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The concept of bidirectional operators and its application to the modeling of microstructures

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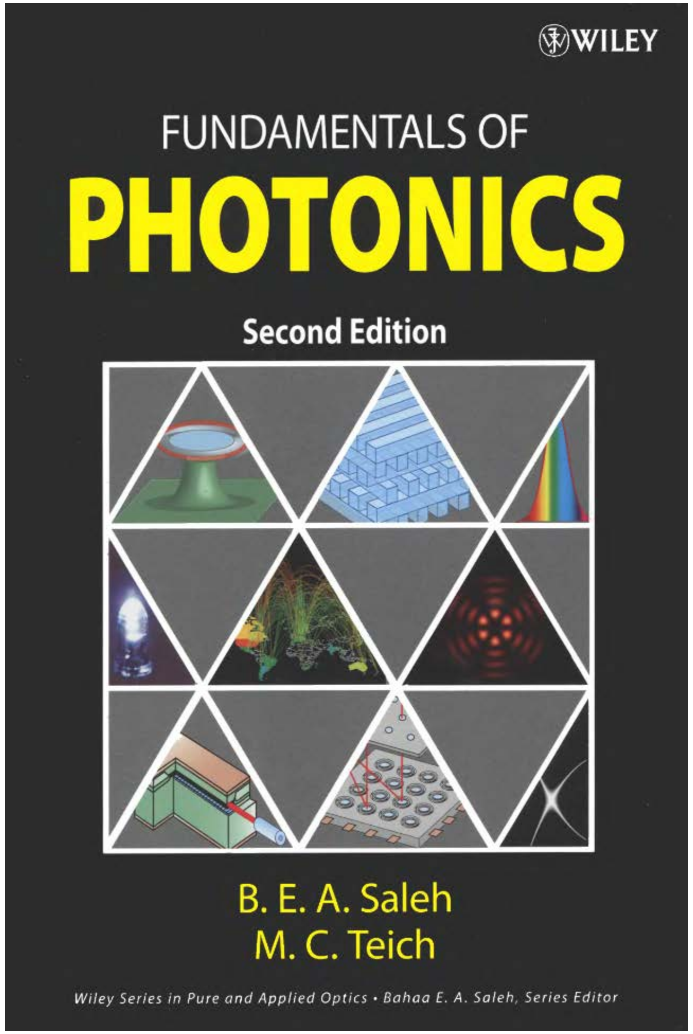
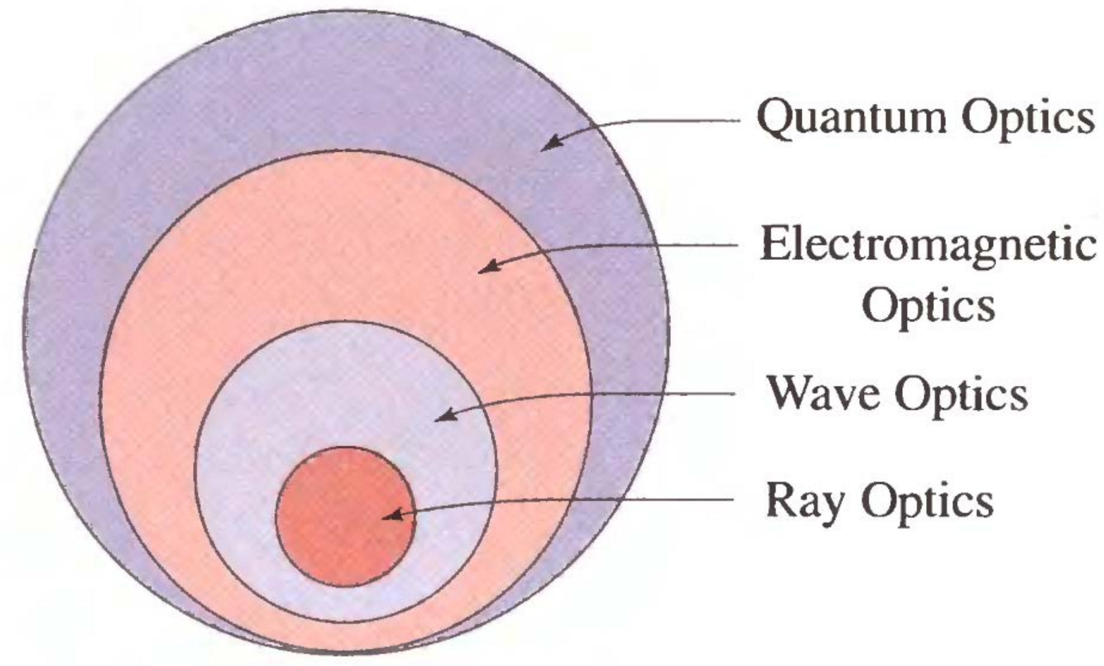
Geometric and diffractive branch of physical optics

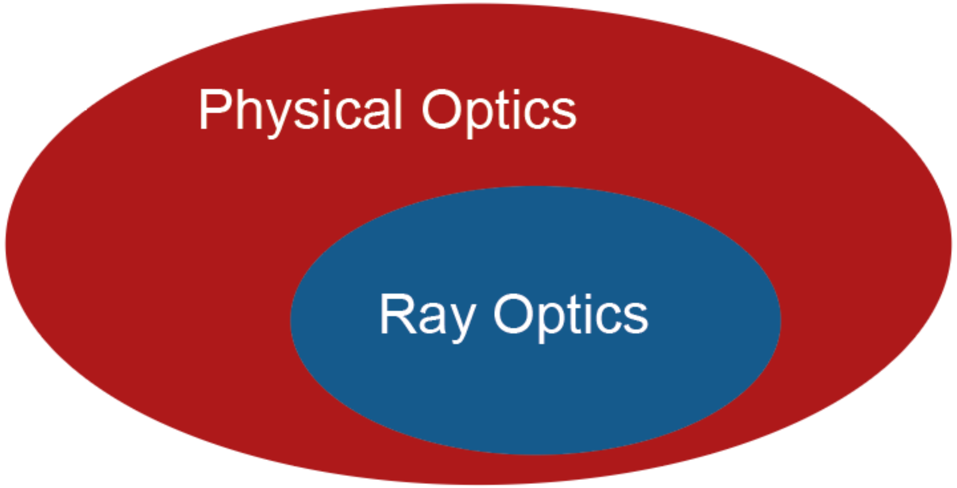
The geometric Fourier transform

Physical and Geometrical Optics: Traditional Understanding





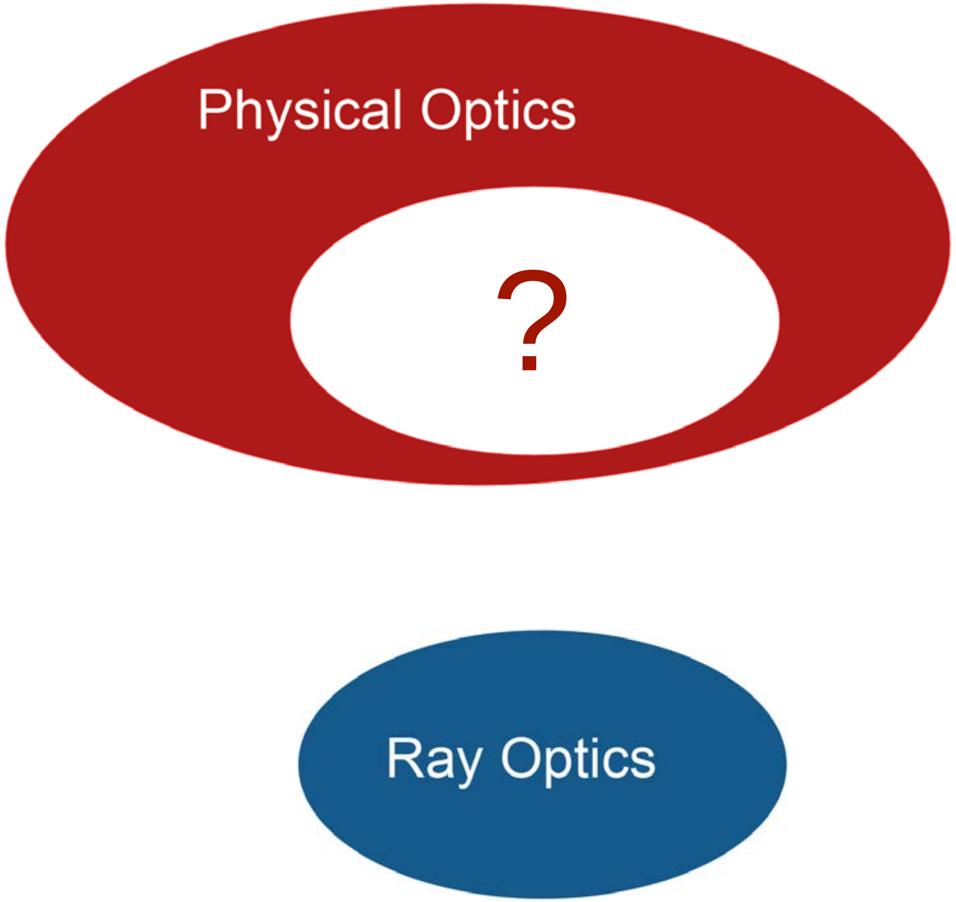
Physical and Geometrical Optics: Traditional Understanding



- Physical optics:
  - Light represented by electromagnetic fields which
  - are governed by Maxwell’s equations.

- Geometrical/ray optics:
  - Light is represented by mathematical rays (with energy flux) which
  - are governed by Fremat’s principle which is mathematically expressed by ray equation.

Physical and Geometrical Optics: Traditional Understanding



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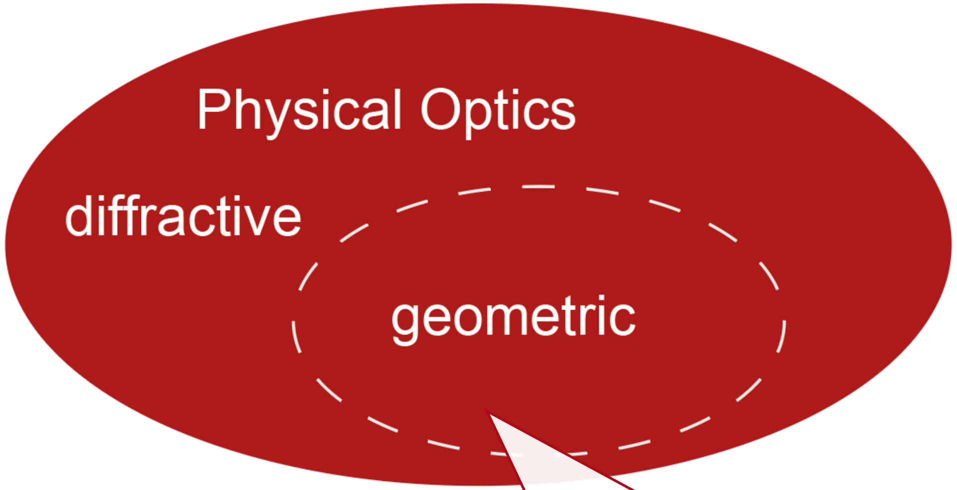
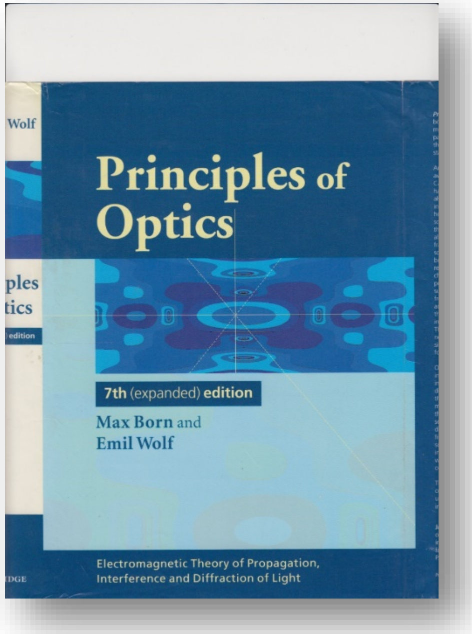
Physical and Geometrical Optics: Unified Theory

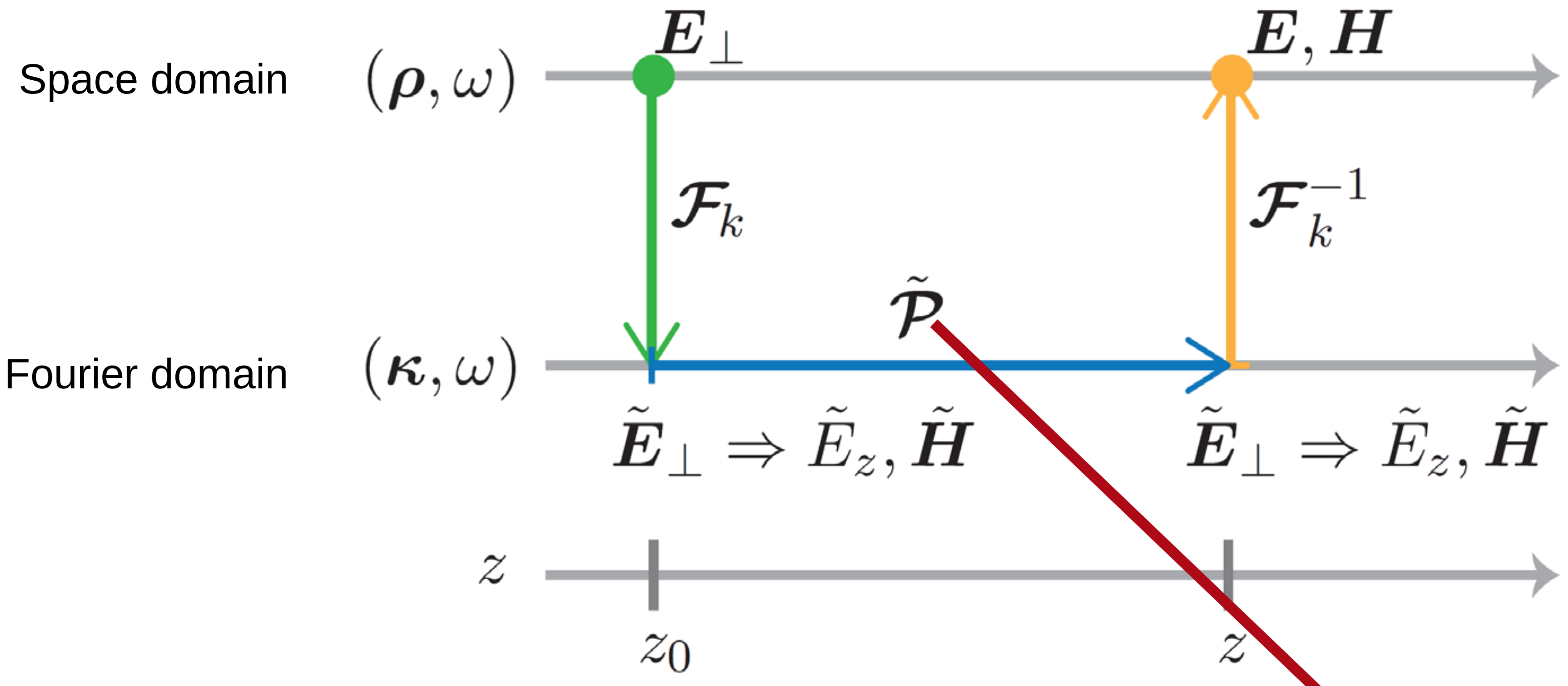


Physical-optics generalization of geometrical optics!?

- Physical optics:
  - Light represented by electromagnetic fields which
  - are governed by Maxwell’s equations.



Field Tracing Diagram: Free-Space Propagation



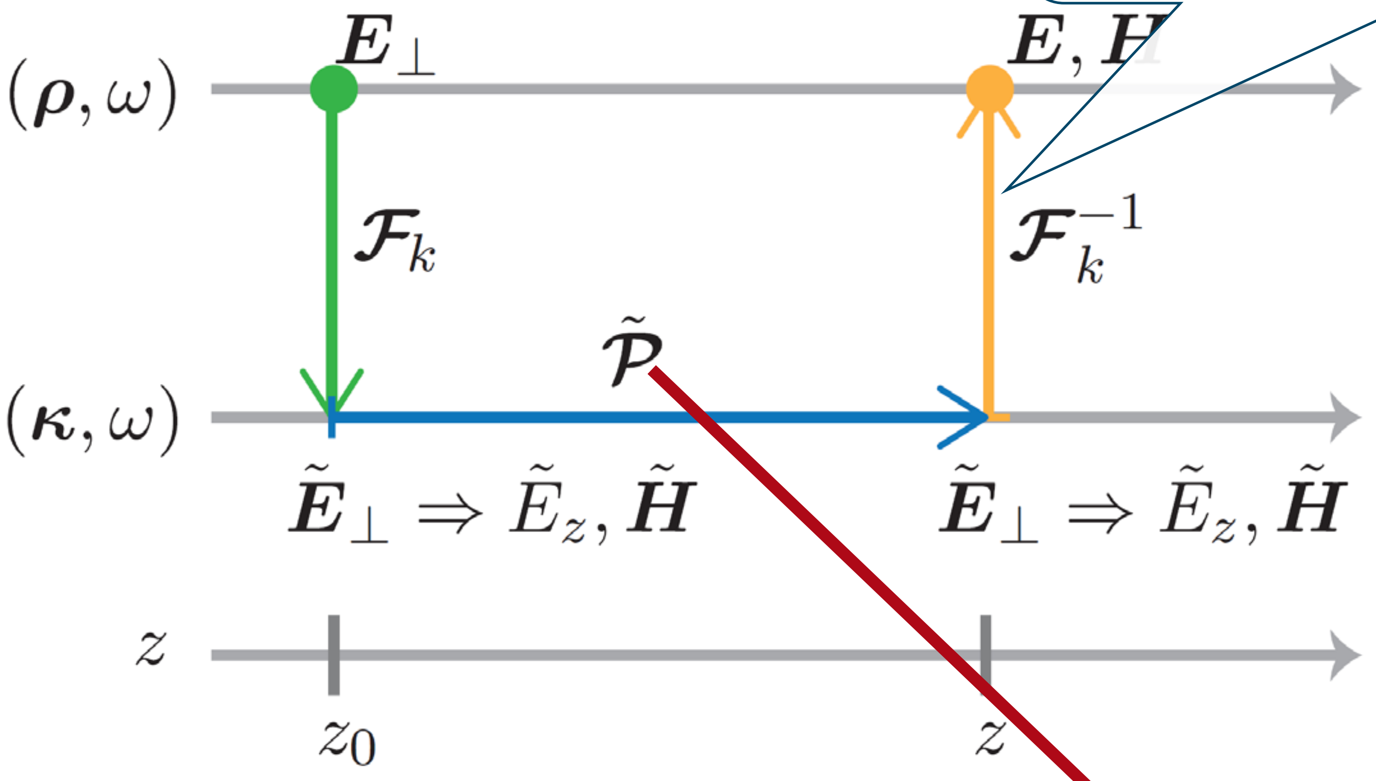
$$\boxed{V_\ell(\boldsymbol{\rho}, z) = \mathcal{F}_k^{-1}\left(\exp(\mathrm{i}\check{k}_z \Delta z)\,(\mathcal{F}_k V_\ell(\boldsymbol{\rho}, z_0))\right)}$$

Field Tracing Diagram: Free-Space Propagation

Characteristics of connection between both domains of utmost importance!

