray Beams	
document code	XRAY.0002	
document version	2.0	
software edition	VirtualLab Fusion Basic	
software version	2021.1 (Build 1.180)	
category	Application Use Case	
further reading	- Single Grating Interferometer for X-Ray Imaging	