Control of Accuracy-Speed Balance for MTF Analysis in Complex Waveguide Devices for AR-Applications
During the design process of waveguide devices in the field of augmented and mixed reality applications (AR/MR), the accurate calculation of the achievable optical performance is one of the major tasks. One very important quantity is, besides the spatial and angular uniformity, the modulation transfer function (MTF), which enables an assessment of the resolution capability of the final device. In this example, we point out the impact of diffraction and coherence effects on the accuracy of the calculated MTF. We further show that an accurate and fast inclusion of these effects necessitates a combination of highly interoperable simulation techniques on a single platform. This also enables the user to seamlessly control the accuracy and speed balance of complex optical systems.
Task Description

**Task:** How to accurately calculate the MTF of a waveguide? What effects have to be considered?

**Layout & initial parameters:**

**Incoupler**
- idealized grating
- 380 nm period
- efficiency 1st order: 50%
- efficiency 0th order: 50%
  (for backside illumination)

**Outcoupler**
- binary grating
- 380 nm period
- height: 50 nm
- fill factor: 50%

**Eye Pupil Expander**
- binary grating
- 268.7 nm period
- height: 50 nm
- fill factor: 50%
Simulation & Setup: Single Platform Interoperability
Connected Modeling Techniques: Source

Light Engine Model
• beam type: plane wave
• beam diameter: 3 mm (circular)
• polarization: linearly polarized
• wavelength: 532 nm
• bandwidth: 0 nm, 1 nm, 10 nm

Available modeling techniques for sources with finite bandwidth (temporal coherence):

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Two different techniques are available for modeling the source in this setup, the advantages and disadvantages of each one will be discussed later in the document.
Each beam interacts with very different kinds of optical components while propagating through the complex system. Therefore, an accurate model requires a seamless interoperability of algorithms to be able to handle all aspects that arise:

1. gratings (incoupler, EPE, outcoupler)
2. free space (propagation inside the glass slab)
3. reflection at surfaces of glass slab
4. region boundaries (at boundaries of a grating)
5. detector (uniformity measurement in eye box)
6. eye model (calculation of PSF & MTF)
## Connected Modeling Techniques: Gratings

1. gratings (incoupler, EPE, outcoupler)
2. free space (propagation inside the glass slab)
3. reflection at surfaces of glass slab
4. region boundaries (at boundaries of a grating)
5. detector (uniformity measurement in eye box)
6. eye model (calculation of PSF & MTF)

Available modeling techniques for periodic micro and nano structures:

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<tbody>
<tr>
<td>Fourier Modal Method (FMM)</td>
<td>None</td>
<td>High</td>
<td>High</td>
<td>Small periods</td>
</tr>
<tr>
<td>Thin Grating Approximation</td>
<td>Large periods &amp; features, thin</td>
<td>High</td>
<td>High</td>
<td>Thickness about wavelength; period &amp; features larger than about ten wavelengths</td>
</tr>
<tr>
<td></td>
<td>Otherwise</td>
<td>Low</td>
<td>High</td>
<td></td>
</tr>
<tr>
<td>FMM in Kogelnik Approximation</td>
<td>Thick volume gratings; Bragg condition</td>
<td>High</td>
<td>Very high</td>
<td>Method is electromagnetic formulation of Kogelnik's approach</td>
</tr>
<tr>
<td></td>
<td>No Bragg condition</td>
<td>Low</td>
<td>Very high</td>
<td></td>
</tr>
</tbody>
</table>

As a rigorous eigenmode solver, the Fourier modal method (also known as rigorous coupled wave analysis, RCWA) provides very high accuracy. Due to the small periods in this setup, the calculation speed is fast. FMM is therefore the best compromise of accuracy and speed.
Available modeling techniques for free-space propagation:

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<tbody>
<tr>
<td>Rayleigh Sommerfeld Integral</td>
<td>None</td>
<td>High</td>
<td>Low</td>
<td>Rigorous solution</td>
</tr>
<tr>
<td>Fourier Domain Techniques</td>
<td>None</td>
<td>High</td>
<td>High</td>
<td>Rigorous mathematical reformulation of RS integral</td>
</tr>
<tr>
<td>Fresnel Integral</td>
<td>Paraxial</td>
<td>High</td>
<td>High</td>
<td>Assumes paraxial light; moderate speed for very short distances</td>
</tr>
<tr>
<td></td>
<td>Non-paraxial</td>
<td>Low</td>
<td>High</td>
<td></td>
</tr>
<tr>
<td>Geometric Propagation</td>
<td>Low diffraction</td>
<td>High</td>
<td>Very high</td>
<td>Neglects diffraction effects</td>
</tr>
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<td>Otherwise</td>
<td>Low</td>
<td>Very high</td>
<td></td>
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</table>

