FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

SPIE Optical Systems Design 2018, Frankfurt, Germany

Physical-Optics Modeling for Optical Components Made out of Birefringent Materials

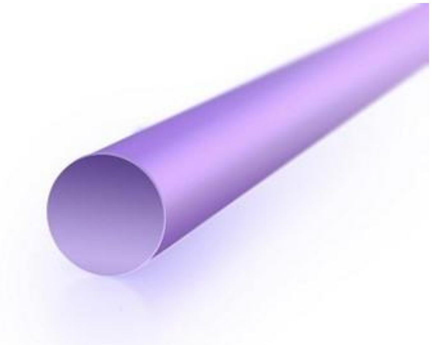
Site Zhang[1,2], Christian Hellmann[2,3], and Frank Wyrowski[1]

[1] Applied Computational Optics Group, Friedrich Schiller University Jena, Jena, Germany

[2] LightTrans International UG, Jena, Germany

[3] Wyrowski Photonics UG, Jena, Germany

Motivation



YAG crystal rod
(http://www.altechna.com)

BBO crystal slabs
(http://www.altechna.com)

quartz waveplates
(http://www.altechna.com)

Wollaston prism
(https://www.thorlabs.com)

- Optical birefringence / anisotropy is a natural properties of many crystal materials, and it also appear as an induced effect from e.g. stress.
- Such effects gives the possibility to control and manipulate light according to the polarization, which is of importance in modern optical research and applications.

Fundamentals – Field in Anisotropic Media

- Maxwell's equations in homogeneous anisotropic media

$$\nabla \times \boldsymbol{E}(\boldsymbol{r},\omega) = \mathrm{i}\omega\mu_0\underline{\mu}(\omega)\boldsymbol{H}(\boldsymbol{r},\omega)$$
$$\nabla \times \boldsymbol{H}(\boldsymbol{r},\omega) = -\mathrm{i}\omega\epsilon_0\underline{\epsilon}(\omega)\boldsymbol{E}(\boldsymbol{r},\omega)$$

permittivity and permeability tensors in 3×3 matrix form ➔ valid for general anisotropy!

- Explicit expression in spatial domain (*x*-domain)

$$
\begin{aligned}
\partial_y H_z(\boldsymbol{r}) - \partial_z H_y(\boldsymbol{r}) &= -\mathrm{i}k_0\left(\epsilon_{11}E_x(\boldsymbol{r}) + \epsilon_{12}E_y(\boldsymbol{r}) + \epsilon_{13}E_z(\boldsymbol{r})\right) \ , \\
\partial_z H_x(\boldsymbol{r}) - \partial_x H_z(\boldsymbol{r}) &= -\mathrm{i}k_0\left(\epsilon_{21}E_x(\boldsymbol{r}) + \epsilon_{22}E_y(\boldsymbol{r}) + \epsilon_{23}E_z(\boldsymbol{r})\right) \ , \\
\partial_x H_y(\boldsymbol{r}) - \partial_y H_x(\boldsymbol{r}) &= -\mathrm{i}k_0\left(\epsilon_{31}E_x(\boldsymbol{r}) + \epsilon_{32}E_y(\boldsymbol{r}) + \epsilon_{33}E_z(\boldsymbol{r})\right) \ , \\
\partial_y E_z(\boldsymbol{r}) - \partial_z E_y(\boldsymbol{r}) &= \mathrm{i}k_0\left(\mu_{11}H_x(\boldsymbol{r}) + \mu_{12}H_y(\boldsymbol{r}) + \mu_{13}H_z(\boldsymbol{r})\right) \ , \\
\partial_z E_x(\boldsymbol{r}) - \partial_x E_z(\boldsymbol{r}) &= \mathrm{i}k_0\left(\mu_{21}H_x(\boldsymbol{r}) + \mu_{22}H_y(\boldsymbol{r}) + \mu_{23}H_z(\boldsymbol{r})\right) \ , \\
\partial_x E_y(\boldsymbol{r}) - \partial_y E_x(\boldsymbol{r}) &= \mathrm{i}k_0\left(\mu_{31}H_x(\boldsymbol{r}) + \mu_{32}H_y(\boldsymbol{r}) + \mu_{33}H_z(\boldsymbol{r})\right) \ .
\end{aligned}
$$

Using Fourier transform to obtain the expression in the conjugated domain

$$\tilde{V}(\boldsymbol{\kappa}, z) = \mathcal{F}_k V(\boldsymbol{\rho}, z)$$

Fundamentals – Field in Anisotropic Media

- Maxwell's equations in homogeneous anisotropic media

$$\nabla \times \boldsymbol{E}(\boldsymbol{r},\omega) = \mathrm{i}\omega\mu_0\underline{\boldsymbol{\mu}}(\omega)\boldsymbol{H}(\boldsymbol{r},\omega)$$
$$\nabla \times \boldsymbol{H}(\boldsymbol{r},\omega) = -\mathrm{i}\omega\epsilon_0\underline{\boldsymbol{\epsilon}}(\omega)\boldsymbol{E}(\boldsymbol{r},\omega)$$

permittivity and permeability tensors in 3×3 matrix form ➔ valid for general anisotropy!

- Explicit expression in spatial frequency domain (*k*-domain)

$$\mathrm{i}k_y\tilde{H}_z(\boldsymbol{\kappa},z) - \mathrm{d}_z\tilde{H}_y(\boldsymbol{\kappa},z) = -\mathrm{i}k_0\left(\epsilon_{11}\tilde{E}_x(\boldsymbol{\kappa},z) + \epsilon_{12}\tilde{E}_y(\boldsymbol{\kappa},z) + \epsilon_{13}\tilde{E}_z(\boldsymbol{\kappa},z)\right) ,$$
$$\mathrm{d}_z\tilde{H}_x(\boldsymbol{\kappa},z) - \mathrm{i}k_x\tilde{H}_z(\boldsymbol{\kappa},z) = -\mathrm{i}k_0\left(\epsilon_{21}\tilde{E}_x(\boldsymbol{\kappa},z) + \epsilon_{22}\tilde{E}_y(\boldsymbol{\kappa},z) + \epsilon_{23}\tilde{E}_z(\boldsymbol{\kappa},z)\right) ,$$
$$\boxed{\mathrm{i}k_x\tilde{H}_y(\boldsymbol{\kappa},z) - \mathrm{i}k_y\tilde{H}_x(\boldsymbol{\kappa},z) = -\mathrm{i}k_0\left(\epsilon_{31}\tilde{E}_x(\boldsymbol{\kappa},z) + \epsilon_{32}\tilde{E}_y(\boldsymbol{\kappa},z) + \epsilon_{33}\tilde{E}_z(\boldsymbol{\kappa},z)\right) ,}$$
$$\mathrm{i}k_y\tilde{E}_z(\boldsymbol{\kappa},z) - \mathrm{d}_z\tilde{E}_y(\boldsymbol{\kappa},z) = \mathrm{i}k_0\left(\mu_{11}\tilde{H}_x(\boldsymbol{\kappa},z) + \mu_{12}\tilde{H}_y(\boldsymbol{\kappa},z) + \mu_{13}\tilde{H}_z(\boldsymbol{\kappa},z)\right) ,$$
$$\mathrm{d}_z\tilde{E}_x(\boldsymbol{\kappa},z) - \mathrm{i}k_x\tilde{E}_z(\boldsymbol{\kappa},z) = \mathrm{i}k_0\left(\mu_{21}\tilde{H}_x(\boldsymbol{\kappa},z) + \mu_{22}\tilde{H}_y(\boldsymbol{\kappa},z) + \mu_{23}\tilde{H}_z(\boldsymbol{\kappa},z)\right) ,$$
$$\boxed{\mathrm{i}k_x\tilde{E}_y(\boldsymbol{\kappa},z) - \mathrm{i}k_y\tilde{E}_x(\boldsymbol{\kappa},z) = \mathrm{i}k_0\left(\mu_{31}\tilde{H}_x(\boldsymbol{\kappa},z) + \mu_{32}\tilde{H}_y(\boldsymbol{\kappa},z) + \mu_{33}\tilde{H}_z(\boldsymbol{\kappa},z)\right) .}$$

Fundamentals – Field in Anisotropic Media

- Eigenvalue problem

$$\frac{\mathrm{d}}{\mathrm{d}z}\begin{pmatrix}\tilde{E}_x\\ \tilde{E}_y\\ \tilde{H}_x\\ \tilde{H}_y\end{pmatrix} = \mathrm{i}k_0\begin{pmatrix}\tilde{\Omega}_{11} & \tilde{\Omega}_{12} & \tilde{\Omega}_{13} & \tilde{\Omega}_{14}\\ \tilde{\Omega}_{21} & \tilde{\Omega}_{22} & \tilde{\Omega}_{23} & \tilde{\Omega}_{24}\\ \tilde{\Omega}_{31} & \tilde{\Omega}_{32} & \tilde{\Omega}_{33} & \tilde{\Omega}_{34}\\ \tilde{\Omega}_{41} & \tilde{\Omega}_{42} & \tilde{\Omega}_{43} & \tilde{\Omega}_{44}\end{pmatrix}\begin{pmatrix}\tilde{E}_x\\ \tilde{E}_y\\ \tilde{H}_x\\ \tilde{H}_y\end{pmatrix}$$

Electromagnetic field properties in anisotropic media are carried in the modes.

