17.05.2018 – SPIE Optical Systems Design

The Gouy Phase Shift Reinterpreted Via the Geometric Fourier Transform

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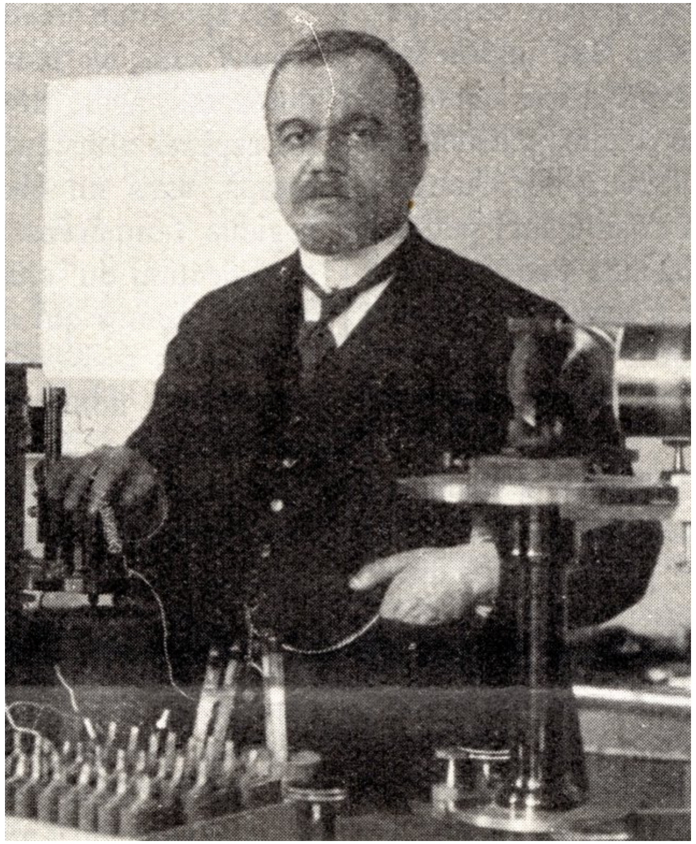
Discovery of the Gouy phase



L. G. Gouy, from Wikipedia

Louis Georges Gouy theoretically predicted and experimentally observed the effect that would go on to bear his name at the end of the 19th century:

Definition of the Gouy phase

π phase shift accumulated by any convergent field as it travels from minus infinity to plus infinity which cannot be explained by the optical path length traversed by each of the rays conforming the wavefront in the ray representation of the light.

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Gaussian beam:

$$V(x,y,z) \propto \exp\left(\frac{-r^2}{\omega^2} + \mathrm{i}\,k_z + \mathrm{i}\,\frac{\pi\left(x^2+y^2\right)}{\lambda\,R} - \mathrm{i}\,\arctan\frac{z}{z_R}\right)$$

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Spherical wave:

$$\begin{cases} V^{\mathrm{con}}(x,y,z) \propto -\exp(\mathrm{i}\,kr) \equiv \exp(\mathrm{i}\,\pi + \mathrm{i}\,kr) \\ V^{\mathrm{div}}(x,y,z) \propto \exp(\mathrm{i}\,kr) \end{cases}$$

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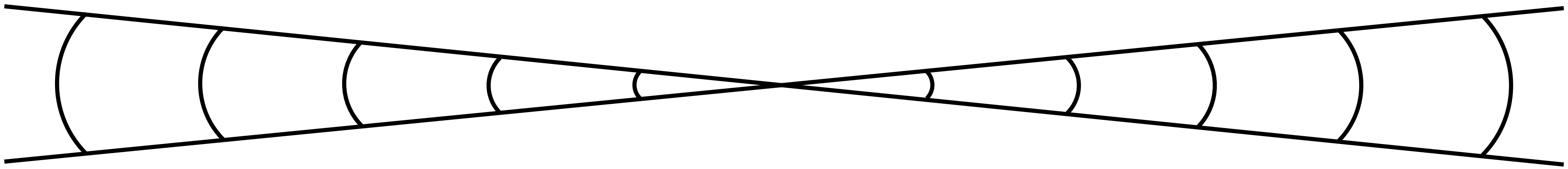
The Gouy phase in literature

- Main audience for Gouy phase effect is academic
- Steady stream of publications on the topic since its discovery
- Many of them provide new explanations or interpretations:
  - Comparison of geometrical and wave-optics techniques (Boyd, 1980)
  - In-depth analysis of the mathematics behind phenomenon, placing it in thermodynamic context (Berry, 1984; Subbarao, 1995)
  - Analysis through light momenta (Feng and Winful, 2001)
  - Study through quantum-mechanical concepts (Hariharan and Robinson, 1996)
  - Explanation via generalisation of geometrical-optics concepts (Yang and Winful, 2006)
  - Study of the effect of aberrations—astigmatism in particular (Visser and Wolf, 2010)

Confusion surrounding the Gouy phase

- When considering a focussed beam, the tendency is to automatically revert to the ray description of light.



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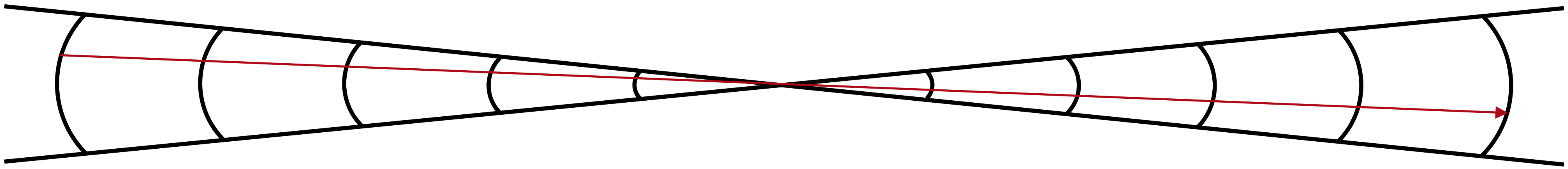
- Assumption: rays are well defined throughout



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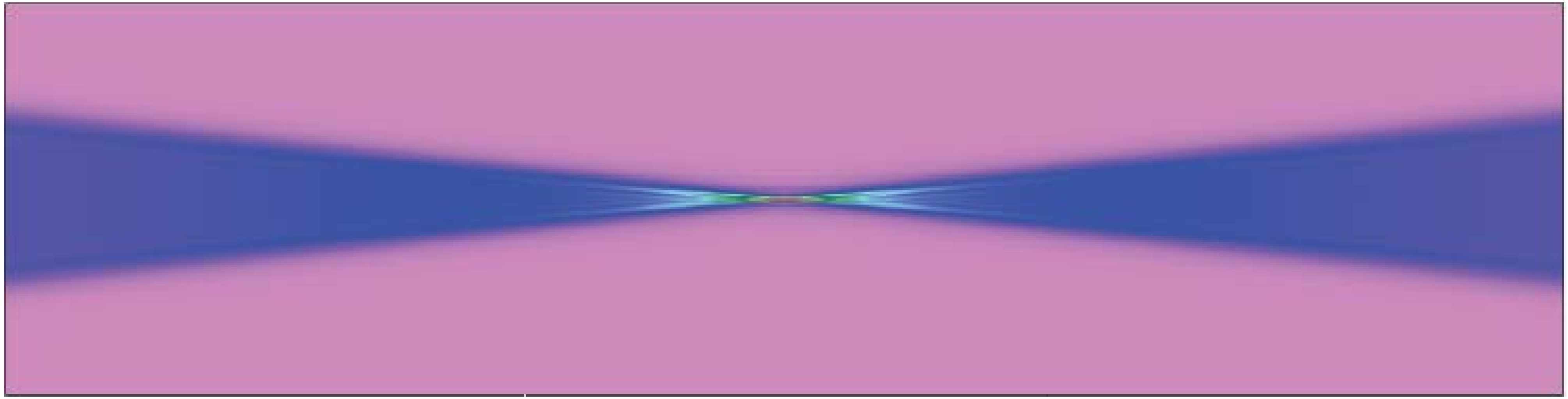

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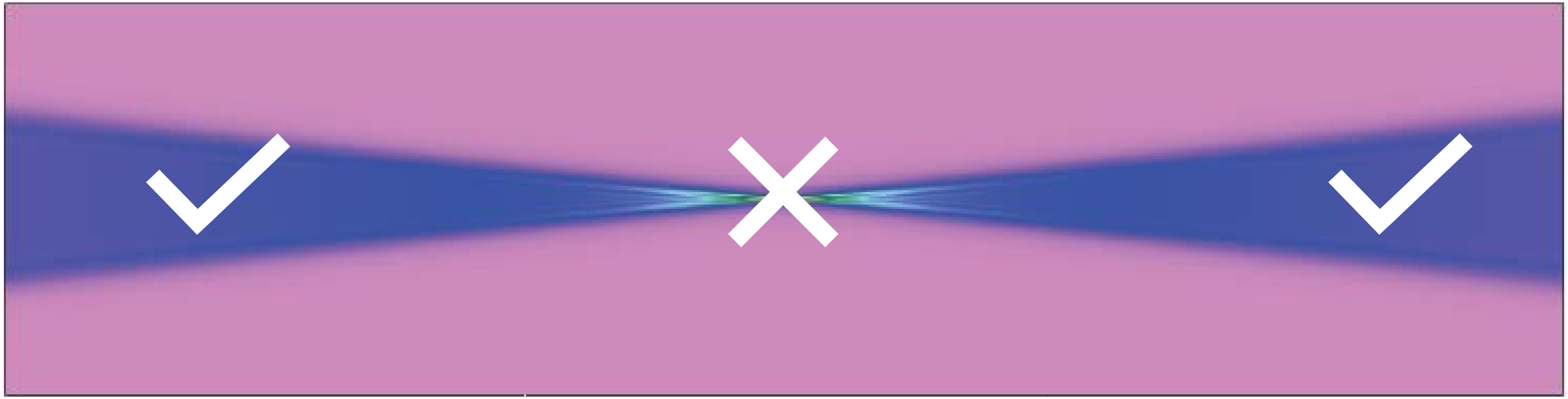