Space domain

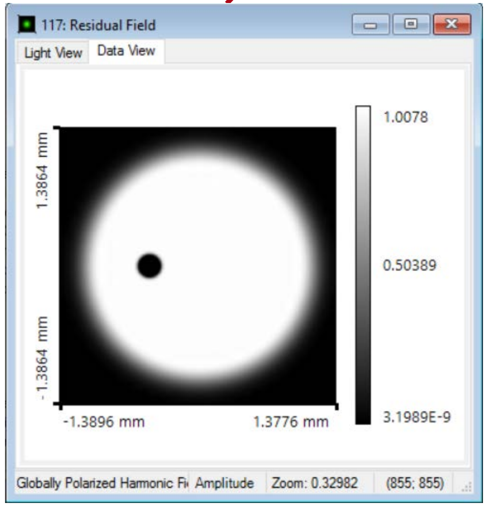
Fourier domain

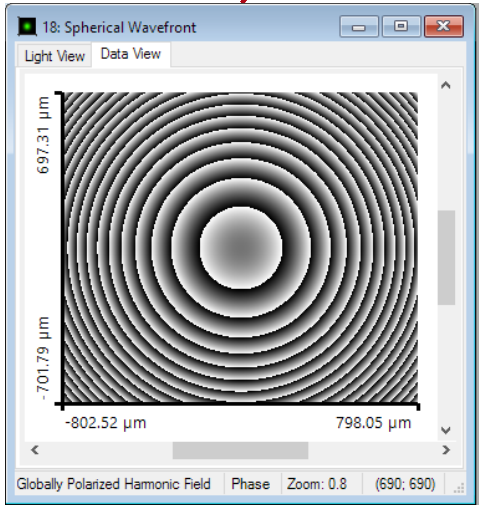


$$\boxed{V_\ell(\boldsymbol{\rho},z)=\mathcal{F}_k^{-1}\left(\exp\left(\mathrm{i}\check{k}_z\Delta z\right)\left(\mathcal{F}_k V_\ell(\boldsymbol{\rho},z_0)\right)\right)}$$

Example Spherical Field with Stop

$$V_\ell(\boldsymbol{\rho},z,\omega) = |V_\ell(\boldsymbol{\rho},z,\omega)| \exp\left(\mathrm{i}\varphi_\ell(\boldsymbol{\rho},z,\omega)\right) \exp\left(\mathrm{i}\psi(\boldsymbol{\rho},z,\omega)\right)$$

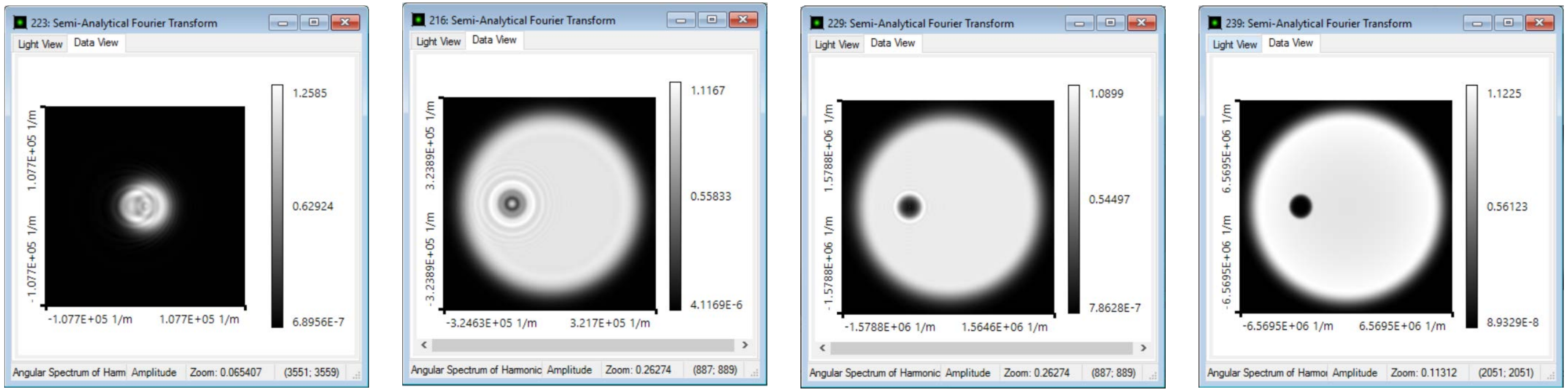




Results of Fourier Transform

$$V_\ell(\boldsymbol{\rho},z,\omega) = |V_\ell(\boldsymbol{\rho},z,\omega)|\exp(\mathrm{i}\varphi_\ell(\boldsymbol{\rho},z,\omega))\boxed{\exp(\mathrm{i}\psi(\boldsymbol{\rho},z,\omega))}$$

Decreasing radius of cruvature;
increasing NA



Results of Fourier Transform

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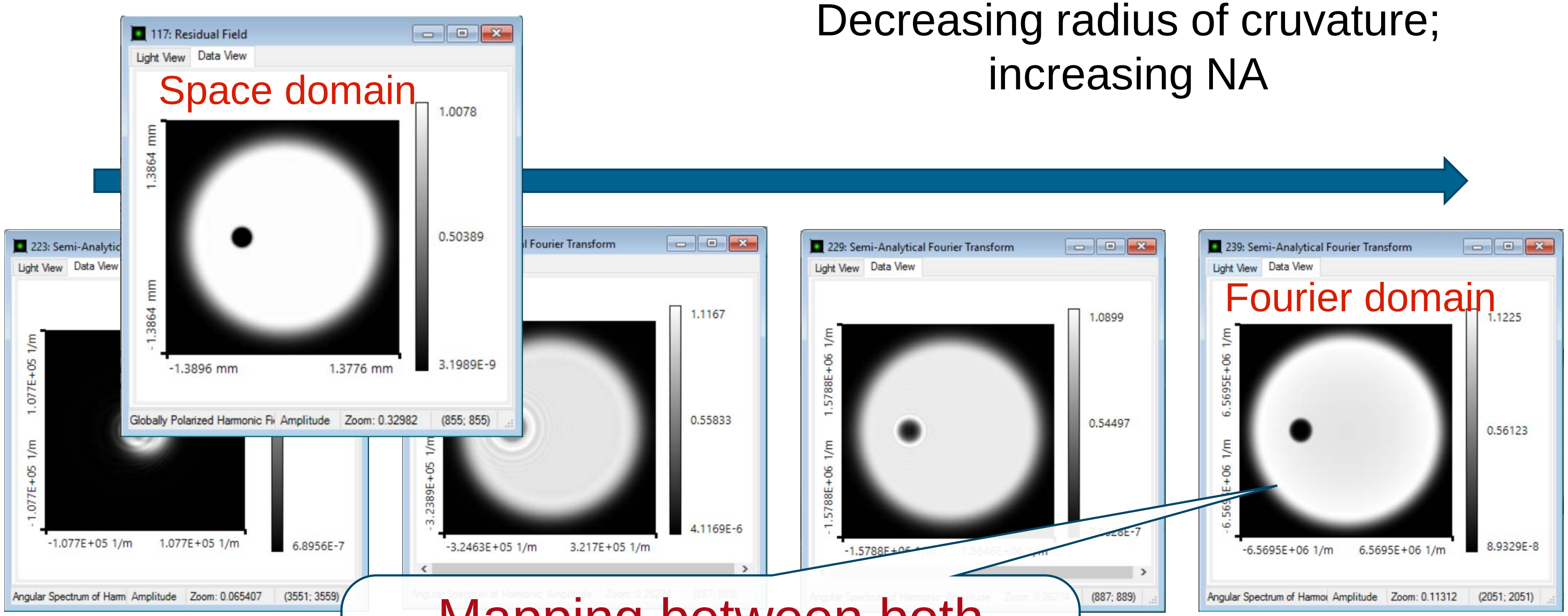
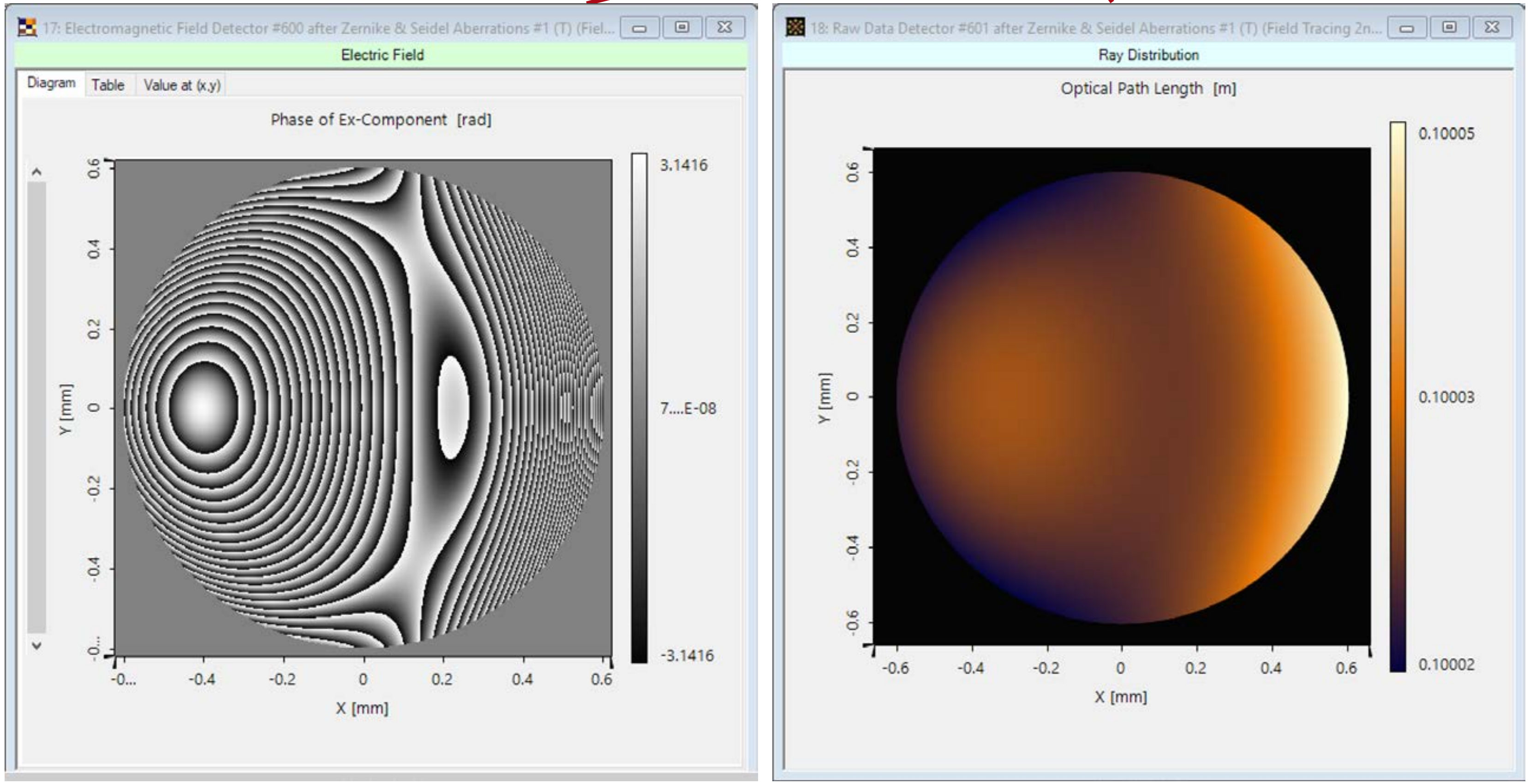


Mapping between both domains: geometric Fourier transform

General Example with Aberrations

$$V_\ell(\boldsymbol{\rho},z,\omega) = |V_\ell(\boldsymbol{\rho},z,\omega)| \exp\left(\mathrm{i}\varphi_\ell(\boldsymbol{\rho},z,\omega)\right) \exp\left(\mathrm{i}\psi(\boldsymbol{\rho},z,\omega)\right)$$



Field Zones by Mathematical Definition

- Geometric Fourier transform valid in specified modeling accuracy

↓

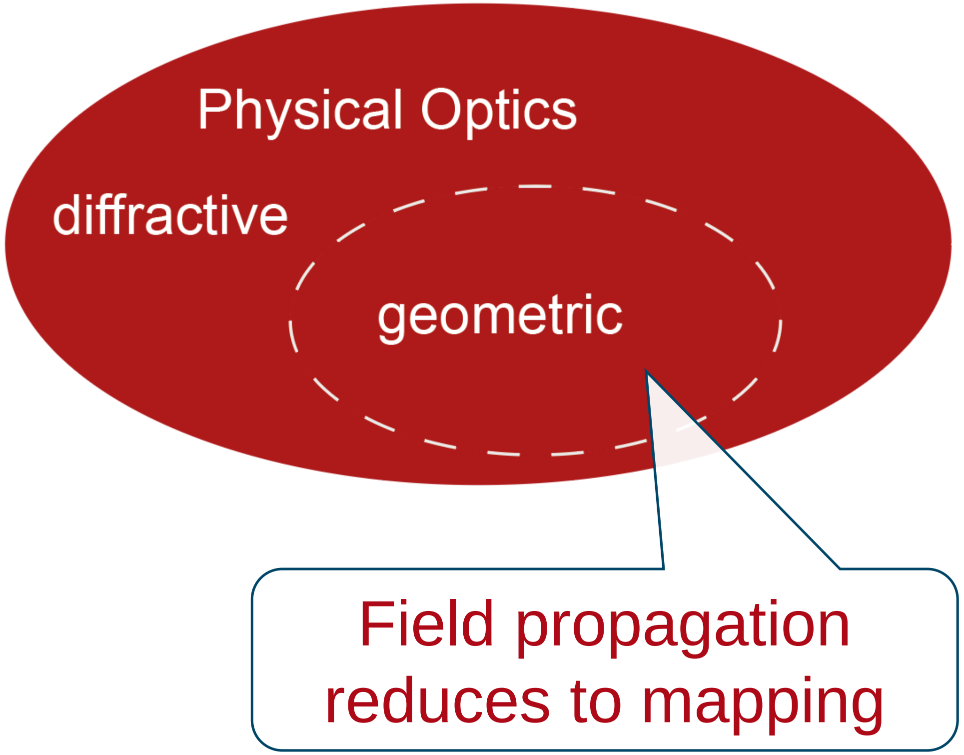
- Field on reference plane is in **geometric field zone**

- Rigorous Fourier transform required in specified modeling accuracy

↓

- Field on reference plane is in **diffractive field zone**

Physical and Geometrical Optics: Unified Theory



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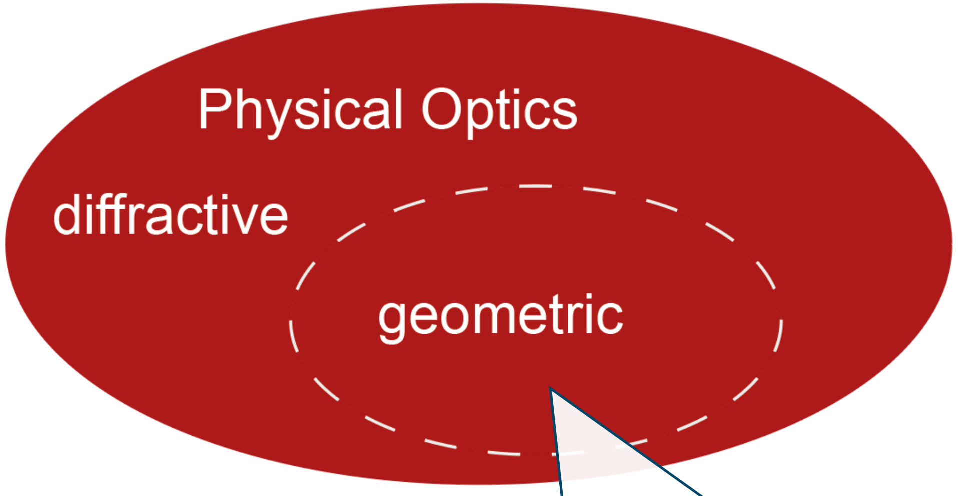
Physical and Geometrical Optics: Unified Theory



Necessary condition: Fields must be in geometric zone wherever need for Fourier transform!

- Physical optics:
  - Light represented by electromagnetic fields which
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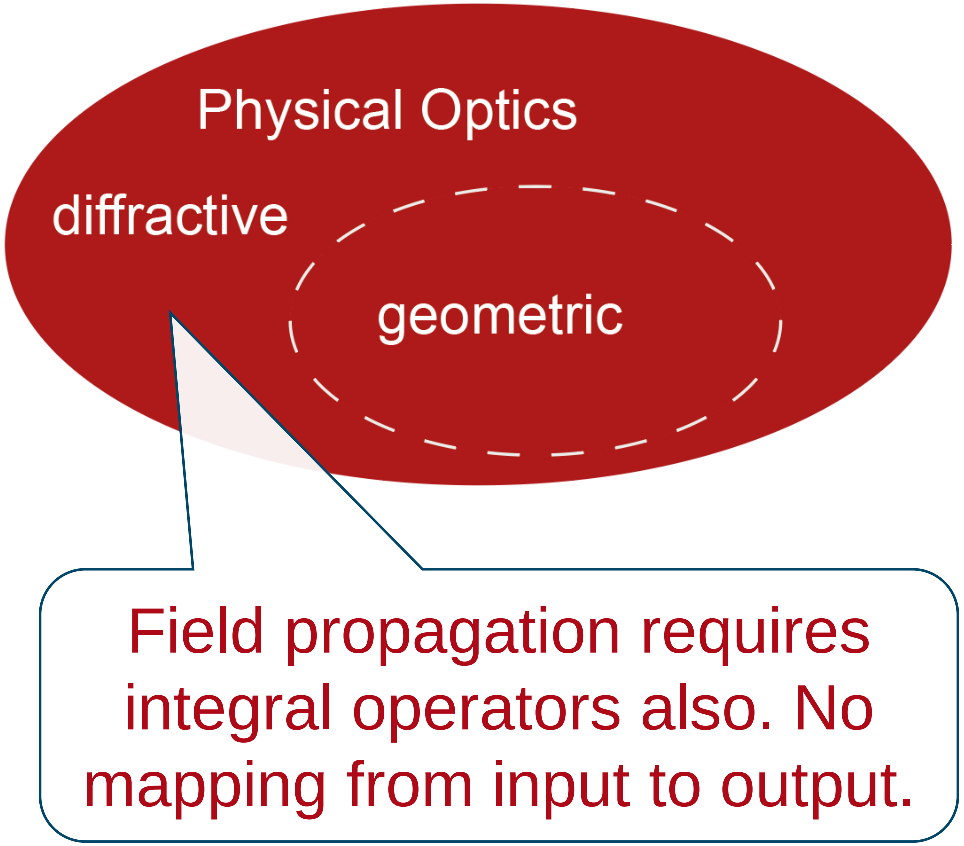
Physical and Geometrical Optics: Unified Theory