- Field solutions in anisotropic media

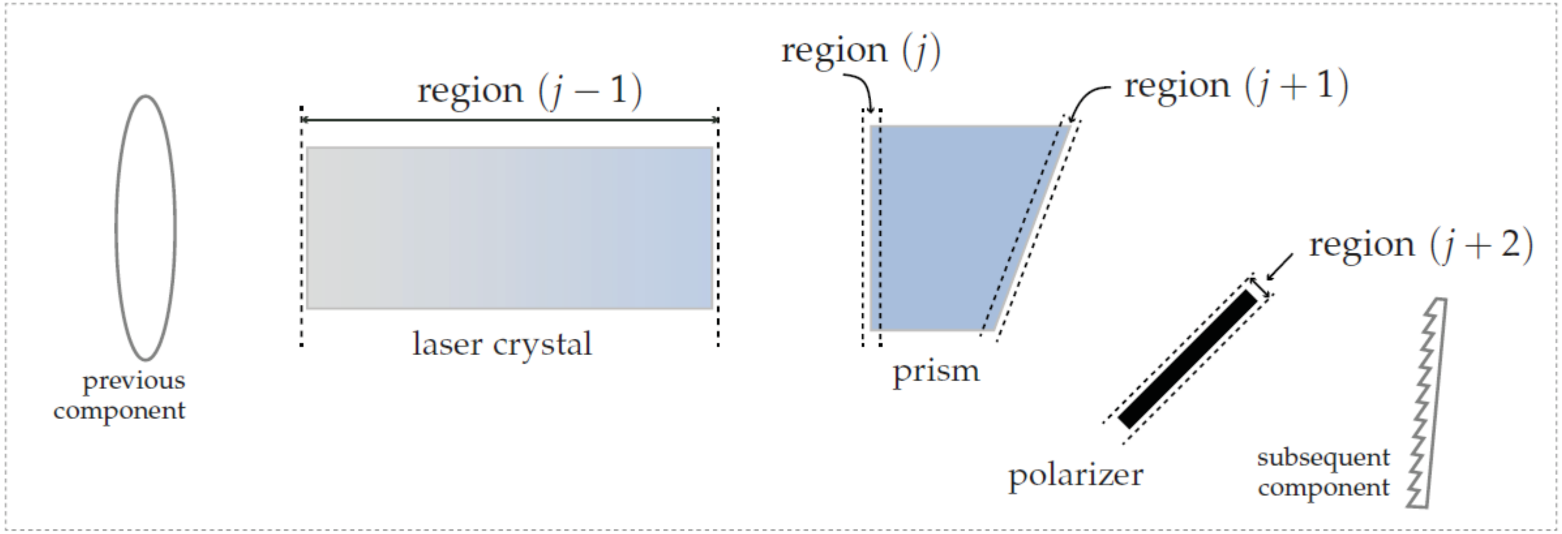
$$\begin{pmatrix}\tilde{E}_x\\ \tilde{E}_y\\ \tilde{H}_x\\ \tilde{H}_y\end{pmatrix} = \begin{pmatrix}\tilde{W}^{\mathrm{I}}_{1,+} & \tilde{W}^{\mathrm{II}}_{1,+} & \tilde{W}^{\mathrm{I}}_{1,-} & \tilde{W}^{\mathrm{II}}_{1,-}\\ \tilde{W}^{\mathrm{I}}_{2,+} & \tilde{W}^{\mathrm{II}}_{2,+} & \tilde{W}^{\mathrm{I}}_{2,-} & \tilde{W}^{\mathrm{II}}_{2,-}\\ \tilde{W}^{\mathrm{I}}_{3,+} & \tilde{W}^{\mathrm{II}}_{3,+} & \tilde{W}^{\mathrm{I}}_{3,-} & \tilde{W}^{\mathrm{II}}_{3,-}\\ \tilde{W}^{\mathrm{I}}_{4,+} & \tilde{W}^{\mathrm{II}}_{4,+} & \tilde{W}^{\mathrm{I}}_{4,-} & \tilde{W}^{\mathrm{II}}_{4,-}\end{pmatrix}\begin{pmatrix}e^{\boxed{ik^{\mathrm{I}}_{z+}}z} & 0 & 0 & 0\\ 0 & e^{\boxed{ik^{\mathrm{II}}_{z+}}z} & 0 & 0\\ 0 & 0 & e^{\boxed{ik^{\mathrm{I}}_{z-}}z} & 0\\ 0 & 0 & 0 & e^{\boxed{ik^{\mathrm{II}}_{z-}}z}\end{pmatrix}\begin{pmatrix}\tilde{C}^{\mathrm{I}}_{+}\\ \tilde{C}^{\mathrm{II}}_{+}\\ \tilde{C}^{\mathrm{I}}_{-}\\ \tilde{C}^{\mathrm{II}}_{-}\end{pmatrix}.$$

eigenvectors

eigenvalues

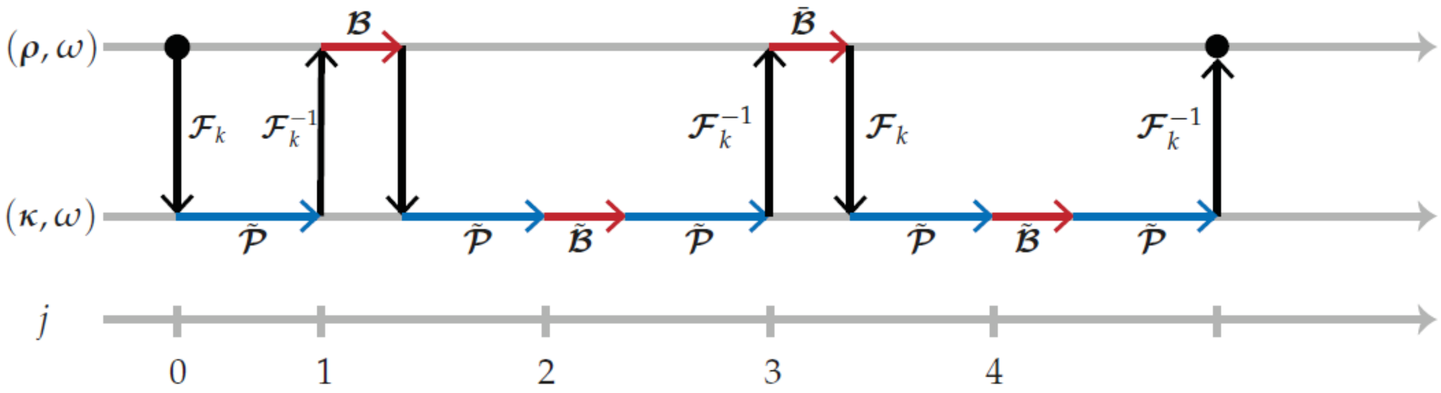
mode coefficients

Field Tracing Concept



A complete system is split into regions, and proper field tracing operators are applied for each region.

Field tracing operators can be defined and applied in different domains; Fourier transform is used to connect different domains.

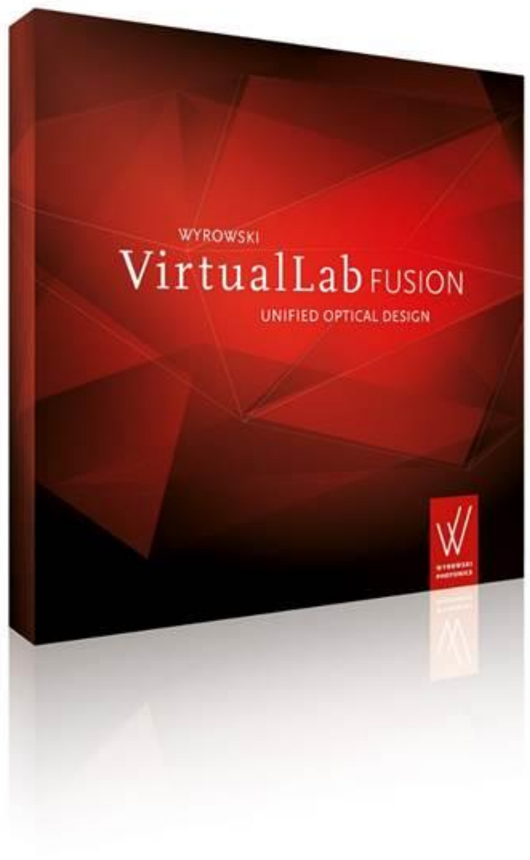


Numerical Implementation

- The corresponding field tracing operators for components made out of anisotropic media are implemented in the physical optics simulation and design software **VirtualLab Fusion**, by using its programming interface.

- Several simulation examples can be downloaded from www.LightTrans.com

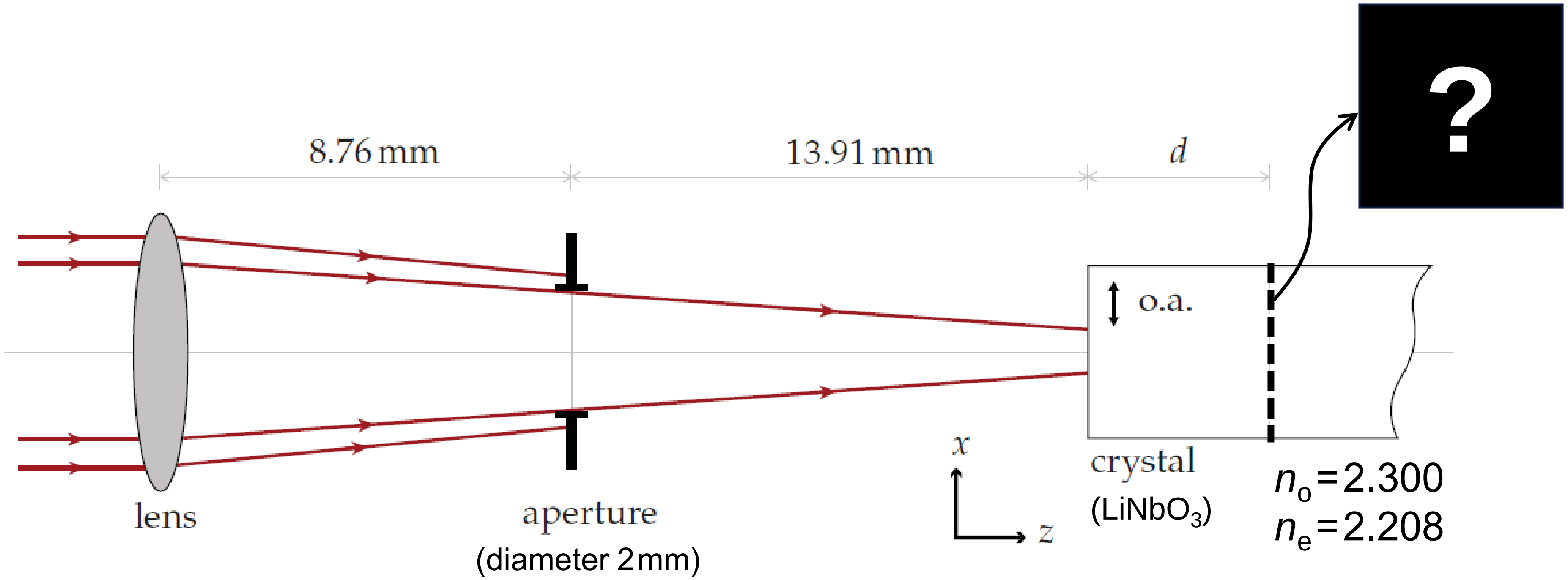
- Visit the LightTrans at OPTATEC **booth G63** for more information.