Confusion surrounding the Gouy phase

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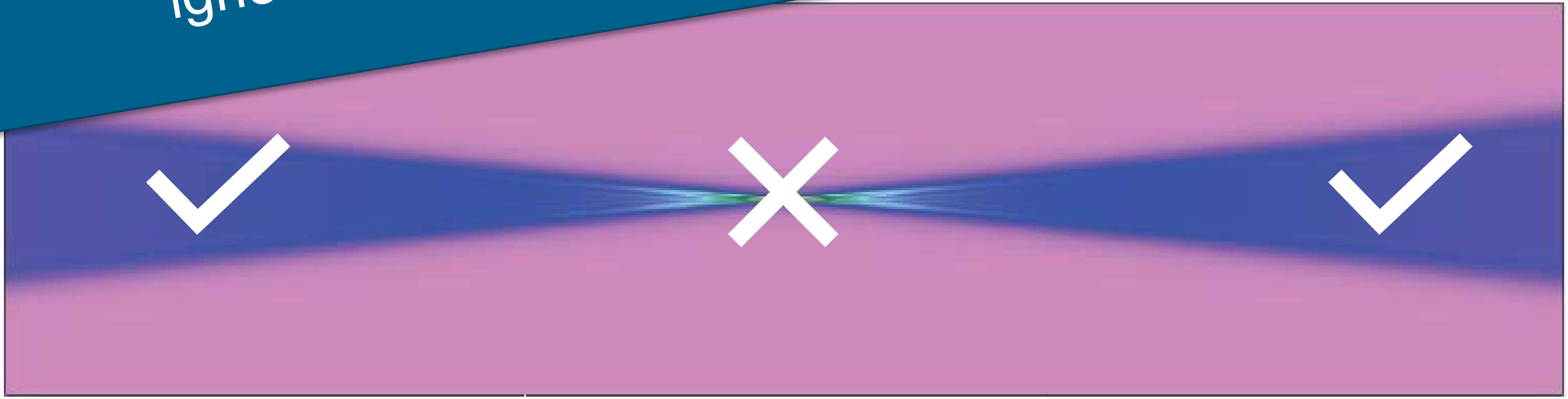
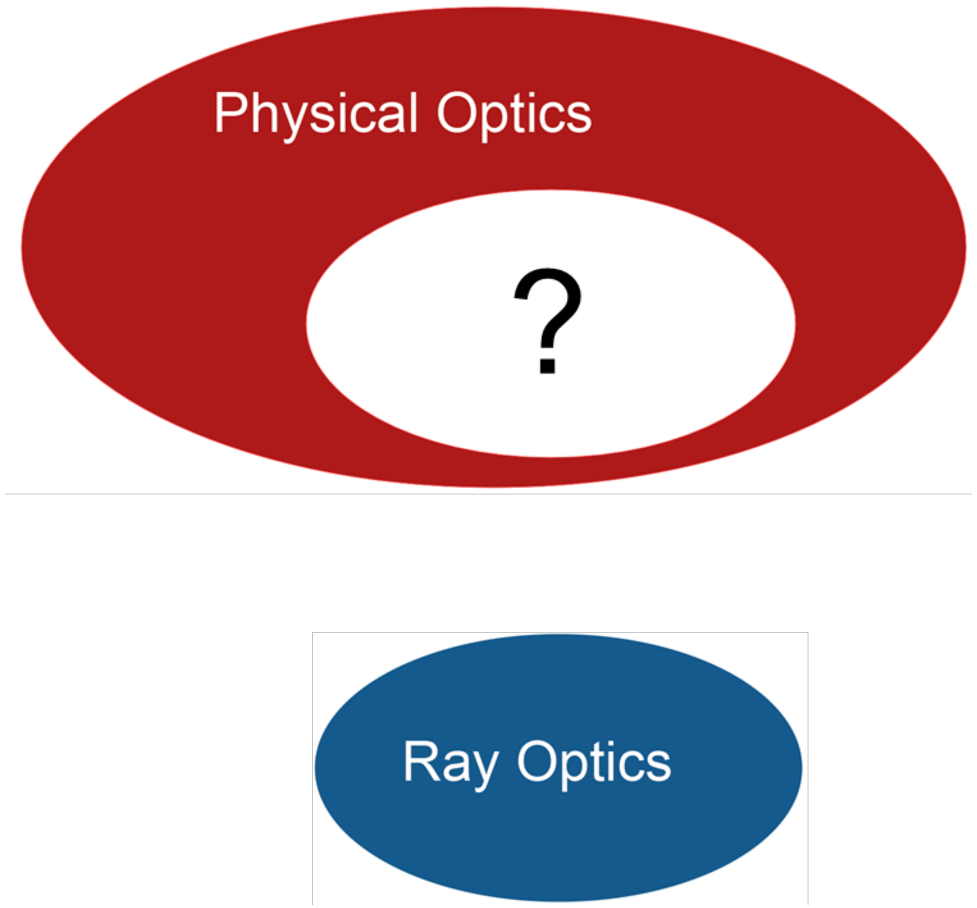
Could there be a way of modelling the beam making use of geometric behaviour in front of and behind focus, but without ignoring the diffractive behaviour in the focal zone?

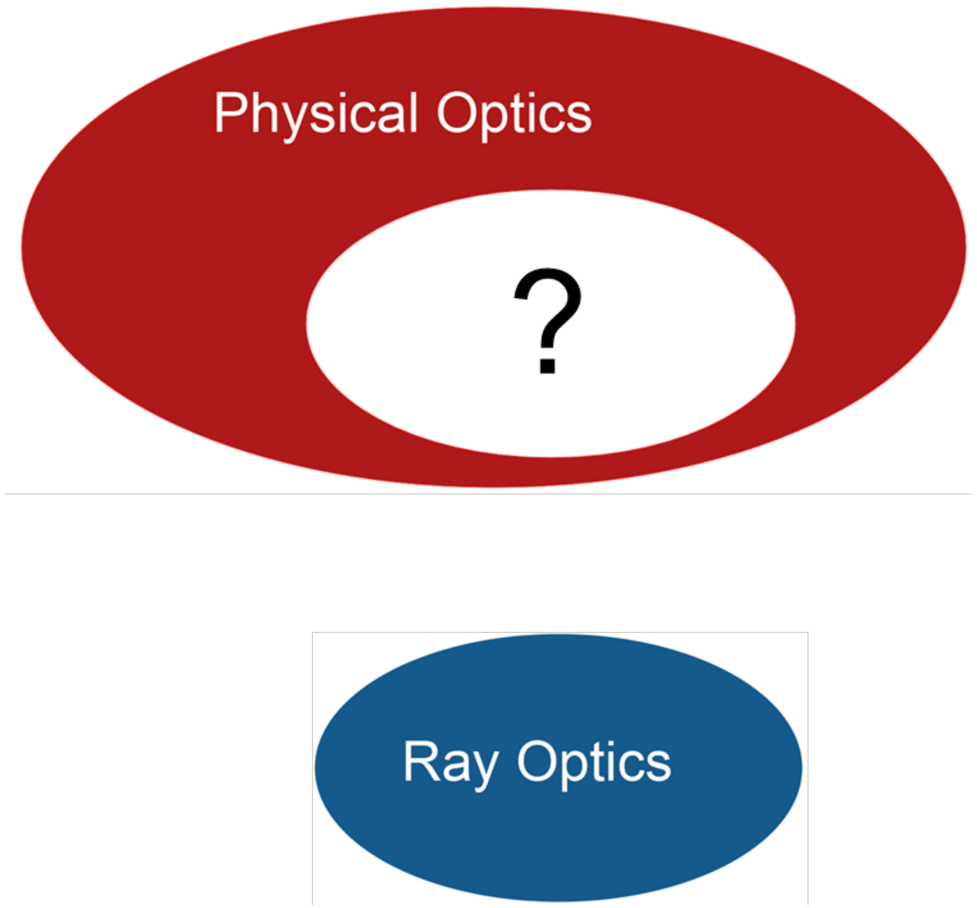


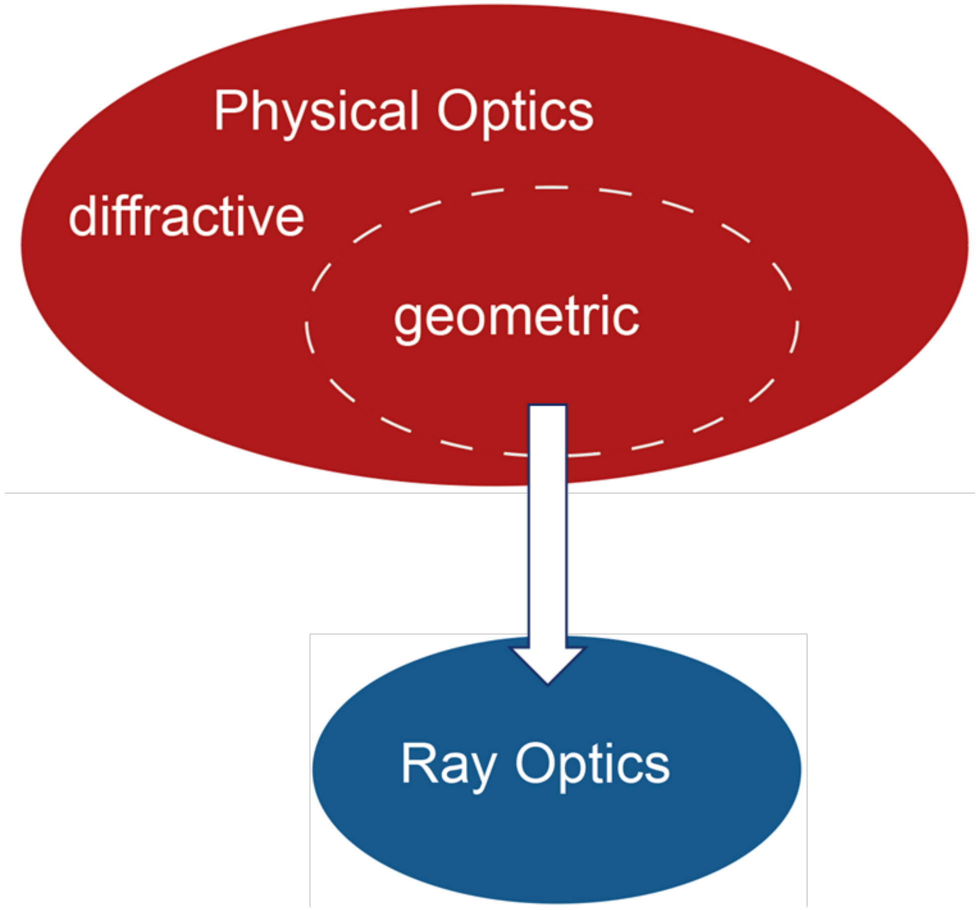
Irreconcilable descriptions of light in optics