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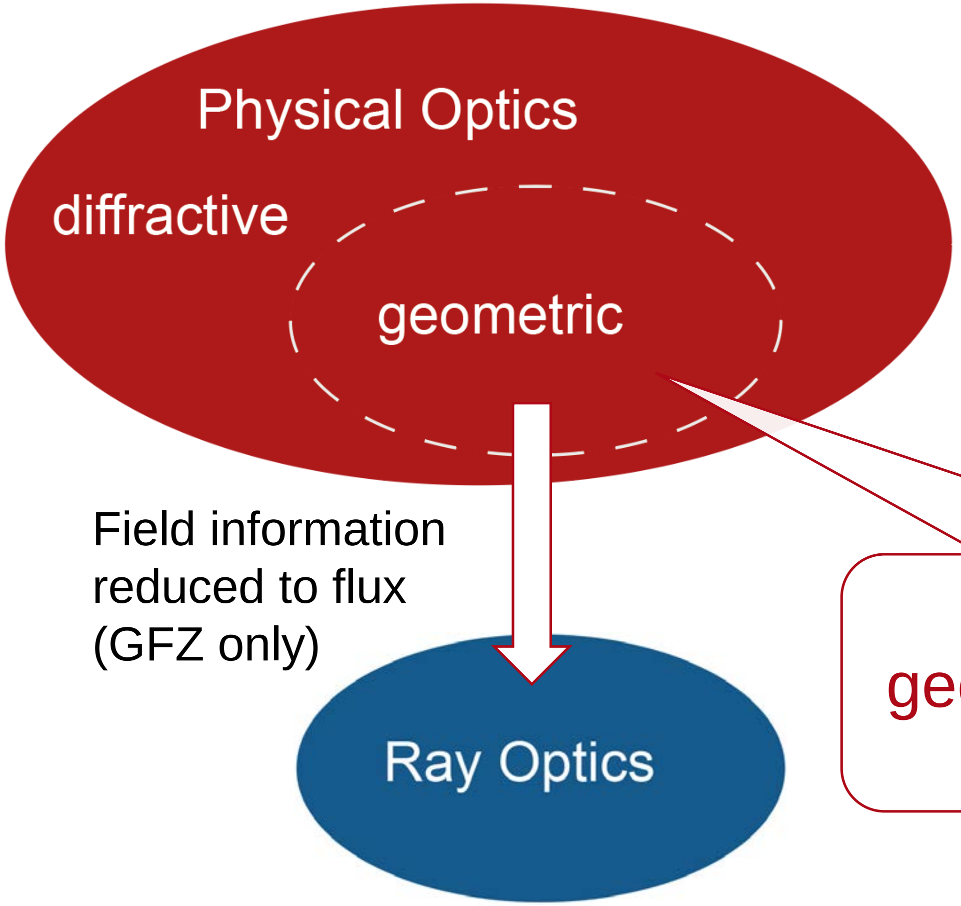
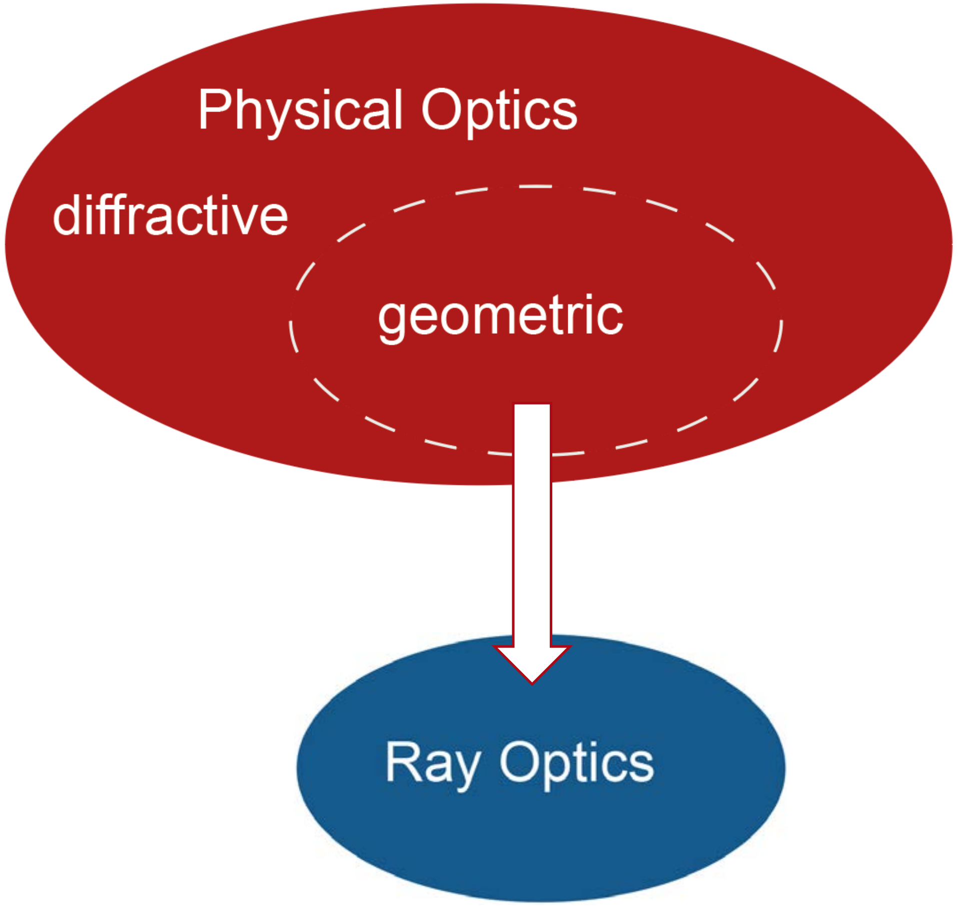
Physical and Geometrical Optics: Unified Theory



- Physical optics:
  - Light represented by electromagnetic fields which
  - are governed by Maxwell’s equations
  - Transition between diffractive/geometric branch fully specified and controlled by mathematical concept of geometric FT.

Physical optics in geometric zones is at least as fast as ray tracing!

Physical and Geometrical Optics: Unified Theory

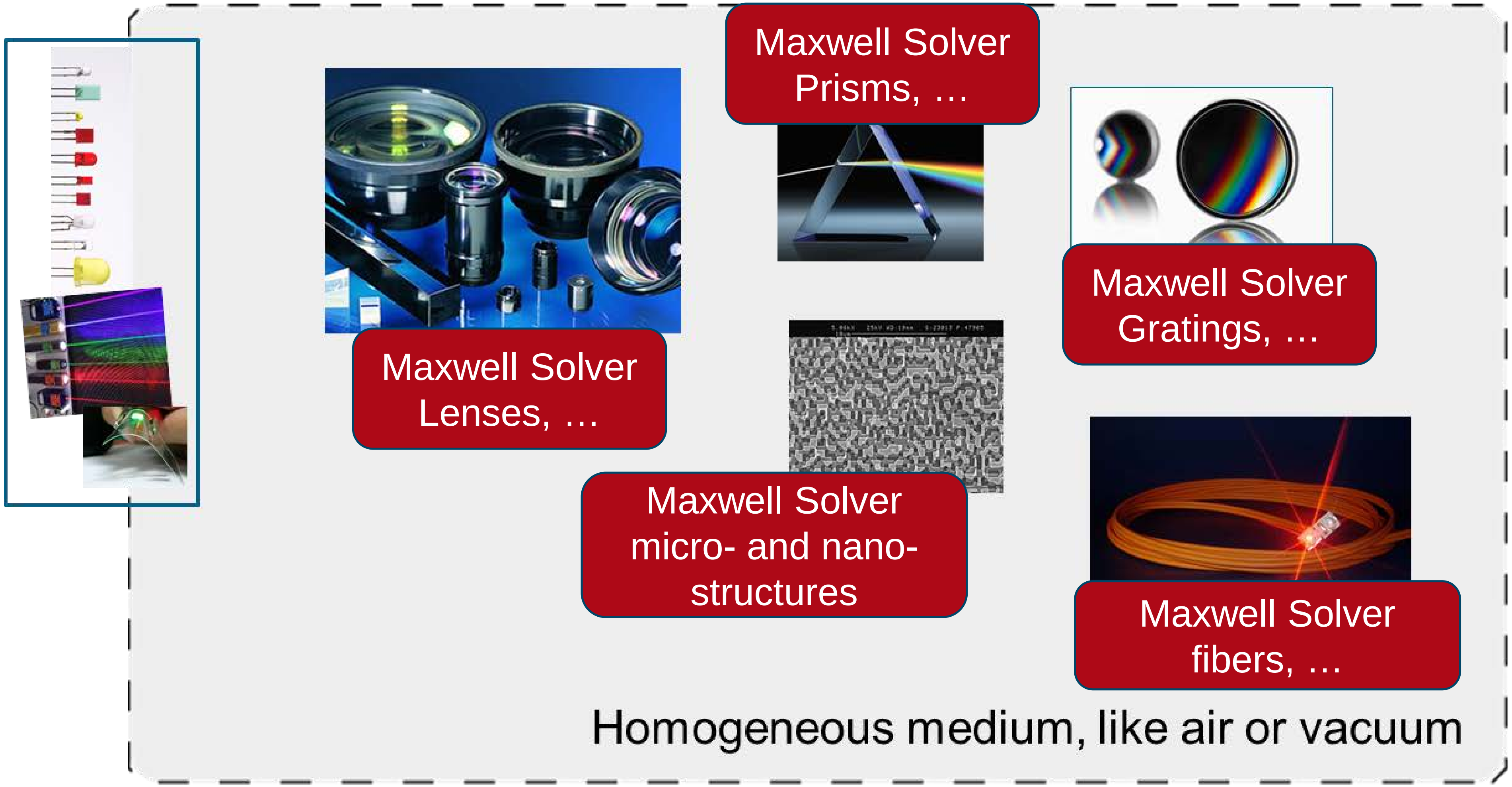


- VirtualLab Fusion manages the transition between diffractive and geometric branches of physical optics automatically (steady development).

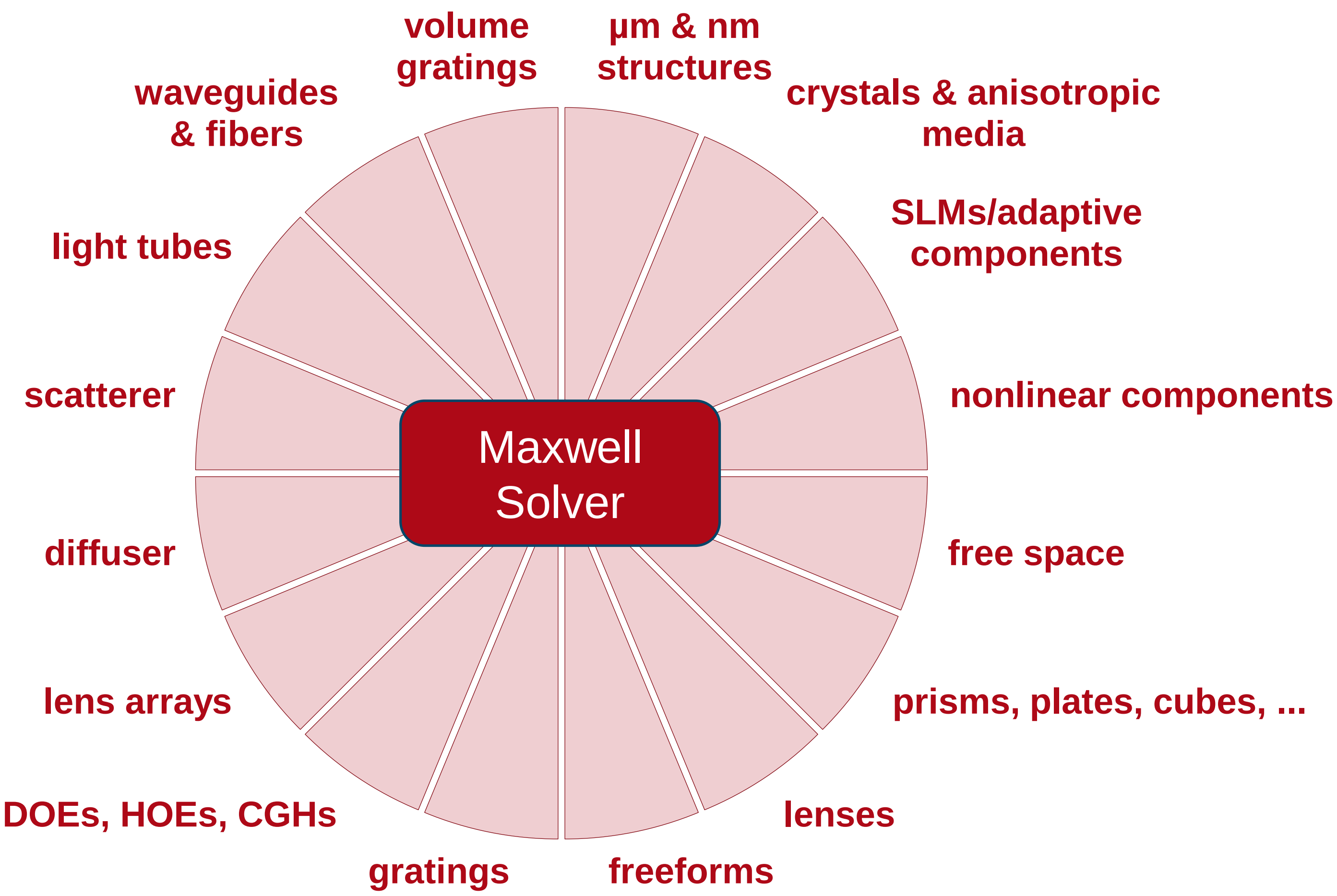
Non-sequential coupling of regional Maxwell Solver

Introduction of bidirectional operators

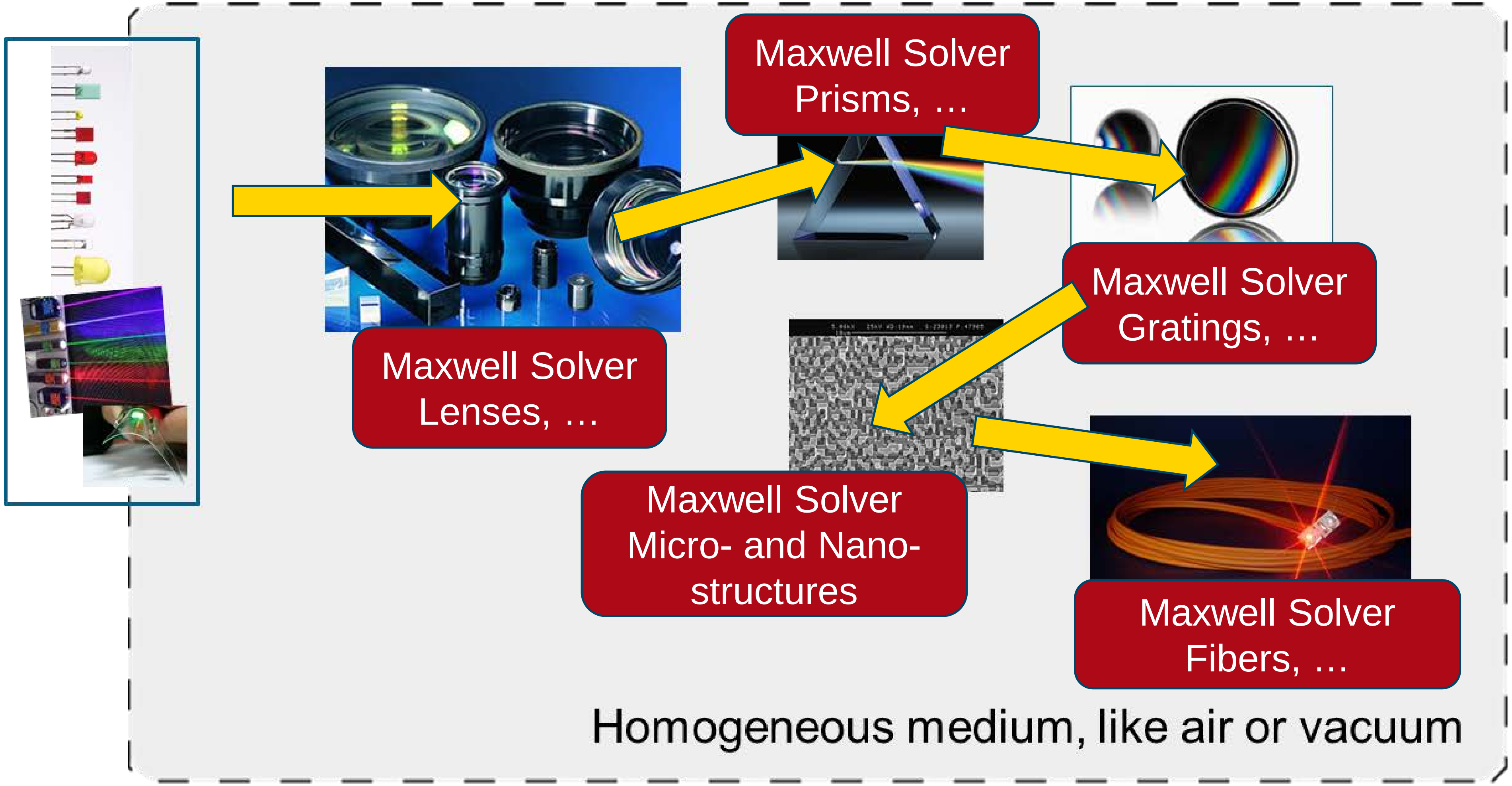
Physical-Optics System Modeling: Regional Maxwell Solver



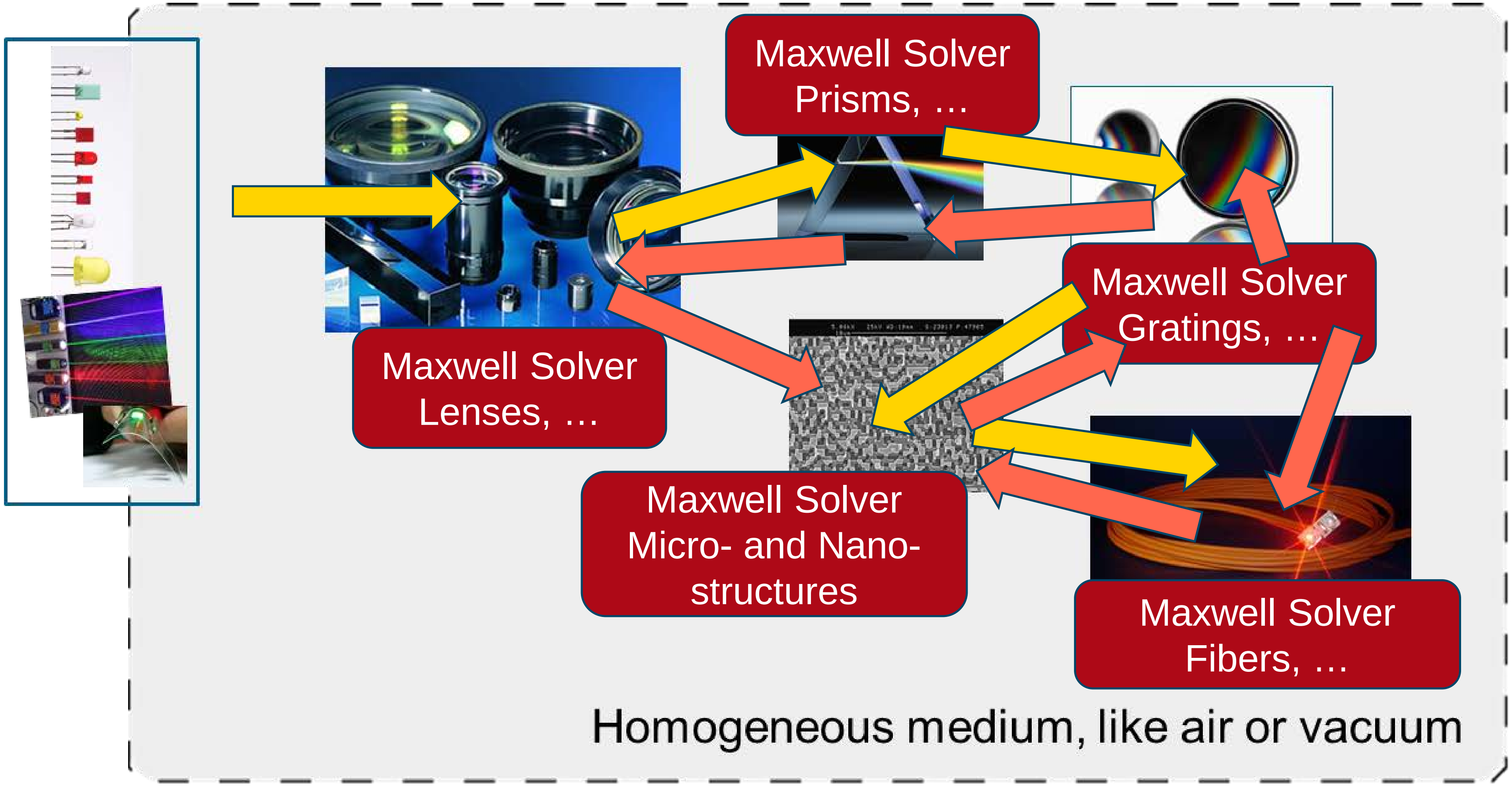
Physical-Optics System Modeling: Regional Maxwell Solver



Sequential Connection of Regional Maxwell Solver



Non-Sequential Connection of Regional Maxwell Solver



Fast Physical Optics by Field Tracing

In **Fast Physical Optics** we comply with the following strategies:

1. Tearing: The optical system is decomposed into various regions in which different types of specialized Maxwell solvers are applied.