Example: Focusing into Uniaxial Crystal

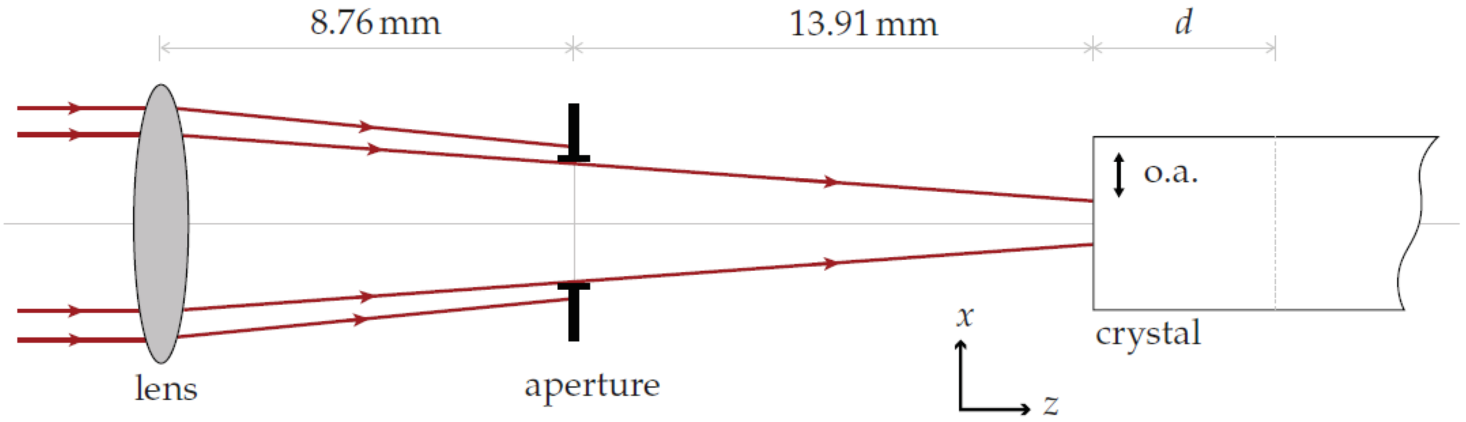
observe focused field distributions inside $LiNbO_3$ crystal, at different depths

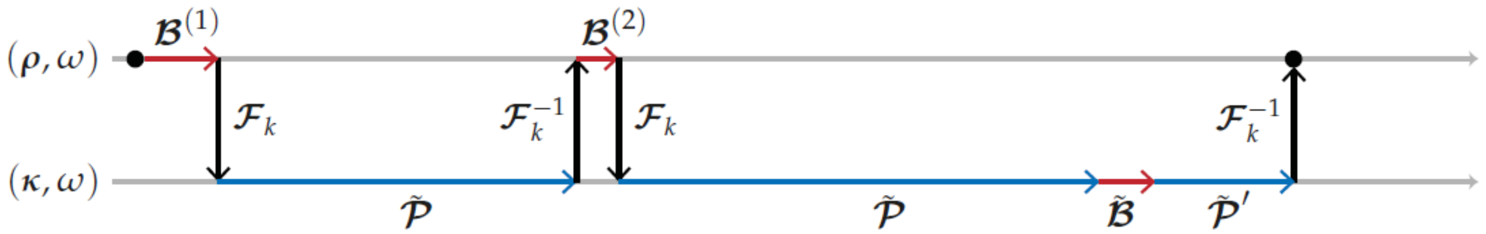
input field
- plane wave
- wavelength 633 nm
- diameter 4 mm
- linearly polarized along *x* direction



Reference: [Jain 2009] Jain *et al.*, J. Opt. Soc. Am. A **26**, 691-698 (2009)

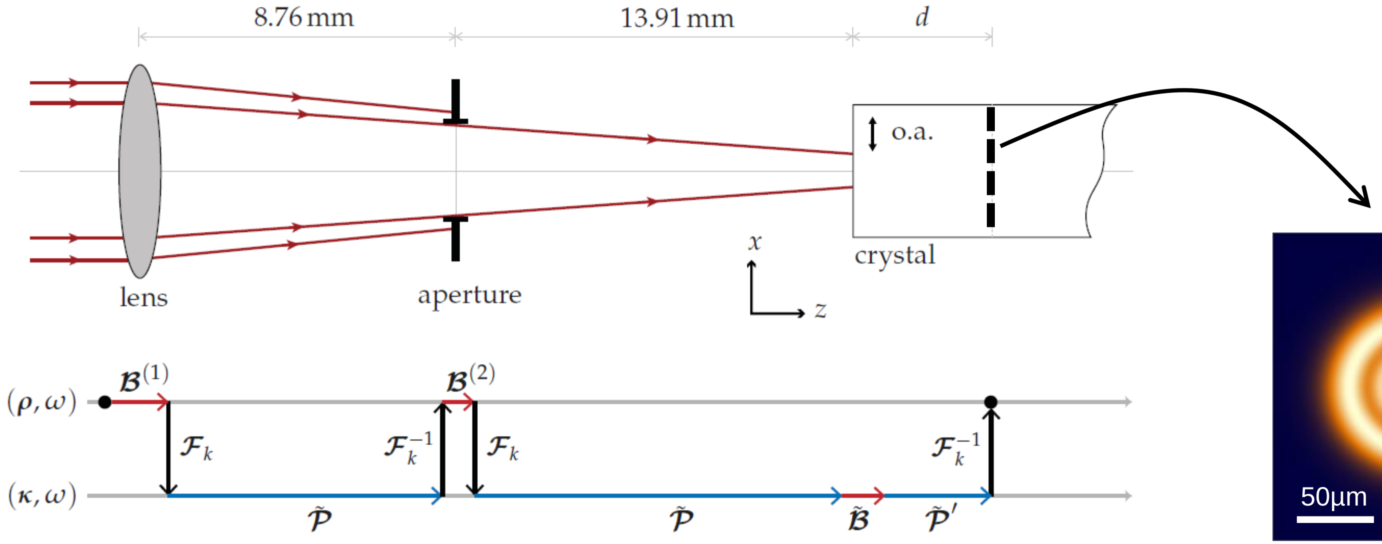
Example: Focusing into Uniaxial Crystal



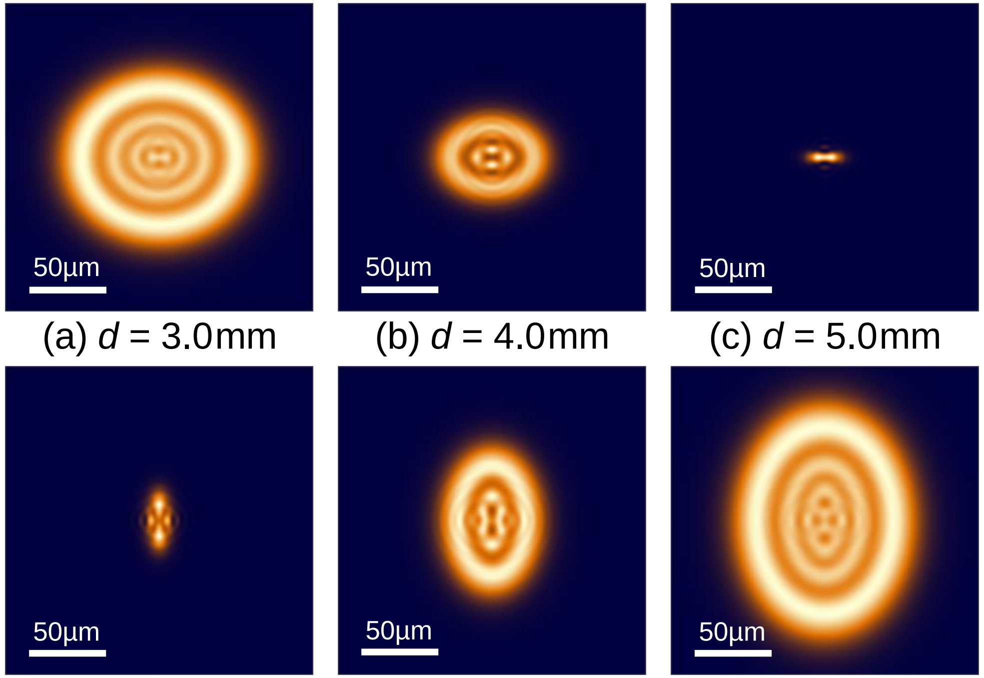


Operator	Description
$\mathcal{B}^{(1)}$	idealized lens model
$\mathcal{B}^{(2)}$	idealized aperture model
$\tilde{\mathcal{P}}$ / $\tilde{\mathcal{P}}'$	free-space propagation (isotropic / anisotropic)
$\tilde{\mathcal{B}}$	planar surface operator