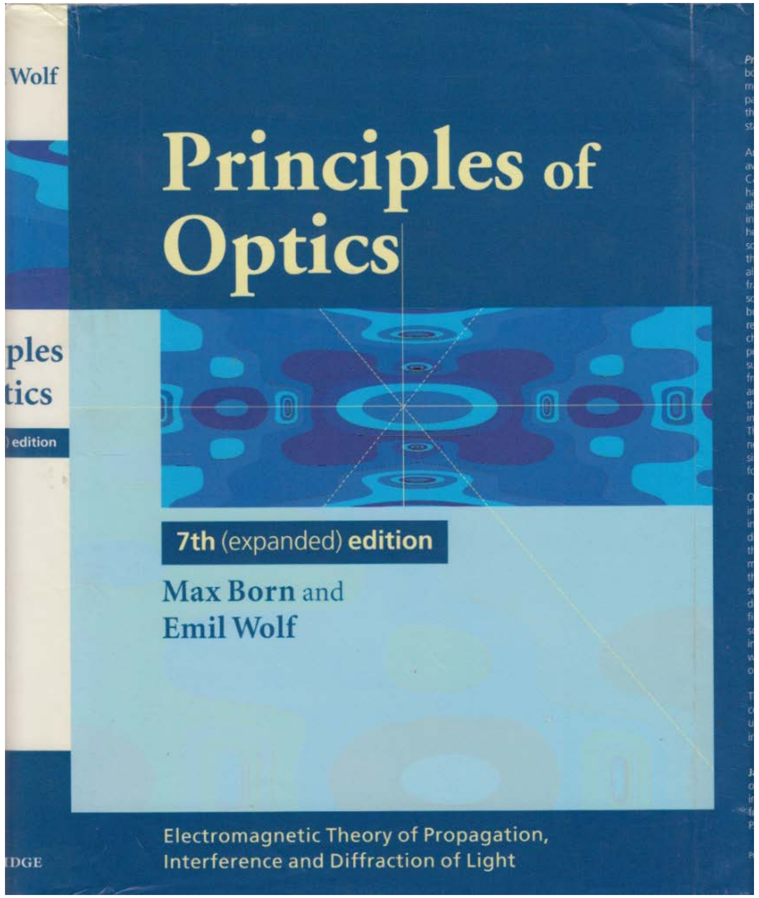
- It is generally accepted, in theory, that geometrical and physical optics correspond to two different levels of approximation to describe the behaviour of the same entity: light.
- However, due to the irreconcilable nature of the mathematical objects used to represent light in the two disciplines (rays vs. vector fields), this is, in practice, far from the truth, especially for optical simulations.

Unified theory of physical optics



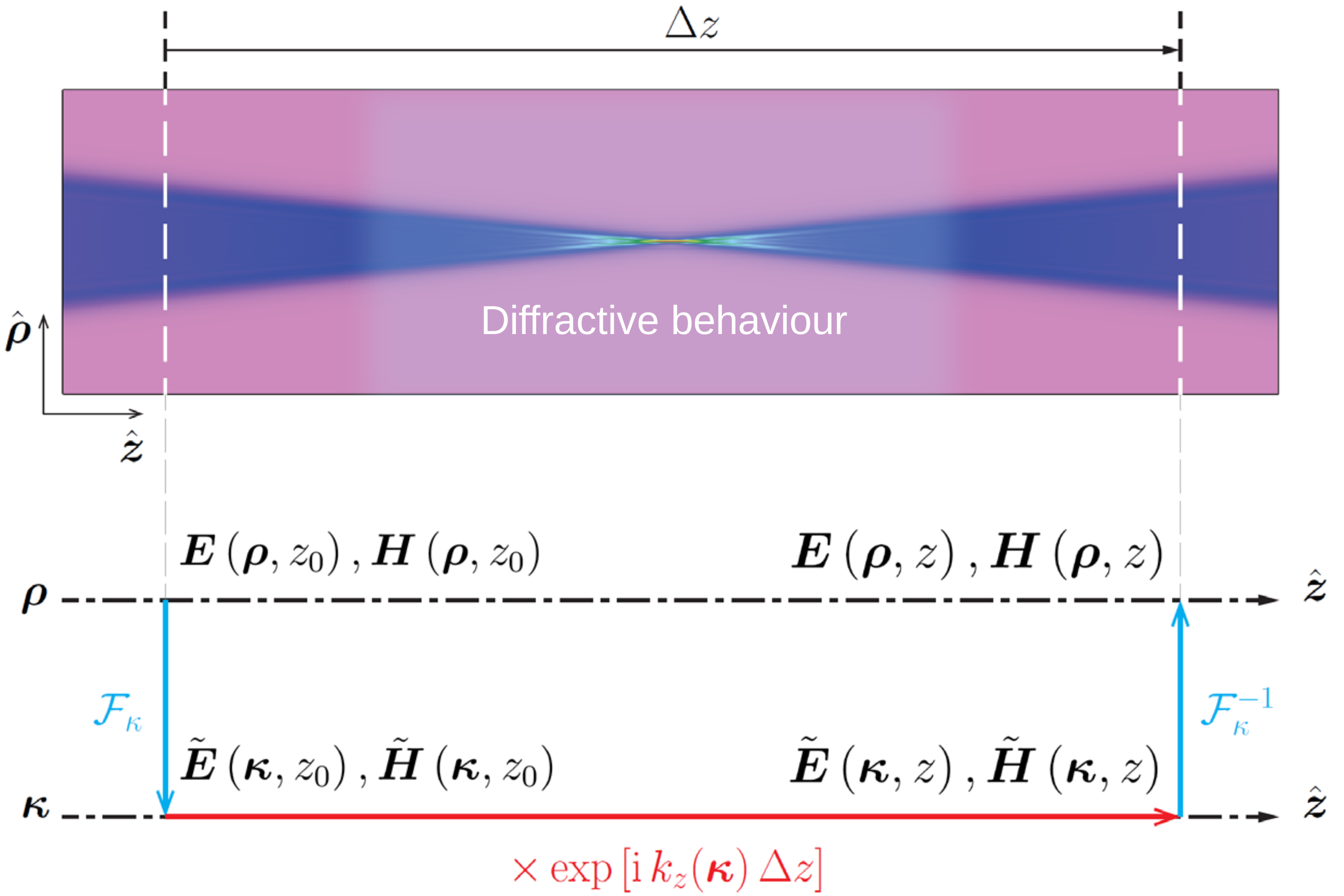


Geometrical Optics of Electromagnetic Fields



"According to traditional terminology, one understands by geometrical optics this approximate picture of energy propagation, using the concept of rays and wave-fronts. In other words polarization properties are excluded. The reason for this restriction is undoubtedly due to the fact that the simple laws of geometrical optics concerning rays and wave-fronts were known from experiments long before the electromagnetic theory of light was established. It is, however, possible, and from our point of view quite natural, to extend the meaning of geometrical optics to embrace also certain geometrical laws relating to the propagation of the 'amplitude vectors' E and H."

Free-space propagation in physical optics: SPW



Note:

$\boldsymbol{\rho} := (x, y)$

$\boldsymbol{\kappa} := (k_x, k_y)$

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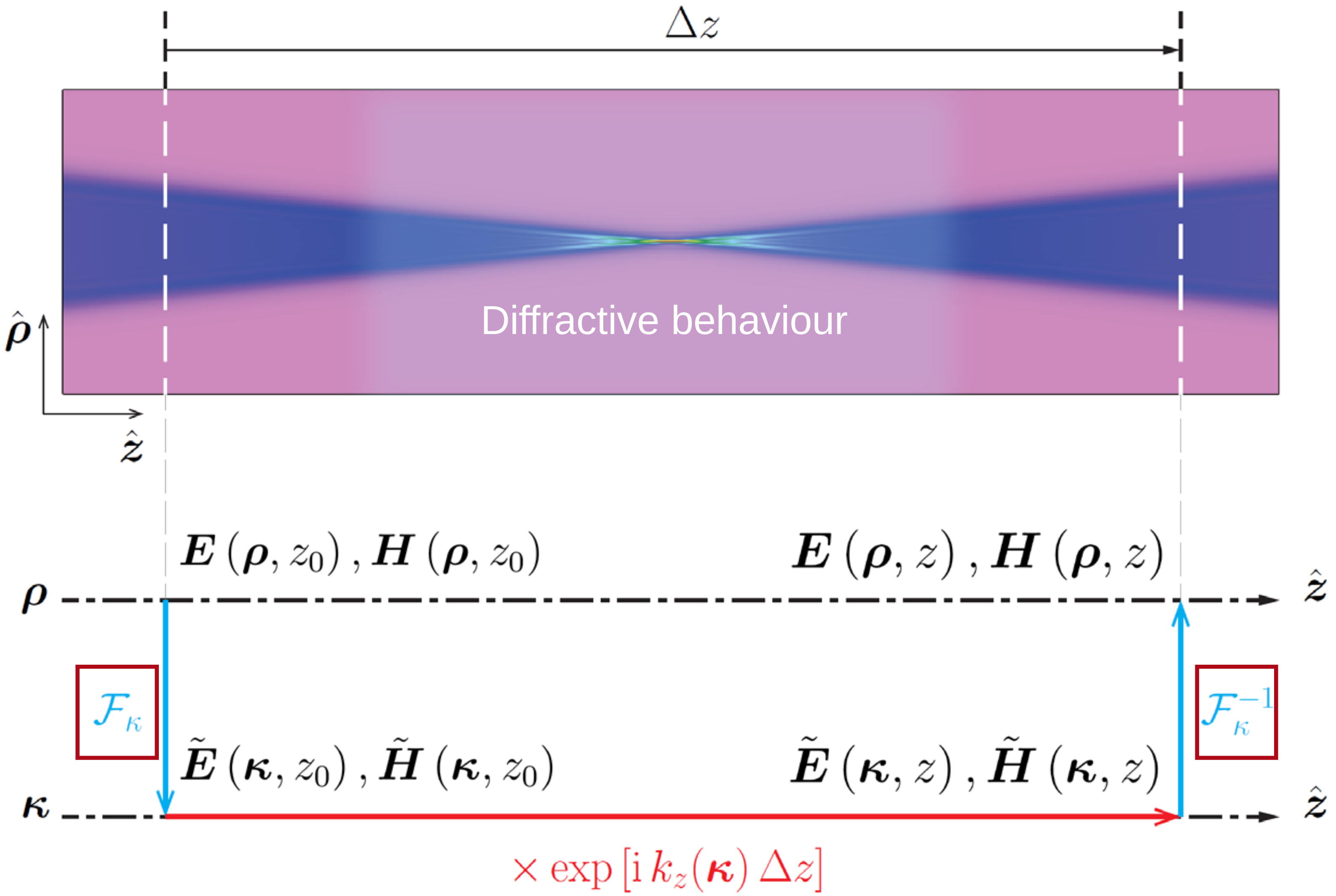

