SPIE Paper 10694-21 11:10-11:30

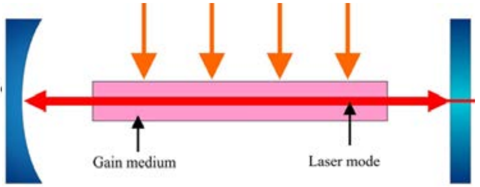
Fast Propagation of Electromagnetic Fields through Graded-Index (GRIN) Media

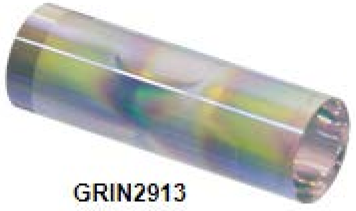
Huiying Zhong[1,2], Site Zhang[1,2], Rui Shi[1], Christian Hellmann[3], Frank Wyrowsk[1]

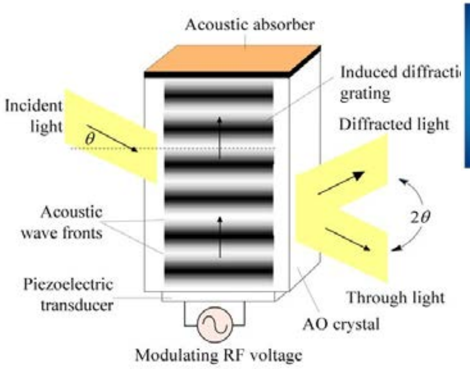
1. Friedrich Schiller University Jena
2. LightTrans International UG
3. Wyrowski Photonics UG

Introduction: GRIN Media in Real Life

- Graded-Index (GRIN) meida are widely used for modeling different situations
  - Applications
    - multi-mode fiber
    - optical lenses
    - acousto-optical modulators
  - Undesired situations
    - stress or heating induced GRIN variations
    - turbulence in air





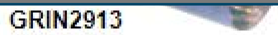


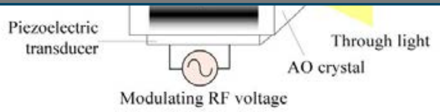
 Pictures comes from [1-4]

Introduction: GRIN Media in Real Life

- Graded-Index (GRIN) meida are widely used for modeling different situations
  - Applications
    - multi-mode fiber
    - optical lenses
    - acousto-optical modulators
  - Undesired situations
    - stress or heating induced GRIN variations

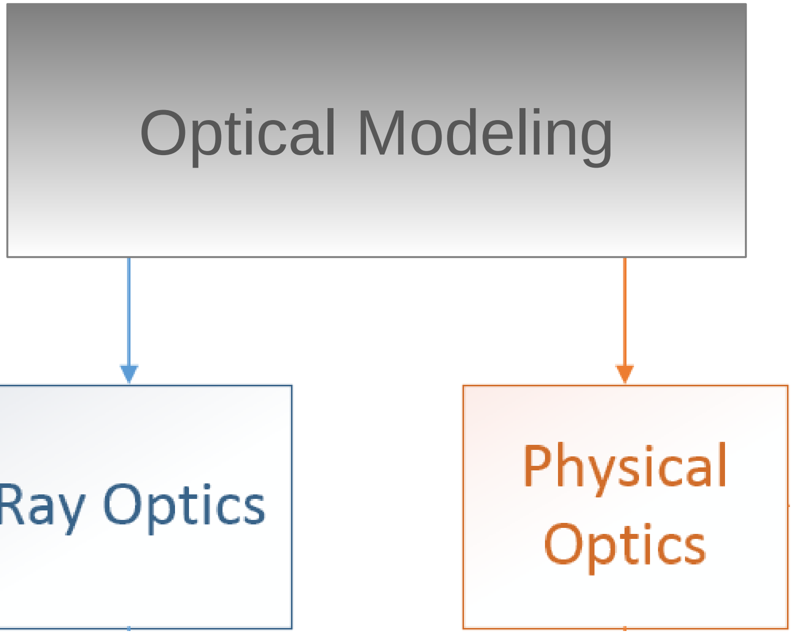
Task: How to model light propagation in GRIN media?





 Pictures comes from [1-4]

Ray and Physical Optics



Pierre de Fermat
(1601 - 1665)

James Clerk Maxwell
(1831-1879)

GRIN Media: Ray Optics

Optical Modeling

Ray Optics

Physical Optics

Pierre de Fermat
(1601 - 1665)

James Clerk Maxwell
(1831-1879)

Ray equation for GRIN media

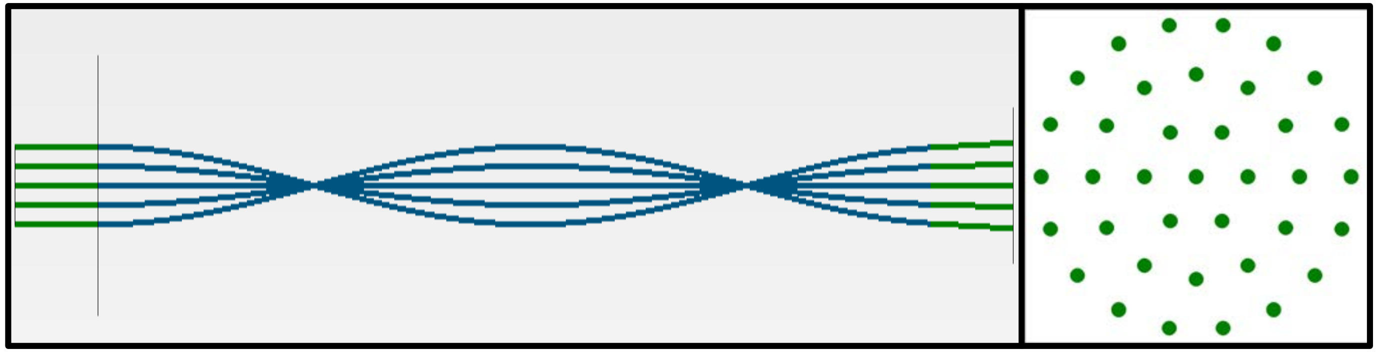
$$\frac{d}{ds}\left[n(\boldsymbol{r})\frac{d\boldsymbol{r}}{ds}\right] = \boldsymbol{\nabla} n(\boldsymbol{r})$$

Solution of the Ray Equation

Ray equation for GRIN media

$$\frac{d}{ds}\left[n(\boldsymbol{r})\frac{d\boldsymbol{r}}{ds}\right] = \nabla n(\boldsymbol{r})$$

Runge-Kutta methods



[5] A. Sharma *et al.* Tracing rays through graded-index media: a new method. *Appl. Opt.*, **21**(6): 984-987 **1982**

Solution of the Ray Equation

Ray equation for GRIN media

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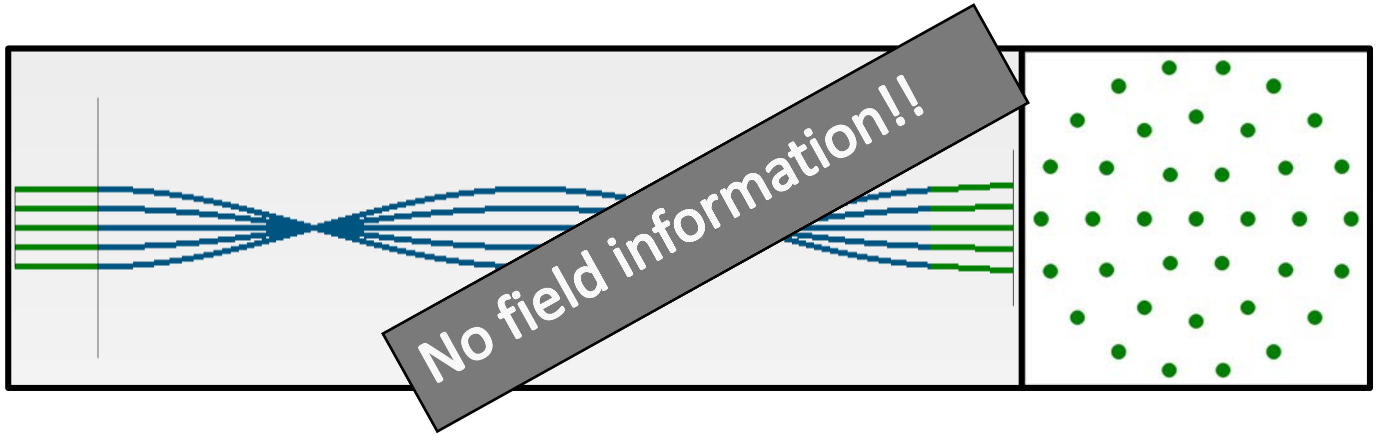
Runge-Kutta methods



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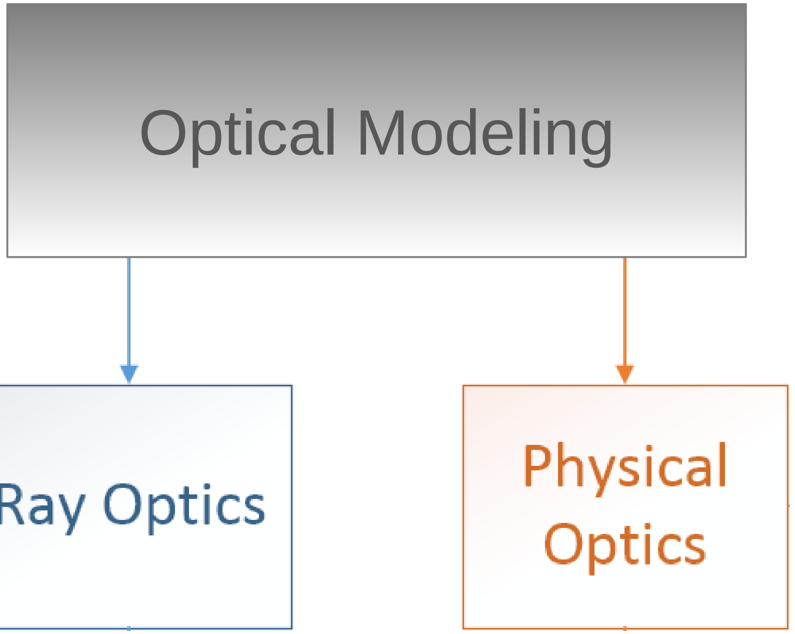
GRIN Media: Physical Optics



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Solution of Maxwell Equations

- Mode solvers for highly symmetric structure
  ➔ Not valid when symmetry decreases
  ➔ Numerical effort is high when size of structure increase
- Universal Maxwell solvers: Finite element method (FEM) and Fourier modal method (FMM) + perfectly matched layers (PMLs)
  ➔ Numerical effort is quite high when size of structure increase

Develop a fast approach to model electromagnetic field through GRIN media!

Field Representation

A rigorous representation of the electromagnet fields

$$E(\mathbf{r},\omega) = E_0(\mathbf{r},\omega)\exp[i\psi(\mathbf{r},\omega)], \quad (1)$$

$$H(\mathbf{r},\omega) = H_0(\mathbf{r},\omega)\exp[i\psi(\mathbf{r},\omega)]. \quad (2)$$

$\psi(\boldsymbol{r},\omega)$ is a common phase function, extracted from electromagnetic field. No approximation!

Geometric Field Zone

A rigorous representation of the electromagnet fields

$$E(r,\omega) = E_0(r,\omega)\exp[i\psi(r,\omega)], \quad (1)$$

$$H(r,\omega) = H_0(r,\omega)\exp[i\psi(r,\omega)]. \quad (2)$$

For some EM-field, we could found a proper $\psi(\boldsymbol{r},\omega)$ so that the field behaviour is dominated by the phase part. More specifically,

- $\boldsymbol{E}_0(\boldsymbol{r},\omega)$ and $\boldsymbol{H}_0(\boldsymbol{r},\omega)$ varies slowly
- $\psi(\boldsymbol{r},\omega)$ varies much faster

➡ Geometric field zone

Geometric Field Equations

$$E(r,\omega) = E_0(r,\omega)\exp[i\psi(r,\omega)], \quad (1)$$

$$H(r,\omega) = H_0(r,\omega)\exp[i\psi(r,\omega)]. \quad (2)$$

Maxwell Eqs.

$$\nabla \times E(r,\omega) = i\omega\mu_0 H(r,\omega),$$
$$\nabla \times H(r,\omega) = -i\omega\epsilon_0 n^2(r,\omega)E(r,\omega),$$
$$\nabla \cdot [n^2(r,\omega)E(r,\omega)] = 0,$$
$$\nabla \cdot H(r,\omega) = 0,$$

$$\nabla \times E_0(r) + i\nabla\psi(r) \times E_0(r) - i\omega\mu_0 H_0(r) = 0,$$
$$\nabla \times H_0(r) + i\nabla\psi(r) \times H_0(r) + i\omega\epsilon_0 n^2(r)E_0(r) = 0,$$
$$\nabla \cdot E_0(r) + iE_0(r) \cdot \nabla\psi(r) + E_0(r) \cdot \nabla \ln n^2(r) = 0,$$
$$\nabla \cdot H_0(r) + iH_0(r) \cdot \nabla\psi(r) = 0.$$

geometric field approximation

Geometric field Eqs.

$$\nabla\psi(r) \times E_0(r) = \omega\mu_0 H_0(r),$$
$$\nabla\psi(r) \times H_0(r) = -\omega\epsilon_0 n^2(r)E_0(r),$$
$$\nabla\psi(r) \cdot E_0(r) = 0,$$
$$\nabla\psi(r) \cdot H_0(r) = 0.$$

Solve the Geometric Field Eqs. for GRIN Media

- To solve geometric field equations for GRIN media, we get solution of how geometric field propagating through GRIN media.