Example: Focusing into Uniaxial Crystal



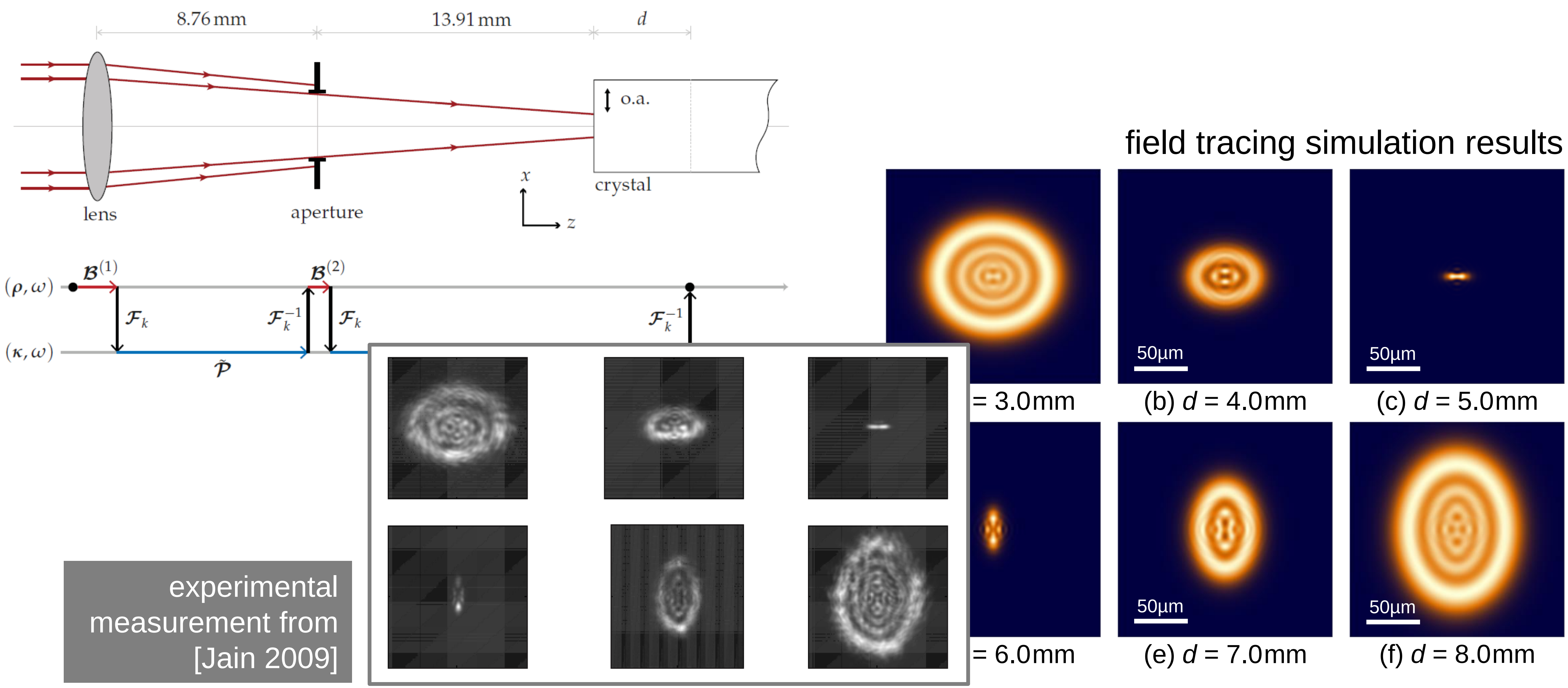
field tracing simulation results



(a) d = 3.0mm (b) d = 4.0mm (c) d = 5.0mm

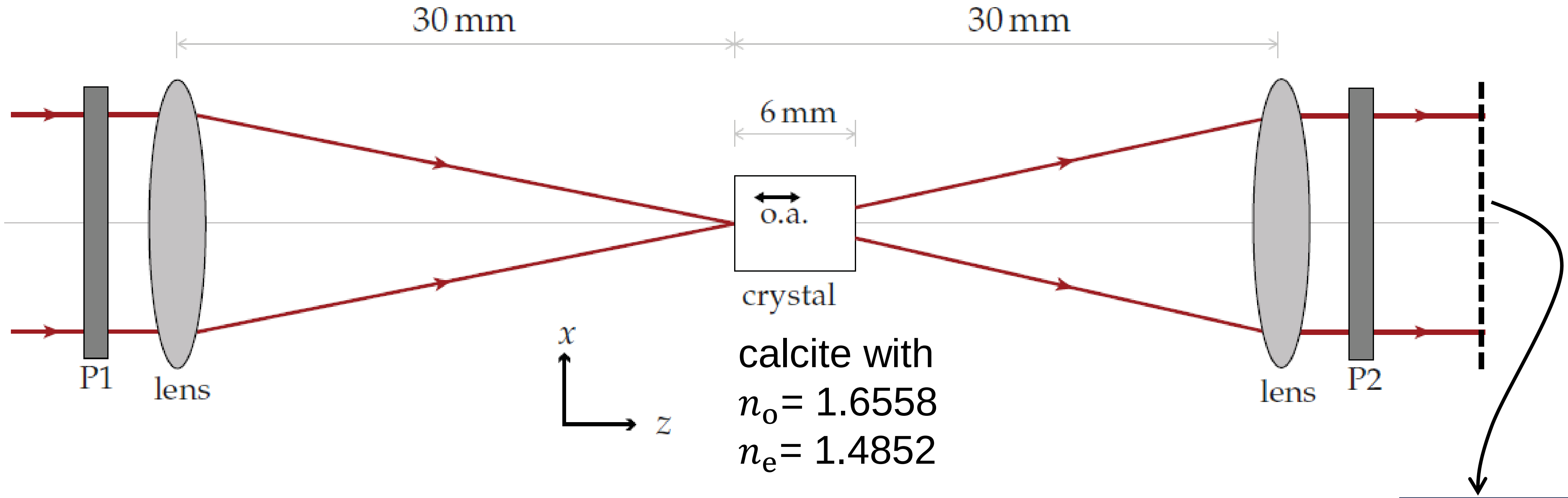
(d) d = 6.0mm (e) d = 7.0mm (f) d = 8.0mm

Example: Focusing into Uniaxial Crystal



Example: Polarization Conversion

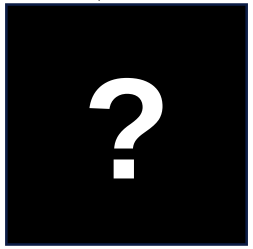
input field
- fundamental Gaussian field
- wavelength 633 nm
- diameter 3 mm
- linearly polarized along *x* direction



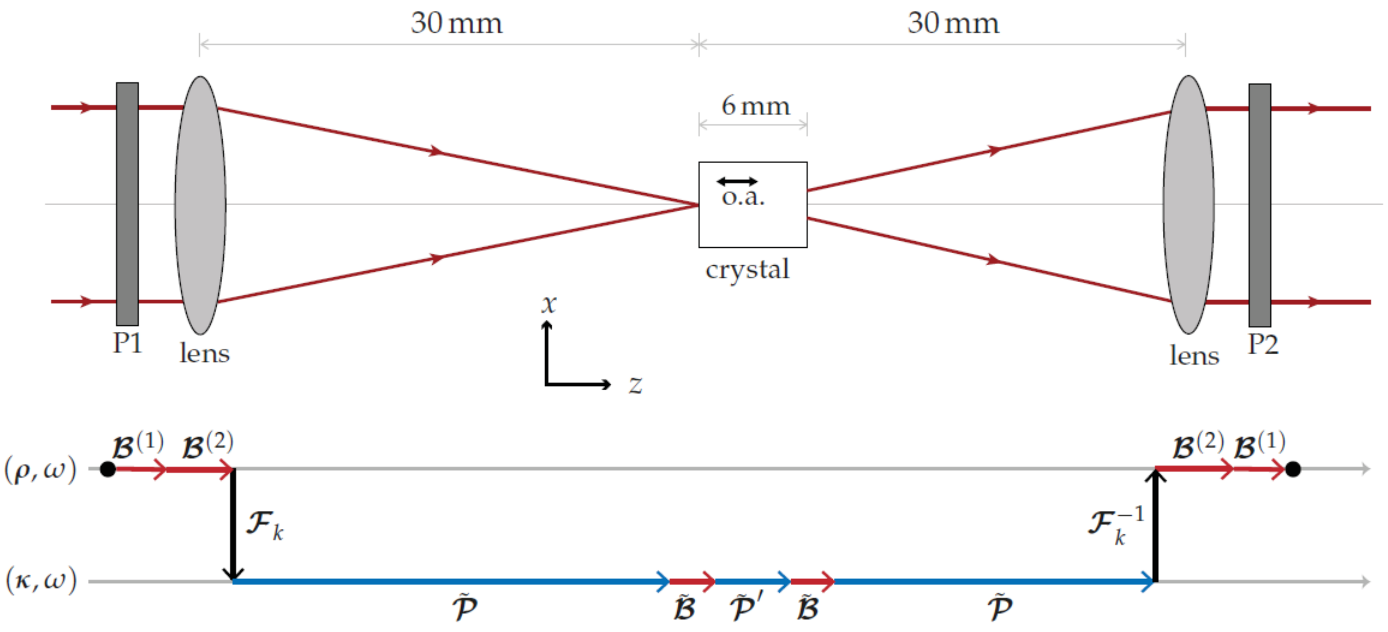
calcite with
n_o = 1.6558
n_e = 1.4852

Reference: [Izdebskaya 2009] Izdebskaya *et al.*, Opt. Express **17**, 18196-18208 (2009)

calculate field behind polarizer P2, when it is either parallel or orthogonal to P1

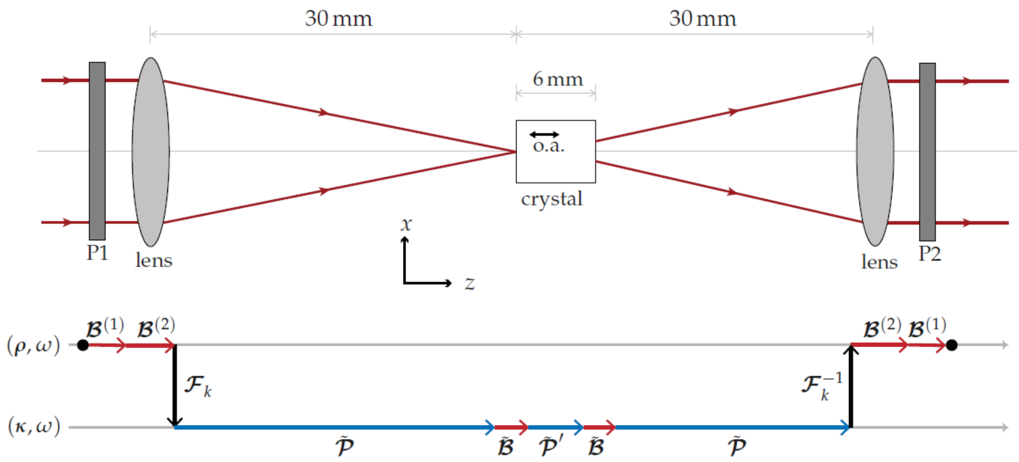


Example: Polarization Conversion



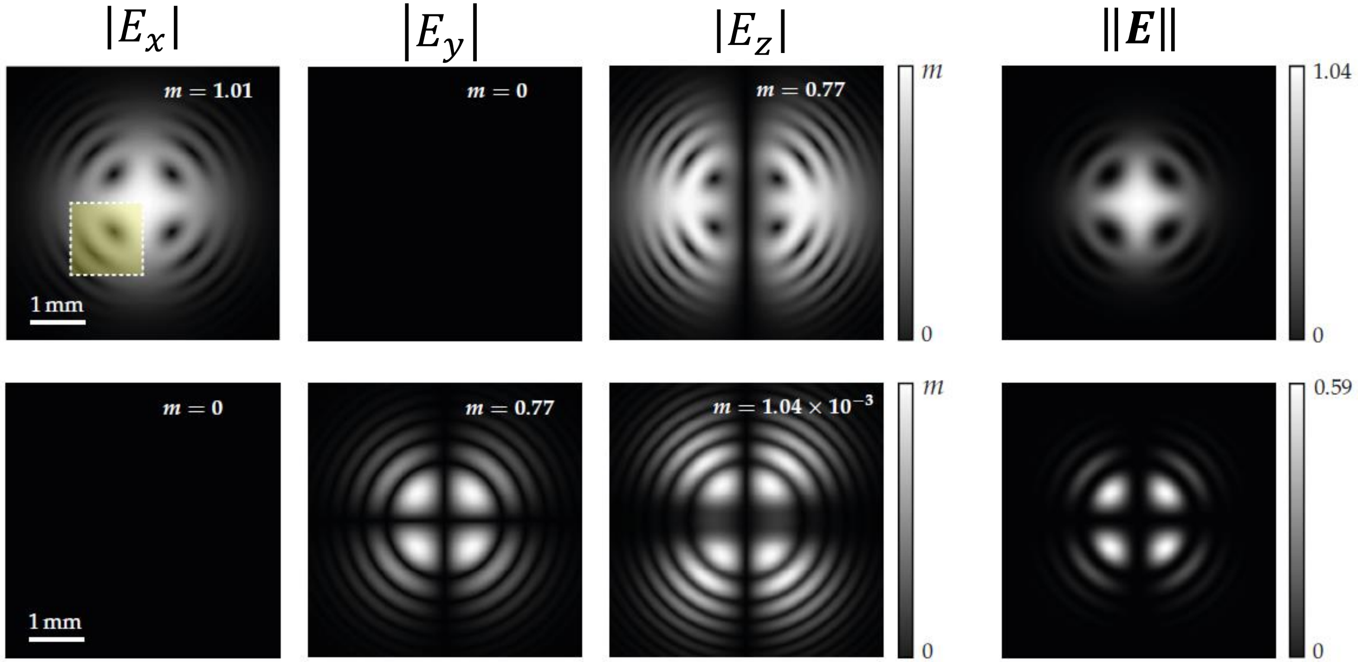
Operator	Description
$\mathcal{B}^{(1)}$	Jones matrix model
$\mathcal{B}^{(2)}$	idealized lens model
$\tilde{\mathcal{P}}$ / $\tilde{\mathcal{P}}'$	free-space propagation (isotropic / anisotropic)
$\tilde{\mathcal{B}}$	planar surface operator