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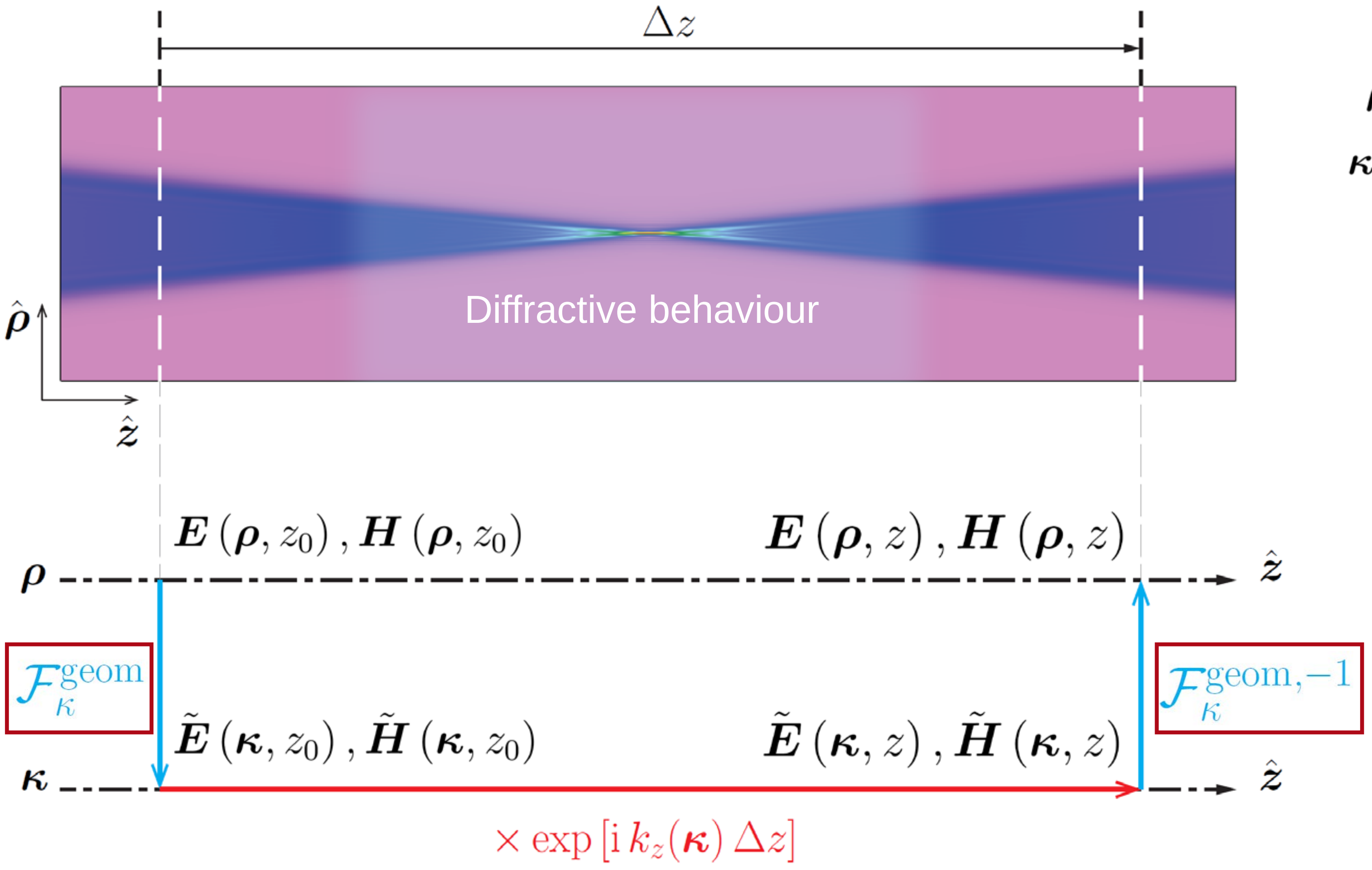

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The geometric Fourier transform

$$V_\ell\left(\boldsymbol{\rho}, z, \omega\right) = \left|V_\ell\left(\boldsymbol{\rho}, z, \omega\right)\right| \, \mathrm{e}^{\mathrm{i}\,\varphi_\ell(\boldsymbol{\rho}, z, \omega)} \, \mathrm{e}^{\mathrm{i}\,\psi(\boldsymbol{\rho}, z, \omega)}$$

$$= U_\ell\left(\boldsymbol{\rho}, z, \omega\right) \, \mathrm{e}^{\mathrm{i}\,\psi(\boldsymbol{\rho}, z, \omega)}$$

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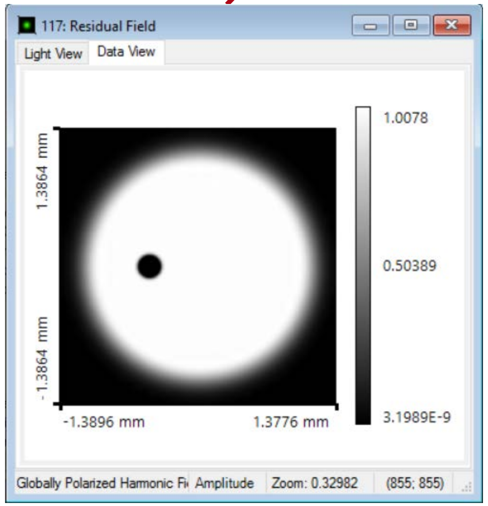
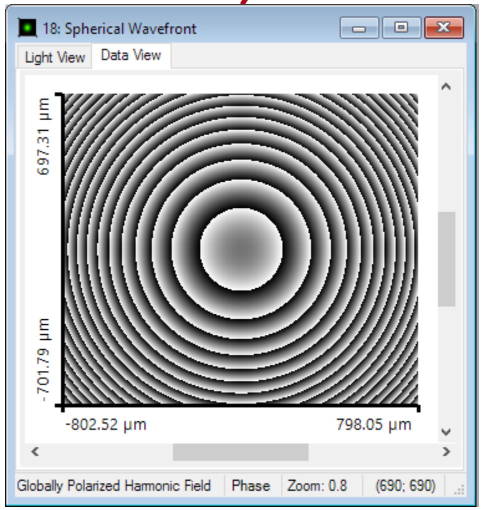
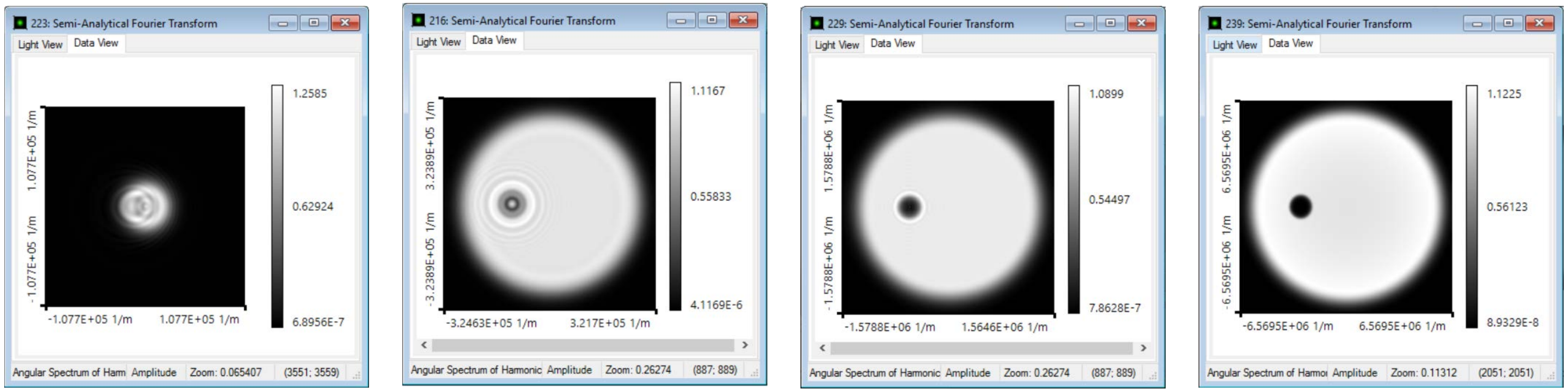




The geometric Fourier transform

$$V_\ell(\boldsymbol{\rho}, z, \omega) = |V_\ell(\boldsymbol{\rho}, z, \omega)|\, \mathrm{e}^{\mathrm{i}\,\varphi_\ell(\boldsymbol{\rho},z,\omega)}\, \boxed{\mathrm{e}^{\mathrm{i}\,\psi(\boldsymbol{\rho},z,\omega)}}$$