Two fast modeling techniques are available to calculate the propagation inside the glass plate:
- **Fourier Domain Techniques** (includes diffraction effects of boundaries and apertures)
- **Geometric Propagation** (neglects diffraction that arises from boundaries and apertures)

In order to choose the adequate technique, the results need to be considered!
Connected Modeling Techniques: Waveguide Surfaces

1. gratings (incoupler, EPE, outcoupler)
2. free space (propagation inside the glass slab)
3. reflection at surfaces of glass slab
4. region boundaries (at boundaries of a grating)
5. detector (uniformity measurement in eye box)
6. eye model (calculation of PSF & MTF)

Available modeling techniques interaction with surfaces:

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<tbody>
<tr>
<td>S matrix</td>
<td>Planar surface</td>
<td>High</td>
<td>Very High</td>
<td>Rigorous model; includes isotropic and birefringent coatings; k-domain</td>
</tr>
<tr>
<td>Local Planar Interface Approximation</td>
<td>Surface not in focal region of beam</td>
<td>High</td>
<td>Very High</td>
<td>Local application of S matrix; LPIA; x-domain</td>
</tr>
</tbody>
</table>

Two modeling techniques are available to calculate the interaction with the surfaces.

Since both are fast, and the Local Planar Interface Approximation allows us in addition to consider curved surfaces (e.g. for tolerance analysis), this technique is chosen.
Available modeling techniques for interaction with region boundaries:

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<tbody>
<tr>
<td>Local application of LPIA and FMM</td>
<td>Region extent not close to a few wavelengths</td>
<td>High</td>
<td>Very High</td>
<td>Beam profile cut along region boundaries; high resolution</td>
</tr>
</tbody>
</table>

With the local application of LPIA and FMM we can take into account the interactions with the boundaries of grating regions.
Light Guide Component

Modeling techniques １ to ４ are combined in the *Light Guide Component*. With this element, grating-based lightguide systems with complex-shaped grating regions can easily be defined. Furthermore, these regions can be equipped with idealized or real grating structures (1D or 2D-periodic) to act as incoupler, outcoupler or exit pupil expanders. More information under:

➡️ Construction of a Light Guide
For the incoupler, outcoupler and eye pupil expander (EPE) real gratings were used. Their Rayleigh matrices and the corresponding diffraction efficiencies are calculated rigorously by applying FMM (RCWA). You can find more information on how to set this up under:

How to Set Up a Lightguide with Real Grating Structures
Connected Modeling Techniques: Detector Eyebox

1. gratings (incoupler, EPE, outcoupler)
2. free-space (propagation inside the glass slab)
3. reflection at surfaces of glass slab
4. region boundaries (at boundaries of a grating)
5. detector (uniformity measurement in eye box)
6. eye model (calculation of PSF & MTF)

5. Full flexibility in detector modeling:
   - Radiometry, e.g., irradiance per FOV or all FOVs, radiance
   - Photometry, e.g., illuminance per FOV or all FOVs, luminance
   - Uniformity measures

6. Eye model for
   - Point spread function (PSF)
   - Modulation Transfer Function (MTF)
Diffraction Inside Waveguide: Irradiance Eye Box

1. grating (incoupler)
2. free-space (propagation inside the glass slab)

Diffraction leads to modulation around the edges caused by interactions with boundaries.

Irradiance in eye box with diffraction:

Simulation Time: 300 s (Number of FFTs: 528)

Irradiance in eye box without diffraction:

Simulation Time: 10 s

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PSF & MTF Analysis – Part #1: Diffraction Effects
PSF and MTF Calculation: One Beam in Pupil

In case of a uniformly filled pupil, the resulting PSF and MTF are rotation symmetric and exhibit a continuous decay. For a smaller beam diameter, the decay of the MTF is stronger, but the general behavior does not change.