Example: Polarization Conversion

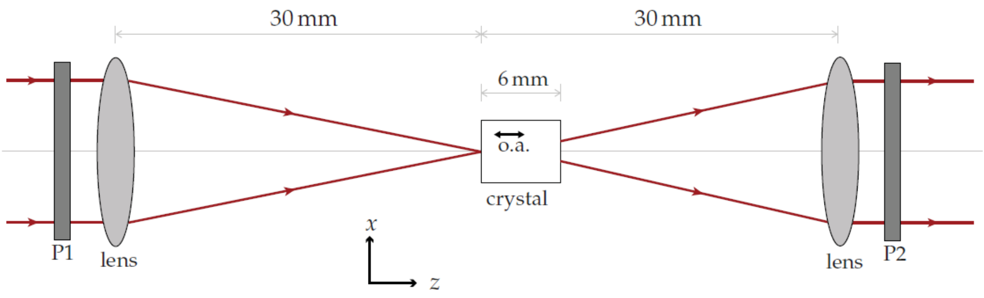


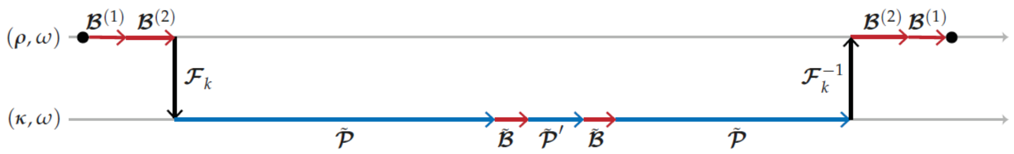
P2 parallel to P1

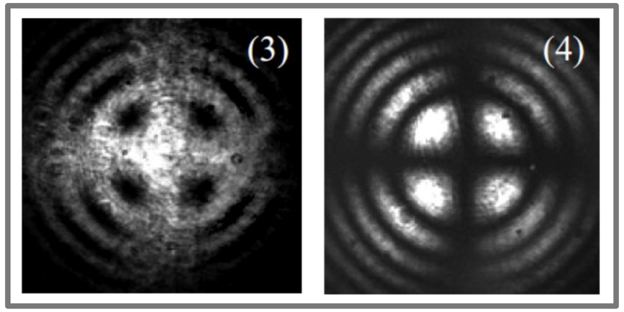
P2 orthogonal to P1



Example: Polarization Conversion

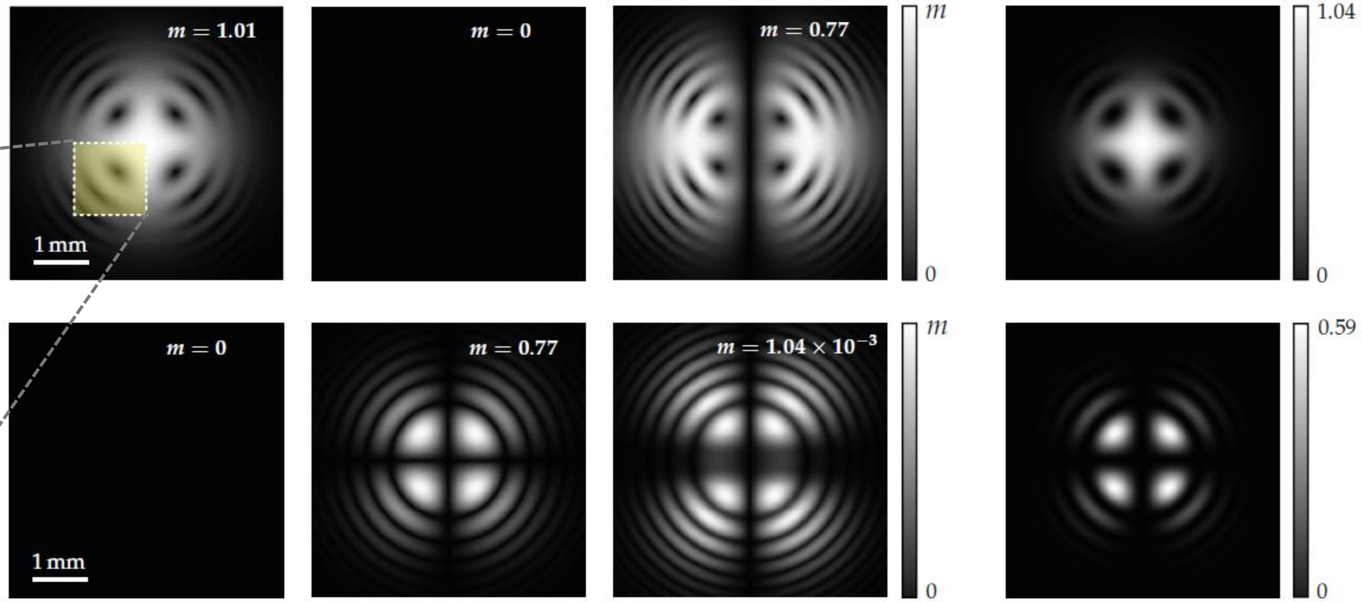


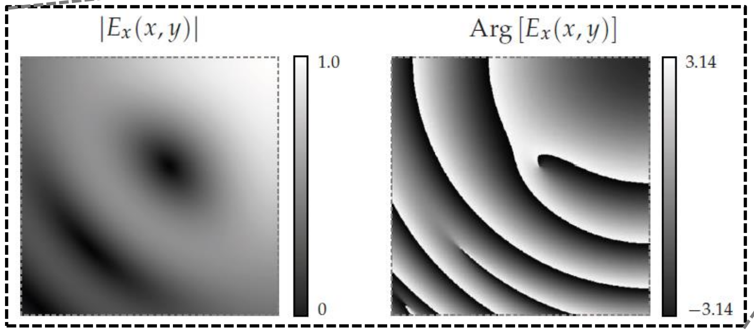




experimental measurement from [Izdebskaya 2009]

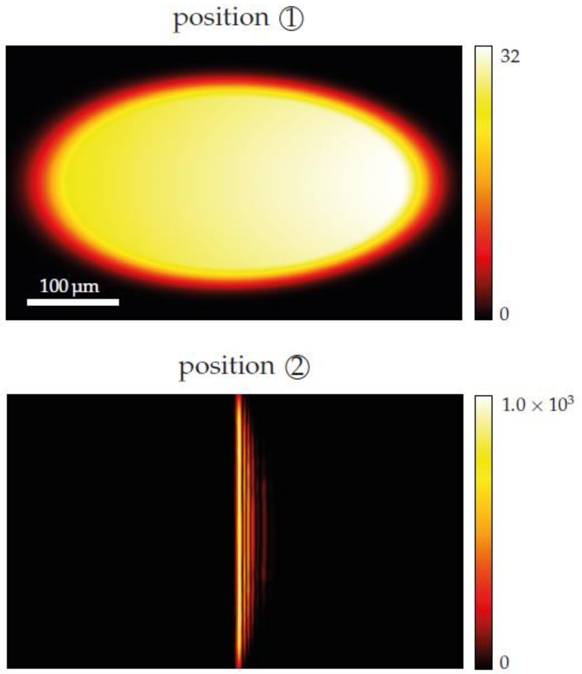
field tracing simulation results



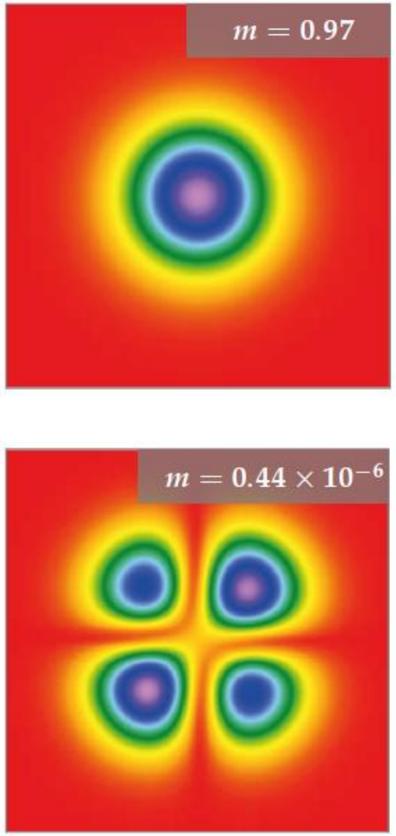


More Examples …

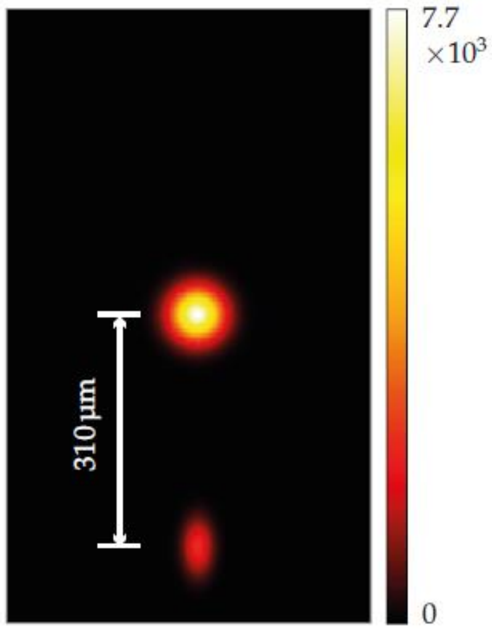
- Wollaston prism



- Stress-birefringence

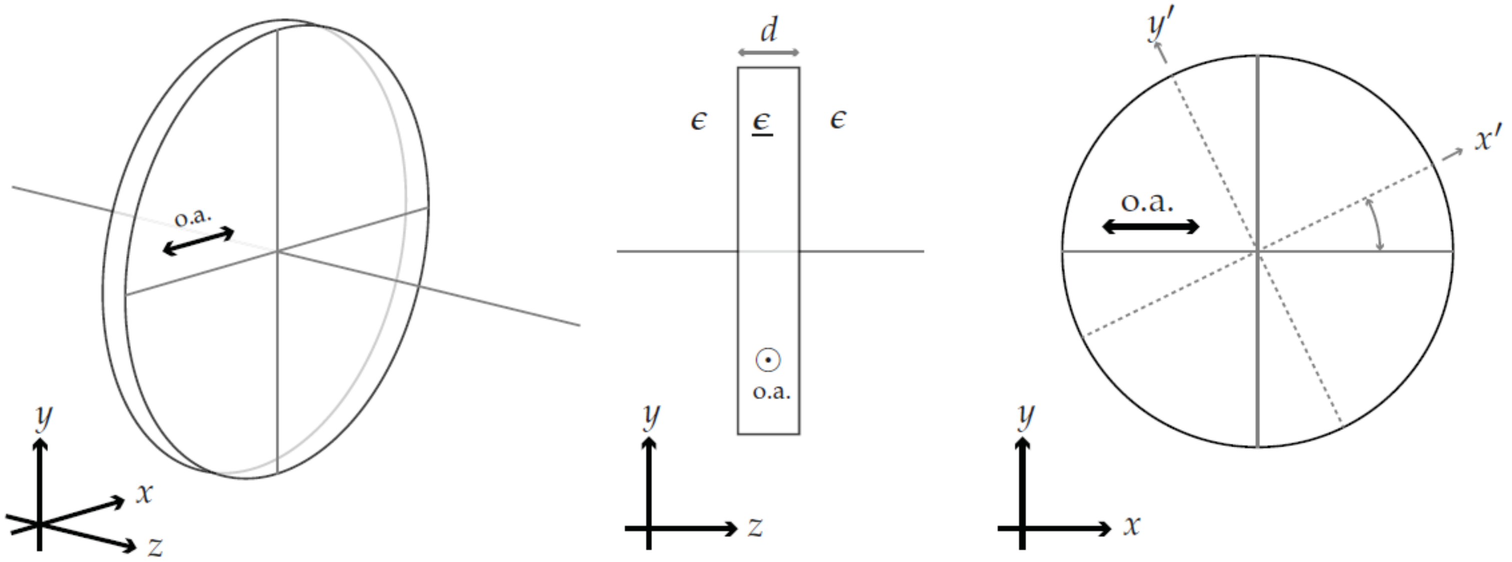


- Walk-off effect



Idealized Non-Paraxial Polarizer

- Uniaxial crystal model for polarizer



(a) three-dimensional view (b) y-z section (c) x-y section

- Uniaxial crystal with optic axis along the *x*-axis

$$\underline{\epsilon} = \begin{pmatrix} \epsilon_e & 0 & 0 \\ 0 & \epsilon_o & 0 \\ 0 & 0 & \epsilon_o \end{pmatrix}$$