Geometric field Eqs.

$$\nabla\psi(\boldsymbol{r}) \times \boldsymbol{E}_0(\boldsymbol{r}) = \omega\mu_0\boldsymbol{H}_0(\boldsymbol{r}),$$
$$\nabla\psi(\boldsymbol{r}) \times \boldsymbol{H}_0(\boldsymbol{r}) = -\omega\epsilon_0 n^2(\boldsymbol{r})\boldsymbol{E}_0(\boldsymbol{r}),$$
$$\nabla\psi(\boldsymbol{r}) \cdot \boldsymbol{E}_0(\boldsymbol{r}) = 0,$$
$$\nabla\psi(\boldsymbol{r}) \cdot \boldsymbol{H}_0(\boldsymbol{r}) = 0.$$

Eikonal Eq.

$$(\nabla\psi(\boldsymbol{r}))^2 = k_0^2 n^2(\boldsymbol{r}).$$

Ray Eq.

$$\frac{d}{ds}[n(\boldsymbol{r})\hat{\mathbf{s}}(\boldsymbol{r})] = \boldsymbol{\nabla} n(\boldsymbol{r}).$$

Eq. of normalized field

$$n(\boldsymbol{r})\frac{d\hat{\boldsymbol{u}}(\boldsymbol{r})}{ds} = -\left[\hat{\boldsymbol{u}}(\boldsymbol{r}) \cdot \frac{\boldsymbol{\nabla} n(\boldsymbol{r})}{n(\boldsymbol{r})}\right] n(\boldsymbol{r})\frac{d\boldsymbol{r}}{ds}$$

 [6] M. Born and E. Wolf, Principles of Optics, Cambridge University press (1999)

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Ray Eq.

$$\frac{d}{ds}[n(\boldsymbol{r})\hat{\mathbf{s}}(\boldsymbol{r})] = \boldsymbol{\nabla} n(\boldsymbol{r}).$$

$\hat{\mathbf{s}}(\boldsymbol{r})$ is ray direction

Eq. of normalized field

$$n(\boldsymbol{r})\frac{d\hat{\boldsymbol{u}}(\boldsymbol{r})}{ds} = -\left[\hat{\boldsymbol{u}}(\boldsymbol{r}) \cdot \frac{\boldsymbol{\nabla} n(\boldsymbol{r})}{n(\boldsymbol{r})}\right] n(\boldsymbol{r})\frac{d\boldsymbol{r}}{ds}$$

$$\boldsymbol{E}_0(\boldsymbol{r}) = ||\boldsymbol{E}_0(\boldsymbol{r})||\hat{\boldsymbol{u}}(\boldsymbol{r})$$

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Solve the Geometric Field Eqs. for GRIN Media

- Rewrite field representation by using gradient theorem

$$\mathbf{E}(\mathbf{r},\omega) = \mathbf{E}_0(\mathbf{r},\omega)\exp[i\psi(\mathbf{r},\omega)], \tag{1}$$

$$\mathbf{H}(\mathbf{r},\omega) = \mathbf{H}_0(\mathbf{r},\omega)\exp[i\psi(\mathbf{r},\omega)]. \tag{2}$$

Solve the Geometric Field Eqs. for GRIN Media

- Rewrite field representation by using gradient theorem

$$\boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}_0(\boldsymbol{r}) \exp\left\{ i \left[\psi(\boldsymbol{r}_0) + \int_{\boldsymbol{r}_0}^{\boldsymbol{r}} \nabla\psi(\boldsymbol{r}) \cdot d\boldsymbol{r} \right] \right\},$$

$$\boldsymbol{H}(\boldsymbol{r}) = \boldsymbol{H}_0(\boldsymbol{r}) \exp\left\{ i \left[\psi(\boldsymbol{r}_0) + \int_{\boldsymbol{r}_0}^{\boldsymbol{r}} \nabla\psi(\boldsymbol{r}) \cdot d\boldsymbol{r} \right] \right\}.$$

Eq. of normalized field

$$n(\boldsymbol{r}) \frac{d\hat{\boldsymbol{u}}(\boldsymbol{r})}{ds} = -\left[\hat{\boldsymbol{u}}(\boldsymbol{r}) \cdot \frac{\boldsymbol{\nabla} n(\boldsymbol{r})}{n(\boldsymbol{r})} \right] n(\boldsymbol{r}) \frac{d\boldsymbol{r}}{ds}$$

Ray Eq.

$$\frac{d}{ds}[n(\boldsymbol{r})\hat{\boldsymbol{s}}(\boldsymbol{r})] = \boldsymbol{\nabla} n(\boldsymbol{r}).$$

Energy conservation

$$\nabla \cdot \langle \boldsymbol{S}(\boldsymbol{r}) \rangle = 0.$$

Solving Ray & Normalized Field Eq.

Eq. of ray path

$$\frac{d}{ds}[n(\boldsymbol{r})\frac{d\boldsymbol{r}}{ds}] = \boldsymbol{\nabla} n(\boldsymbol{r})$$

$$\begin{cases} dt = \frac{ds}{n} \\ \boldsymbol{T}(\boldsymbol{r}) = n(\boldsymbol{r})\frac{d\boldsymbol{r}}{ds} \\ \boldsymbol{D}(\boldsymbol{r}) = n(\boldsymbol{r})\boldsymbol{\nabla} n(\boldsymbol{r}) \end{cases}$$

Eq. of norm. field

$$n(\boldsymbol{r})\frac{d\hat{u}(\boldsymbol{r})}{ds} = -\left[\hat{u}(\boldsymbol{r})\cdot\frac{\boldsymbol{\nabla} n(\boldsymbol{r})}{n(\boldsymbol{r})}\right] n(\boldsymbol{r})\frac{d\boldsymbol{r}}{ds}$$

Runge-Kutta methods

$$\begin{cases} \boldsymbol{A} = \Delta t \boldsymbol{D}(\boldsymbol{r}_i) \\ \boldsymbol{B} = \Delta t \boldsymbol{D}(\boldsymbol{r}_i + \frac{\Delta t}{2}\boldsymbol{T}(\boldsymbol{r}_i) + \frac{1}{8}\Delta t \boldsymbol{A}) \\ \boldsymbol{C} = \Delta t \boldsymbol{D}(\boldsymbol{r}_i + \Delta t \boldsymbol{T}(\boldsymbol{r}_i) + \frac{1}{2}\Delta t \boldsymbol{B}) \\ \boldsymbol{T}(\boldsymbol{r}_{i+1}) = \boldsymbol{T}(\boldsymbol{r}_i) + \frac{1}{6}(\boldsymbol{A} + 4\boldsymbol{B} + \boldsymbol{C}) \end{cases}$$

$$\boldsymbol{r}_{i+1} = \boldsymbol{r}_i + \Delta t\left[\boldsymbol{T}(\boldsymbol{r}_i) + \frac{1}{6}(\boldsymbol{A} + 2\boldsymbol{B})\right]$$

$$\hat{\boldsymbol{s}}(\boldsymbol{r}_{i+1}) = \frac{\boldsymbol{T}(\boldsymbol{r}_{i+1})}{n(\boldsymbol{r}_{i+1})}$$

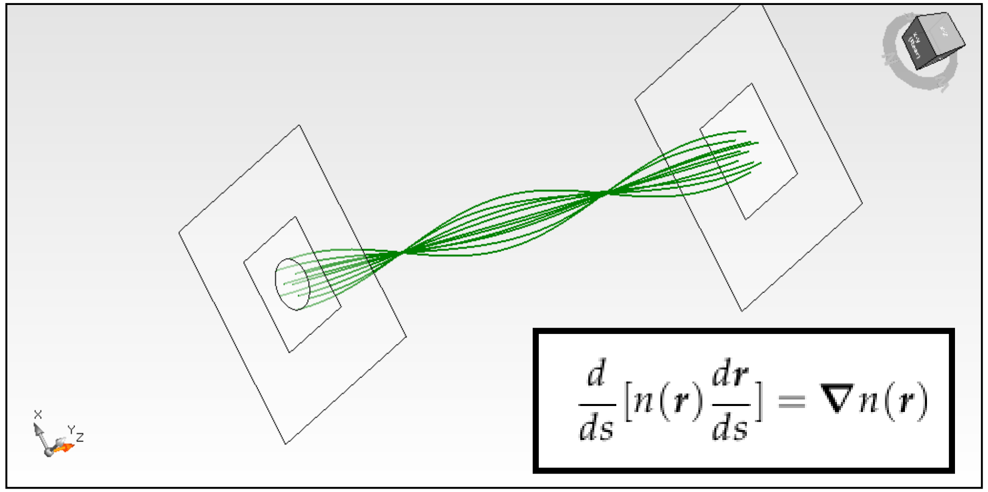
Runge-Kutta methods

$$\begin{cases} \boldsymbol{k}_1 = -\{\hat{\boldsymbol{u}}(\boldsymbol{r}_i)\cdot\frac{\boldsymbol{D}(\boldsymbol{r}_i)}{n^2(\boldsymbol{r}_i)}\}\boldsymbol{T}(\boldsymbol{r}_i) \\ \boldsymbol{k}_2 = -\{[\hat{\boldsymbol{u}}(\boldsymbol{r}_i) + \frac{\Delta t}{2}\boldsymbol{k}_1]\cdot\frac{\boldsymbol{D}(\boldsymbol{r}'_i)}{n^2(\boldsymbol{r}'_i)}\}\boldsymbol{T}(\boldsymbol{r}'_i) \\ \boldsymbol{k}_3 = -\{[\hat{\boldsymbol{u}}(\boldsymbol{r}_i) + \frac{\Delta t}{2}\boldsymbol{k}_2]\cdot\frac{\boldsymbol{D}(\boldsymbol{r}'_i)}{n^2(\boldsymbol{r}'_i)}\}\boldsymbol{T}(\boldsymbol{r}'_i) \\ \boldsymbol{k}_4 = -\{[\hat{\boldsymbol{u}}(\boldsymbol{r}_i) + \Delta t\boldsymbol{k}_3]\cdot\frac{\boldsymbol{D}(\boldsymbol{r}_{i+1})}{n^2(\boldsymbol{r}_{i+1})}\}\boldsymbol{T}(\boldsymbol{r}_{i+1}) \\ \hat{\boldsymbol{u}}_{i+1} = \hat{\boldsymbol{u}}_i + \frac{\Delta t}{6}(\boldsymbol{k}_1 + 2\boldsymbol{k}_2 + 2\boldsymbol{k}_3 + \boldsymbol{k}_4) \end{cases}$$

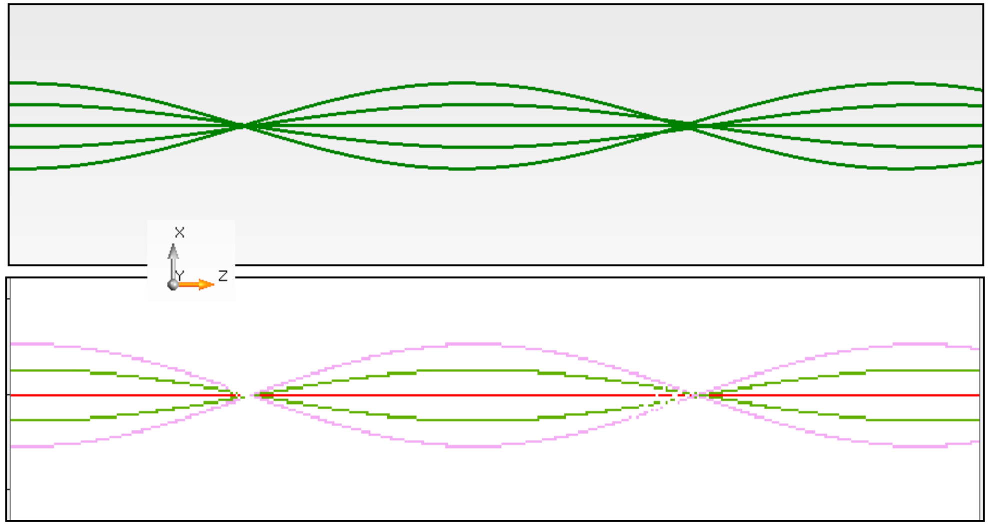
$$\boldsymbol{r}'_i = \boldsymbol{r}(t_i + \frac{\Delta t}{2})$$

[5] A. Sharma *et al.* Tracing rays through graded-index media: a new method. *Appl. Opt.*, **21**(6): 984-987 (1982)

Summary of Geometric Solver in GRIN

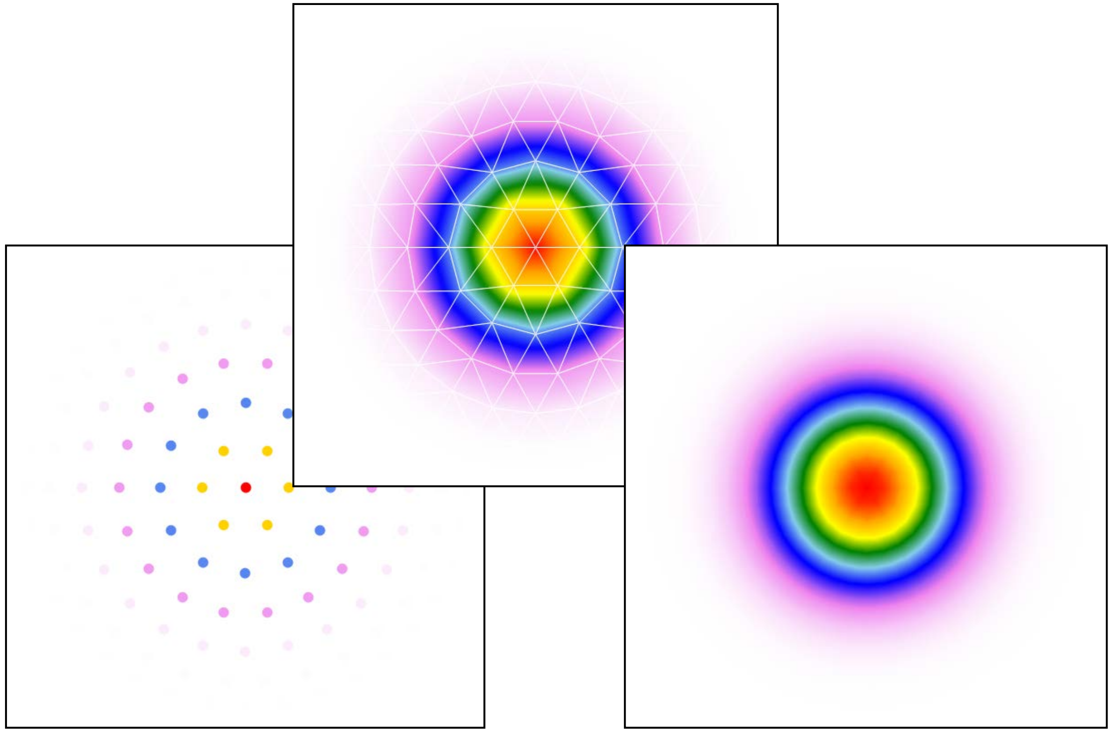


$$\frac{d}{ds}\left[n(\boldsymbol{r})\frac{d\boldsymbol{r}}{ds}\right] = \boldsymbol{\nabla} n(\boldsymbol{r})$$