Decreasing radius of cruvature;
increasing NA



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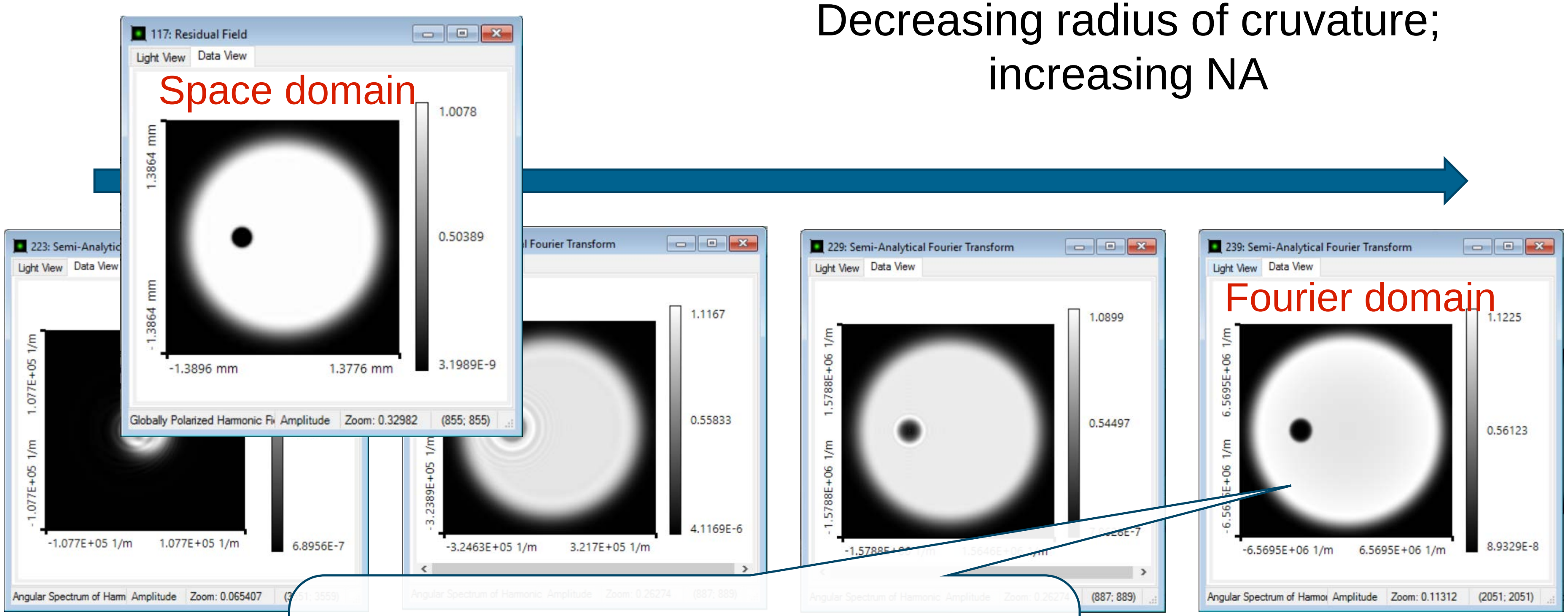
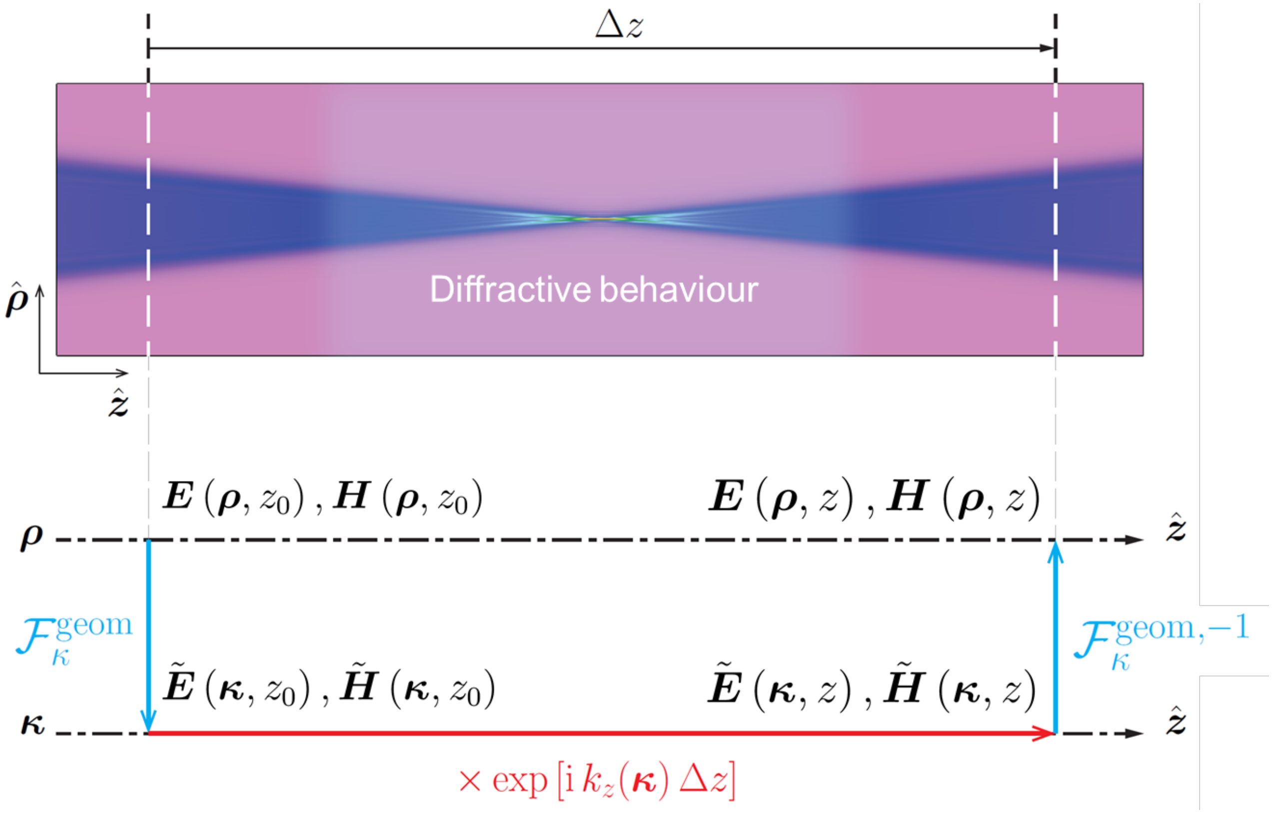


Mapping between both
domains: geometric field zone

Propagation between geometric field zones



$$\operatorname{sign}\left\{\tilde{\beta}^{\text{target}}\left[\boldsymbol{\kappa}(\boldsymbol{\rho}), z\right] \beta^{\text{in}}\left[\boldsymbol{\rho}'(\boldsymbol{\rho}), z_0\right]\right\}$$

$$\boxed{\beta(\boldsymbol{\rho}) = f\left(\frac{\partial^2 \psi}{\partial x_{ij}^2}\right)}$$

$$V_\ell(\boldsymbol{\rho}, z) = \beta\left[\boldsymbol{\rho}'(\boldsymbol{\rho}), z_0\right] \tilde{\beta}\left[\boldsymbol{\kappa}(\boldsymbol{\rho}), z\right] \mathrm{e}^{\mathrm{i}\left\{\frac{k_0^2 \check{n}^2}{\check{k}_z[\boldsymbol{\kappa}(\boldsymbol{\rho})]} \Delta z\right\}} V_\ell\left[\boldsymbol{\rho}'(\boldsymbol{\rho}), z_0\right]$$

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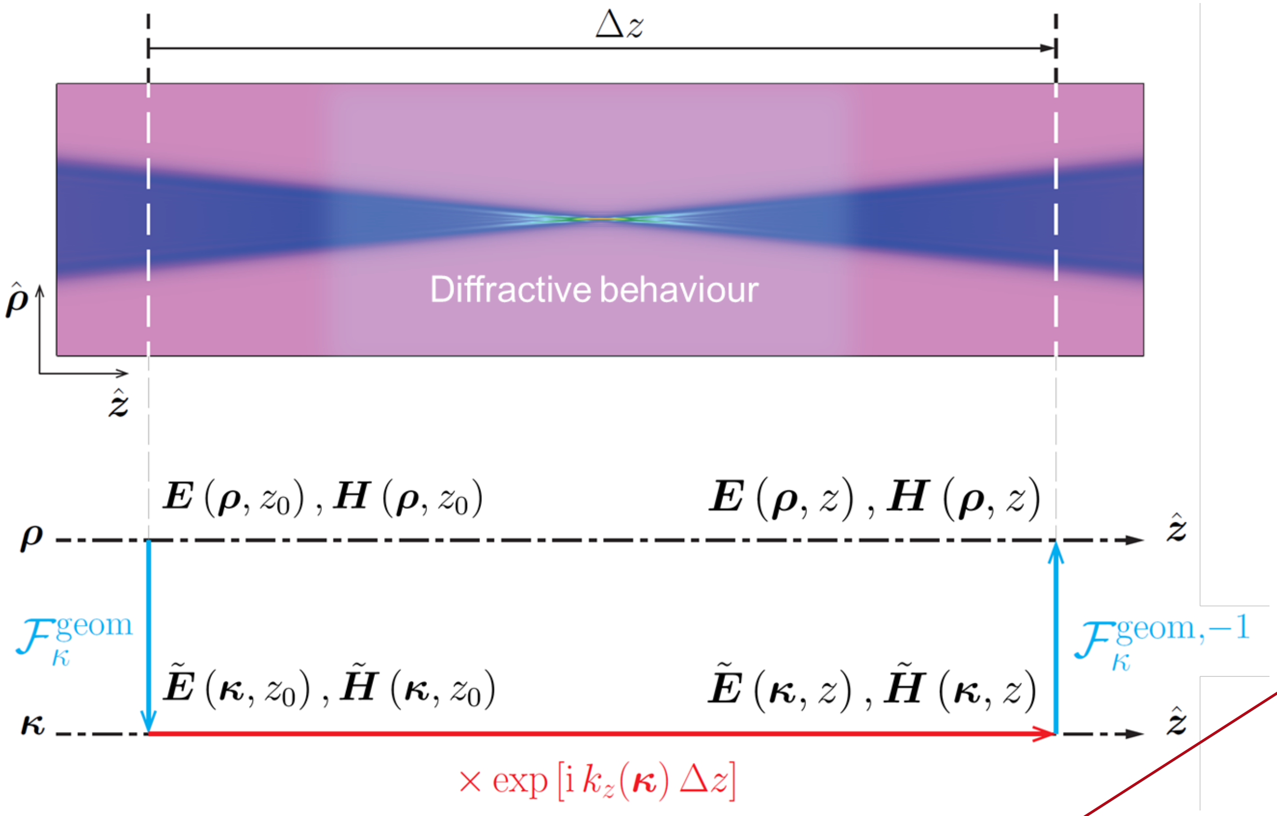

Gouy phase!