Idealized Non-Paraxial Polarizer

- Field solution inside polarizer / uniaxial crystal plate

$$\begin{pmatrix} \tilde{E}_x(\boldsymbol{\kappa}, z) \\ \tilde{E}_y(\boldsymbol{\kappa}, z) \\ \tilde{H}_x(\boldsymbol{\kappa}, z) \\ \tilde{H}_y(\boldsymbol{\kappa}, z) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ \tilde{W}_B & \tilde{W}_D & \tilde{W}_B & -\tilde{W}_D \\ 0 & 1 & 0 & 1 \\ \tilde{W}_C & \tilde{W}_B & -\tilde{W}_C & \tilde{W}_B \end{pmatrix} \begin{pmatrix} \tilde{C}_+^{\mathrm{TM}} \exp(\tilde{\gamma}^{\mathrm{TM}} z) \\ \tilde{C}_+^{\mathrm{TE}} \exp(\tilde{\gamma}^{\mathrm{TE}} z) \\ \tilde{C}_-^{\mathrm{TM}} \exp(-\tilde{\gamma}^{\mathrm{TM}} z) \\ \tilde{C}_-^{\mathrm{TE}} \exp(-\tilde{\gamma}^{\mathrm{TE}} z) \end{pmatrix}$$

transverse magnetic (TM)

transverse electric (TE)

with $\tilde{\gamma}^{\mathrm{TM}}(\boldsymbol{\kappa}) = \mathrm{i}k_0\sqrt{\epsilon_\mathrm{e} - n_y^2 - n_x^2\epsilon_\mathrm{e}/\epsilon_\mathrm{o}}$, $\tilde{\gamma}^{\mathrm{TE}}(\boldsymbol{\kappa}) = \mathrm{i}k_0\sqrt{\epsilon_\mathrm{o} - n_x^2 - n_y^2}$

- B-operator for idealized polarizer, with respect to modes

$$\begin{pmatrix} \tilde{C}_+^{\mathrm{out,TM}}(\boldsymbol{\kappa}, z^{\mathrm{out}}) \\ \tilde{C}_+^{\mathrm{out,TE}}(\boldsymbol{\kappa}, z^{\mathrm{out}}) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \tilde{C}_+^{\mathrm{in,TM}}(\boldsymbol{\kappa}, z^{\mathrm{in}}) \\ \tilde{C}_+^{\mathrm{in,TE}}(\boldsymbol{\kappa}, z^{\mathrm{in}}) \end{pmatrix}$$

Idealized Non-Paraxial Polarizer

- Field solution inside polarizer / uniaxial crystal plate

$$\begin{pmatrix} \tilde{E}_x(\boldsymbol{\kappa}, z) \\ \tilde{E}_y(\boldsymbol{\kappa}, z) \\ \tilde{H}_x(\boldsymbol{\kappa}, z) \\ \tilde{H}_y(\boldsymbol{\kappa}, z) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ \tilde{W}_B & \tilde{W}_D & \tilde{W}_B & -\tilde{W}_D \\ 0 & 1 & 0 & 1 \\ \tilde{W}_C & \tilde{W}_B & -\tilde{W}_C & \tilde{W}_B \end{pmatrix} \begin{pmatrix} \tilde{C}_+^{\mathrm{TM}} \exp(\tilde{\gamma}^{\mathrm{TM}} z) \\ \tilde{C}_+^{\mathrm{TE}} \exp(\tilde{\gamma}^{\mathrm{TE}} z) \\ \tilde{C}_-^{\mathrm{TM}} \exp(-\tilde{\gamma}^{\mathrm{TM}} z) \\ \tilde{C}_-^{\mathrm{TE}} \exp(-\tilde{\gamma}^{\mathrm{TE}} z) \end{pmatrix}$$

transverse magnetic (TM)

transverse electric (TE)

with $\tilde{\gamma}^{\mathrm{TM}}(\boldsymbol{\kappa}) = \mathrm{i}k_0\sqrt{\epsilon_\mathrm{e} - n_y^2 - n_x^2\epsilon_\mathrm{e}/\epsilon_\mathrm{o}}\,, \quad \tilde{\gamma}^{\mathrm{TE}}(\boldsymbol{\kappa}) = \mathrm{i}k_0\sqrt{\epsilon_\mathrm{o} - n_x^2 - n_y^2}$

- B-operator for idealized polarizer, with respect to field components

$$\begin{pmatrix} \tilde{E}_x^{\mathrm{out}}(\boldsymbol{\kappa}, z^{\mathrm{out}}) \\ \tilde{E}_y^{\mathrm{out}}(\boldsymbol{\kappa}, z^{\mathrm{out}}) \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ -\tilde{W}_B^{\mathrm{iso}}(\boldsymbol{\kappa}) & 1 \end{pmatrix} \begin{pmatrix} \tilde{E}_x^{\mathrm{in}}(\boldsymbol{\kappa}, z^{\mathrm{in}}) \\ \tilde{E}_y^{\mathrm{in}}(\boldsymbol{\kappa}, z^{\mathrm{in}}) \end{pmatrix}$$

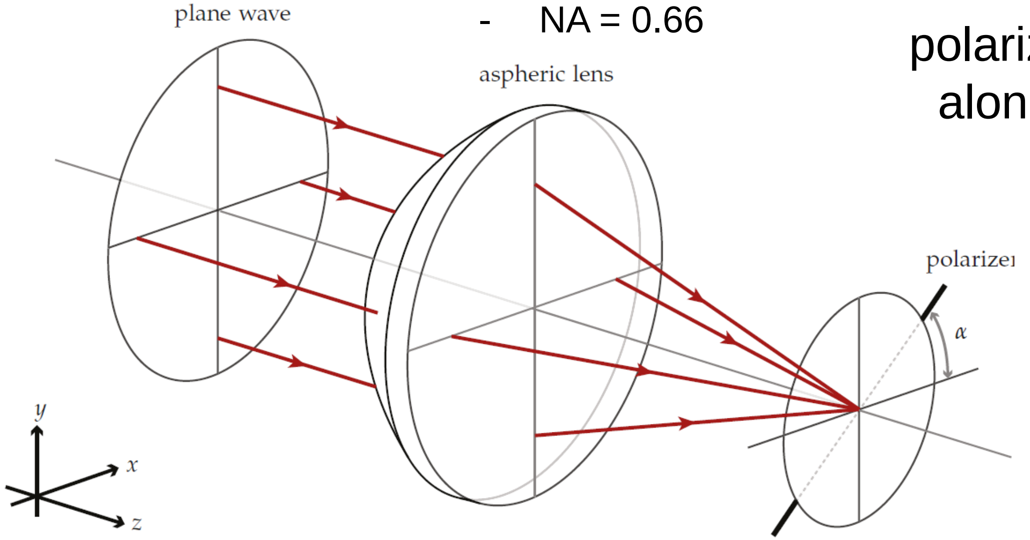
$$\tilde{W}_B^{\mathrm{iso}}(\boldsymbol{\kappa}) = \frac{k_x k_y}{-k_0^2\epsilon + k_x^2}$$

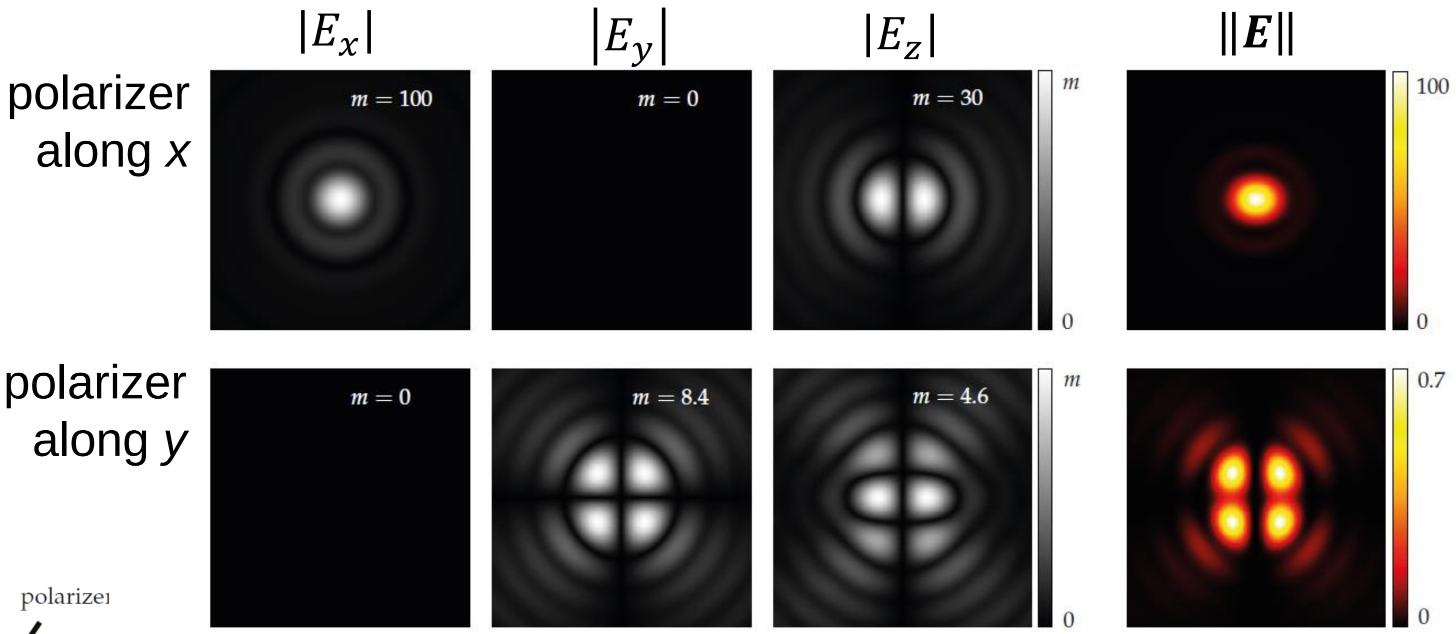
angular dependent polarization crosstalk