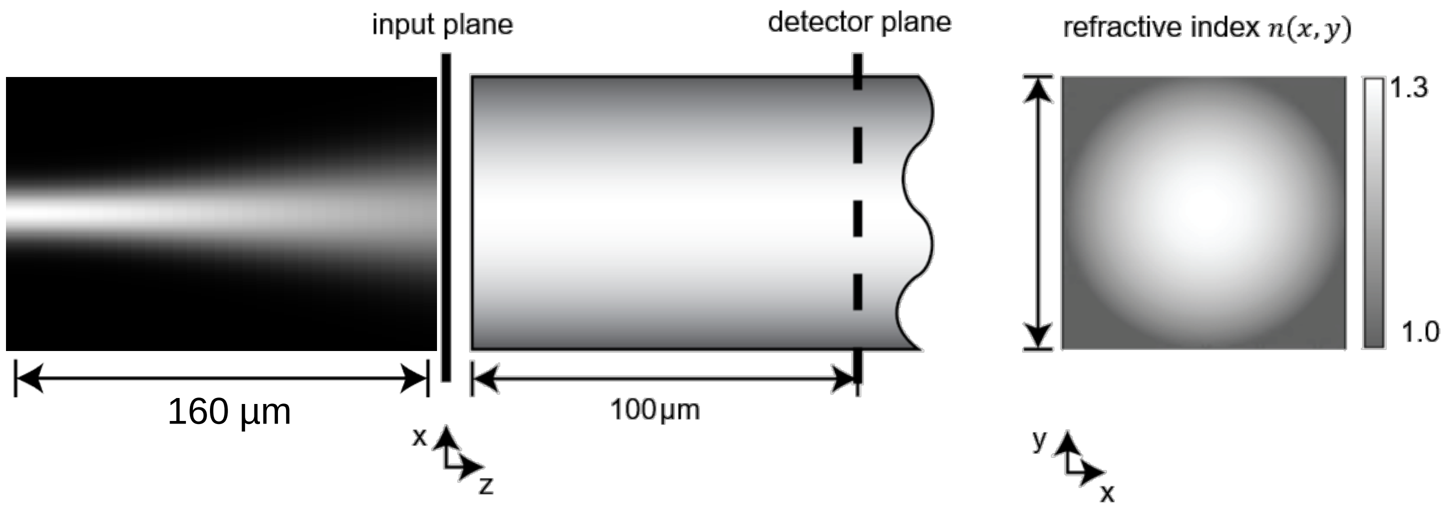
$$n(\boldsymbol{r})\frac{d\hat{\boldsymbol{u}}(\boldsymbol{r})}{ds} = -\left[\hat{\boldsymbol{u}}(\boldsymbol{r}) \cdot \frac{\boldsymbol{\nabla} n(\boldsymbol{r})}{n(\boldsymbol{r})}\right] n(\boldsymbol{r})\frac{d\boldsymbol{r}}{ds}$$

$$\nabla \cdot < \boldsymbol{S}(\boldsymbol{r}) >= 0.$$



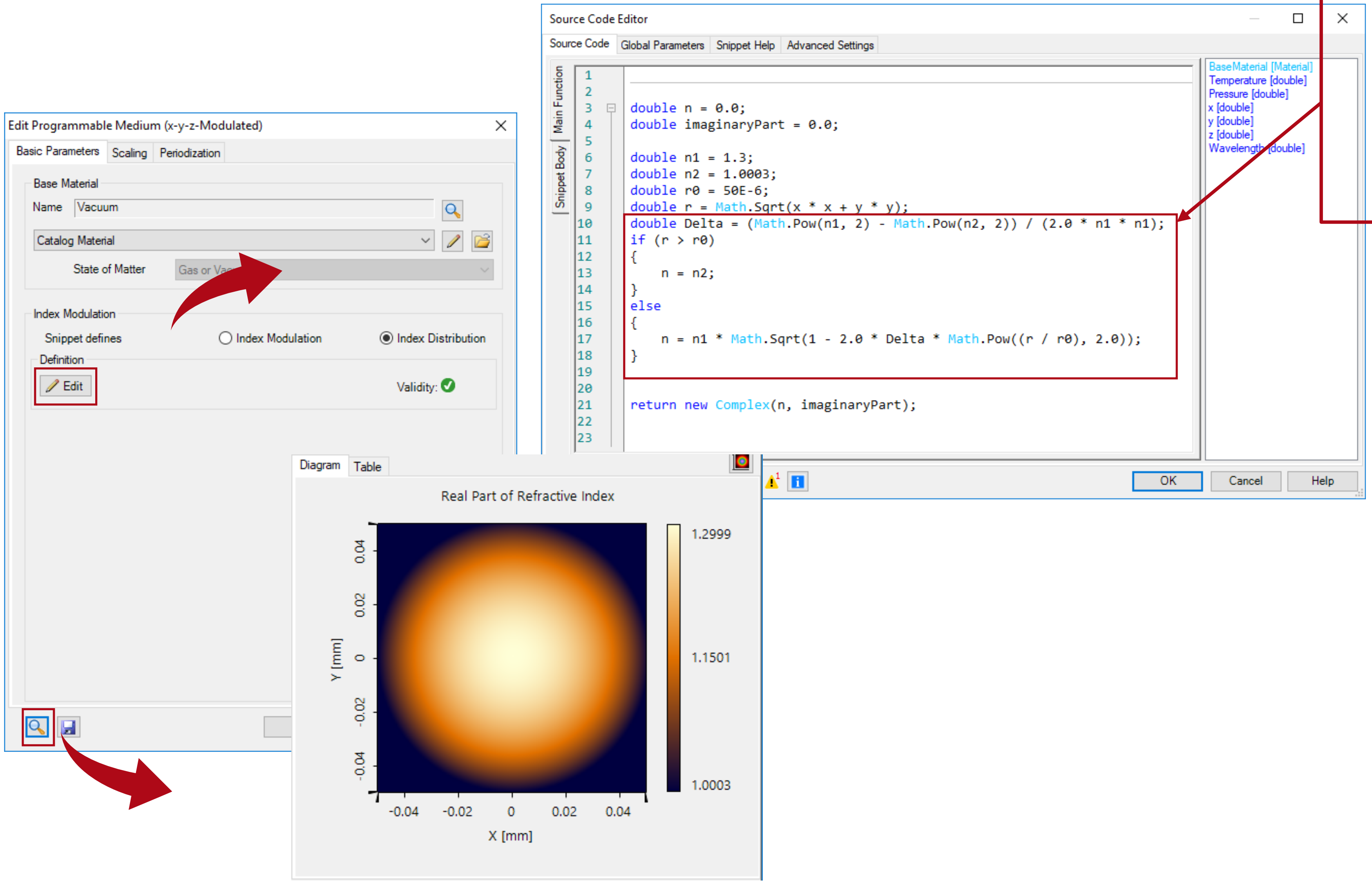
[7] Huiying Zhong et al, "Fast propagation of electromagnetic fields through graded-index media," J. Opt. Soc. Am. A 35, 661-668 (2018)

Example: Multimode Fiber



- ray propagation through a GRIN fiber
- electromagnetic field propagation through a GRIN fiber by
  - a rigorous Maxwell solver, the Fourier Modal Method (FMM) with Perfectly Matched Layers (PMLs)
  - our newly developed very fast approximated Maxwell solver

Implementation of Refractive Index Distribution



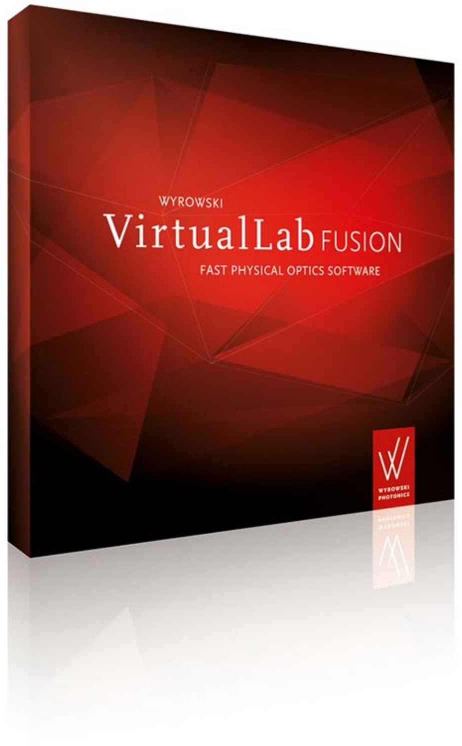
$$n(r) = n_1\sqrt{1-\frac{2\Delta r^2}{r_0^2}} \text{ with } \Delta = \frac{n_1^2-n_2^2}{2n_1^2}$$