2. Interconnection: The solutions per region are connected through non-sequential field tracing to solve Maxwell's equations in the entire system.

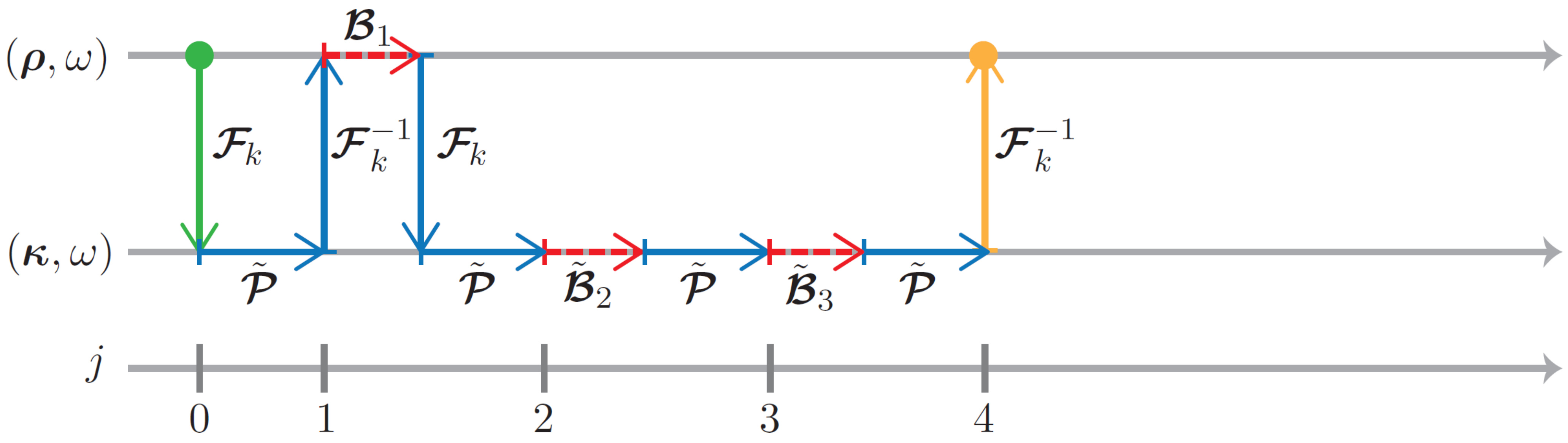
Non-Sequential Optical Field Tracing

Michael Kuhn, Frank Wyrowski, and Christian Hellmann

Kuhn, M.; Wyrowski, F. & Hellmann, C. (2012), Non-sequential optical field tracing, *in* T. Apel & O. Steinbach, ed., 'Finite Element Methods and Applications', Springer-Verlag, Berlin, , pp. 257-274.

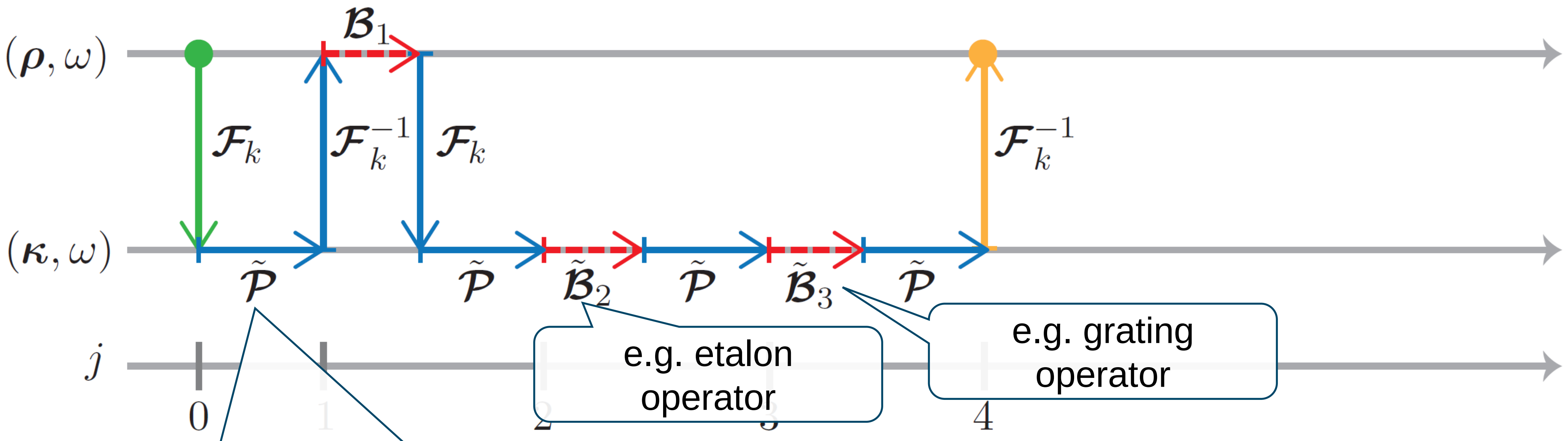
Non-sequential Field Tracing

- Lightpaths through system are investigated
- Each lightpath is evaluated by field tracing



Field Tracing Diagrams: Examples

e.g. aperture modeling: multiplication with aperture fucntion



Rigorous propagation in k-domain:

$$\tilde{V}^{\text{out}}(\boldsymbol{\kappa},z)=\tilde{V}^{\text{in}}(\boldsymbol{\kappa},z_0)\times\exp\left(\mathrm{i}\check{k}_z(\kappa)\Delta z\right)$$

Field Tracing Operators: Space- and k-Domain

- The free-space operator equation is given by

$$\boldsymbol{E}_{\perp}^{\text{out}}(\boldsymbol{\rho},\omega) = \boldsymbol{\mathcal{P}}\boldsymbol{E}_{\perp}^{\text{in}}(\boldsymbol{\rho},\omega)$$

and in matrix form by

$$\begin{pmatrix} E_x^{\text{out}}(\boldsymbol{\rho},\omega) \\ E_y^{\text{out}}(\boldsymbol{\rho},\omega) \end{pmatrix} = \begin{pmatrix} \mathcal{P} & 0 \\ 0 & \mathcal{P} \end{pmatrix} \begin{pmatrix} E_x^{\text{in}}(\boldsymbol{\rho},\omega) \\ E_y^{\text{in}}(\boldsymbol{\rho},\omega) \end{pmatrix}.$$

- Analogously we have for the B-operator

$$\boldsymbol{E}_{\perp}^{\text{out}}(\boldsymbol{\rho},\omega) = \boldsymbol{\mathcal{B}}\boldsymbol{E}_{\perp}^{\text{in}}(\boldsymbol{\rho},\omega)$$

and in matrix form

$$\begin{pmatrix} E_x^{\text{out}}(\boldsymbol{\rho},\omega) \\ E_y^{\text{out}}(\boldsymbol{\rho},\omega) \end{pmatrix} = \begin{pmatrix} \mathcal{B}_{xx} & \mathcal{B}_{xy} \\ \mathcal{B}_{yx} & \mathcal{B}_{yy} \end{pmatrix} \begin{pmatrix} E_x^{\text{in}}(\boldsymbol{\rho},\omega) \\ E_y^{\text{in}}(\boldsymbol{\rho},\omega) \end{pmatrix}.$$

- The free-space operator equation in k-domain is given by

$$\tilde{\boldsymbol{E}}_{\perp}^{\text{out}}(\boldsymbol{\kappa},\omega) = \tilde{\boldsymbol{\mathcal{P}}}\tilde{\boldsymbol{E}}_{\perp}^{\text{in}}(\boldsymbol{\kappa},\omega)$$

with $\boldsymbol{\kappa} = (k_x, k_y)$. In matrix form we have

$$\begin{pmatrix} \tilde{E}_x^{\text{out}}(\boldsymbol{\kappa},\omega) \\ \tilde{E}_y^{\text{out}}(\boldsymbol{\kappa},\omega) \end{pmatrix} = \begin{pmatrix} \tilde{\mathcal{P}} & 0 \\ 0 & \tilde{\mathcal{P}} \end{pmatrix} \begin{pmatrix} \tilde{E}_x^{\text{in}}(\boldsymbol{\kappa},\omega) \\ \tilde{E}_y^{\text{in}}(\boldsymbol{\kappa},\omega) \end{pmatrix}.$$

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Bidirectional Operator: k-Domain

- For the bidirectional operator in k-domain we have per matrix element the integral of the form

$$\tilde{V}^{\text{out}}(k_x, k_y) = \int_{K^2} \tilde{B}(k_x, k_y, k'_x, k'_y)\tilde{V}^{\text{in}}(k'_x, k'_y)\,\mathrm{d}k'_x\,\mathrm{d}k'_y\,.$$

  with he integral kernel $\tilde{B}(k_x, k_y, k'_x, k'_y)$.

$$\begin{pmatrix} \tilde{\mathcal{B}}_{xx} & \tilde{\mathcal{B}}_{xy} \\ \tilde{\mathcal{B}}_{yx} & \tilde{\mathcal{B}}_{yy} \end{pmatrix}$$

- Since $\tilde{V}^{\text{in}}(k'_x, k'_y)$ can be understood as a decomposition into (inhomogeneous) plane waves, the bidirectional operator provides the plane-waves response on one incident plane wave.
- The integral operator reduces to a multiplication for stratified media and gratings per order (with k-value mapping).

Bidirectional Operator: k-Domain

- For the bidirectional operator in k-domain trix element the integral of the form

Bidirectional operator
Generalization of Bidirectional Scattering Distribution Function (BSDF).

$$\tilde{V}^{\text{out}}(k_x, k_y) = \int_{K^2} \tilde{B}(k_x, k_y, k'_x, k'_y)\tilde{V}^{\text{in}}(k'_x, k'_y)\,\mathrm{d}k'_x\,\mathrm{d}k'_y\,.$$

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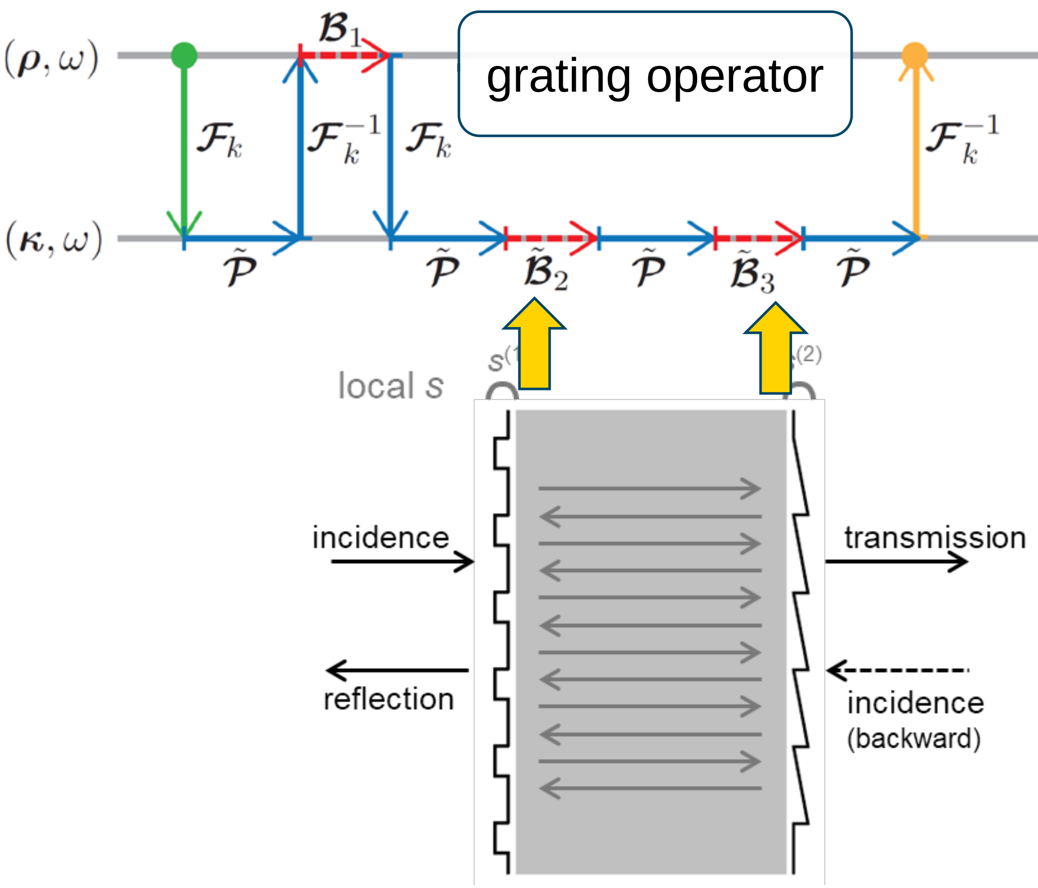
- Since $\tilde{V}^{\text{in}}(k'_x, k'_y)$ can be understood as a decomposition into (inhomogeneous) plane waves, the bidirectional operator provides the plane-waves response on one incident plane wave.
- The integral operator reduces to a multiplication for stratified media and gratings per order (with k-value mapping).