Idealized Non-Paraxial Polarizer

input field
- plane wave
- wavelength 633 nm
- linearly polarized along x direction

aspheric lens
- Edmund No. 49113
- NA = 0.66

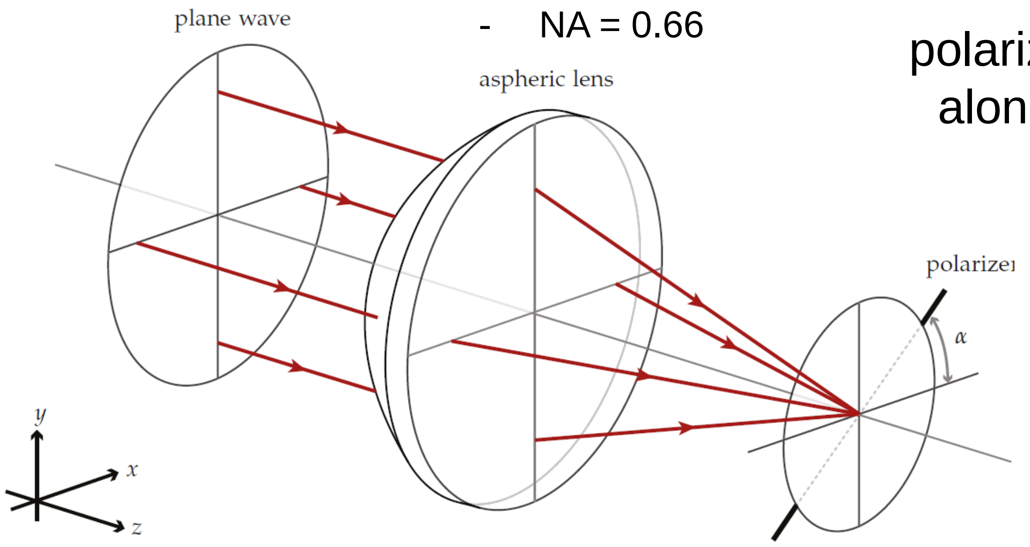


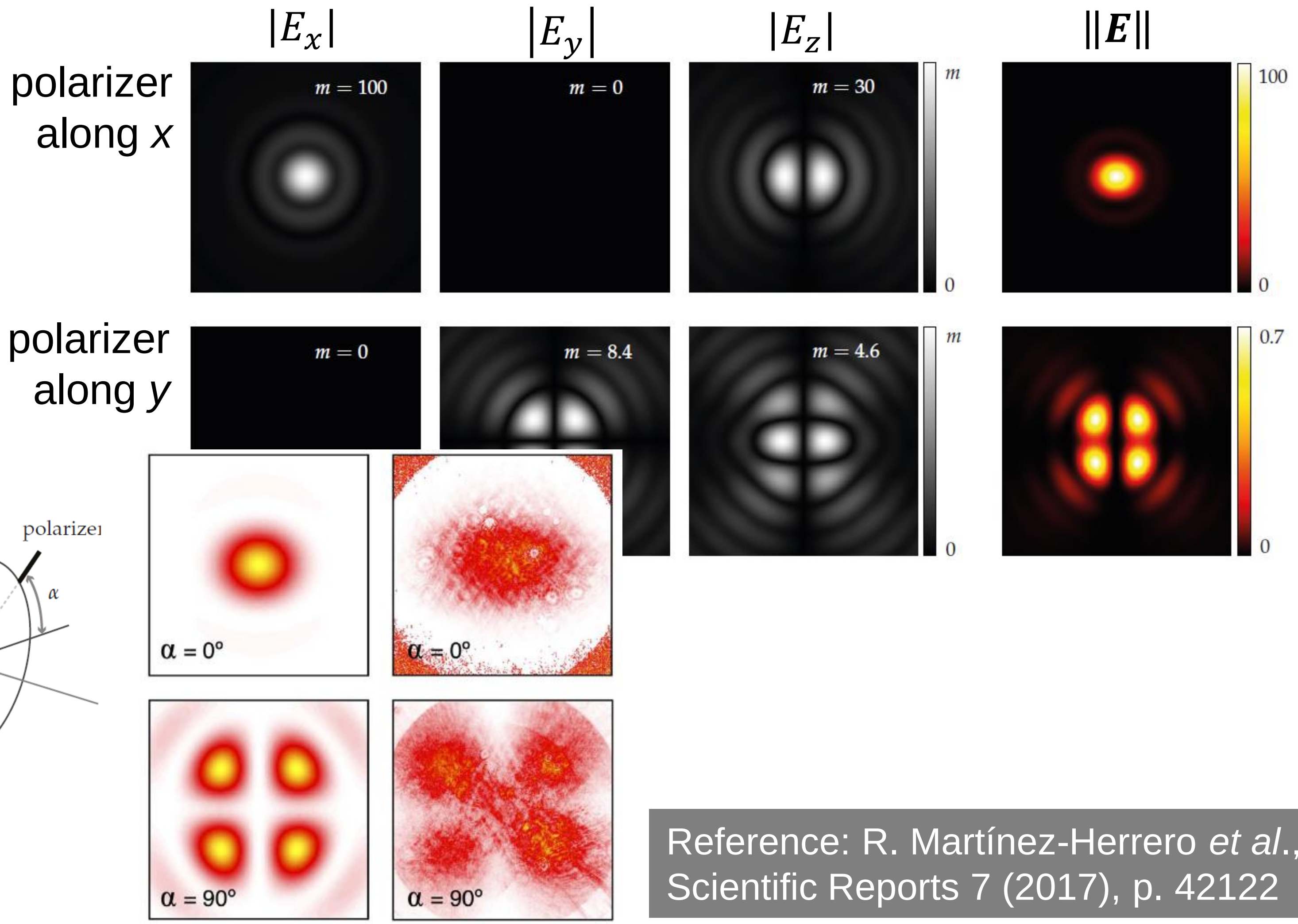


Idealized Non-Paraxial Polarizer

input field
- plane wave
- wavelength 633 nm
- linearly polarized along *x* direction

aspheric lens
- Edmund No. 49113
- NA = 0.66





Reference: R. Martínez-Herrero *et al.*, Scientific Reports 7 (2017), p. 42122

Metalens based on Birefringent Nanofin Structure

Reference: M. Khorasaninejad et al., Science 352 (6290), 1190 (2016)

- B-operator for a rotated nanofin structure

$$\begin{pmatrix} \tilde{E}_x^{\mathrm{out}}(\boldsymbol{\kappa},\omega) \\ \tilde{E}_y^{\mathrm{out}}(\boldsymbol{\kappa},\omega) \end{pmatrix} = \begin{pmatrix} \tilde{B}_{xx}(\boldsymbol{\kappa},\omega) & \tilde{B}_{xy}(\boldsymbol{\kappa},\omega) \\ \tilde{B}_{yx}(\boldsymbol{\kappa},\omega) & \tilde{B}_{yy}(\boldsymbol{\kappa},\omega) \end{pmatrix} \begin{pmatrix} \tilde{E}_x^{\mathrm{in}}(\boldsymbol{\kappa},\omega) \\ \tilde{E}_y^{\mathrm{in}}(\boldsymbol{\kappa},\omega) \end{pmatrix}$$

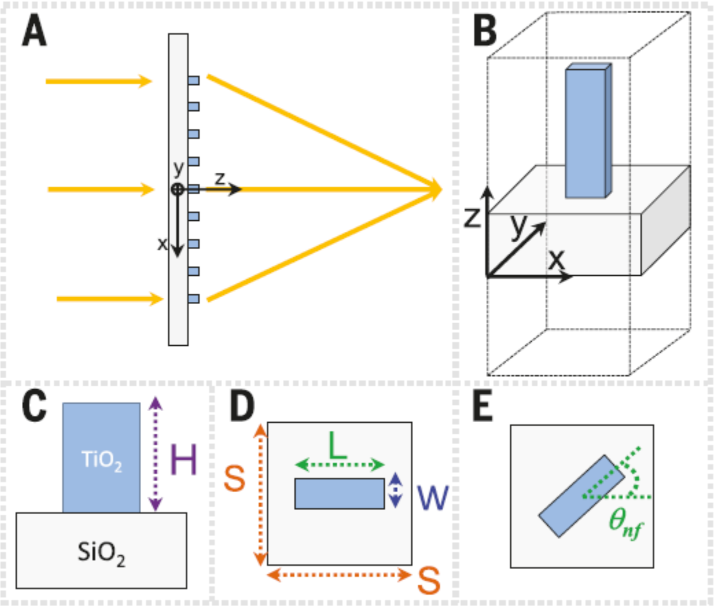
$$= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{S}_{xx}^{++}(\boldsymbol{\kappa},\omega) & \tilde{S}_{xy}^{++}(\boldsymbol{\kappa},\omega) \\ \tilde{S}_{yx}^{++}(\boldsymbol{\kappa},\omega) & \tilde{S}_{yy}^{++}(\boldsymbol{\kappa},\omega) \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

- Ideally, each nanofin is supposed to work as a half-wave plate, i.e.

$$\begin{pmatrix} \tilde{S}_{xx}^{++}(\boldsymbol{\kappa},\omega) & \tilde{S}_{xy}^{++}(\boldsymbol{\kappa},\omega) \\ \tilde{S}_{yx}^{++}(\boldsymbol{\kappa},\omega) & \tilde{S}_{yy}^{++}(\boldsymbol{\kappa},\omega) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \exp(\pm i\pi) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- That leads to the B-operator in form of

$$\begin{pmatrix} \tilde{B}_{xx} & \tilde{B}_{xy} \\ \tilde{B}_{yx} & \tilde{B}_{yy} \end{pmatrix} = \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix} = \begin{pmatrix} \cos\psi & \sin\psi \\ \sin\psi & -\cos\psi \end{pmatrix}$$



Metalens based on Birefringent Nanofin Structure

Reference: M. Khorasaninejad et al., Science 352 (6290), 1190 (2016)

- B-operator for a rotated nanofin structure

$$\begin{pmatrix} \tilde{E}_x^{\mathrm{out}}(\boldsymbol{\kappa},\omega) \\ \tilde{E}_y^{\mathrm{out}}(\boldsymbol{\kappa},\omega) \end{pmatrix} = \begin{pmatrix} \tilde{B}_{xx}(\boldsymbol{\kappa},\omega) & \tilde{B}_{xy}(\boldsymbol{\kappa},\omega) \\ \tilde{B}_{yx}(\boldsymbol{\kappa},\omega) & \tilde{B}_{yy}(\boldsymbol{\kappa},\omega) \end{pmatrix} \begin{pmatrix} \tilde{E}_x^{\mathrm{in}}(\boldsymbol{\kappa},\omega) \\ \tilde{E}_y^{\mathrm{in}}(\boldsymbol{\kappa},\omega) \end{pmatrix}$$