here

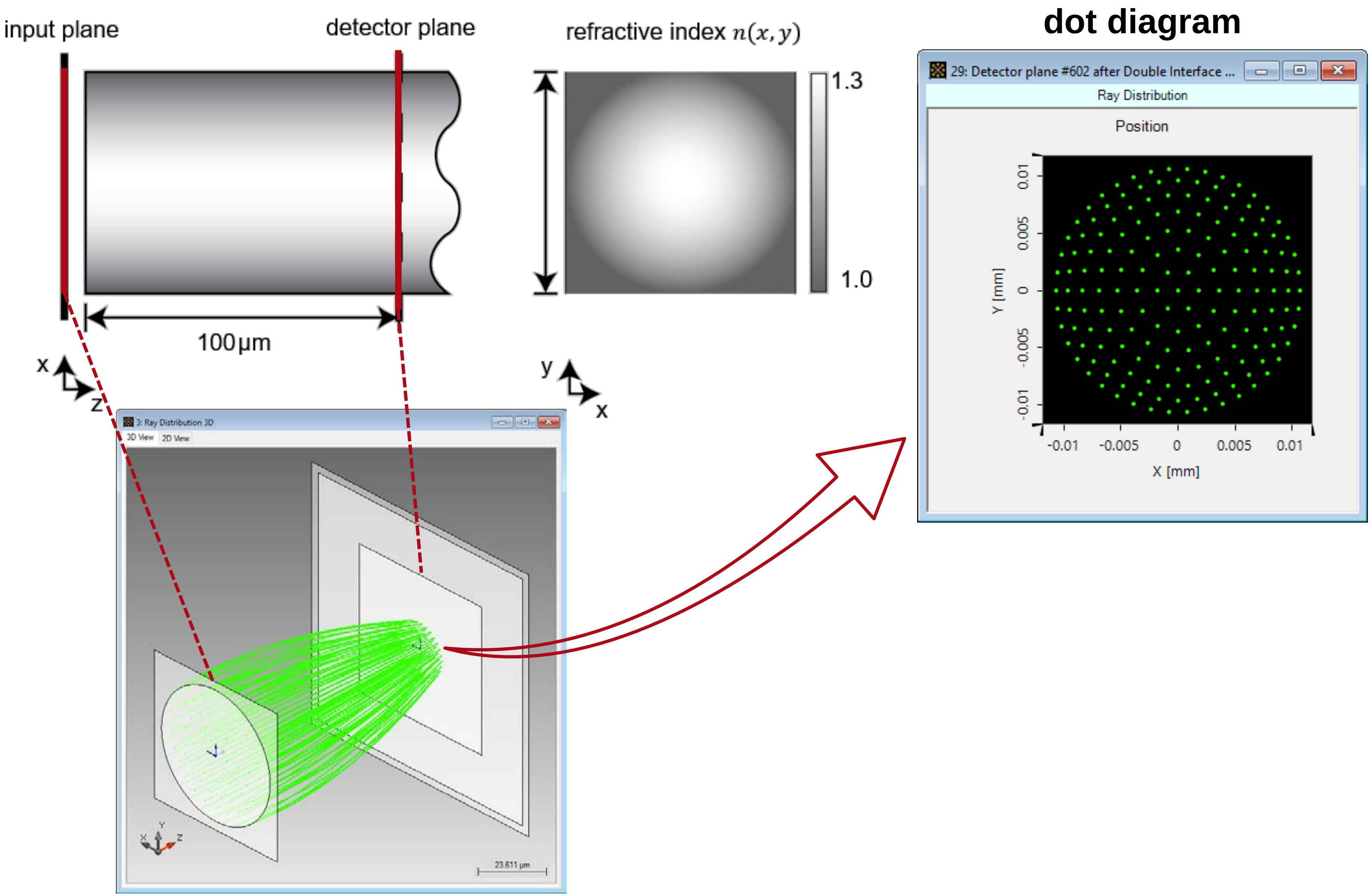
$r_0 = 50\ \mu\text{m}$

$n_1 = 1.3$

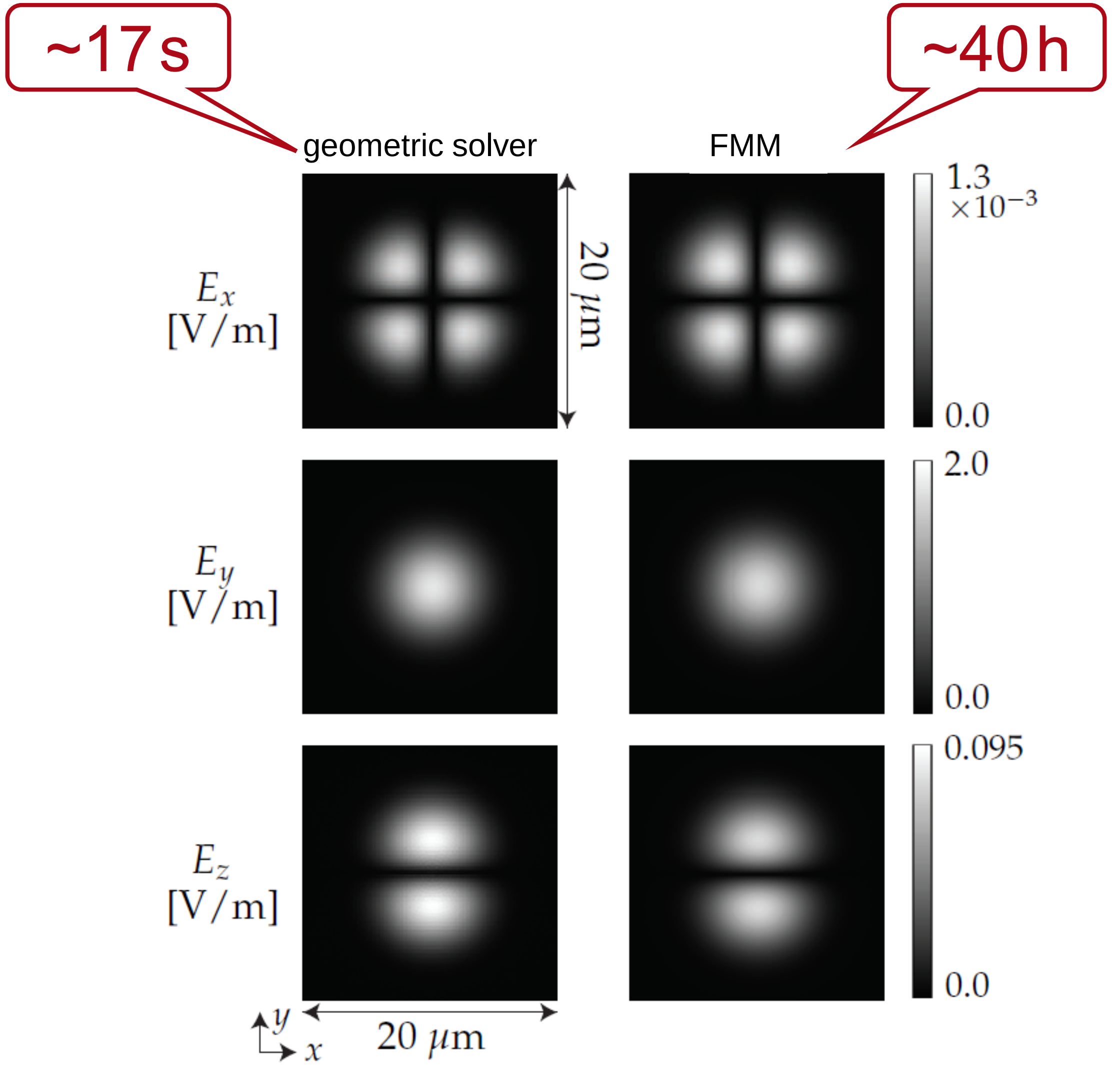
$n_2 = 1.0$



Results: 3D System Ray Tracing

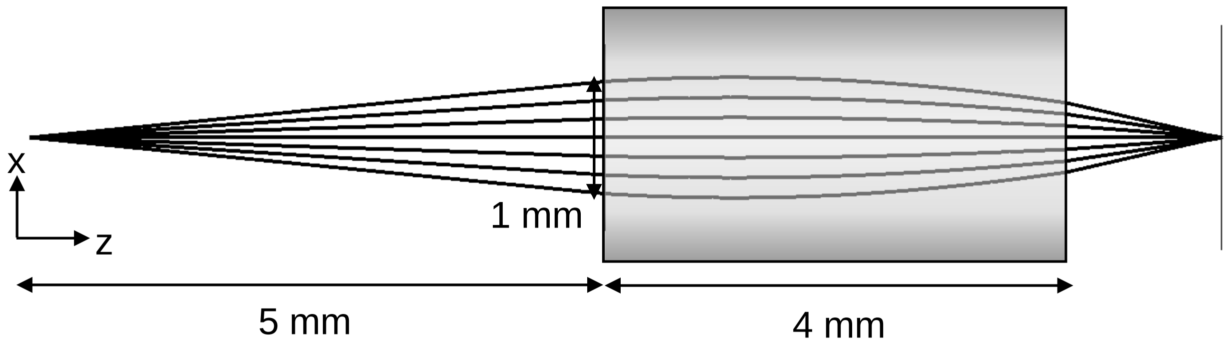


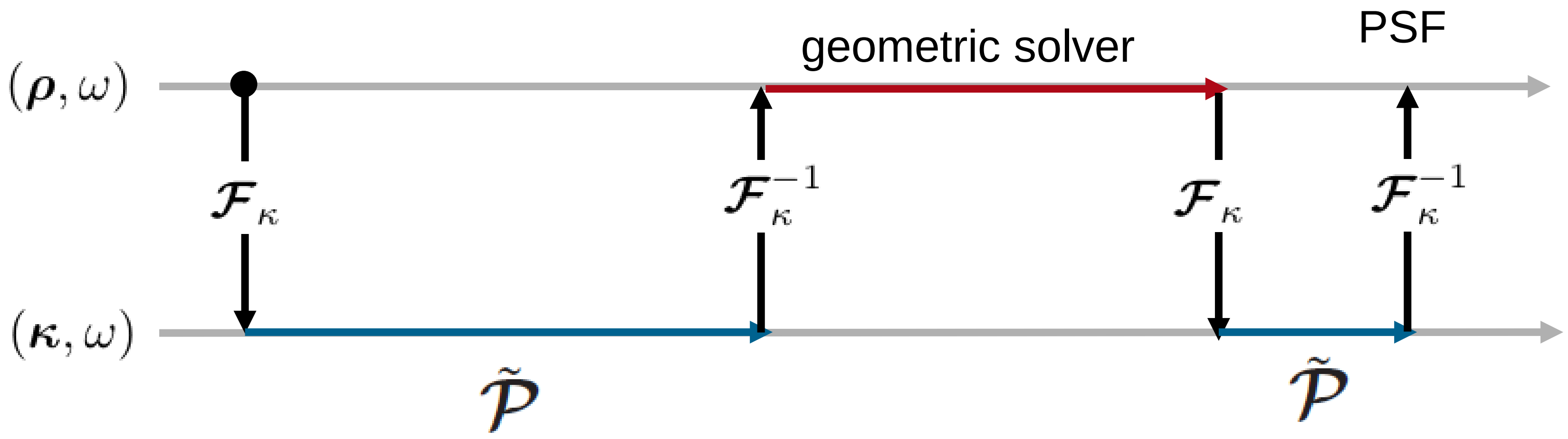
Results: Our Fast Approach vs FMM



Deviation between results of both approaches is ~1%

Example: PSF of GRIN Lens





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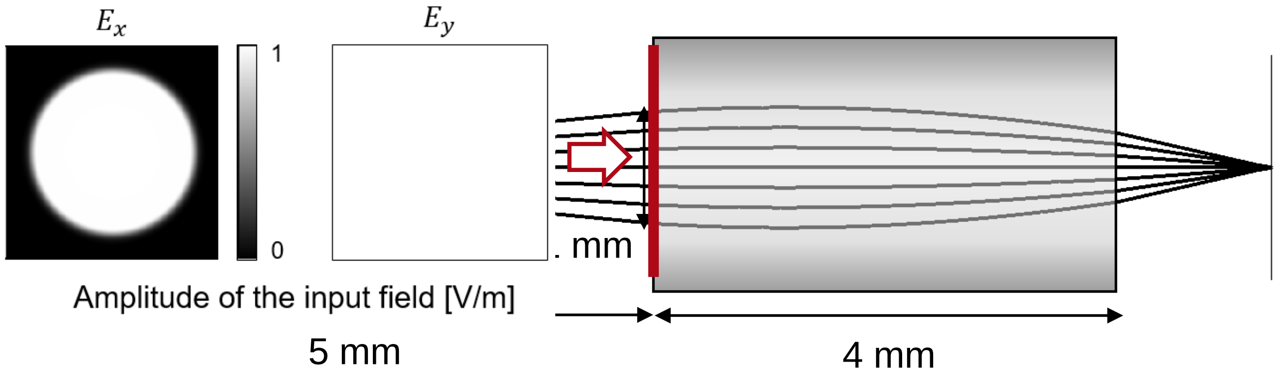
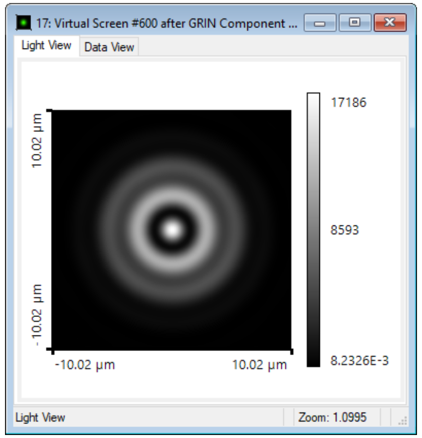
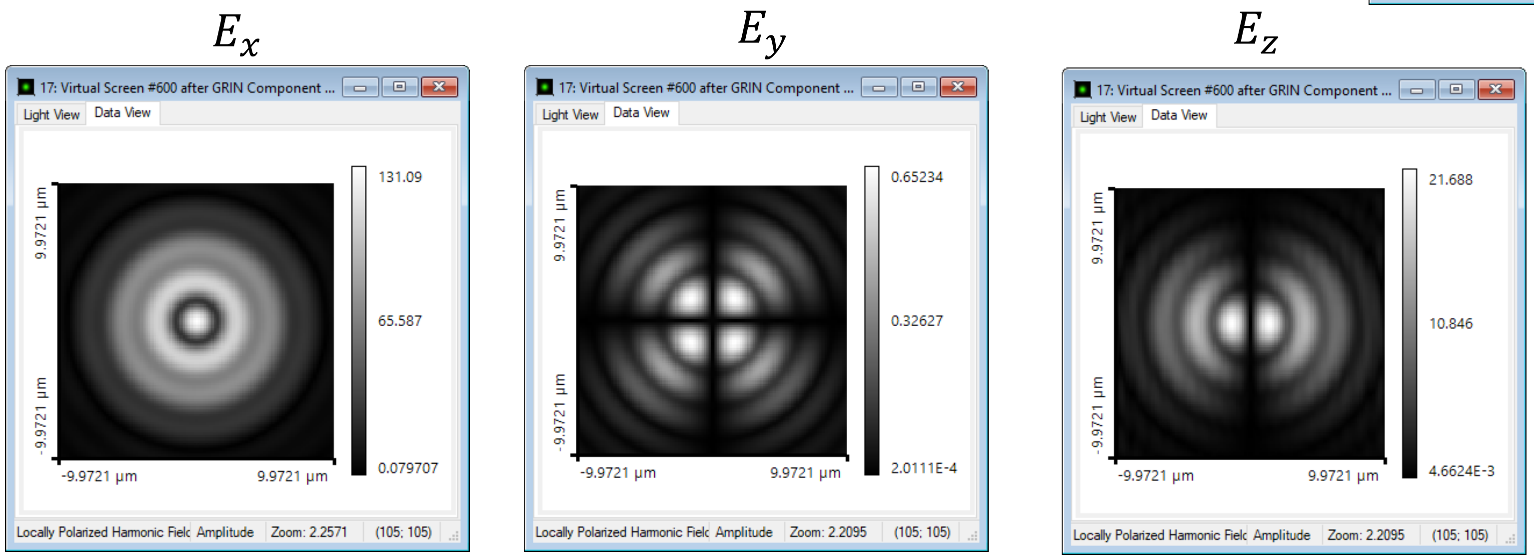


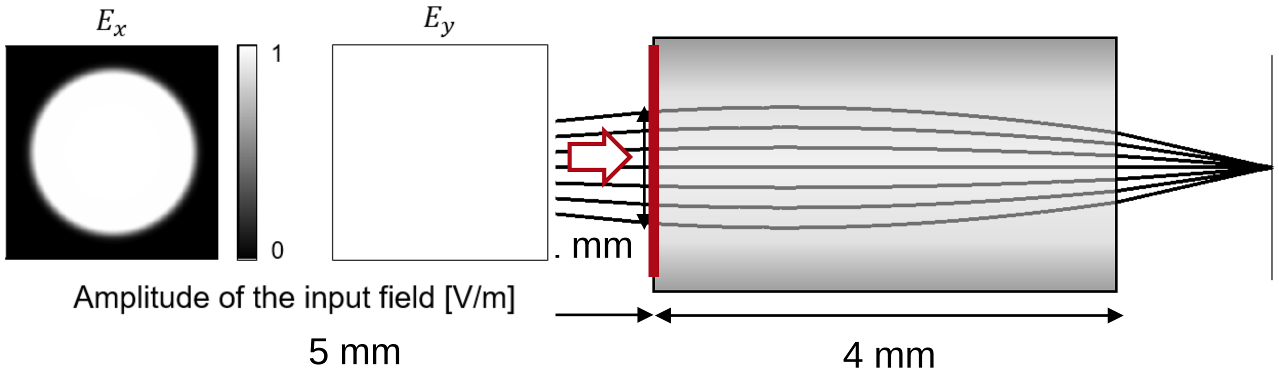
PSF



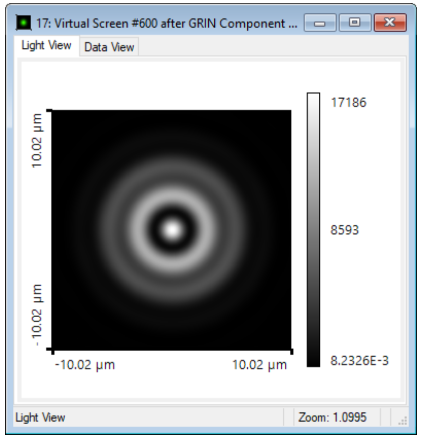


Amplitude of the image [V/m]

