Grazing-Incidence Focusing Mirrors for X-Ray Beams
Grazing-incidence reflective optics is widely used in x-ray beamline, in particular Kirkpatrick-Baez elliptical mirror systems [A. Verhoeven, et al., *Journal of Synchrotron Radiation* 27.5 (2020): 1307-1319]. Focusing is accomplished using two physically separated elliptical mirrors to focus two dimensions of the beam. The incoming x-rays can be focused by the system down to nanometer-scale spot size. This system is built up in VirtualLab Fusion and the focal field is calculated.
Modeling Task

fundamental Gaussian
- wavelength 173 pm
- waist radius 4.44 x 4.44 µm

focusing elliptical mirror
- grazing-incidence angle 0.172°
- material gold

- The elliptical mirrors will be designed analytically.
- The diffraction pattern is to be calculated.

Analytical Design of the Elliptical Mirror

- The cylinder elliptical surface height profile $z(x)$ is to be calculated, when we know
  - distance between source and the mirror center $F_1$
  - distance between image/focus and the mirror center $F_2$
  - grazing-incidence angle $\theta$

- first elliptical mirror
  - $F_1 = 50$ m
  - $F_2 = 400$ mm
  - $\theta = 0.172^\circ$

- second elliptical mirror
  - $F_1 = 50.2$ m
  - $F_2 = 200$ mm
  - $\theta = 0.172^\circ$

When focal length $F_1$, $F_2$ and grazing angle $\theta$ are decided, the height profile $z(x)$ then can be calculated and programming.
Analytical Design of the Elliptical Mirror

- To calculate height function $z(x)$, two equations need to be considered.
  - Elliptical equation
    \[ \frac{x'^2}{a^2} + \frac{z'^2}{b^2} = 1 \]
  - Coordinate transform
    \[
    \begin{pmatrix}
    x' \\
    z'
    \end{pmatrix} = \begin{pmatrix}
    \cos \phi & -\sin \phi \\
    \sin \phi & \cos \phi
    \end{pmatrix} \begin{pmatrix}
    x \\
    z
    \end{pmatrix} + \begin{pmatrix}
    x_0 \\
    z_0
    \end{pmatrix}
    \]

- The final height function $z(x)$ is
  \[
  z(x) = (z' - z_0) \cos \phi - (x' - x_0) \sin \phi
  \]
  with $\phi = \arctan \left( -\frac{b^2}{a^2} \frac{x_0}{z_0} \right)$, $x_0 = \frac{F_1^2 - F_2^2}{4c}$, and
  \[
  z_0 = -b \sqrt{1 - x_0^2/a^2}
  \]

When focal length $F_1, F_2$ and grazing angle $\theta$ are decided, the height profile $z(x)$ then can be calculated and programming.
Analytical Design of the Elliptical Mirror

- $a$ and $b$ can be calculated by $F_1$, $F_2$ and $\theta$
  \[
  a = \frac{F_1 + F_2}{2}
  \]
  \[
  c = \frac{1}{2}\sqrt{\frac{F_1^2 + F_2^2}{F_1^2 + F_2^2} - 2F_1 F_2 \cos(\pi - 2\theta)}
  \]
  \[
  b = \sqrt{a^2 - c^2}
  \]

- Parameter $z'$ and $x'$ are
  \[
  z'(x') = -b\sqrt{1 - \frac{x'^2}{a^2}}, \quad x'(x) = \frac{-n + \sqrt{n^2 - 4mt}}{2m}
  \]

where $m$, $n$, $t$ are

\[
  m = \cos \phi^2 + \frac{b^2}{a^2} \sin \phi
  \]
\[
  n = -2 \cos \phi (x + x_0 \cos \phi + z_0 \sin \phi)
  \]
\[
  t = (x + x_0 \cos \phi + z_0 \sin \phi)^2 - b^2 \sin^2 \phi
  \]
Electric Field and Energy Density at Focal Plane

- The vectorial focus field is calculated.
- The size of diffraction pattern is at nanometer-scale.

fundamental Gaussian
- wavelength 173 pm
- waist radius 4.44 x 4.44 µm
Peek into VirtualLab Fusion

- Flexible definition of customized interface
- Convenient definition of position and orientation
Workflow in VirtualLab Fusion

• Set up input Gaussian field
  – Basic Source Models [Tutorial Video]

• Set the position and orientation of components
  – LPD II: Position and Orientation [Tutorial Video]

• Programmable the elliptical interface
  – How to Work with the Programmable Interface & Example (Spherical Surface) [Use Case]
VirtualLab Fusion Technologies

- free space prisms, plates, cubes, ...
- lenses & freeforms
- gratings
- apertures & boundaries
- diffractive, Fresnel, meta lenses
- HOE, CGH, DOE
- micro lens & freeform arrays
- SLM & adaptive components
- diffractive beam splitters
- scatterer
- waveguides & fibers
- crystals & anisotropic components
- nonlinear components
- free space

Field Solver

# idealized component
<table>
<thead>
<tr>
<th>title</th>
<th>Grazing-Incidence Focusing Mirrors for X-Ray Beams</th>
</tr>
</thead>
<tbody>
<tr>
<td>document code</td>
<td>MISC.0089</td>
</tr>
<tr>
<td>version</td>
<td>1.0</td>
</tr>
<tr>
<td>edition</td>
<td>VirtualLab Fusion Basic</td>
</tr>
<tr>
<td>software version</td>
<td>2020.2 (Build 1.116)</td>
</tr>
<tr>
<td>category</td>
<td>Application Use Case</td>
</tr>
<tr>
<td>further reading</td>
<td>- <a href="#">Single Grating Interferometer for X-Ray Imaging</a></td>
</tr>
</tbody>
</table>
MISC.0089, X-Ray Imaging, Kirkpatrick–Baez Mirrors, KB Mirrors, Focal Spot
Kirkpatrick-Baez mirrors focus the grazing-incident x-ray field into a nanometer-scale spot. This use case shows Kirkpatrick-Baez mirrors' analytical design procedure and the diffraction pattern in the focal region.
fundamental Gaussian
- wavelength 173 pm
- waist radius 4.44 x 4.44 μm