Eye model:
- Pupil diameter: $D_{\text{pupil}} = 4 \text{ mm}$
- Ideal lens: $f_{\text{ideal}} = 16.452 \text{ mm}$

MTF x-profile

4 mm

3 mm
PSF and MTF Calculation: Multiple Beams in Pupil

For a pupil filled with multiple beams, which are affected by numerous interactions with region boundaries, the resulting (two dimensional) PSF and MTF are much more complex and no longer rotationally symmetric.

Eye model:
- Pupil diameter: $D^{\text{eye}} = 4 \text{ mm}$
- Ideal lens: $f^{\text{eye}} = 16.452 \text{ mm}$

Irradiance in Eyebox

Eye Pupil

Irradiance in Pupil

Eye Pupil

PSF

MTF
In case the geometric propagation technique is chosen, diffraction effects introduced by apertures and region boundaries are neglected. However, the resulting (two dimensional) PSF and MTF exhibit a complex behavior and are not longer rotationally symmetric.
In contrast to the MTF of a uniformly filled aperture, the cross-sections along x and y show different profiles and no monotonous decay anymore.

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By applying a rigorous propagation technique, such as VirtualLab’s Fourier domain techniques, diffraction effects introduced by apertures and region boundaries can be considered. The resulting PSF and MTF are very different from the non-diffractive case.

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PSF and MTF Calculation: with Diffraction in Waveguide

In contrast to the MTF of a uniformly filled aperture, the cross-sections along x and y show different profiles with some oscillations. Moreover, the resulting MTF is very different compared to the result without including diffraction effects.

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MTF

MTF x-profile

MTF y-profile
PSF & MTF Analysis – Part #2: Temporal Coherence
PSF and MTF Calculation: Frequency Model

In order to model the bandwidth of a source rigorously, the bandwidth must be sampled properly. The sampled frequencies (or wavelengths) then must be propagated through the system separately. This leads to a higher computational effort and calculation time.

For example, a source possessing a Gaussian spectrum (FWHM 10 nm) requires a sampling of 1 pm in this application. This leads to over 10000 simulations. Alternatively, it is sufficient to model one single free spectral range (FSR, 60 pm) with 60 wavelength samples (sampling distance again 1 pm).

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Gaussian spectrum (FWHM 10 nm)
PSF and MTF Calculation: Frequency Model (w/o Diffraction)

Results for Frequency Model:

Irradiance in eye box

\[ \Delta \lambda = 0 \text{ nm} \]

\[ \Delta \lambda = 1 \text{ nm} \]

\[ \Delta \lambda = 10 \text{ nm} \]

Dispersion leads to a smearing of the overlapping beams

The coherence properties of the light engine influence the MTF!

Simulation Time: 4.5 min
PSF and MTF Calculation: Frequency Model (with Diffraction)

Results for Frequency Model:

\[ \Delta \lambda = 10 \text{ nm} \]

Only the combination of including coherence and diffraction effects provides an accurate result!
PSF and MTF Calculation: Time Model

An alternative modeling technique is the analysis of the travel time of a single wavelength (e.g. peak wavelength of the spectrum). The comparison of the travel time and the coherence time (or length) allows us to distinguish whether the light must be treated coherently or incoherently. For example, a bandwidth of 10 nm (FHWM) leads to a coherence time of 60.1 fs and 18.0 µm coherence length.

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PSF and MTF Calculation: Time Model (w/o Diffraction)

Results for Time Model:

Irradiance in eye box

Cross-section of MTF (along x-direction)

- $\Delta \lambda = 0$ nm
- $\Delta \lambda = 1$ nm
- $\Delta \lambda = 10$ nm

Dispersion effect not included!

The coherence properties of the light engine influence the MTF!

Simulation Time: 7 sec
PSF and MTF Calculation: Time Model (with Diffraction)

Results for Frequency Model:
\[ \Delta \lambda = 10 \text{ nm} \]

Excellent accuracy-speed balance for MTF calculation!

Simulation Time: 2.5 min
Summary & Conclusion
Irradiances in Pupil and MTFs

\[ \Delta \lambda = 10 \text{ nm} \]

- Partially coherent, with diffraction
- Partially coherent, without diffraction
- Coherent, uniform, 4 mm
- Coherent, uniform, 3 mm
Conclusion:
For the adequate modeling of the optical performance, such as PSF and MTF, of grating-based waveguides, diffraction and coherence effects must be considered! In order to be able to model complex optical systems accurately and fast, highly interoperable modeling techniques on a single platform are a necessity. This also allows for a detailed control of the accuracy and speed balance.