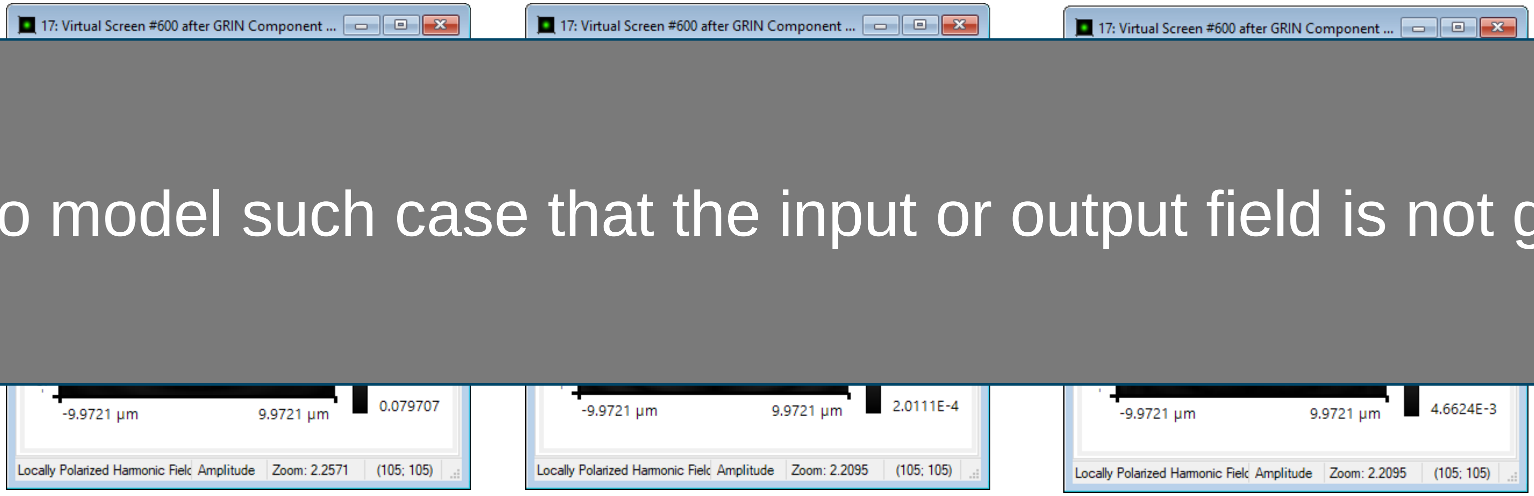
Example: PSF of GRIN Lens



PSF



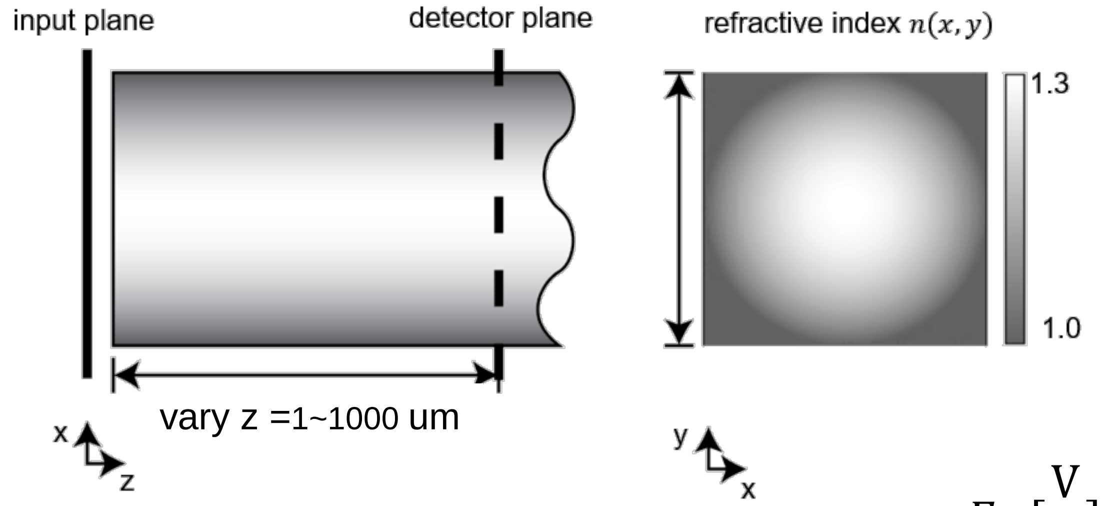
E_x E_y E_z

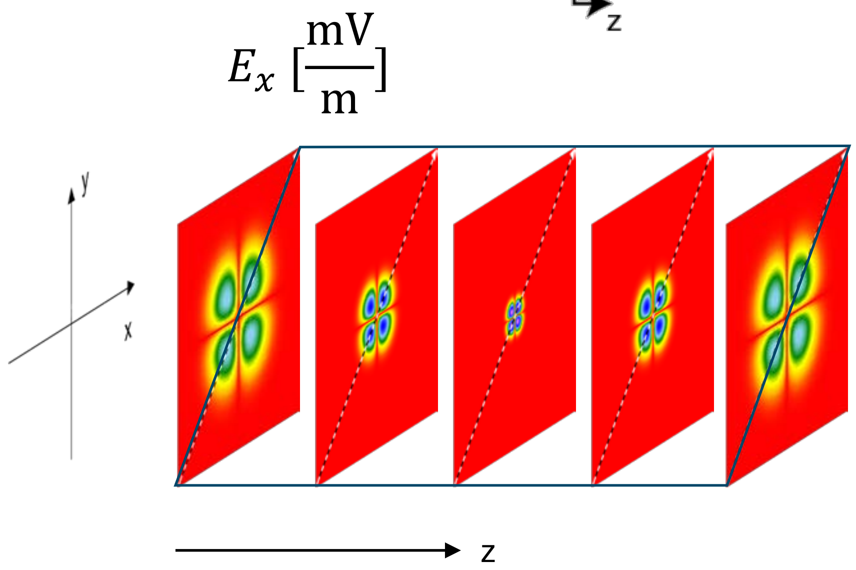


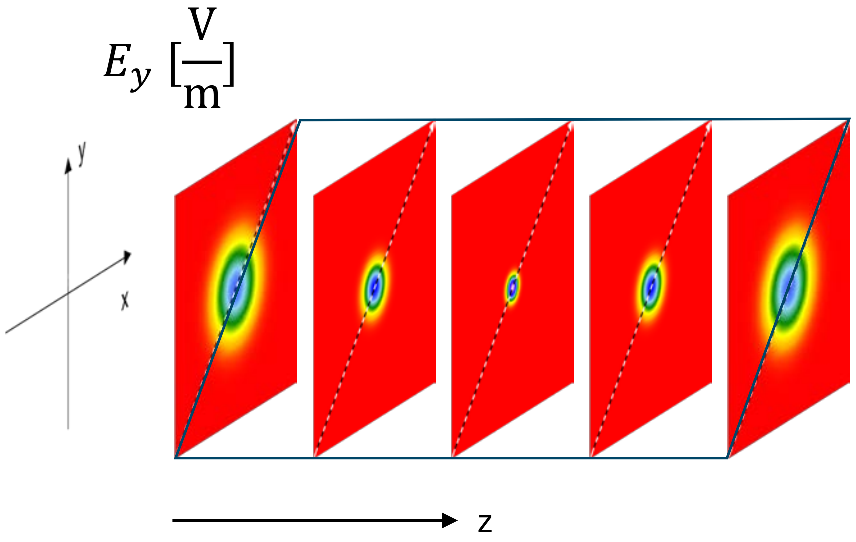
How to model such case that the input or output field is not geometric field?

Amplitude of the image [V/m]

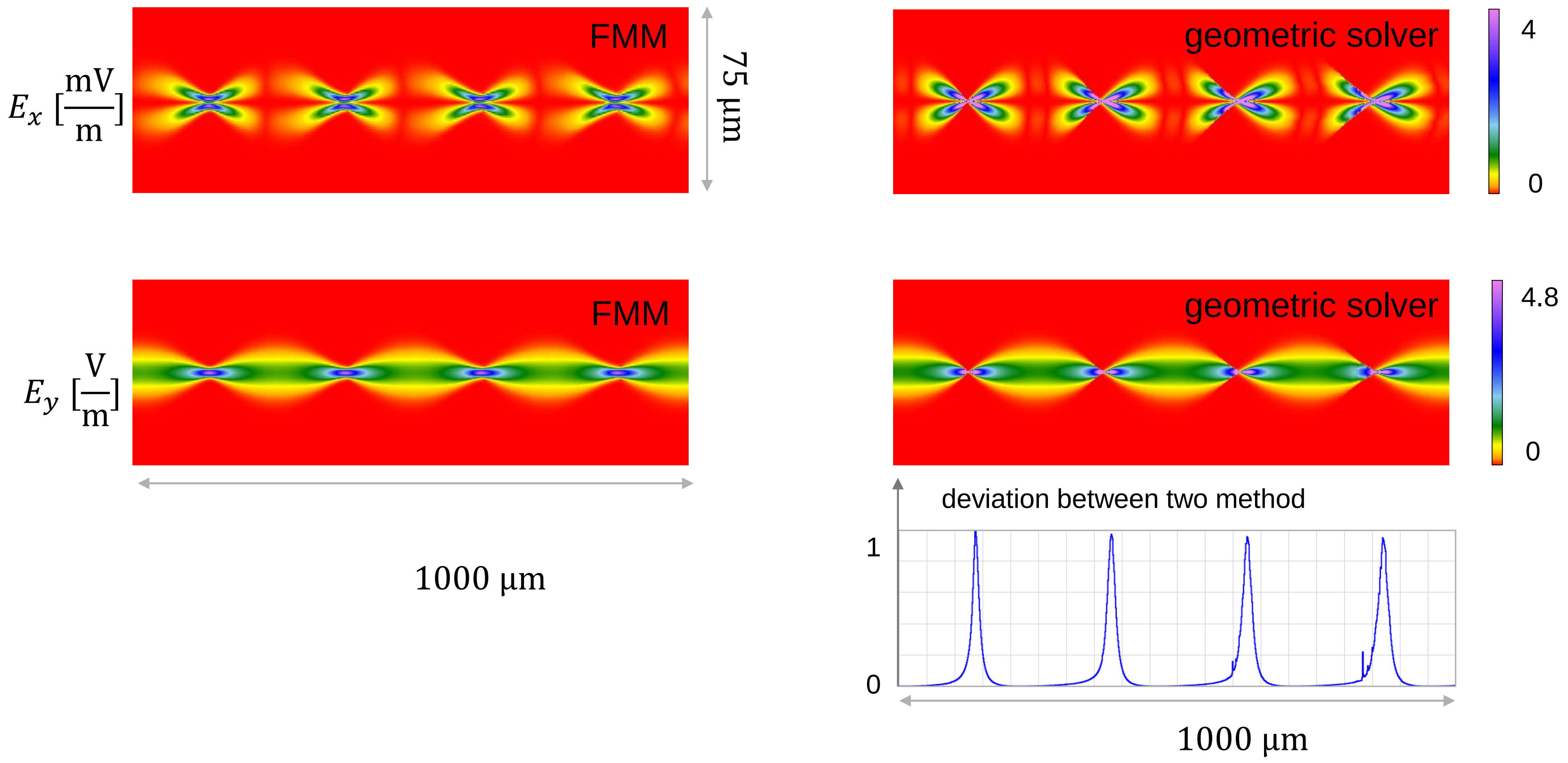
More Exploration of the Fast Solver





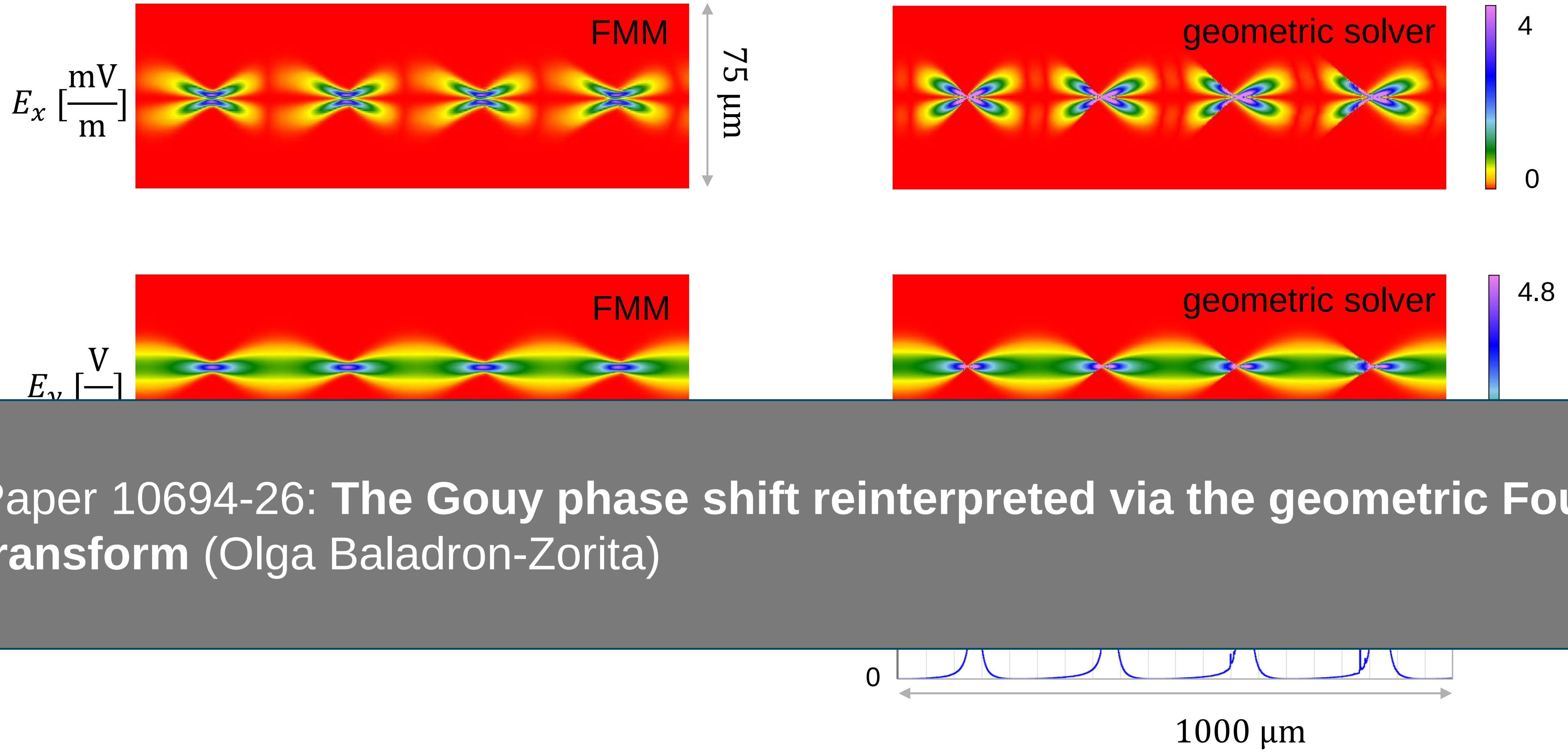


Result



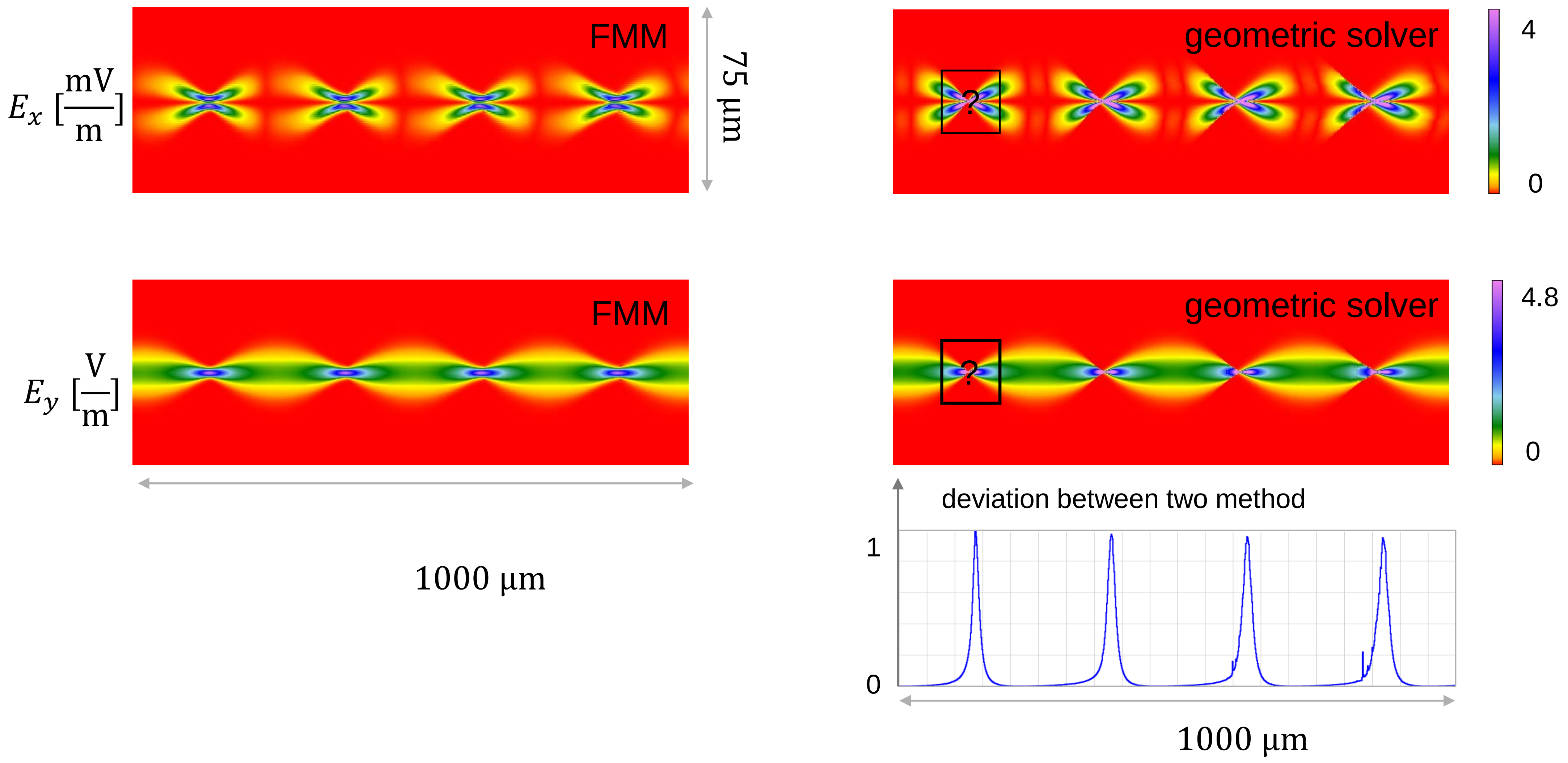
FMM
75 µm
$E_x\ [\frac{mV}{m}]$
geometric solver
4
0
FMM
$E_y\ [\frac{V}{m}]$
geometric solver
4.8
0
1000 µm
deviation between two method
1
0
1000 µm