Non-sequential coupling of regional Maxwell Solver

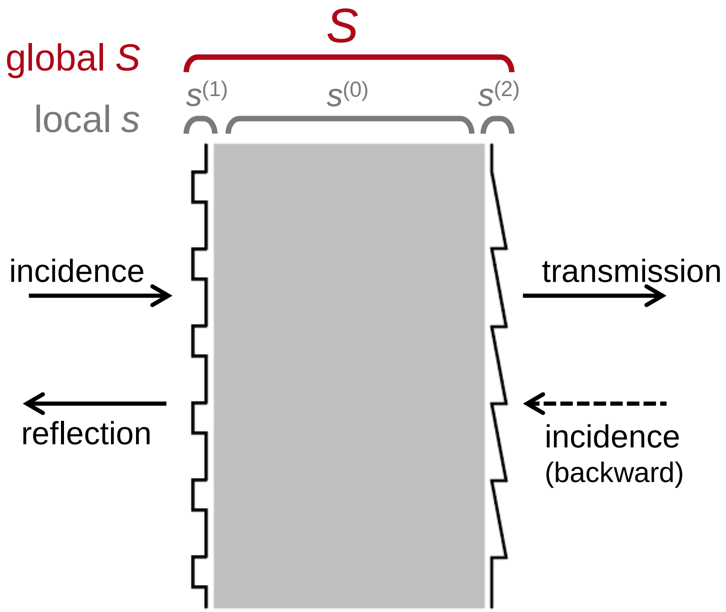
Coupling of FMM solver

Nonsequential Coupling of FMM Solvers



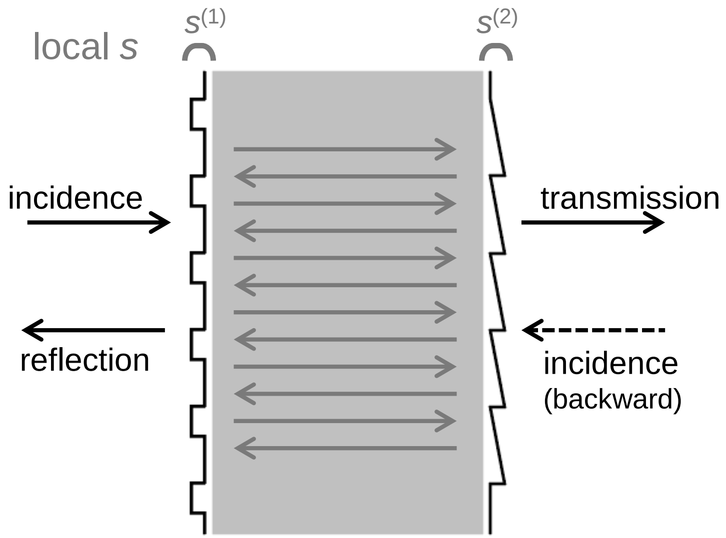
Theory Background

- Global S matrix



  - Recursion with respect to number of regions / layers

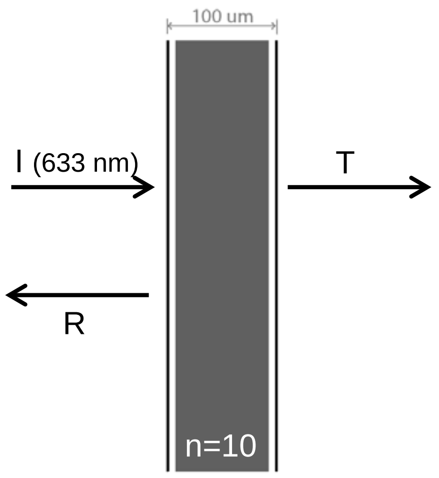
- Non-sequential field tracing

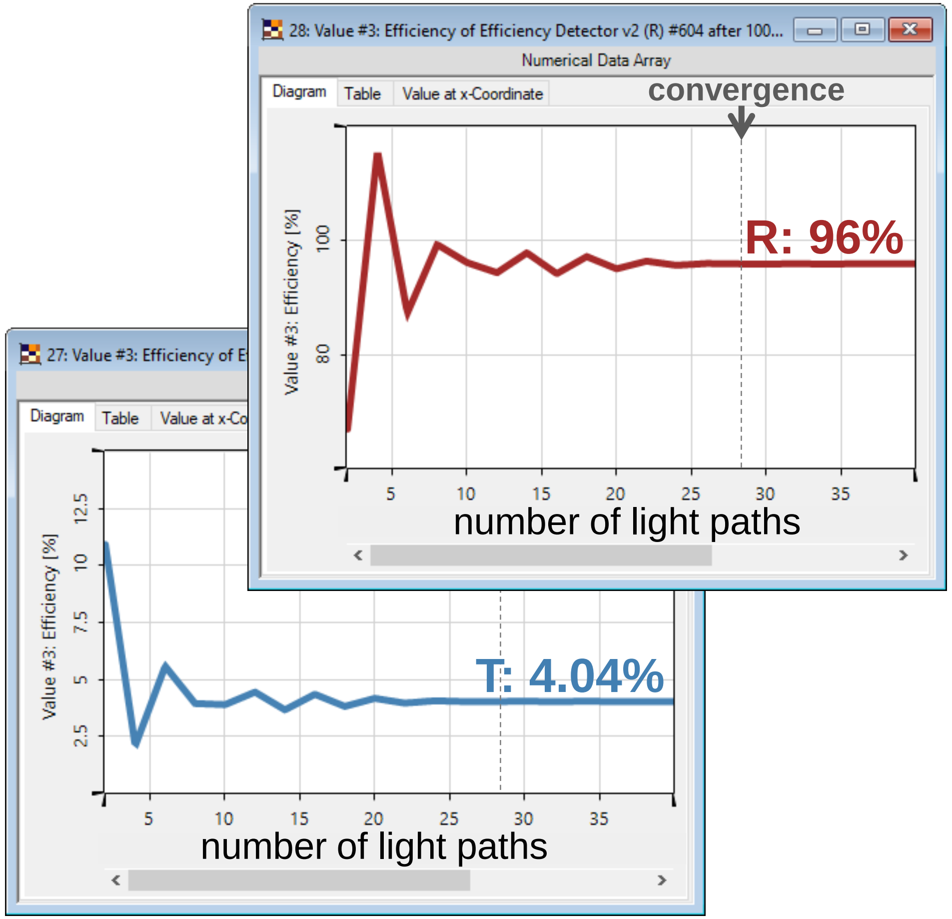


  - Recursion with respect to number of light paths

Planar Surface + Planar Surface

- Structure
- Non-sequential field tracing



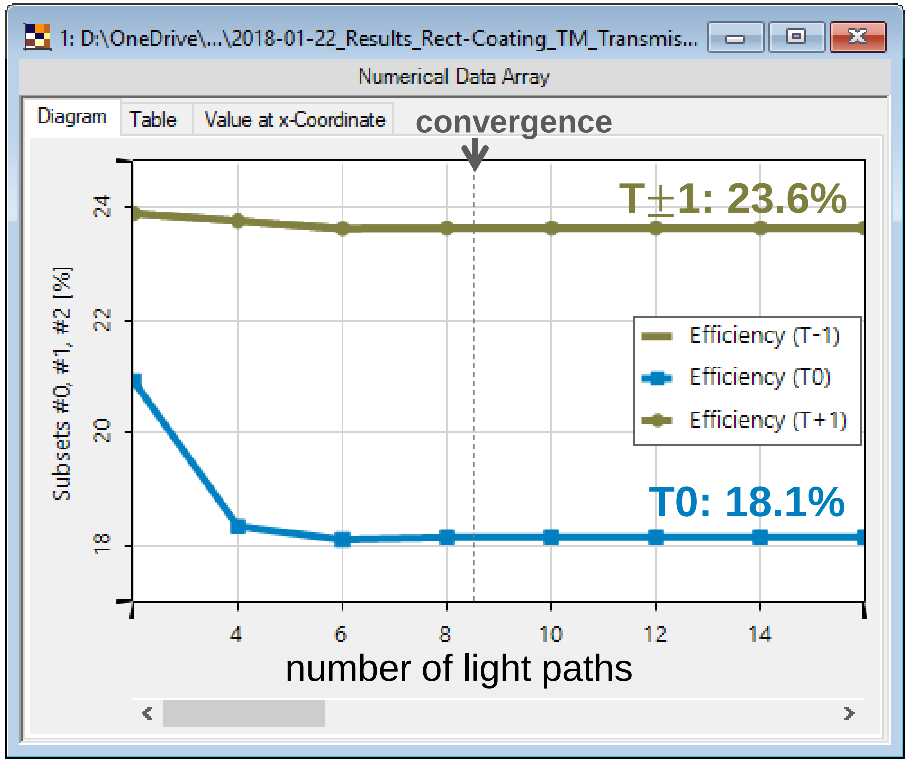


Global S matrix

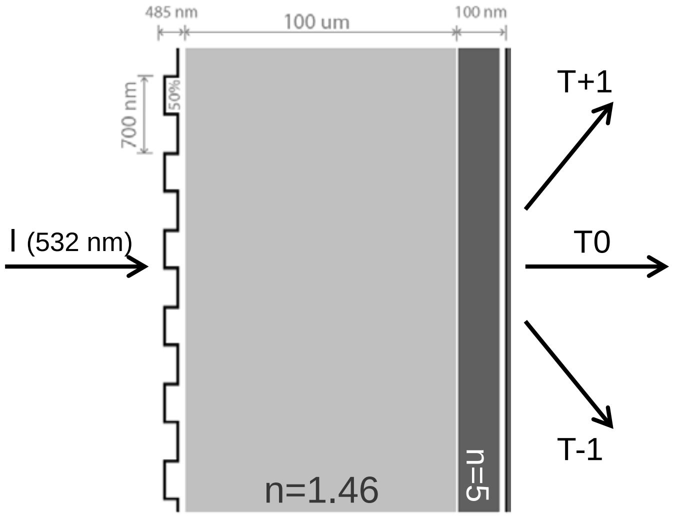
Eff. (T)	Eff. (R)
4.04%	96%

Rectangular Grating + Backside Coating

- Non-sequential field tracing



- ... with backside coating

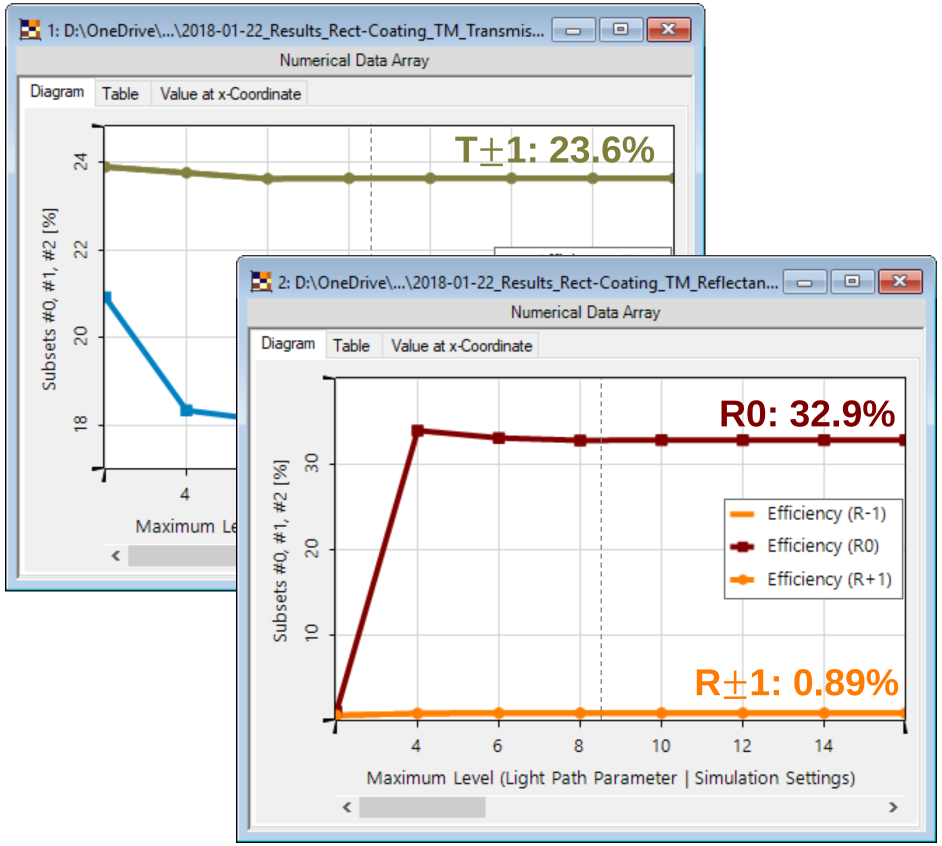


Global S matrix (TM)

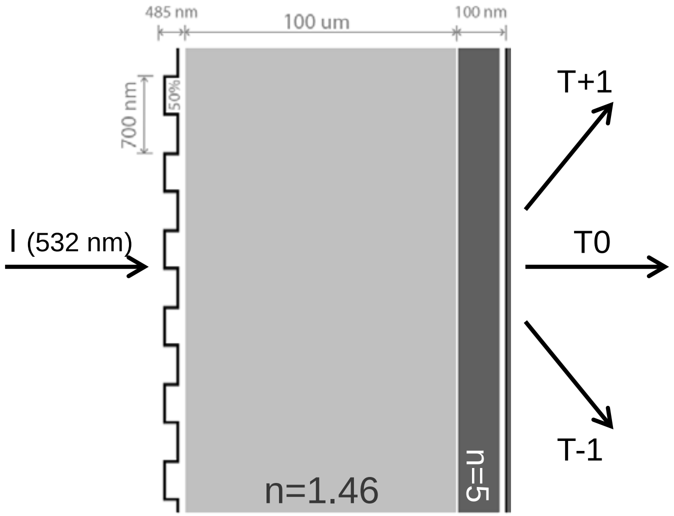
T	Eff.	R	Eff.
±1	23.6%	±1	0.762%
0	18.1%	0	33.1%

Rectangular Grating + Backside Coating

- Non-sequential field tracing



- ... with backside coating



Global S matrix (TM)

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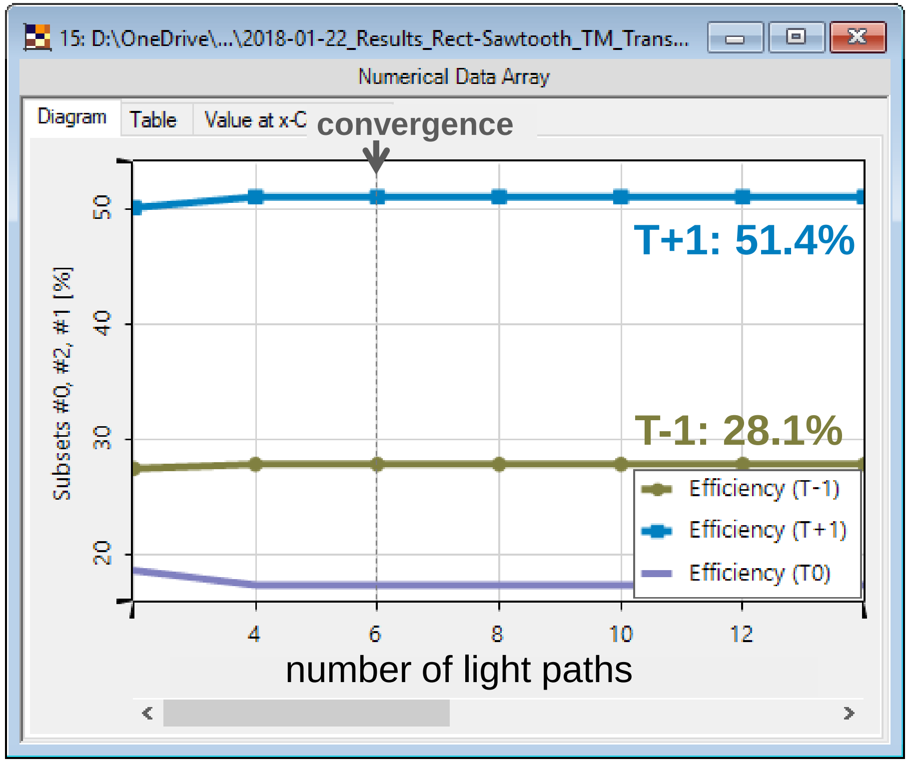
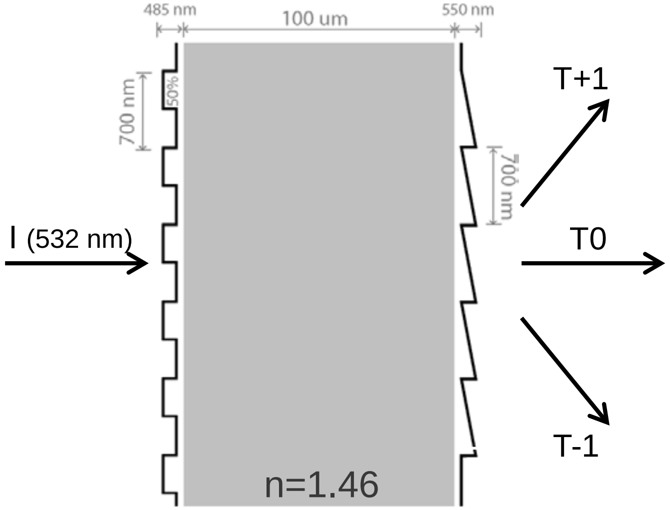
Rectangular + Sawtooth Grating (parallel)

- Non-sequential field tracing



- ... with sawtooth coating

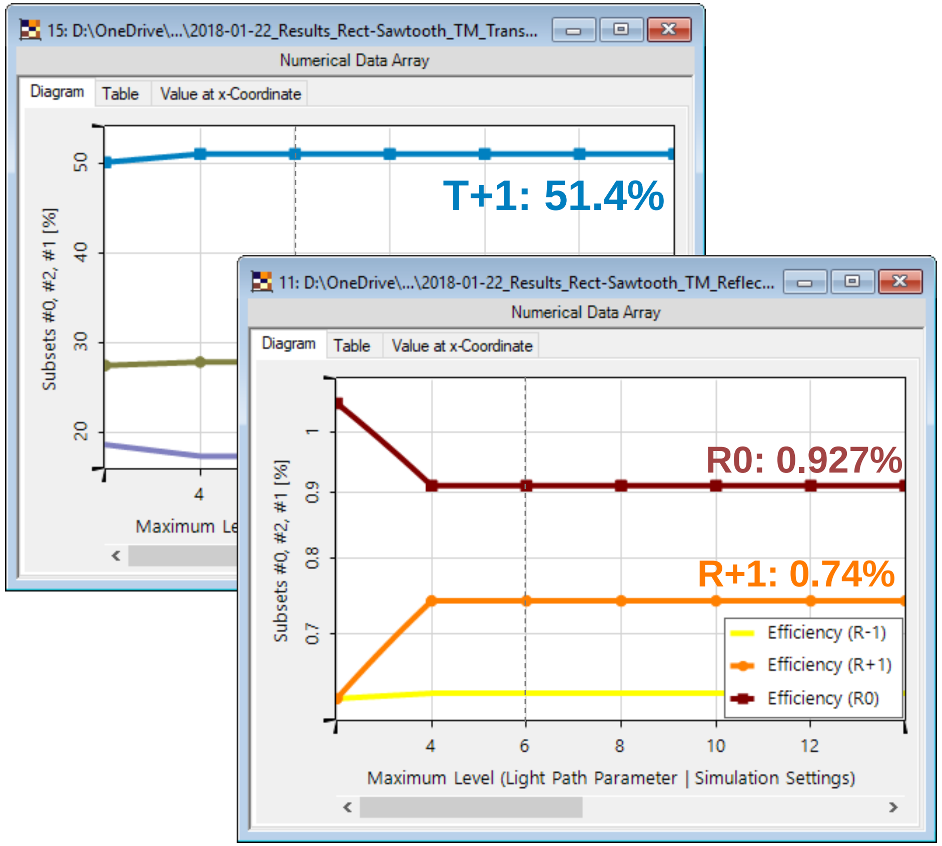


Global S matrix (TM)

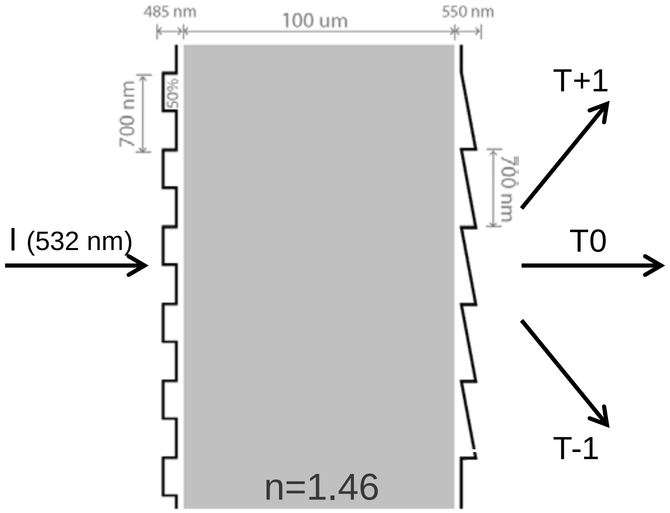
T	Eff.	R	Eff.
-1	28.1%	-1	0.65%
0	18.2%	0	0.923%
+1	51.4%	+1	0.74%

Rectangular + Sawtooth Grating (parallel)

- Non-sequential field tracing



- ... with sawtooth coating



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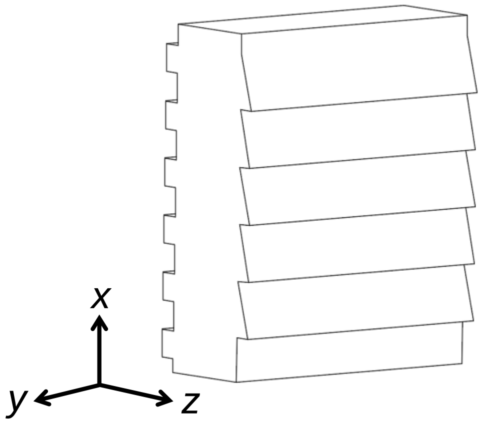
Computational Effort

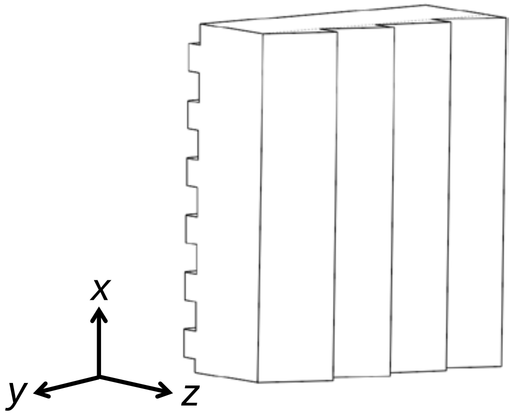
- Parallel gratings



Global S matrix	Non-sequential field tracing
$\sim M^3$ (scaling with number of layers)	$\sim M^3$ (scaling with number of light paths)

with M as the number of diffraction (evanescent included) orders used in calculation