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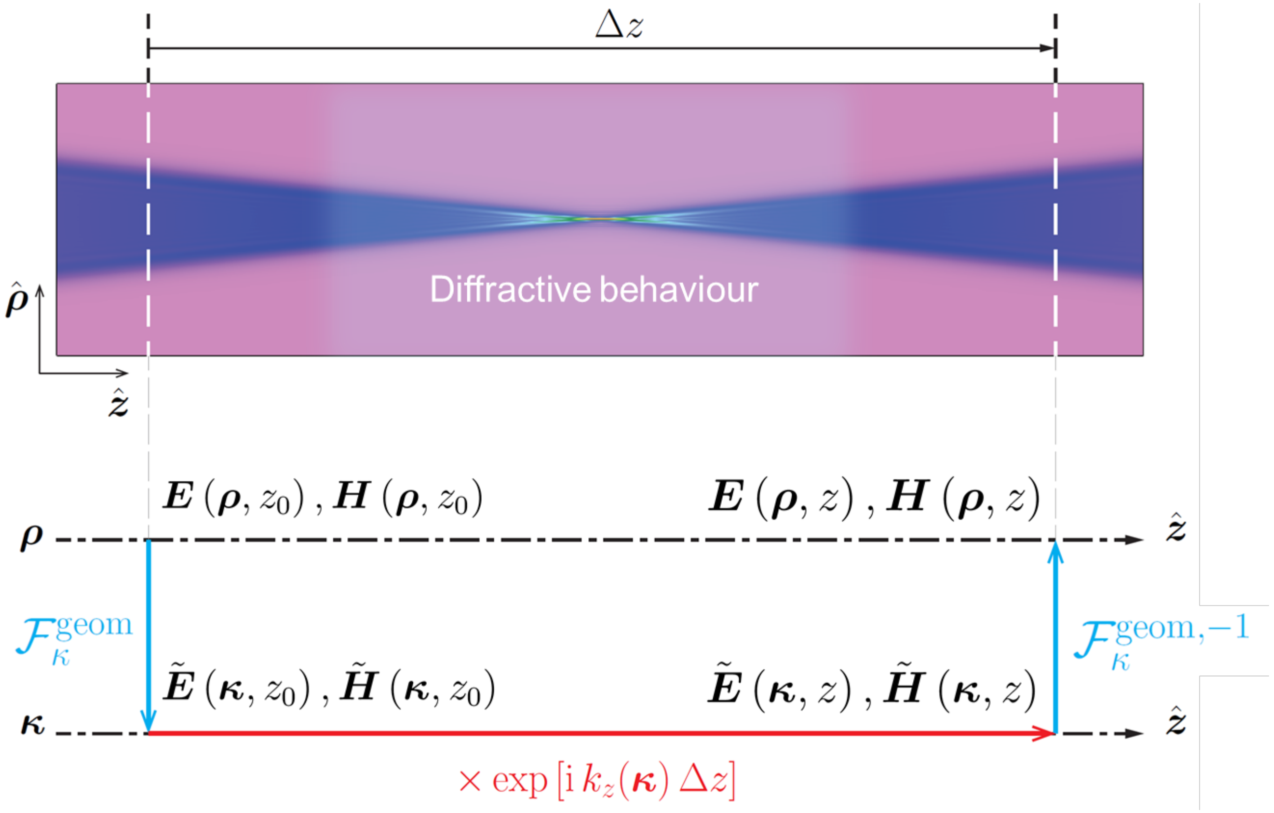
The case of the ideal spherical wave



$$\operatorname{sign}\left\{\tilde{\beta}^{\mathrm{target}}\left[\boldsymbol{\kappa}(\boldsymbol{\rho}), z\right] \beta^{\mathrm{in}}\left[\boldsymbol{\rho}'(\boldsymbol{\rho}), z_0\right]\right\}$$

$$= \operatorname{sign}\left(\frac{R_1}{R_1 + \Delta z}\right)$$

$$V_\ell(\boldsymbol{\rho}, z) = \beta\left[\boldsymbol{\rho}'(\boldsymbol{\rho}), z_0\right] \tilde{\beta}\left[\boldsymbol{\kappa}(\boldsymbol{\rho}), z\right] \mathrm{e}^{\mathrm{i}\left\{\frac{k_0^2 \check{n}^2}{\check{k}_z[\boldsymbol{\kappa}(\boldsymbol{\rho})]} \Delta z\right\}} V_\ell\left[\boldsymbol{\rho}'(\boldsymbol{\rho}), z_0\right]$$

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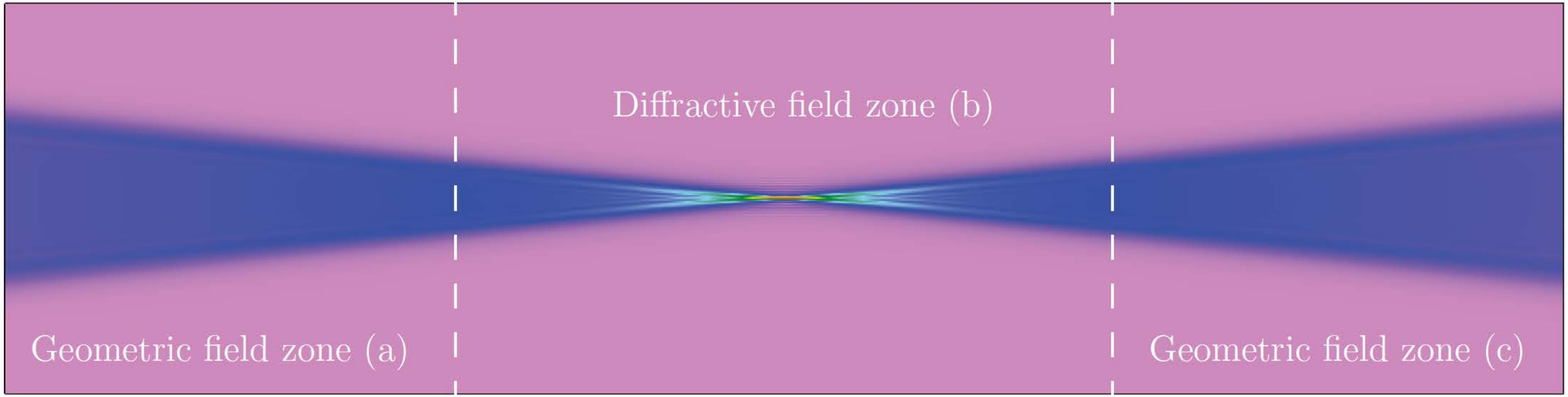
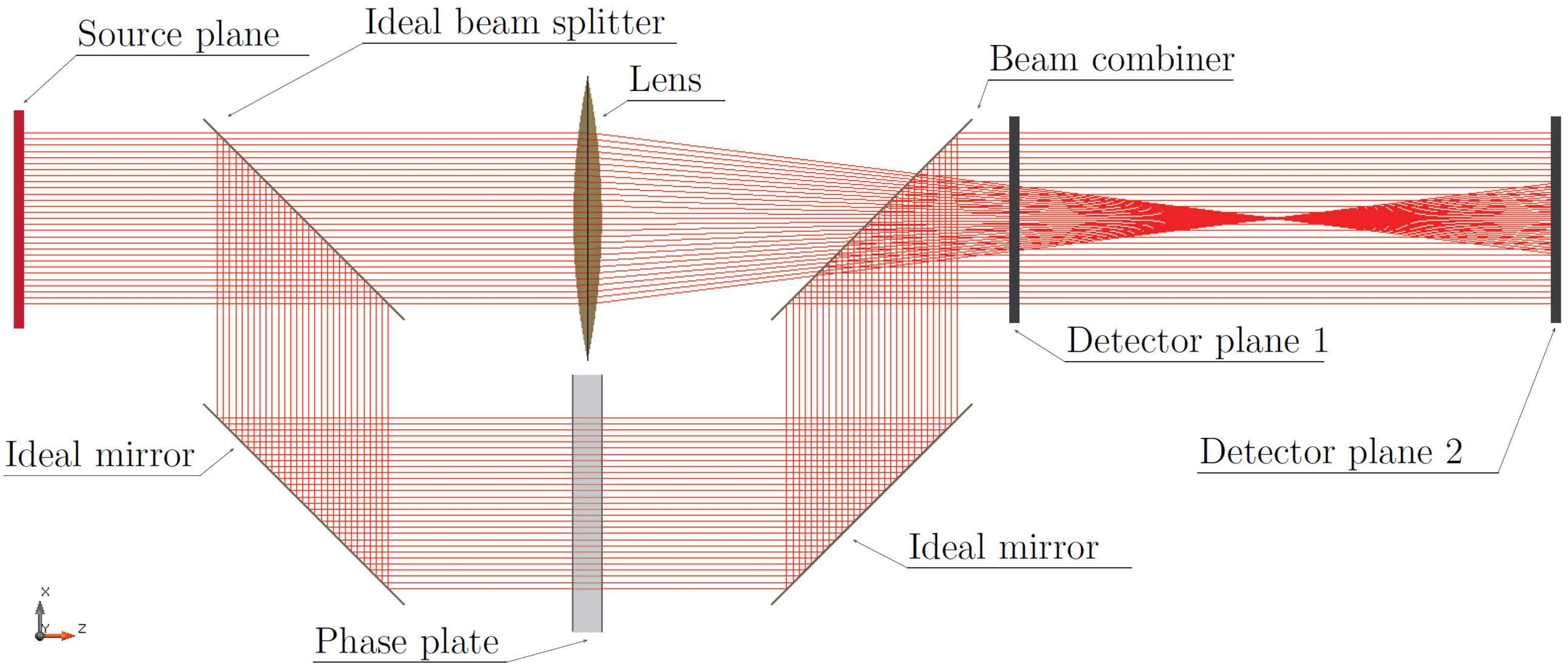