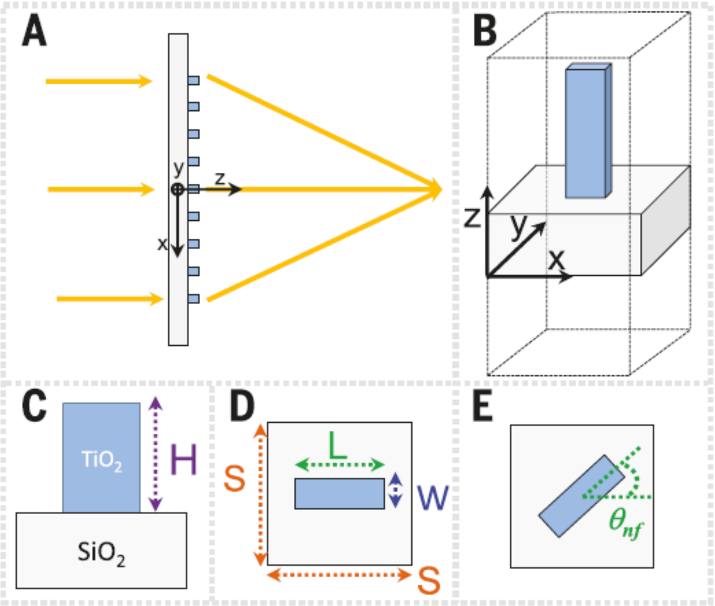
$$= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{S}_{xx}^{++}(\boldsymbol{\kappa},\omega) & \tilde{S}_{xy}^{++}(\boldsymbol{\kappa},\omega) \\ \tilde{S}_{yx}^{++}(\boldsymbol{\kappa},\omega) & \tilde{S}_{yy}^{++}(\boldsymbol{\kappa},\omega) \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

- Ideally, each nanofin is supposed to work as a half-wave plate, i.e.

$$\begin{pmatrix} \tilde{S}_{xx}^{++}(\boldsymbol{\kappa},\omega) & \tilde{S}_{xy}^{++}(\boldsymbol{\kappa},\omega) \\ \tilde{S}_{yx}^{++}(\boldsymbol{\kappa},\omega) & \tilde{S}_{yy}^{++}(\boldsymbol{\kappa},\omega) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \exp(\pm \mathrm{i}\pi) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- That leads to the B-operator in form of

$$\begin{pmatrix} \tilde{B}_{xx} & \tilde{B}_{xy} \\ \tilde{B}_{yx} & \tilde{B}_{yy} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -\mathrm{i} \\ -\mathrm{i} & -1 \end{pmatrix} \exp(\mathrm{i}\psi) + \frac{1}{2} \begin{pmatrix} 1 & \mathrm{i} \\ \mathrm{i} & -1 \end{pmatrix} \exp(-\mathrm{i}\psi)$$



Metalens based on Birefringent Nanofin Structure

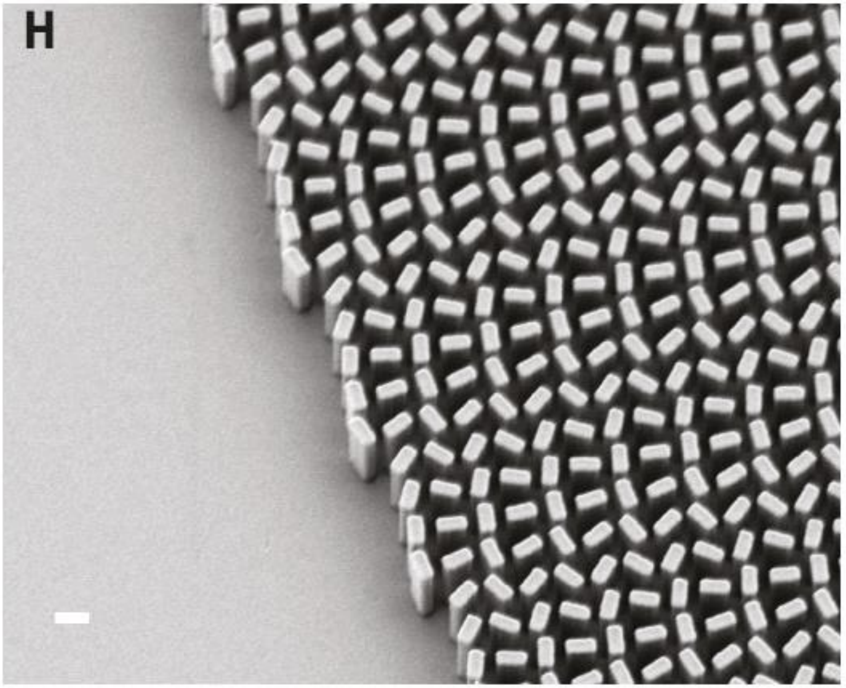
Reference: M. Khorasaninejad et al., Science 352 (6290), 1190 (2016)

- Spatially varying rotation of nanofins, to realize lens function

$$\begin{pmatrix} B_{xx}(\boldsymbol{\rho}) & B_{xy}(\boldsymbol{\rho}) \\ B_{yx}(\boldsymbol{\rho}) & B_{yy}(\boldsymbol{\rho}) \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & -\mathrm{i} \\ -\mathrm{i} & -1 \end{pmatrix} \exp\left(\mathrm{i}\psi(\boldsymbol{\rho})\right) + \frac{1}{2}\begin{pmatrix} 1 & \mathrm{i} \\ \mathrm{i} & -1 \end{pmatrix} \exp\left(-\mathrm{i}\psi(\boldsymbol{\rho})\right)$$

  with

$$\psi(\boldsymbol{\rho}) = \frac{2\pi}{\lambda_\mathrm{d}}\left(f - \sqrt{|\boldsymbol{\rho}|^2 + f^2}\right)$$



- When input field is circularly polarized, i.e.

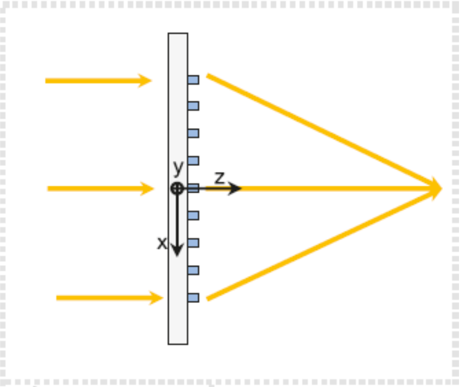
$$\begin{pmatrix} E_x^{\mathrm{in}}(\boldsymbol{\rho},\omega) \\ \tilde{E}_y^{\mathrm{in}}(\boldsymbol{\rho},\omega) \end{pmatrix} = \begin{pmatrix} 1 \\ \mathrm{i} \end{pmatrix}$$

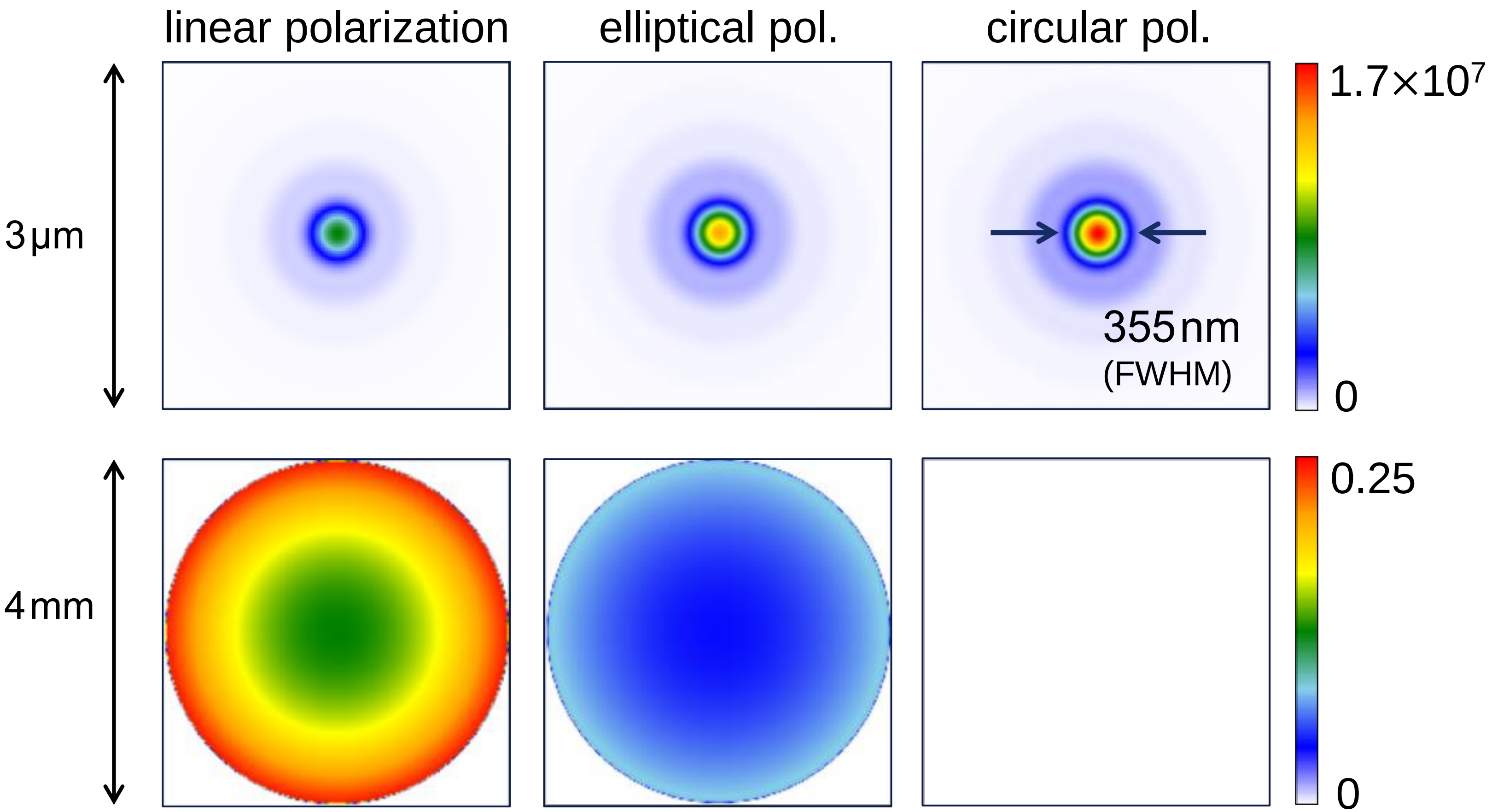
  it can be shown that

$$\begin{pmatrix} E_x^{\mathrm{out}}(\boldsymbol{\rho},\omega) \\ E_y^{\mathrm{out}}(\boldsymbol{\rho},\omega) \end{pmatrix} = \begin{pmatrix} 1 \\ \mathrm{i} \end{pmatrix} \exp\left(\mathrm{i}\psi(\boldsymbol{\rho})\right)$$

Metalens based on Birefringent Nanofin Structure

- Analysis with different input polarizations





Further Information

- **Fast propagation of electromagnetic fields in graded-index media**
  Paper 10694-21
  Time: 11:10 AM - 11:30 AM

- **Semi-analytical Fourier transform and its application to physical-optics modelling**
  Paper 10694-24

- **The Gouy phase shift reinterpreted via the geometric Fourier transform**
  Paper 10694-26
  Time: 2:30 PM - 2:50 PM