- Crossed gratings

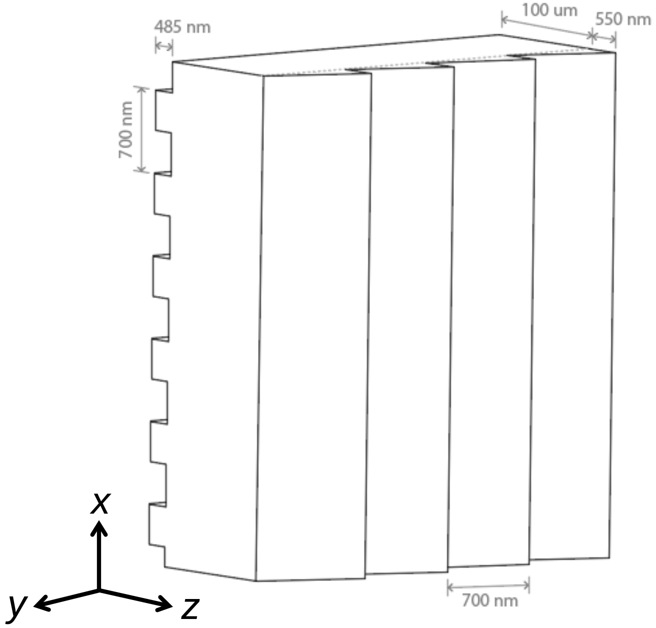


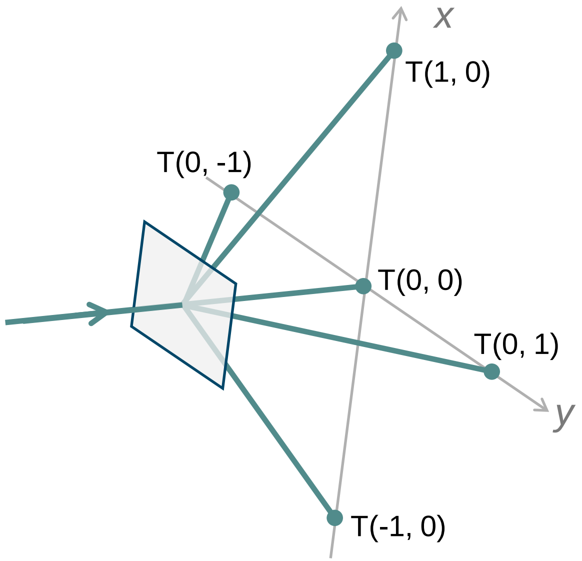
Global S matrix	Non-sequential field tracing
$\boxed{\sim\left(M_x \times M_y\right)^3}$ (scaling with number of layers)	$\sim\left(M_x^3 + M_y^3\right)$ (scaling with number of light paths)

with M_x and M_y as the number of diffraction (evanescent included) orders in both directions

Rectangular + Sawtooth Grating (crossed)

- Structure
  - Front: rectangular grating
    (along *x* direction)
  - Back: sawtooth grating
    (along *y* direction)



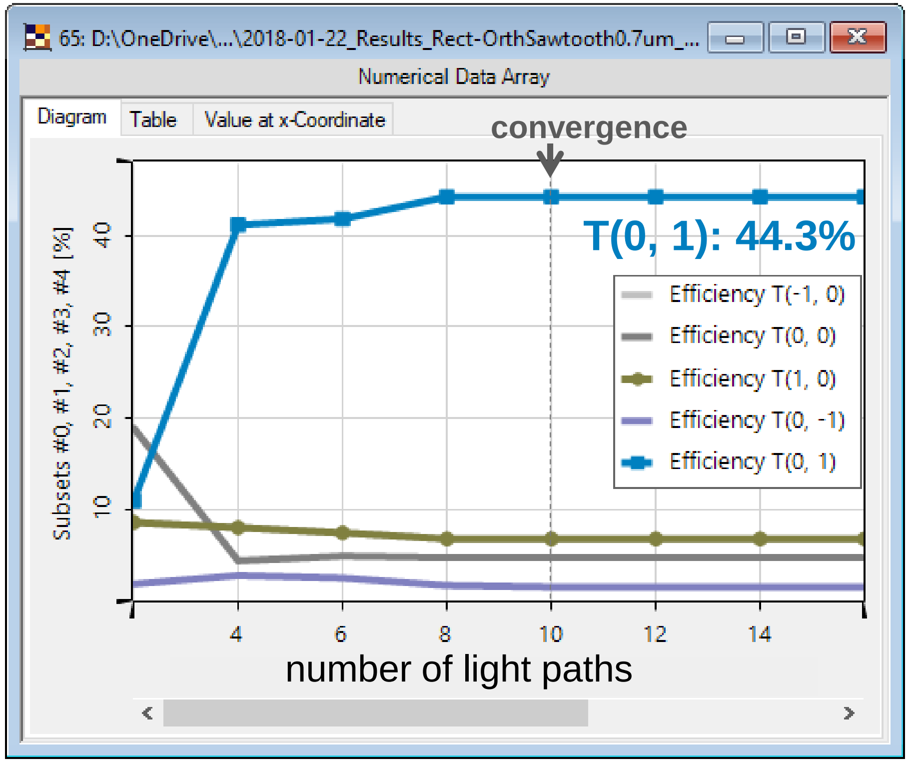


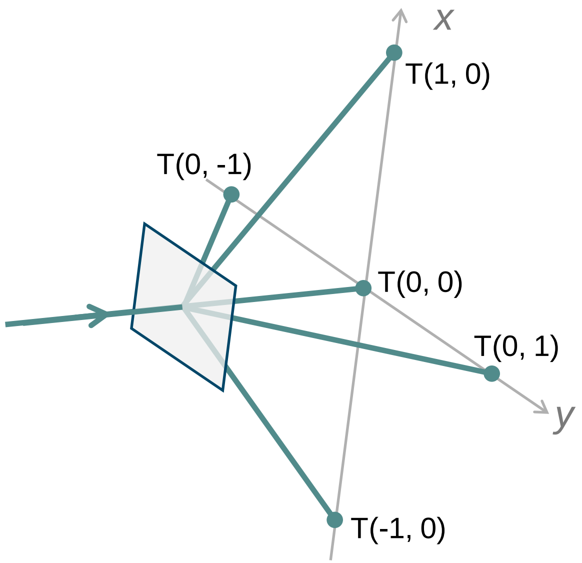
Global S matrix (TM)

T	Eff.	R	Eff.
-1, 0	5.4%	-1, 0	5.7%
0, -1	4.2%	0, -1	5.8%
0, 0	4.5%	0, 0	13.8%
0, 1	44.9%	0, 1	4.6%
1, 0	5.4%	1, 0	5.7%

Rectangular + Sawtooth Grating (crossed)

- Non-sequential field tracing (TM)



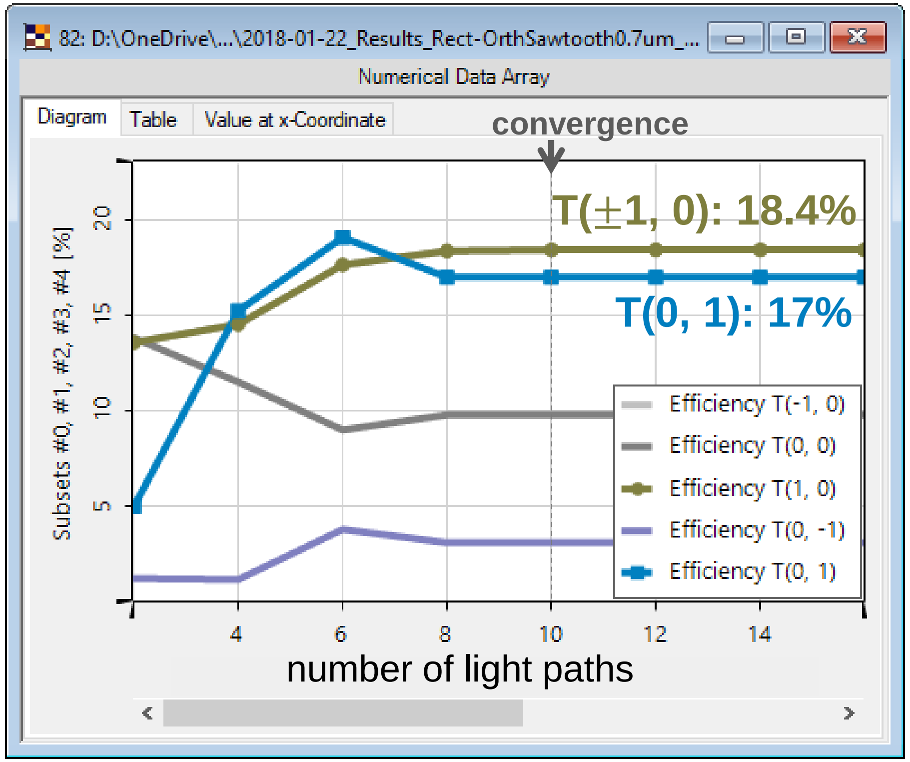


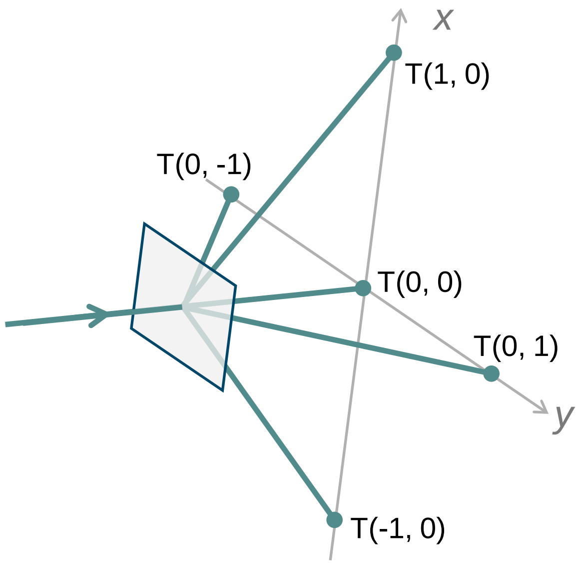
Global S matrix (TM)

T	Eff.	R	Eff.
-1, 0	5.4%	-1, 0	5.7%
0, -1	4.2%	0, -1	5.8%
0, 0	4.5%	0, 0	13.8%
0, 1	**44.9%**	0, 1	4.6%
1, 0	5.4%	1, 0	5.7%

Rectangular + Sawtooth Grating (crossed)

- Non-sequential field tracing (TE)



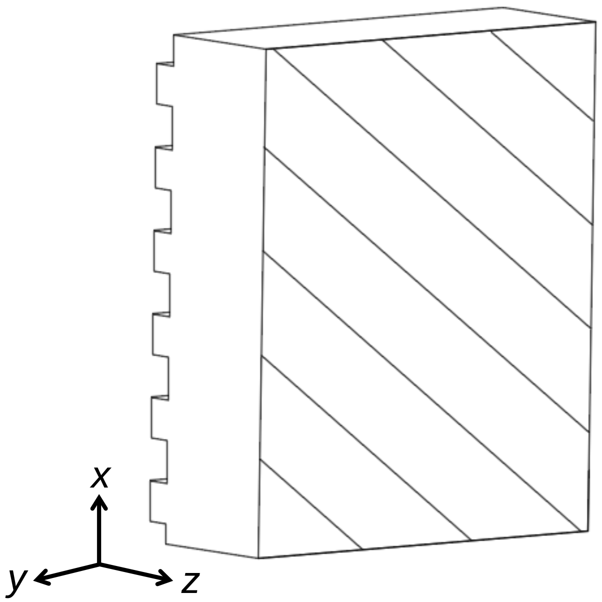


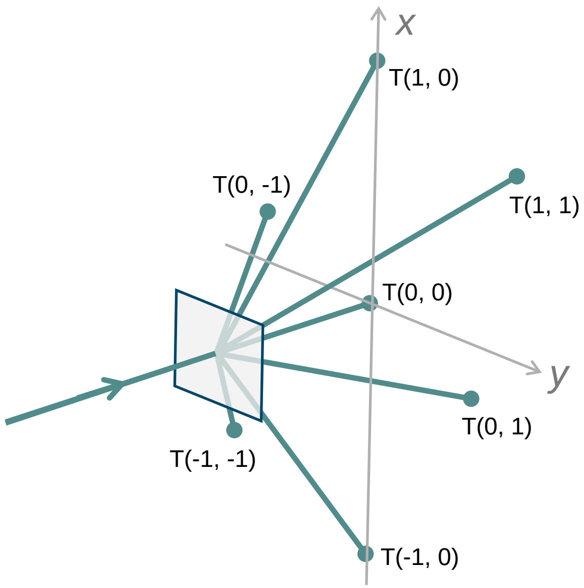
Global S matrix (TE)

T	Eff.	R	Eff.
-1, 0	**18%**	-1, 0	1.1%
0, -1	2.8%	0, -1	0.46%
0, 0	11.9%	0, 0	22.6%
0, 1	**17.1%**	0, 1	6.89%
1, 0	**18%**	1, 0	1.1%

Rectangular + Sawtooth Grating (45° rotated)

- Structure
  - Front: rectangular grating (along *x* direction)
  - Back: sawtooth grating (along *x*-*y* diagonal direction)

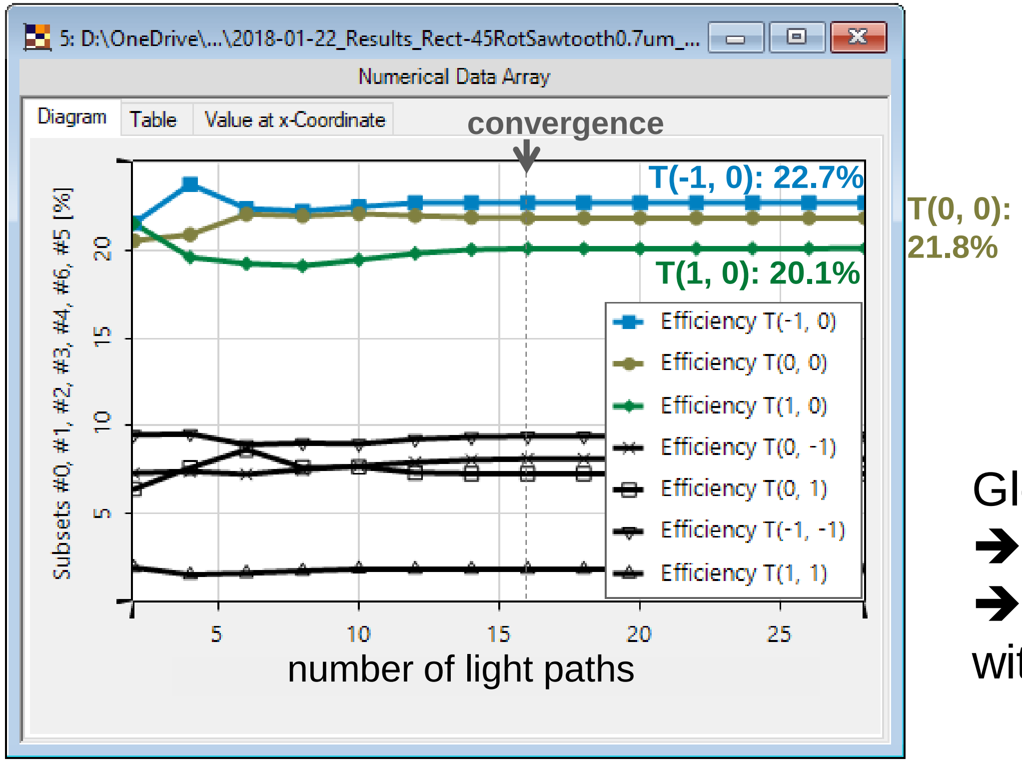


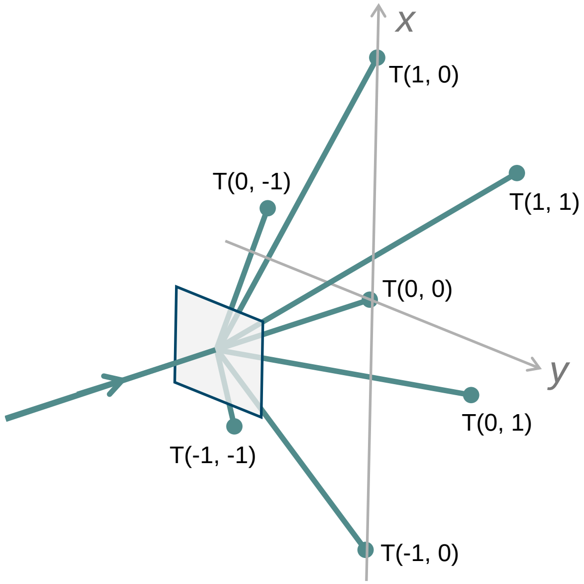


Global S matrix (TM)
➔ No common period!
➔ Huge computational effort even with approximated common period

Rectangular + Sawtooth Grating (45° rotated)

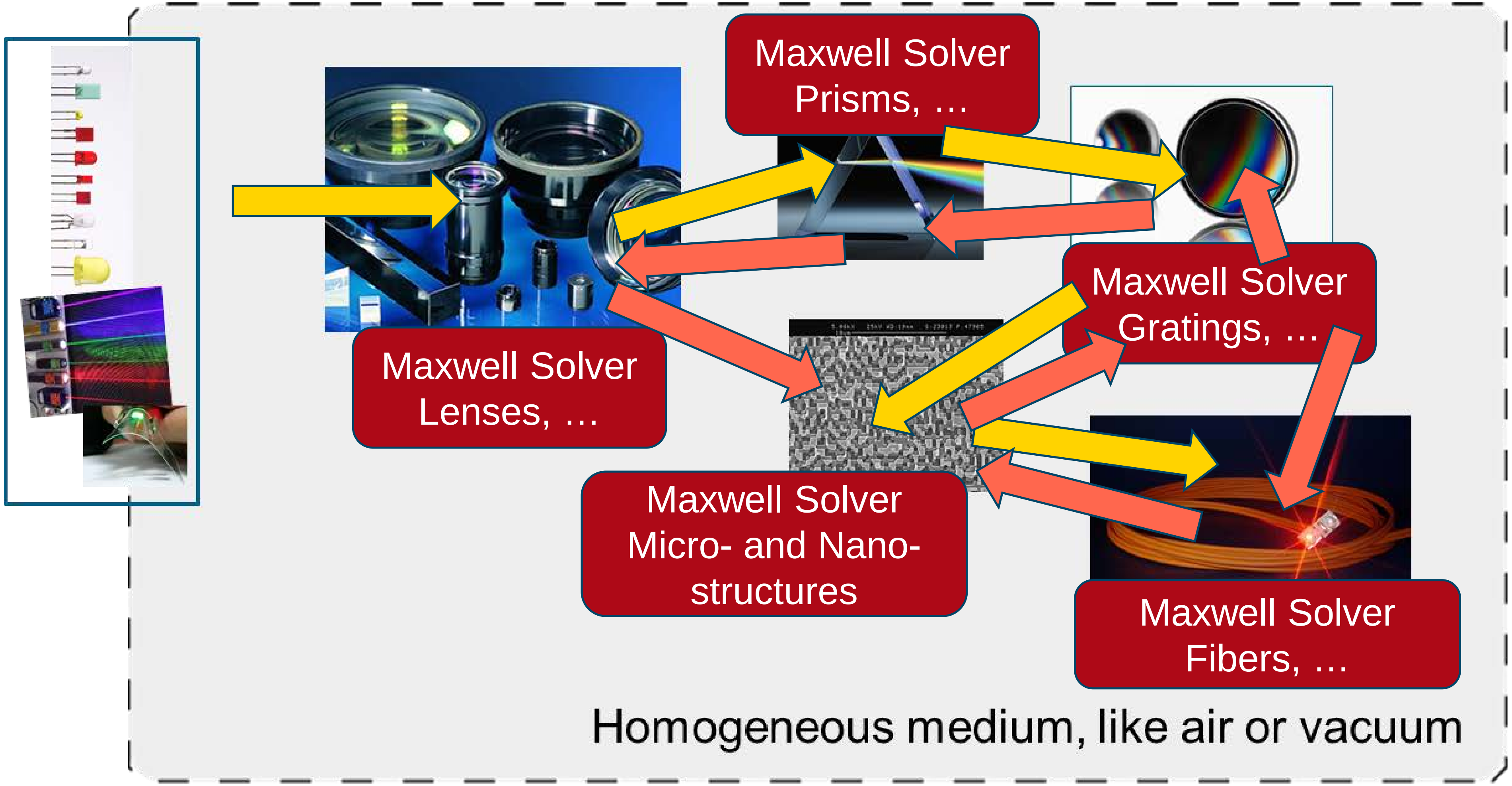
- Non-sequential field tracing (TM)