Result



Paper 10694-26: **The Gouy phase shift reinterpreted via the geometric Fourier transform** (Olga Baladron-Zorita)

Result



FMM
75 µm
geometric solver
4
0
$E_x\ [\frac{mV}{m}]$
FMM
geometric solver
4.8
0
$E_y\ [\frac{V}{m}]$
?
?
1000 µm
deviation between two method
1
0
1000 µm

Concept of B-Operator

- B-operator

$$\boldsymbol{E}^{\mathrm{out}}(\boldsymbol{\kappa}) = \frac{1}{2\pi}\iint \widetilde{\boldsymbol{B}}(\boldsymbol{\kappa},\boldsymbol{\kappa}')\,\boldsymbol{E}^{\mathrm{in}}(\boldsymbol{\kappa}')d\boldsymbol{\kappa}'$$

$$\begin{pmatrix} E_x^{\mathrm{out}}(\boldsymbol{\kappa}) \\ E_y^{\mathrm{out}}(\boldsymbol{\kappa}) \end{pmatrix} = \frac{1}{2\pi} \begin{bmatrix} \tilde{B}_{xx}(\boldsymbol{k},\boldsymbol{k}') & \tilde{B}_{yx}(\boldsymbol{k},\boldsymbol{k}') \\ \tilde{B}_{xy}(\boldsymbol{k},\boldsymbol{k}') & \tilde{B}_{yy}(\boldsymbol{k},\boldsymbol{k}') \end{bmatrix} \begin{pmatrix} E_x^{\mathrm{in}}(\boldsymbol{\kappa}') \\ E_y^{\mathrm{in}}(\boldsymbol{\kappa}') \end{pmatrix}$$

Concept of B-Operator

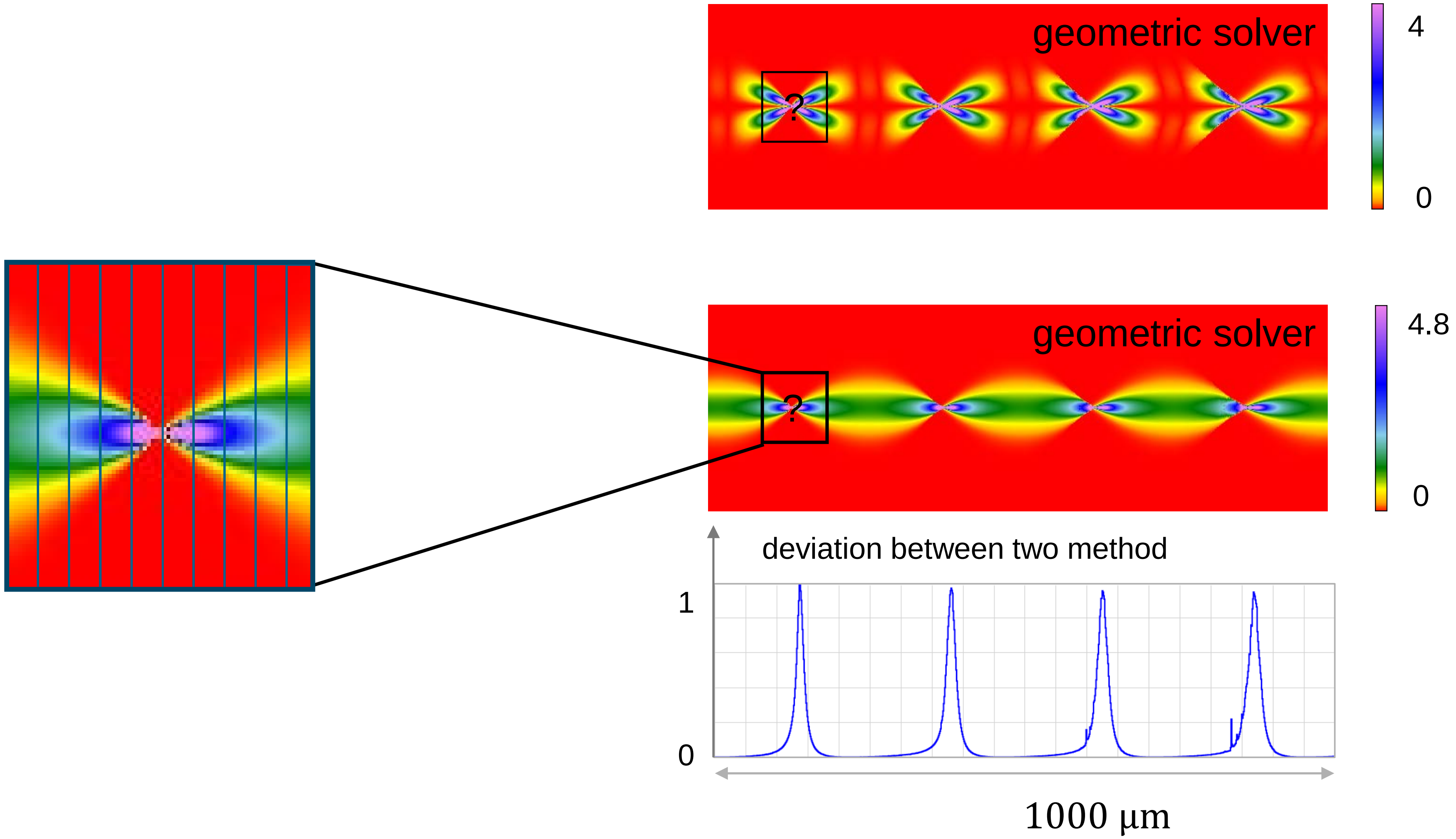
- B-operator

$$\boldsymbol{E}^{\mathrm{out}}(\boldsymbol{\kappa}) = \frac{1}{2\pi}\iint \widetilde{\boldsymbol{B}}(\boldsymbol{\kappa},\boldsymbol{\kappa}')\,\boldsymbol{E}^{\mathrm{in}}(\boldsymbol{\kappa}')d\boldsymbol{\kappa}'$$

$$E_x^{\mathrm{out}}(\boldsymbol{\kappa}) \quad 1 \quad \tilde{B}_{xx}(\boldsymbol{k},\boldsymbol{k}') \quad \tilde{B}_{yx}(\boldsymbol{k},\boldsymbol{k}') \quad E_x^{\mathrm{in}}(\boldsymbol{\kappa}')$$

How to calculate B-matrix?

B-Operator of Slab



B-Operator: Split-Step Method

- Review of split-step method

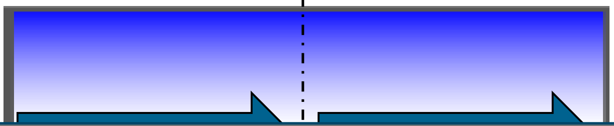
$$E(x,y,z+\Delta z) = \boxed{\exp\left\{\frac{-i\Delta z}{2}\left[\frac{\nabla_\perp^2}{(\nabla_\perp^2+k_0^2)^{1/2}+k_0}\right]\right\}}$$

$$\boxed{\exp(-i\Delta z \chi)}$$

$$\boxed{\exp\left\{\frac{-i\Delta z}{2}\left[\frac{\nabla_\perp^2}{(\nabla_\perp^2+k_0^2)^{1/2}+k_0}\right]\right\}} E(x,y,z)$$

$$\chi(x,y) = k_0\left[\frac{n(x,y)}{n_0} - 1\right]$$

Thin element approximation



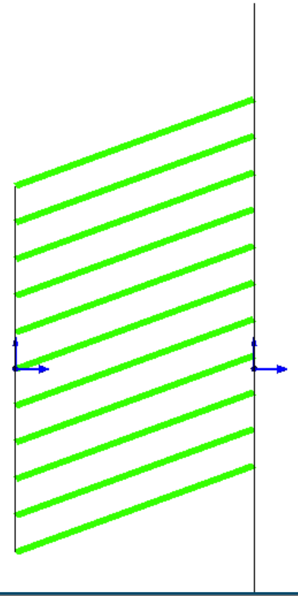
so the slab should be quite thin. Linear operation in spatial domain.

[8] M. D. Feit and J. A. Fleck, “Light propagation in graded-index optical fibers,” *Appl. Opt.* **17**,3990–3998 (1978).

B-Operator: Wave Propagation Method

- Review of wave propagation method

$$\tilde{\boldsymbol{E}}^{\text{out}}(\boldsymbol{\kappa}) = \frac{1}{2\pi}\iint \tilde{\boldsymbol{E}}^{\text{in}}(\boldsymbol{\kappa}')e^{ik_z(\boldsymbol{\kappa}')\Delta z}\delta(\boldsymbol{\kappa}' - \boldsymbol{\kappa})\, d\boldsymbol{\kappa}'$$



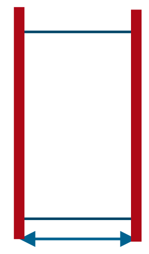
- Elementary B-operator is Parabasal TEA

Neglect the curvature of each ray, so the slab should be quite thin!

[9] Brenner, K.-H. & Singer, W. Light propagation through microlenses: a new simulation method *Appl. Opt., OSA,* **1993**, *32*, 4984-4988

B-operator

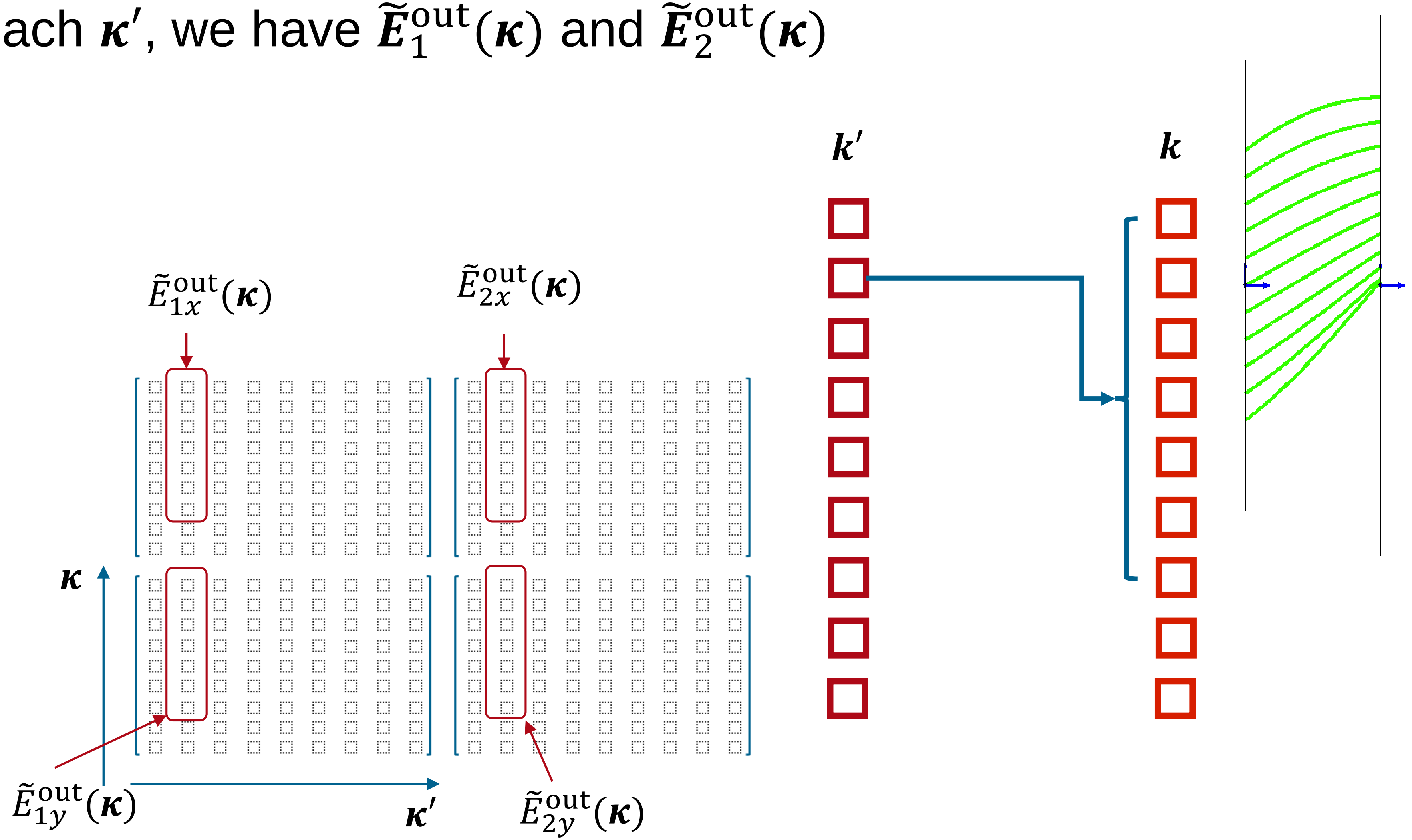
- Input plane, define a grid of $\boldsymbol{\kappa}'$
- For each $\boldsymbol{\kappa}'$
  - Define input plane waves
    - $\boldsymbol{E}_1^{\mathrm{in}}(\boldsymbol{\rho}') = \begin{pmatrix} 1 \\ 0 \\ z_1 \end{pmatrix} \mathrm{e}^{i\boldsymbol{\kappa}'\cdot\boldsymbol{\rho}'}$
    - $\boldsymbol{E}_2^{\mathrm{in}}(\boldsymbol{\rho}') = \begin{pmatrix} 0 \\ 1 \\ z_2 \end{pmatrix} \mathrm{e}^{i\boldsymbol{\kappa}'\cdot\boldsymbol{\rho}'}$
  - Calculate the output field $\boldsymbol{E}_1^{\mathrm{out}}(\boldsymbol{\rho})$and $\boldsymbol{E}_2^{\mathrm{out}}(\boldsymbol{\rho})$
  - Calculate the spectrum $\widetilde{\boldsymbol{E}}_1^{\mathrm{out}}(\boldsymbol{\kappa})$ and $\widetilde{\boldsymbol{E}}_2^{\mathrm{out}}(\boldsymbol{\kappa})$