R_1	$\Delta z + R_1$	$\text{sign}\left[\frac{R_1}{\Delta z + R_1}\right]$	**Physical Meaning**
< 0	< 0	1	Propagation in convergent geometric zone (case 1)
< 0	> 0	-1	Propagation through focus between geometric zones (case 2)
> 0	> 0	1	Propagation in divergent geometric zone (case 3)

Simulation example: Mach-Zehnder

Simulation with VirtualLab Fusion



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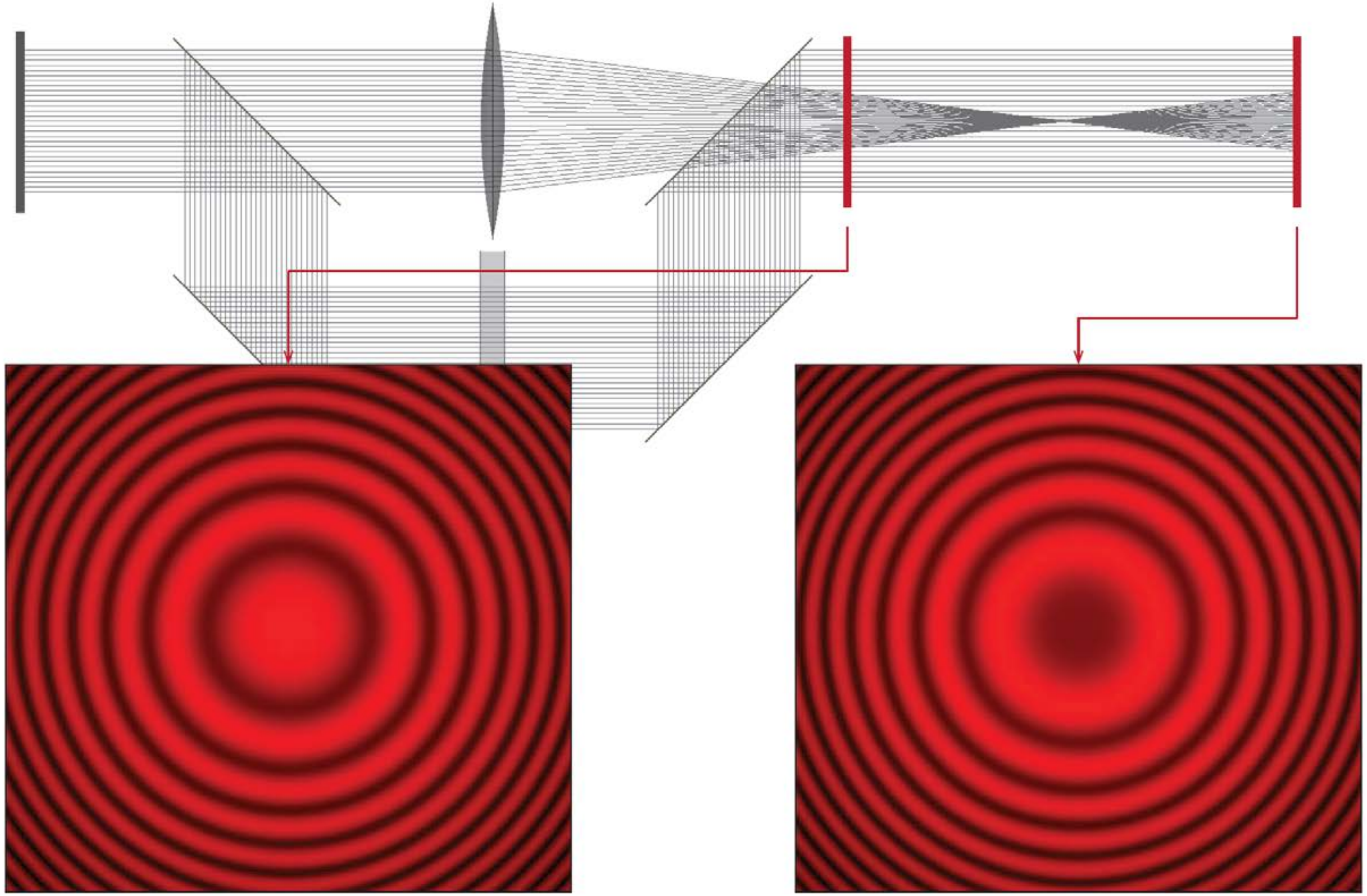

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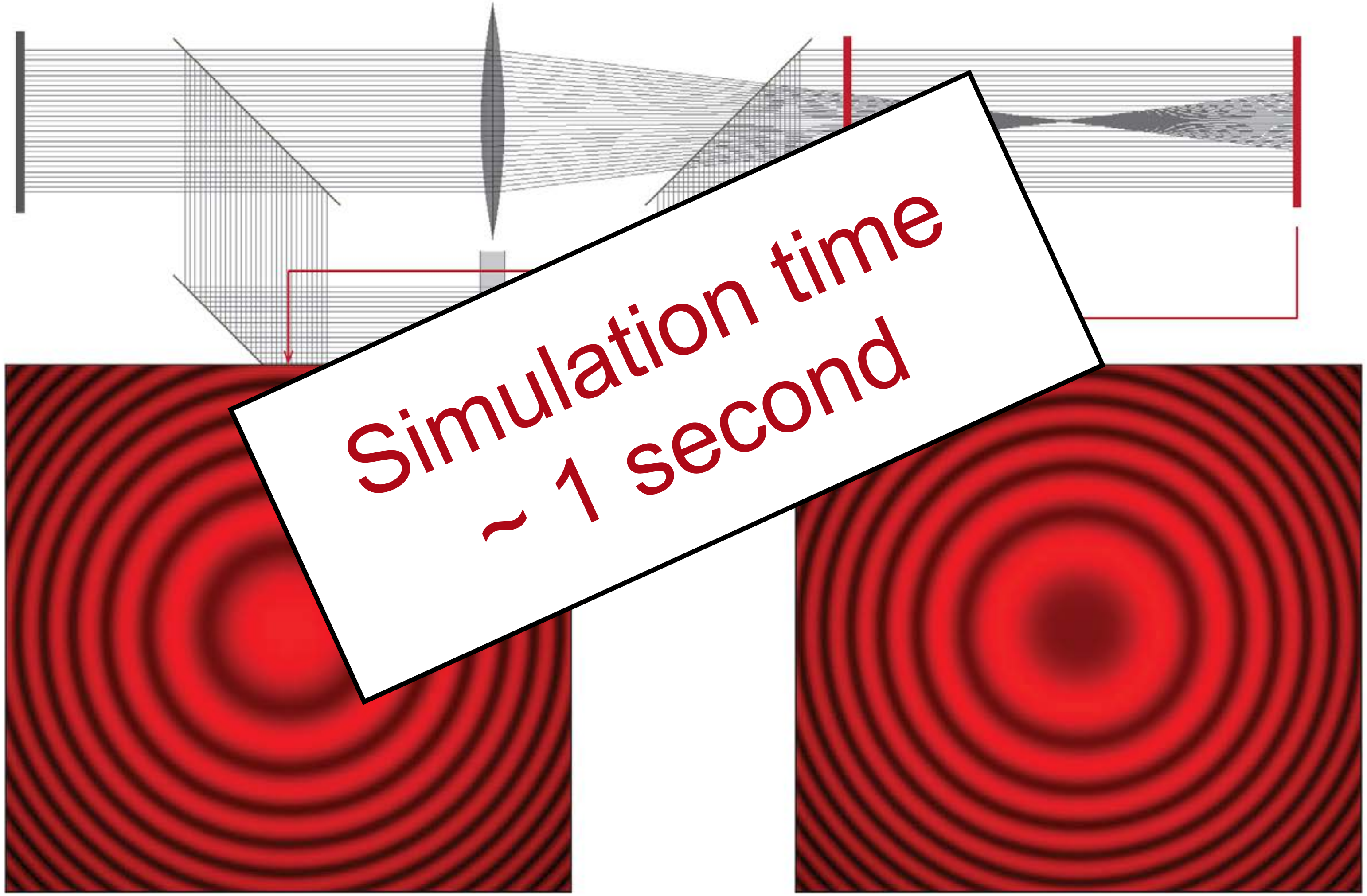