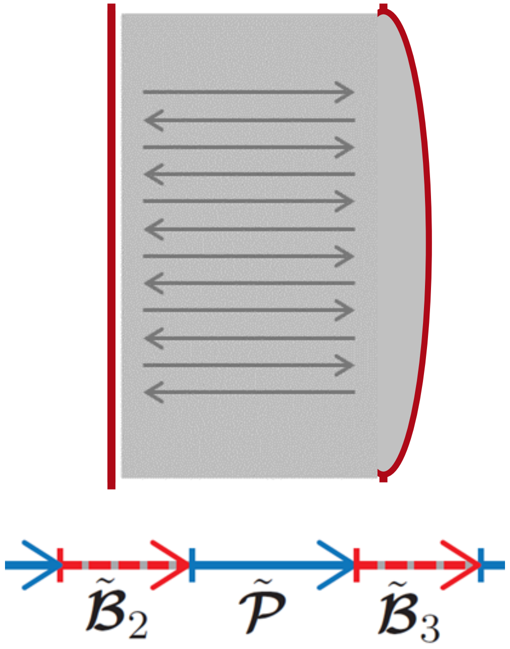


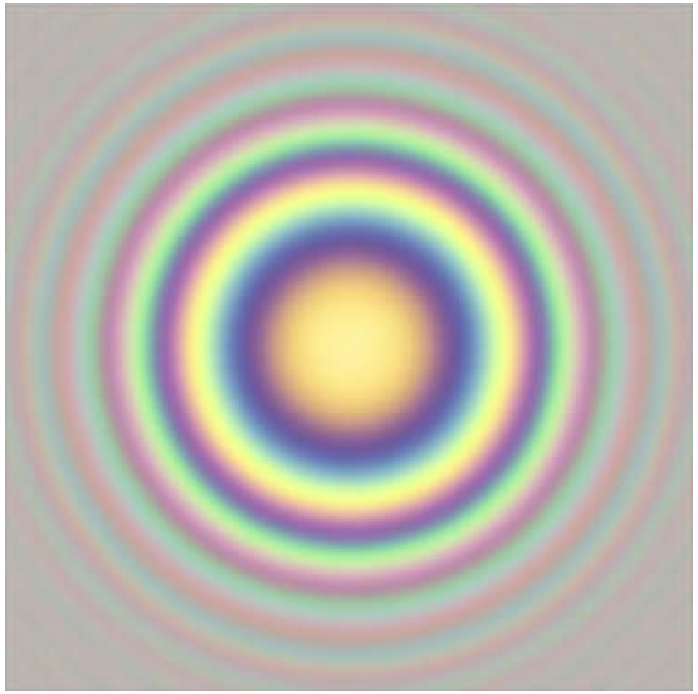
Global S matrix ***NOT*** possible!
➔ No common period
➔ Huge computational effort even with approximated common period

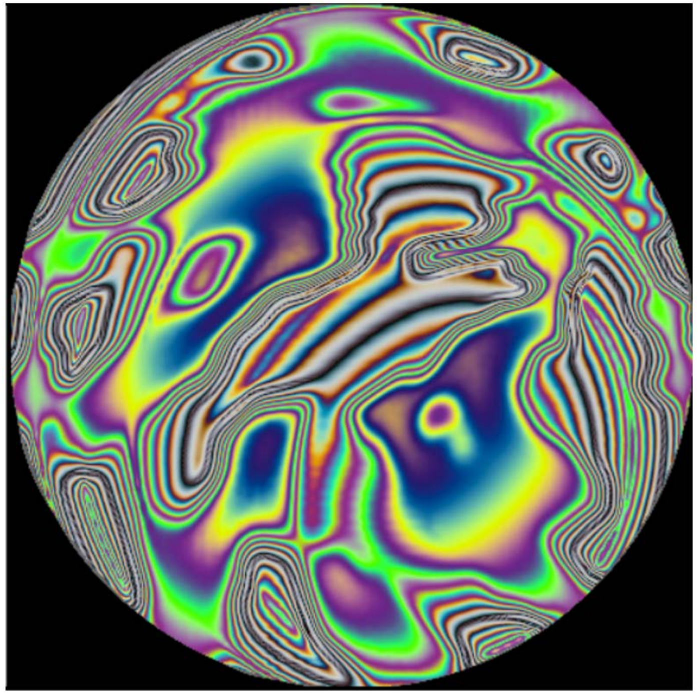
Non-Sequential Connection of Regional Maxwell Solver



Curved Surfaces: Newton Rings

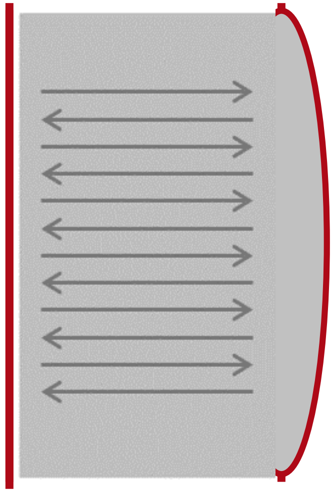


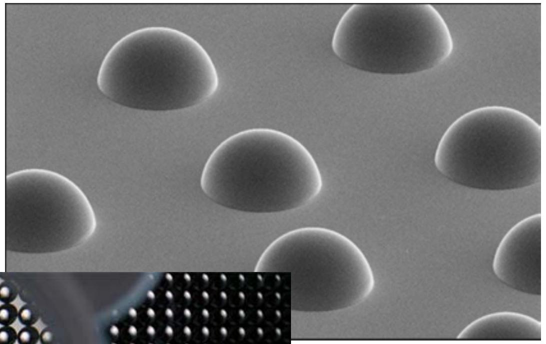




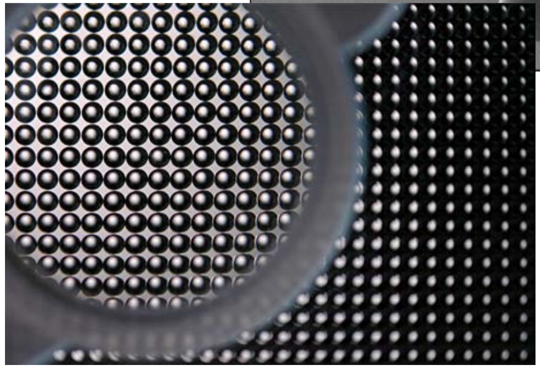
$$\tilde{V}^{\text{out}}(k_x, k_y) = \int_{K^2} \tilde{B}(k_x, k_y, k'_x, k'_y)\,\tilde{V}^{\text{in}}(k'_x, k'_y)\,\mathrm{d}k'_x\,\mathrm{d}k'_y$$

Curved Surfaces: Lenses and Freeform Surfaces





www.temicon.com



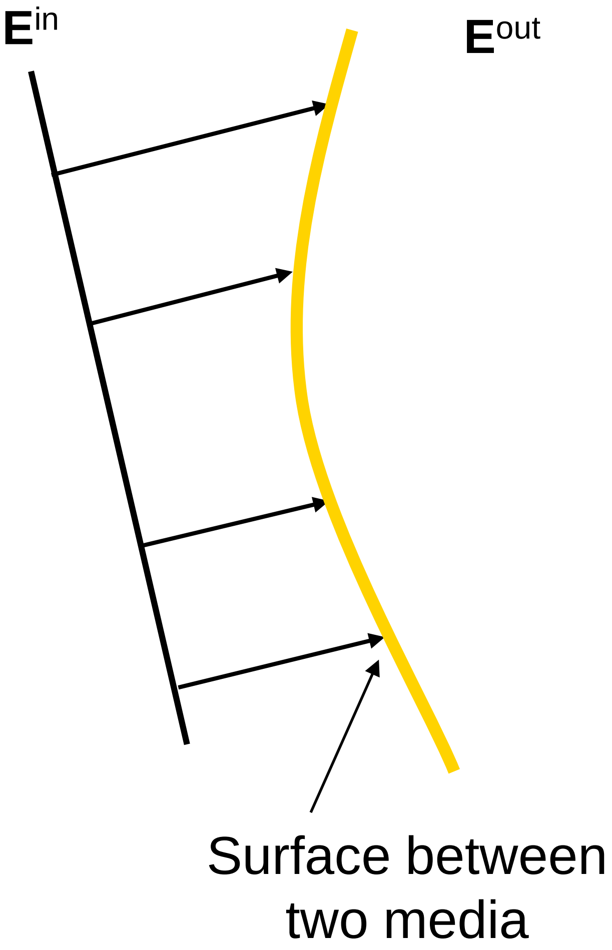
www.itwissen.info

$$\tilde{V}^{\text{out}}(k_x,k_y) = \int_{K^2} \tilde{B}(k_x,k_y,k'_x,k'_y)\,\tilde{V}^{\text{in}}(k'_x,k'_y)\,\mathrm{d}k'_x\,\mathrm{d}k'_y$$

B operator for curved surfaces

Local plane interface approach (LPIA)

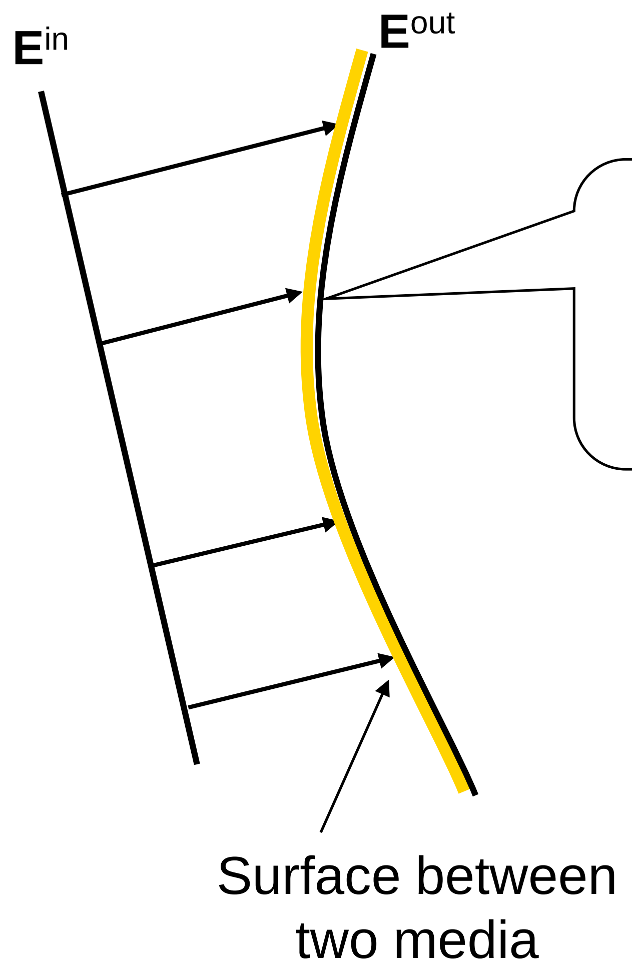
Local Plane Interface Approximation (LPIA)



- For each k-value the bidirectional operator in space domain is the response of the surface on a plane wave.
- Response is obtained by local satisfaction of boundary condition.

$$\tilde{V}^{\mathrm{out}}(k_x, k_y) = \int_{K^2} \tilde{B}(k_x, k_y, k'_x, k'_y)\,\tilde{V}^{\mathrm{in}}(k'_x, k'_y)\,\mathrm{d}k'_x\,\mathrm{d}k'_y$$

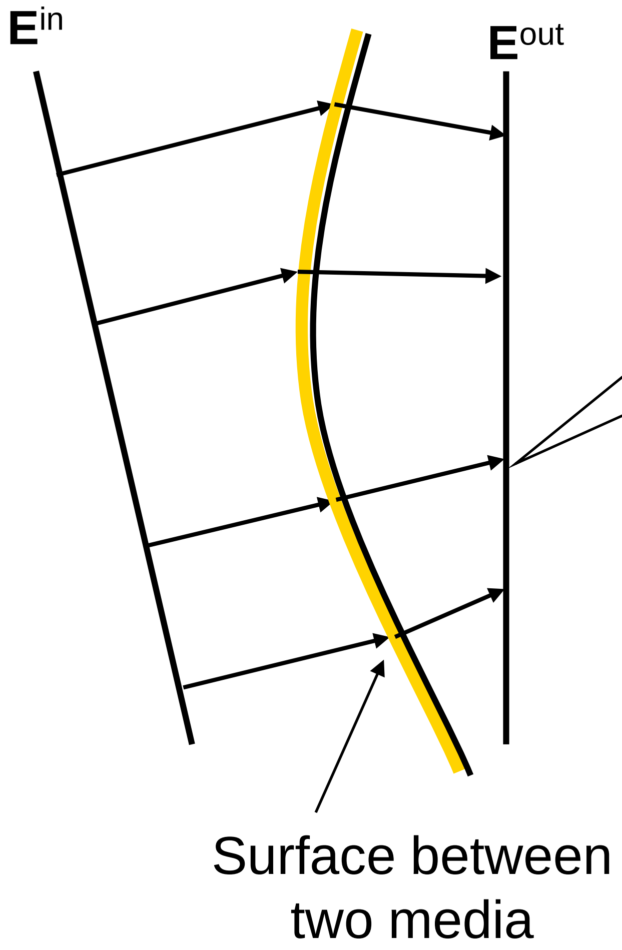
Local Plane Interface Approximation (LPIA)



LPIA: Local satisfaction of boundary condition under the assumption of local plane waves and **local plane interface.**

$$\tilde{V}^{\text{out}}(k_x, k_y) = \int_{K^2} \tilde{B}(k_x, k_y, k'_x, k'_y)\, \tilde{V}^{\text{in}}(k'_x, k'_y)\, \mathrm{d}k'_x\, \mathrm{d}k'_y$$

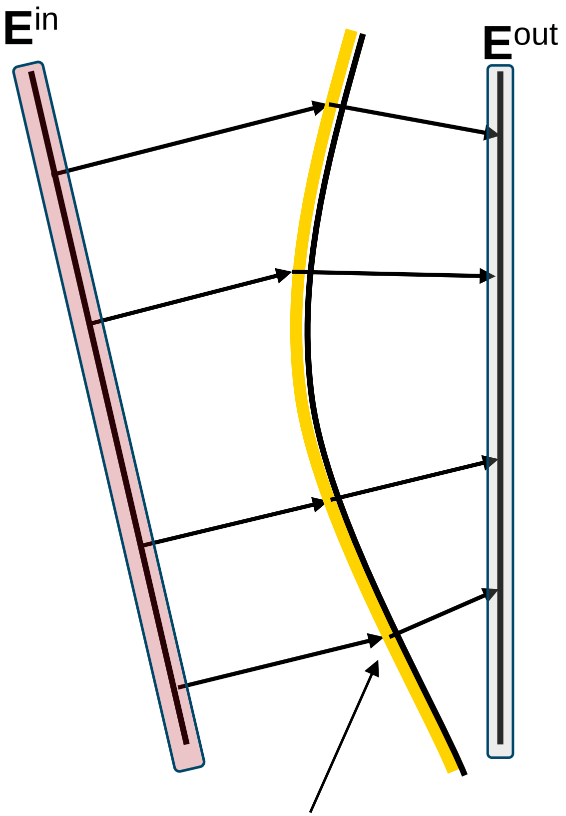
Local Plane Interface Approximation + Propagation



Further propagation into suitable reference plane by suitable free-space propagation, e.g. geometric field propagation.

$$\tilde{V}^{\text{out}}(k_x,k_y) = \int_{K^2} \tilde{B}(k_x,k_y,k'_x,k'_y)\,\tilde{V}^{\text{in}}(k'_x,k'_y)\,\mathrm{d}k'_x\,\mathrm{d}k'_y$$