with $\boldsymbol{\rho}' = (x', y')$

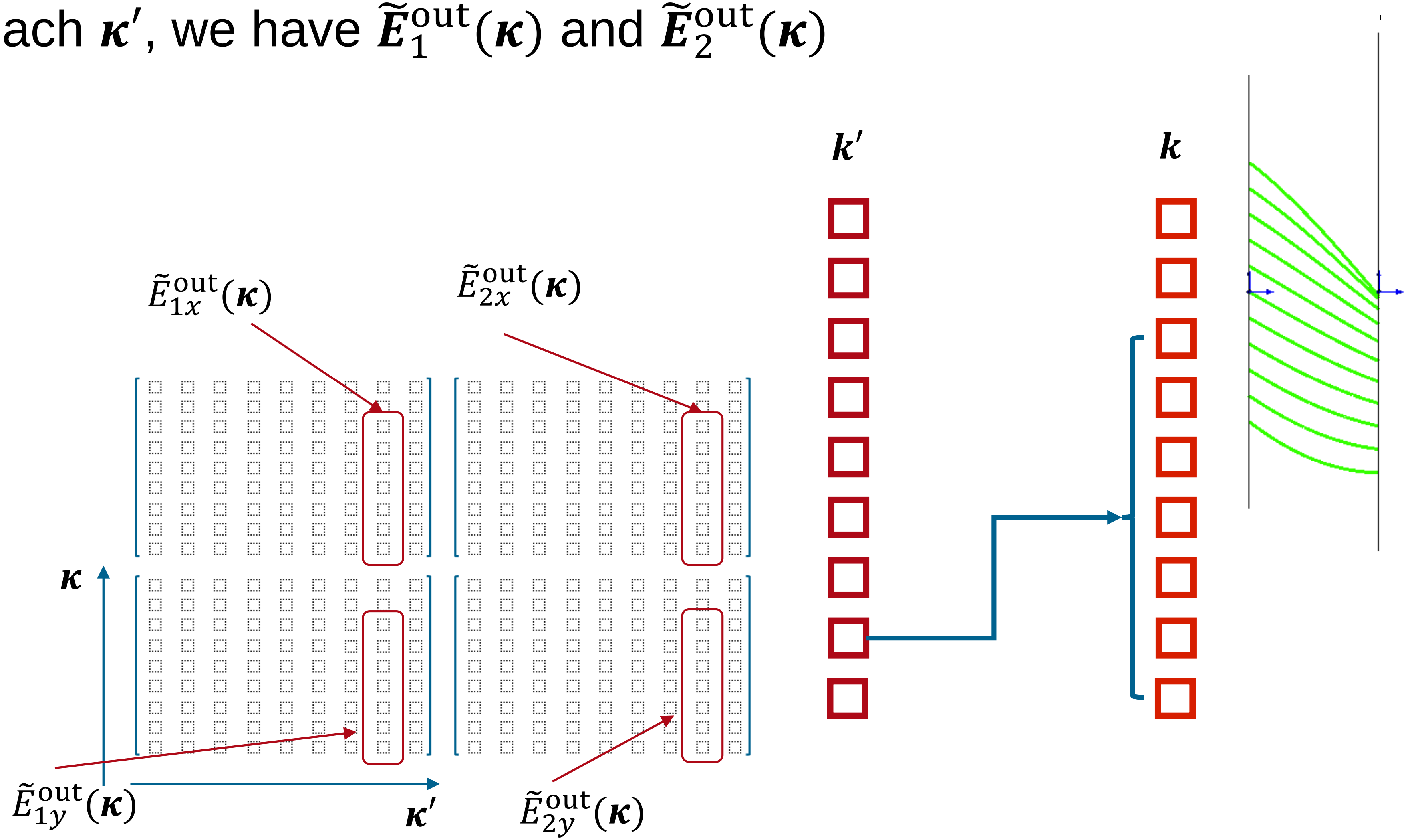
B-Operator

- For each $\boldsymbol{\kappa}'$, we have $\widetilde{\boldsymbol{E}}_1^{\text{out}}(\boldsymbol{\kappa})$ and $\widetilde{\boldsymbol{E}}_2^{\text{out}}(\boldsymbol{\kappa})$

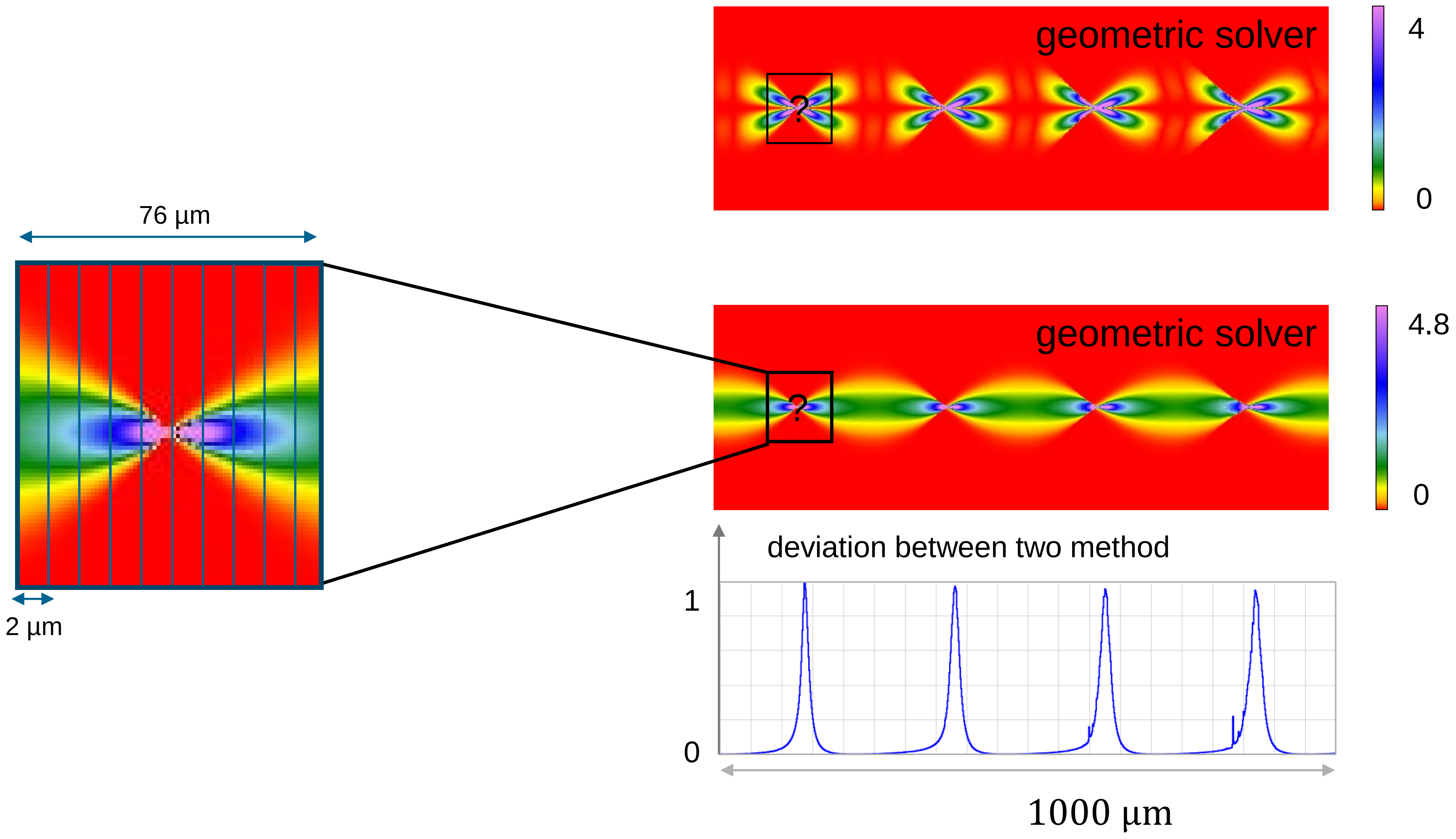


B-Operator

- For each $\boldsymbol{\kappa}'$, we have $\widetilde{\boldsymbol{E}}_1^{\text{out}}(\boldsymbol{\kappa})$ and $\widetilde{\boldsymbol{E}}_2^{\text{out}}(\boldsymbol{\kappa})$

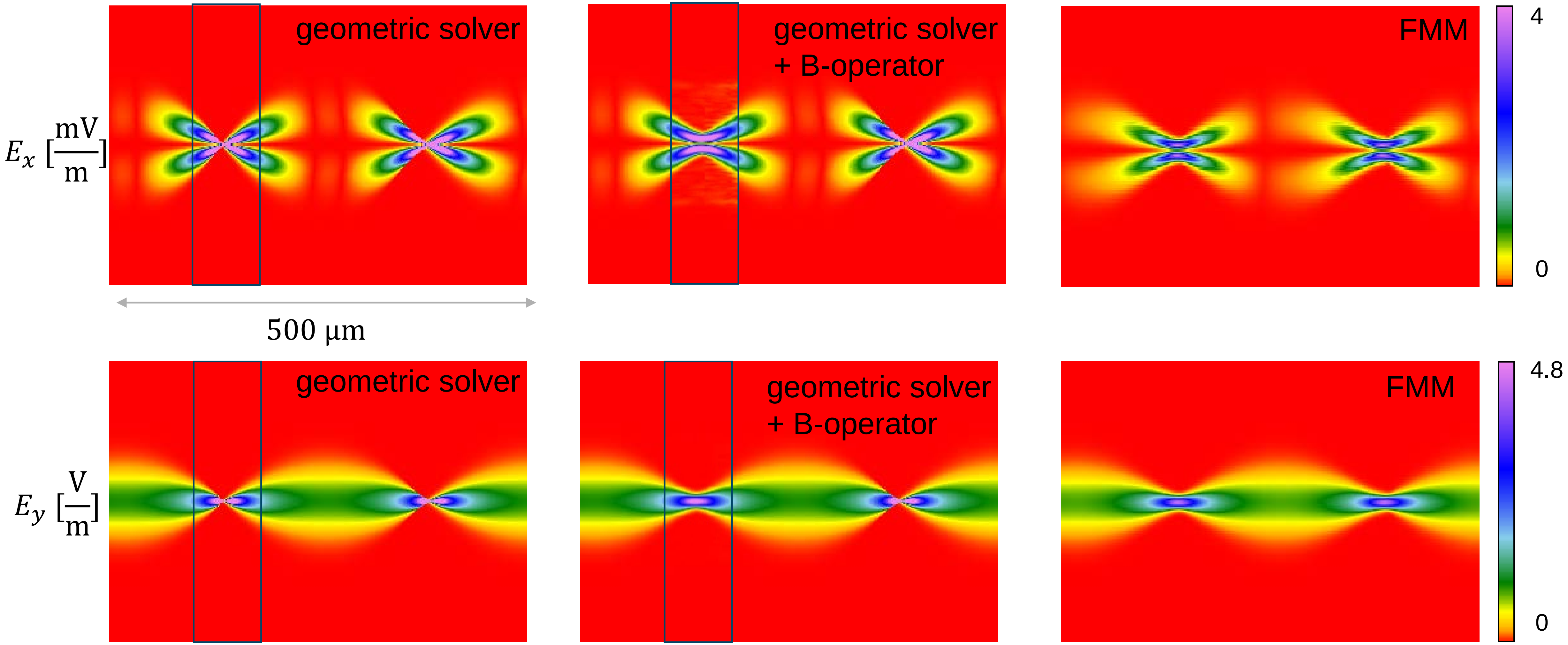


Result

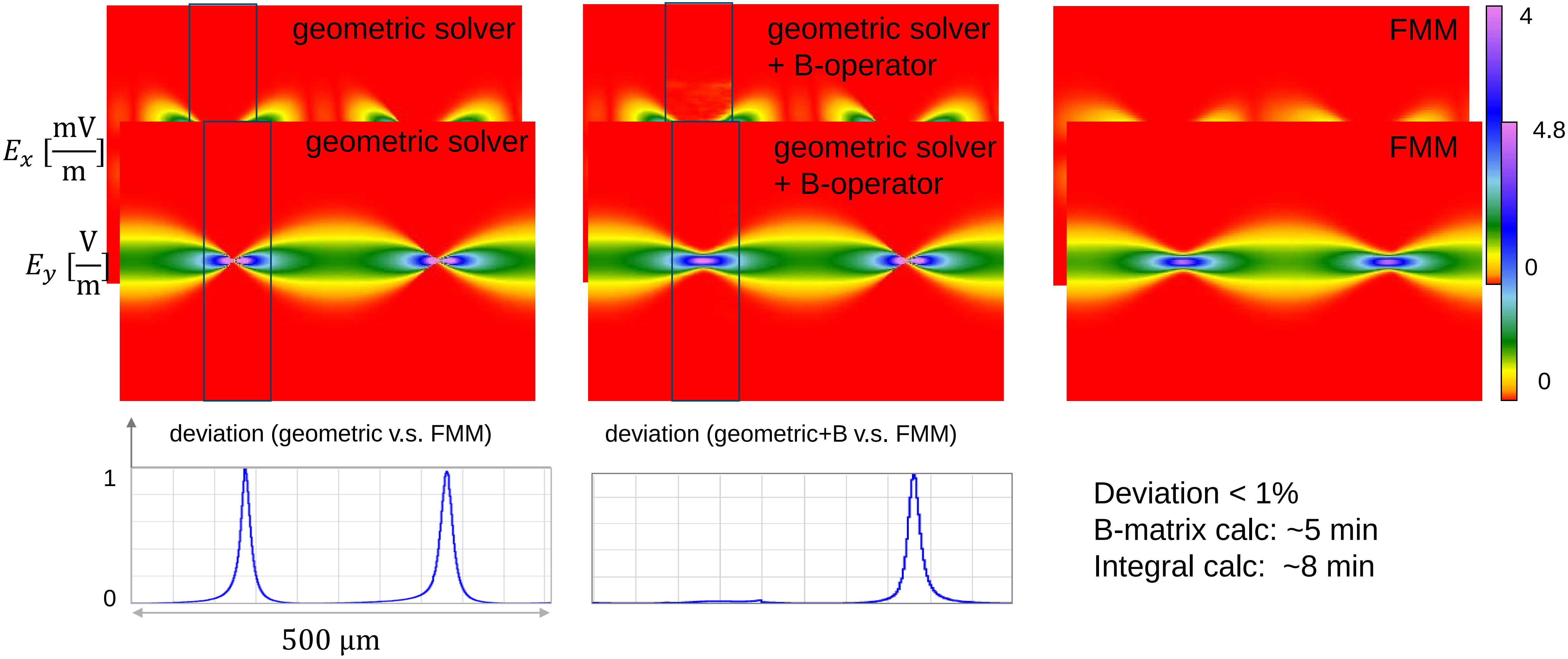


76 µm
2 µm
geometric solver
4
0
geometric solver
4.8
0
?
?
deviation between two method
1
0
1000 µm