Simulation example: multiple reflections in lens

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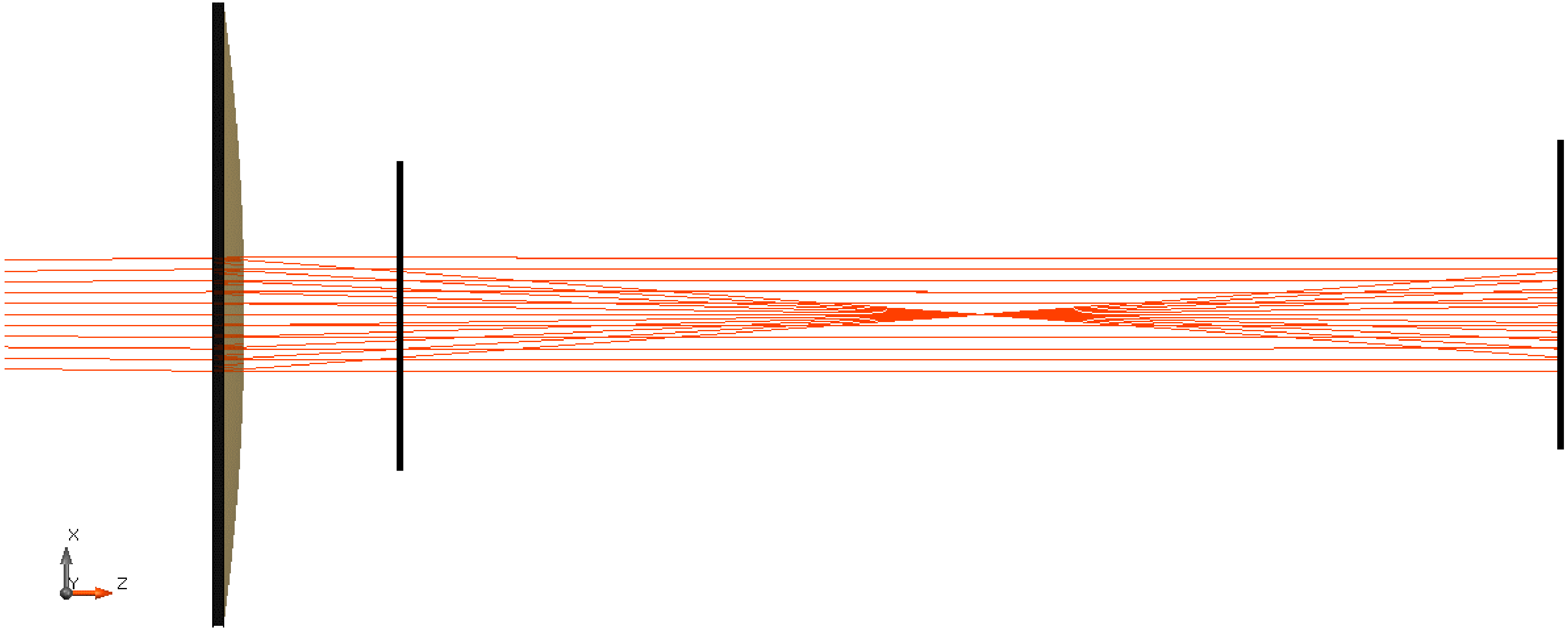

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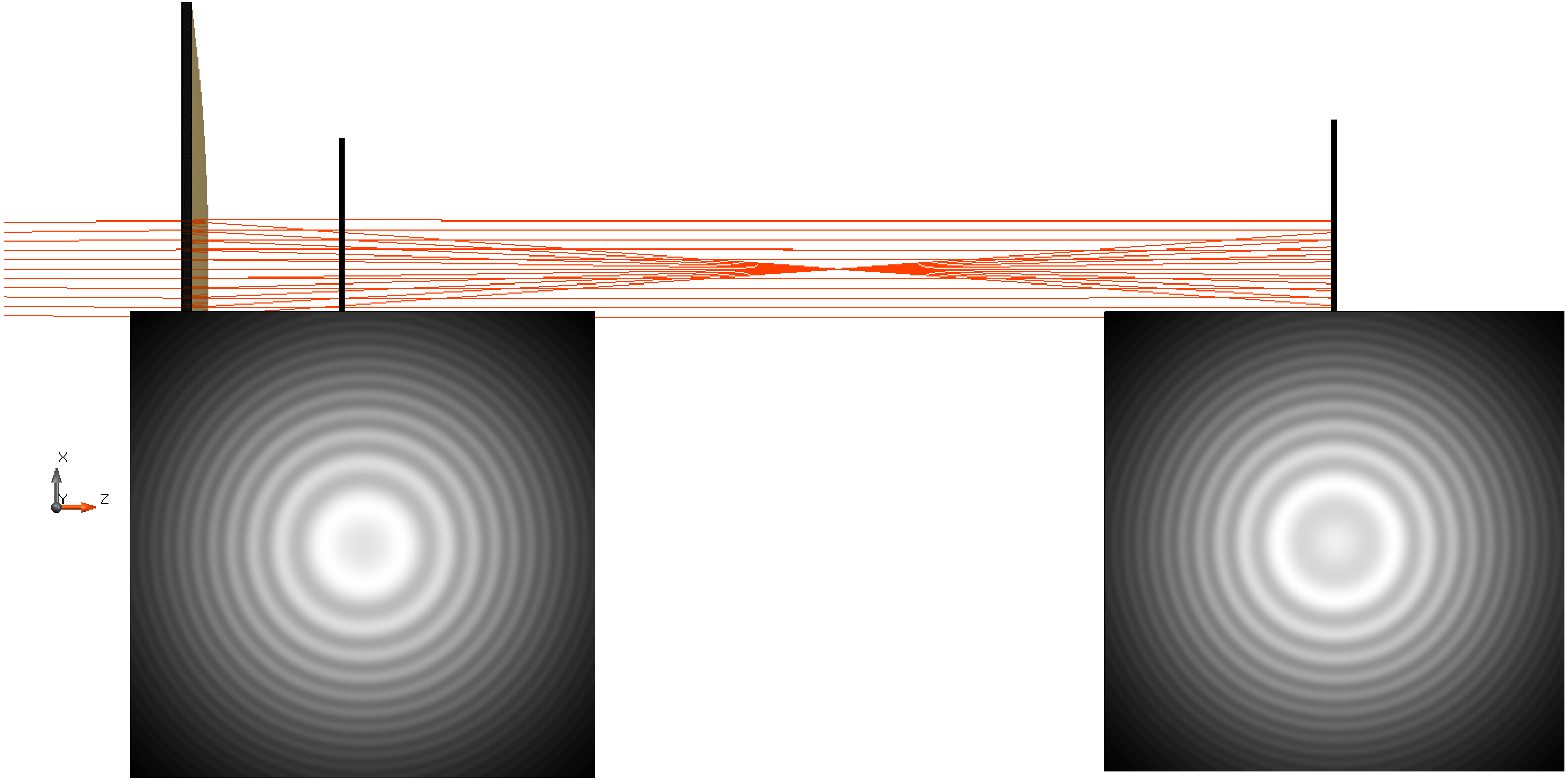

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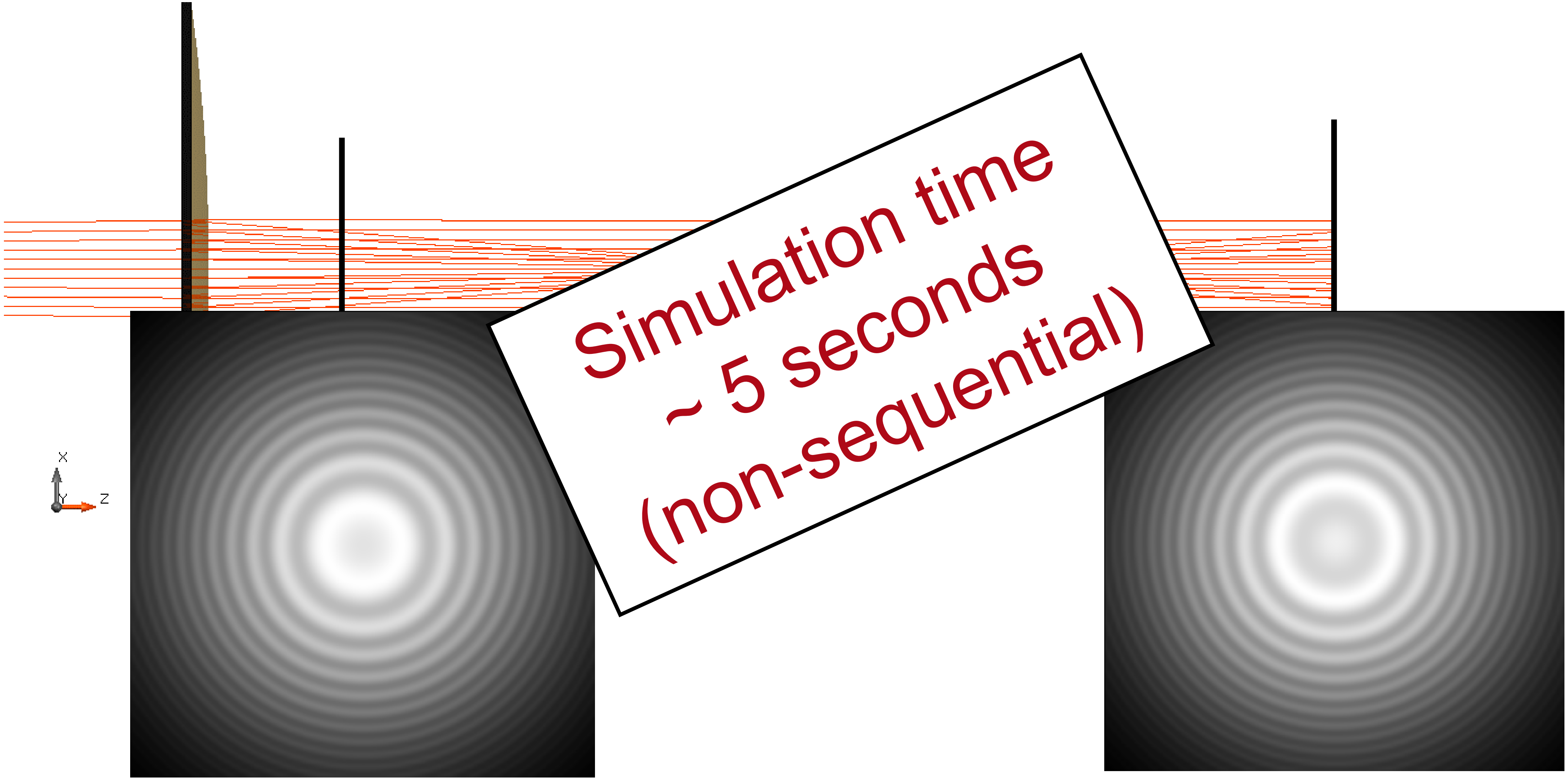

Thank you for your attention!