Diffractive Field: Fiber

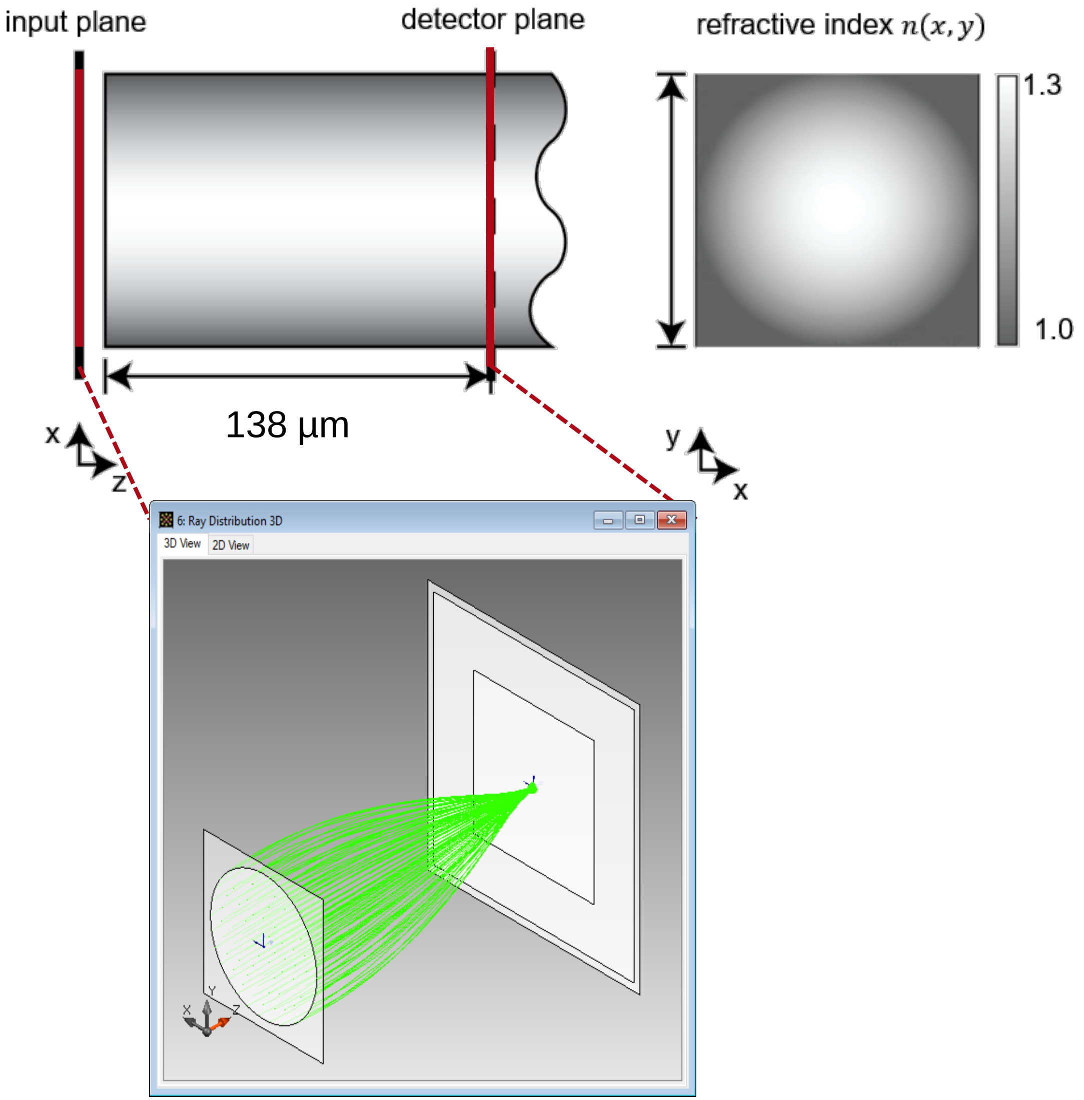


Diffractive Field: Fiber

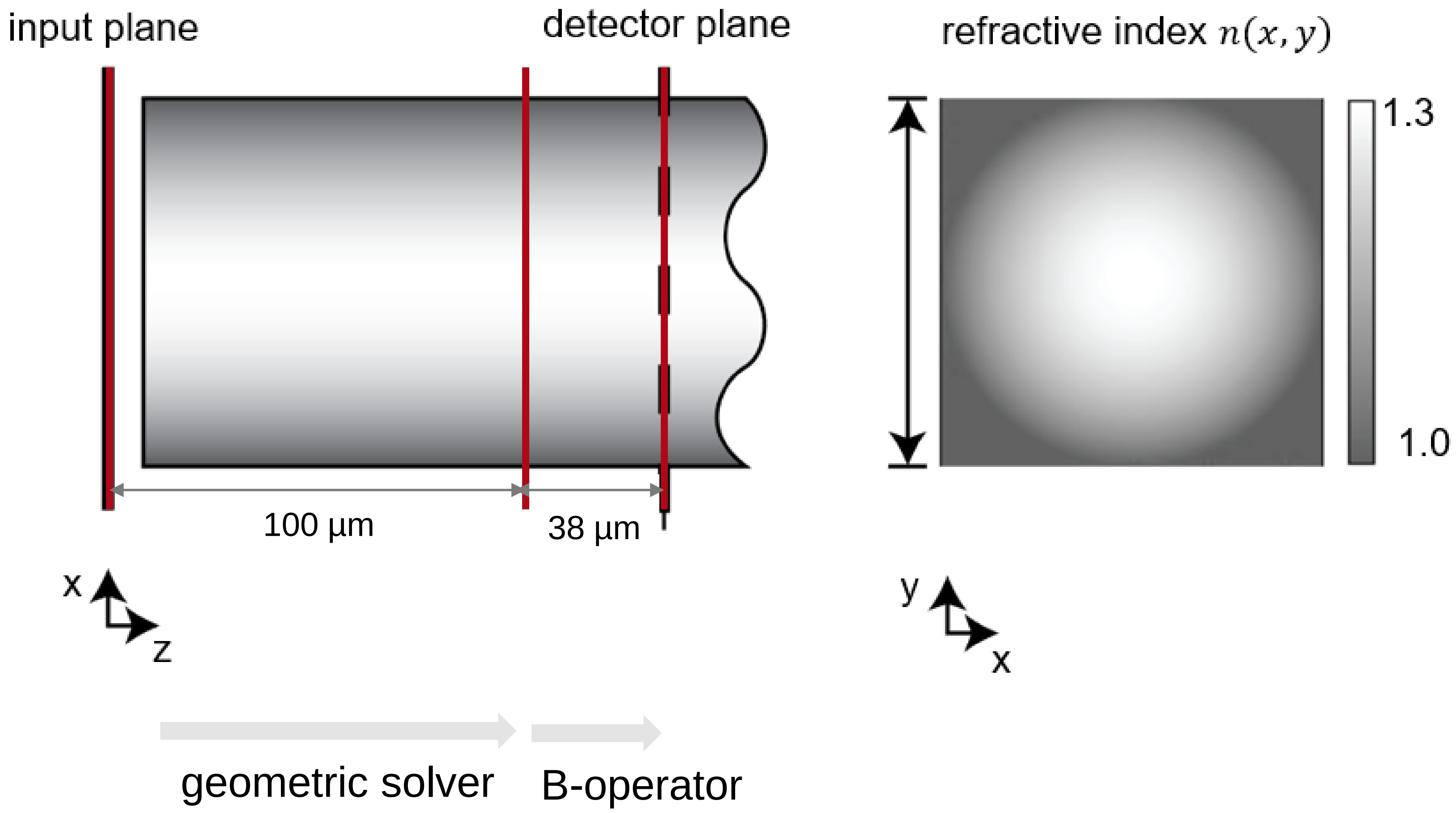


Deviation < 1%
B-matrix calc: ~5 min
Integral calc: ~8 min

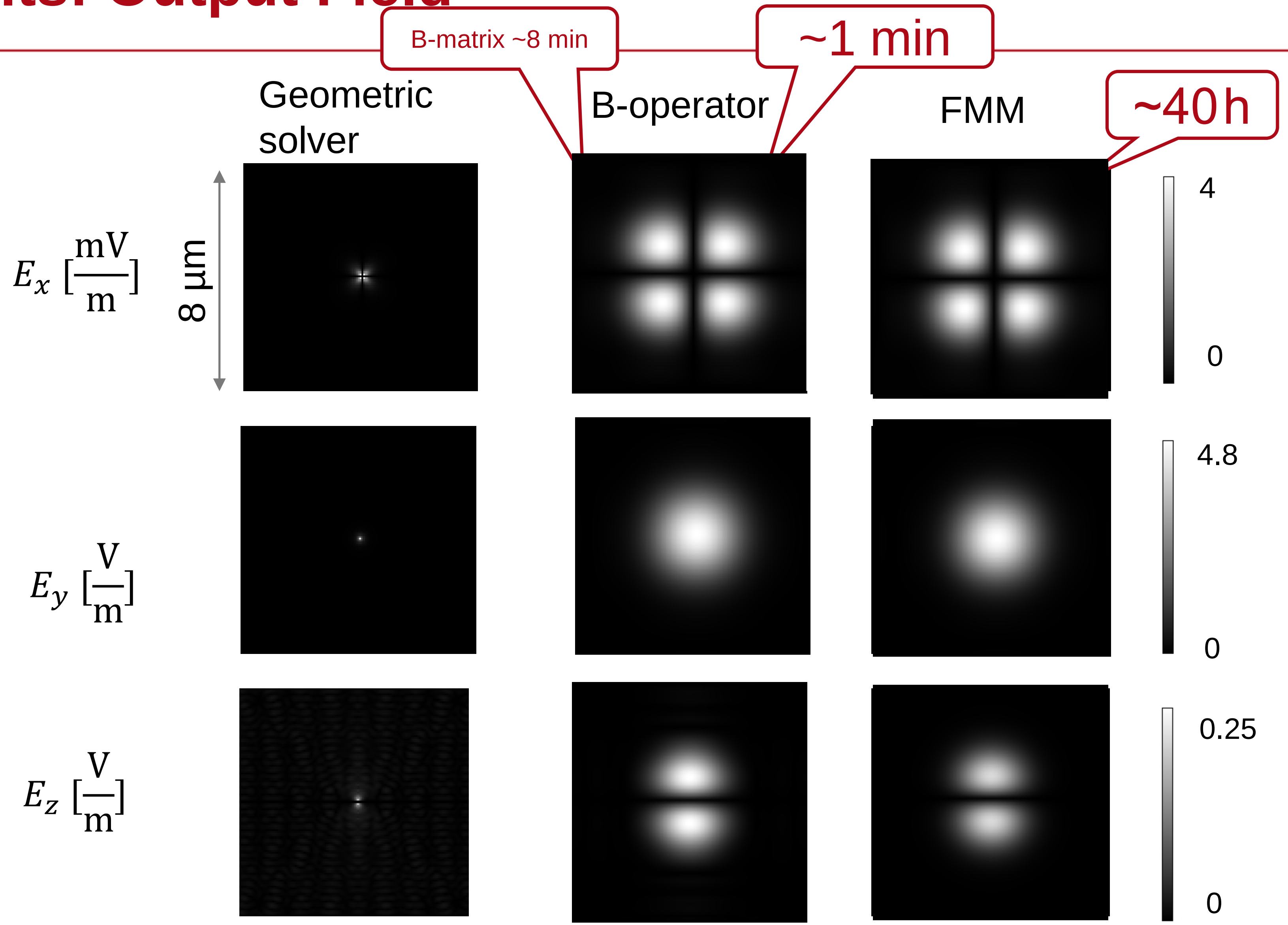
Calculation of Focus in Multimode Fiber



Problem Statement



Results: Output Field



Deviation ~ 1%

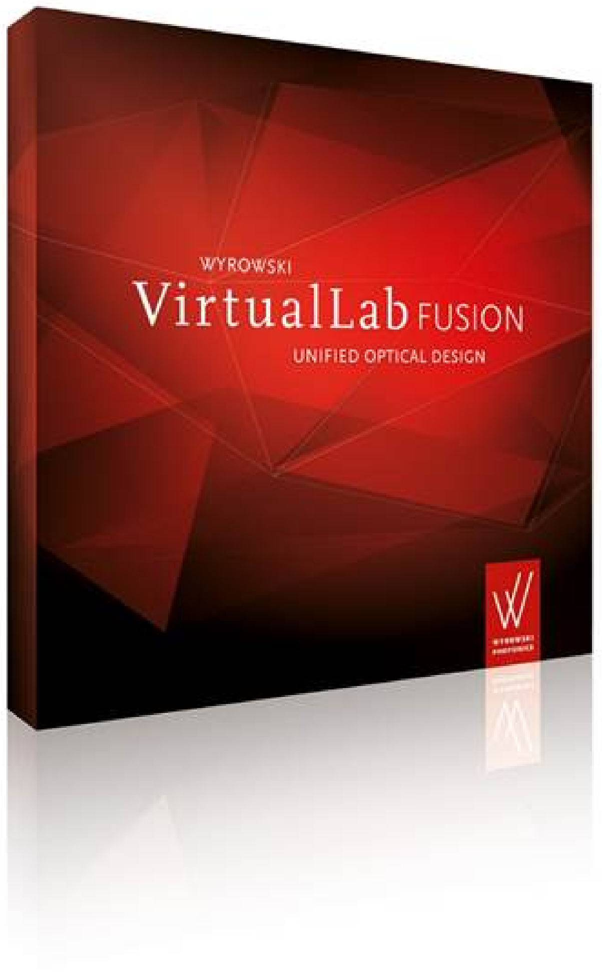
Conclusion and Outlook

- Field propagation through GRIN media
  - Geometric solver
  - B-operator of GRIN media
    - Elementary B: geometric solver
    - What is the elementary B-operator of split-step method and wave propagation method
- Calculation of B-operator need most time.
  - Just calculate $\widetilde{\boldsymbol{B}}(\boldsymbol{\kappa}, \boldsymbol{\kappa}')$ for a few $\boldsymbol{\kappa}'_{\mathbf{0}}$
    - $\widetilde{\boldsymbol{B}}(\boldsymbol{\kappa}, \boldsymbol{\kappa}') =: \widetilde{\boldsymbol{B}}(\boldsymbol{\kappa}, \boldsymbol{\kappa}'_{\mathbf{0}})$ for $\left|\left|\boldsymbol{\kappa}' - \boldsymbol{\kappa}'_{\mathbf{0}}\right|\right| < \boldsymbol{\delta}$
    - $\widetilde{\boldsymbol{B}}(\boldsymbol{\kappa}, \boldsymbol{\kappa}') =: \widetilde{\boldsymbol{B}}(\boldsymbol{\kappa}, \boldsymbol{\kappa}'_{\mathbf{0}} + \boldsymbol{\delta\kappa}')$ with$\boldsymbol{\delta\kappa}' = \boldsymbol{\kappa}' - \boldsymbol{\kappa}'_{\mathbf{0}}$

Thank you!

Implementation

- All algorithms are implemented in the physical optics simulation and design software **VirtualLab Fusion**
- VirtualLab Fusion is developed, following the field tracing concept, by Wyrowski Photonics UG, Jena, Germany
- Visit our booth @**booth B63** for more infomation



Reference

[1] www.thorlab.de

[2] https://lp.uni-goettingen.de

[3] www.bgr.com

[4] slideplayer.com/slide/8658764/

[5] A. Sharma *et al.* Tracing rays through graded-index media: a new method. *Appl. Opt.,* **21**(6): 984-987 (1982)

[6] M. Born and E. Wolf, Principles of Optics, Cambridge University press (1999)

[7] Huiying Zhong et al, Fast propagation of electromagnetic fields through graded-index media, J. Opt. Soc. Am. A 35, 661-668 (2018)

[8] M. D. Feit and J. A. Fleck, “Light propagation in graded-index optical fibers,” *Appl. Opt.* **17**,3990–3998 (1978).

[9] Brenner, K.-H. & Singer, W. Light propagation through microlenses: a new simulation method *Appl. Opt., OSA,* **1993**, *32*, 4984-4988