Local Plane Interface Approximation: B-Operator

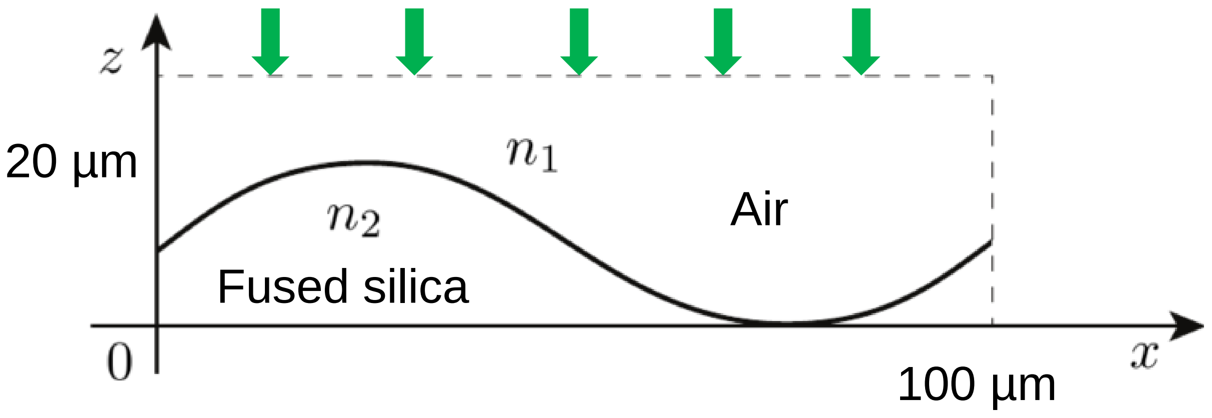


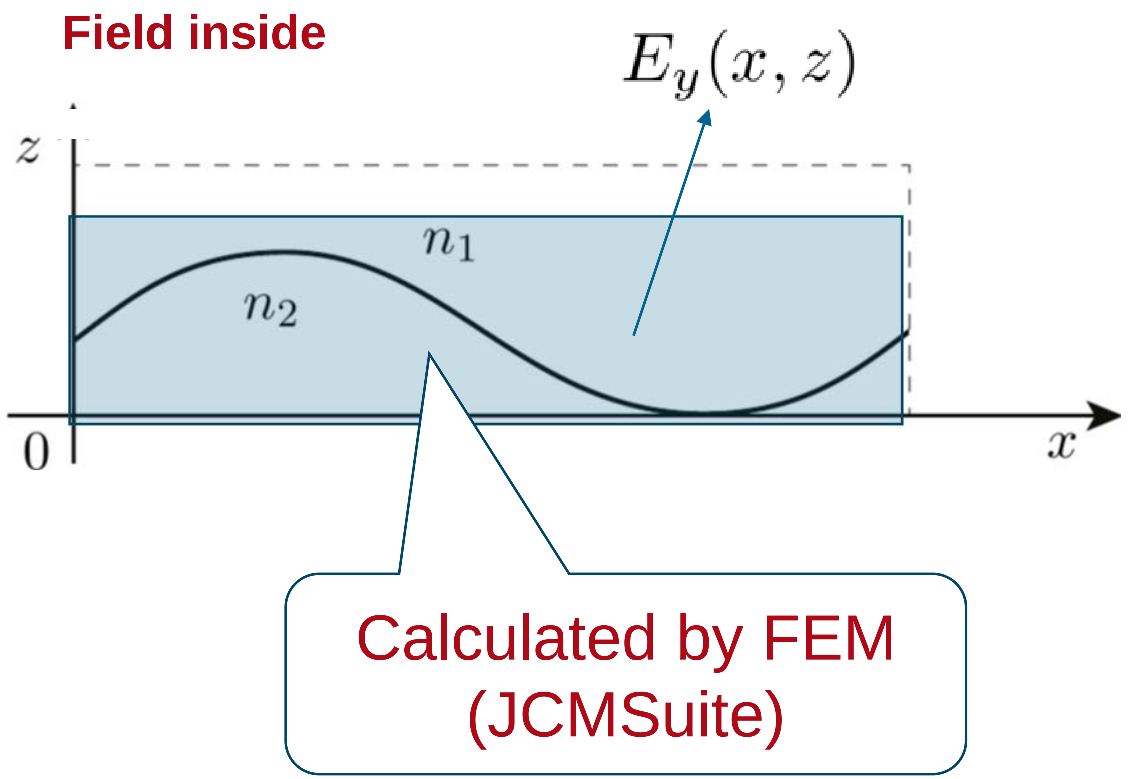
Surface between
two media

Fourier transform of resulting field provides B-operator response for one incident k-value

$$\tilde{V}^{\text{out}}\left(\boxed{k_x, k_y}\right) = \int_{K^2} \tilde{B}\left(\boxed{k_x, k_y}, \boxed{k_x', k_y'}\right) \tilde{V}^{\text{in}}\left(\boxed{k_x', k_y'}\right) \mathrm{d}k_x' \, \mathrm{d}k_y'$$

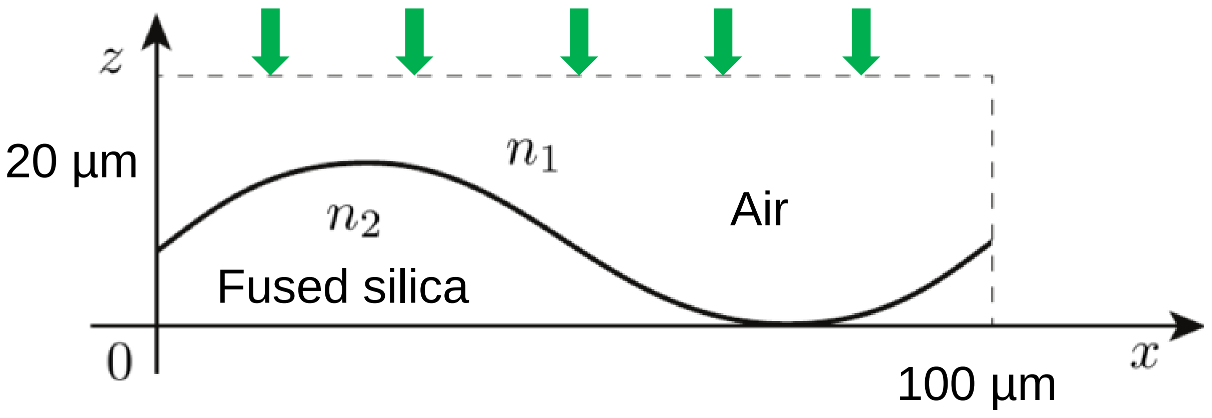
Simulations: Specifications



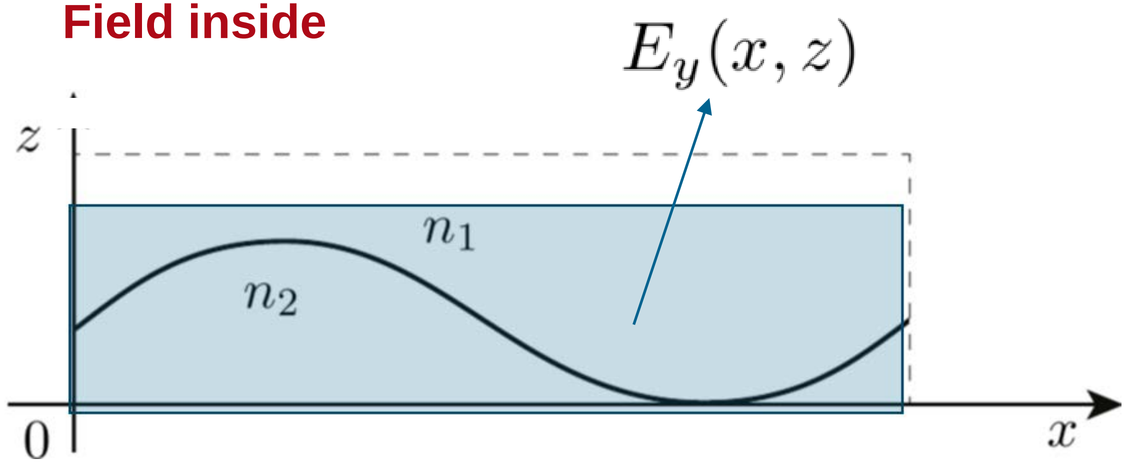


- LPIA with

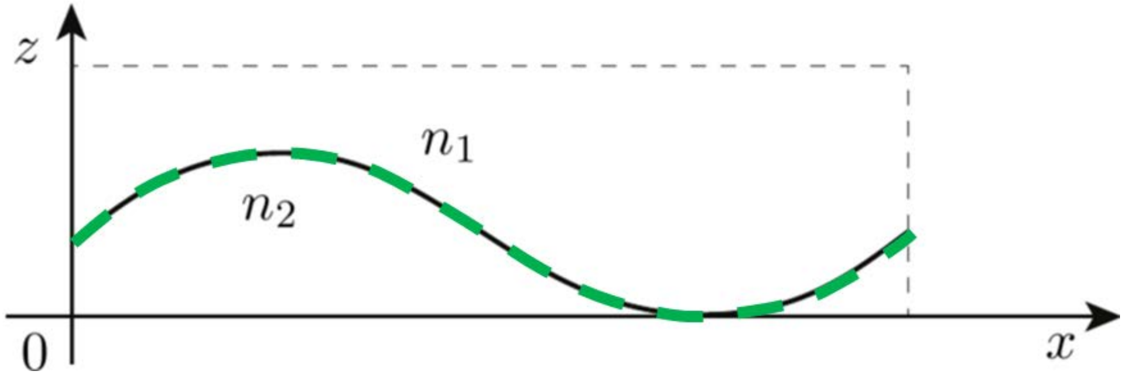
Simulations: Fields Inside + On Surface



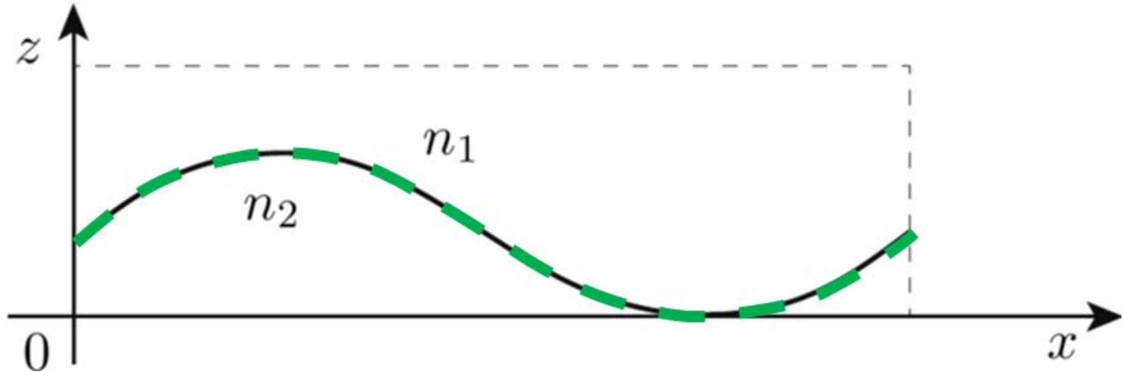
Field inside



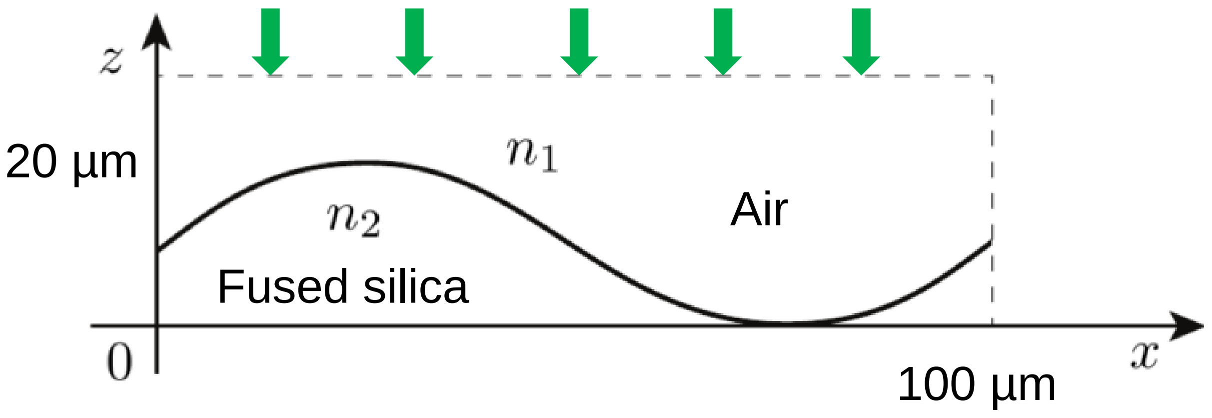
Field on interface: Rigoros



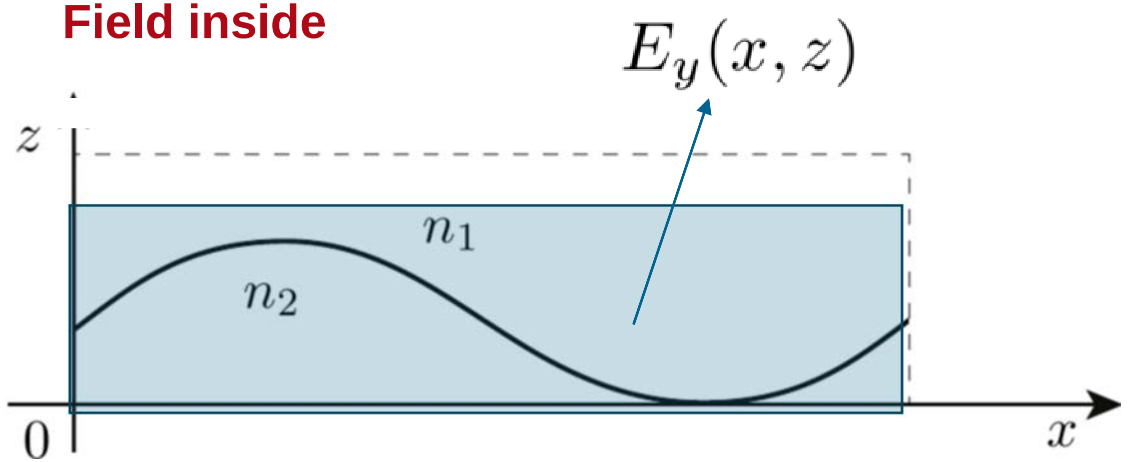
Field on interface: LPIA



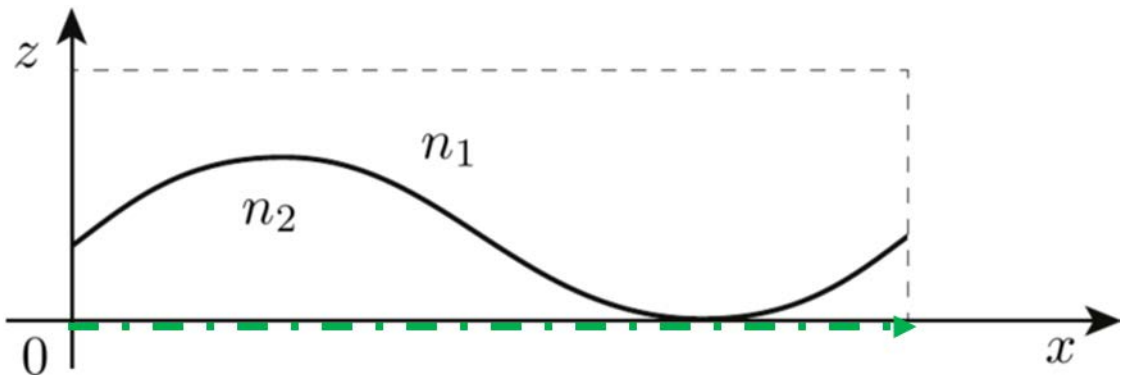
Simulations: Fields Inside + Propagated



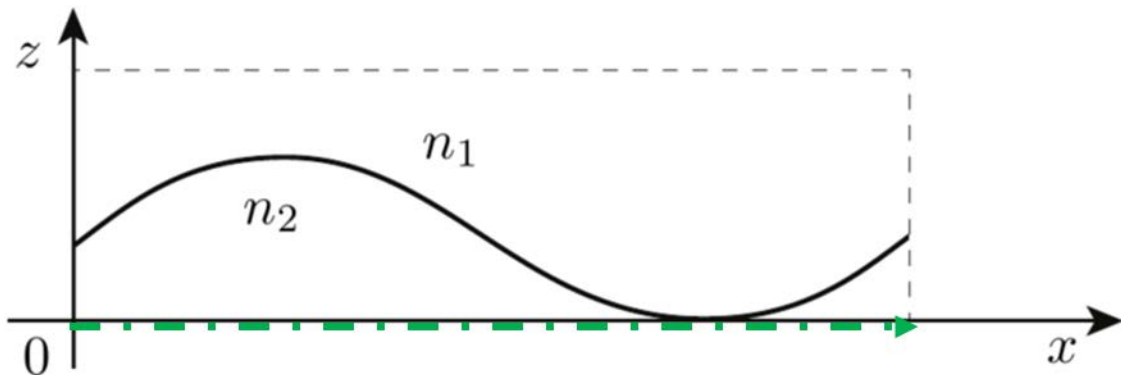
Field inside



Field propagated: Rigoros

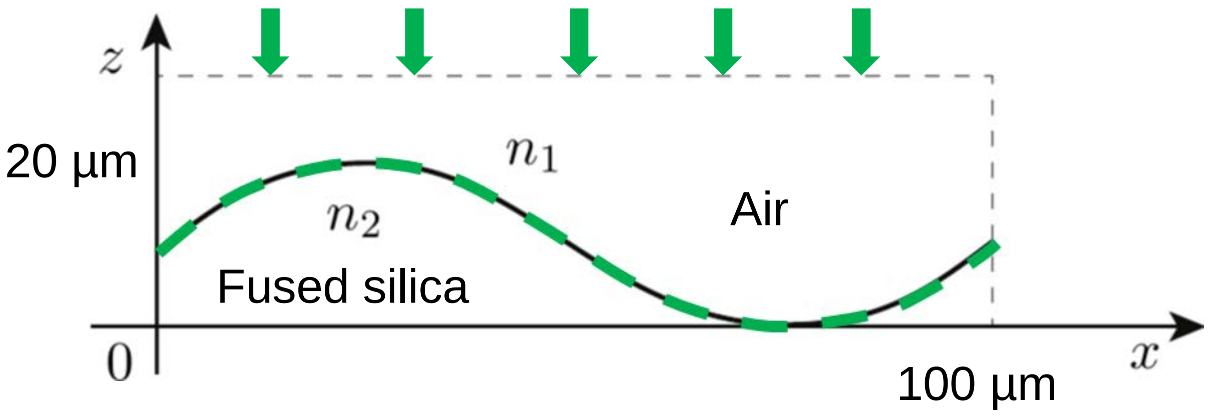


Field propagated: LPIA + Propagation



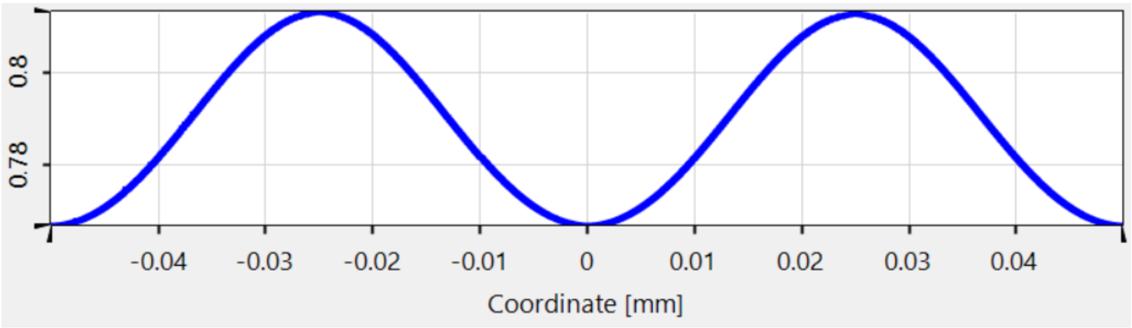
Simulations: Transmitted Field On Surface (TE polarized)

High NA focus!

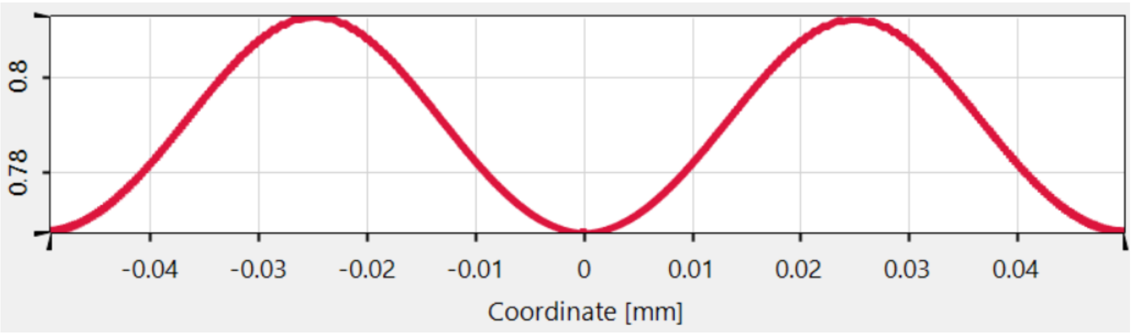


Field inside

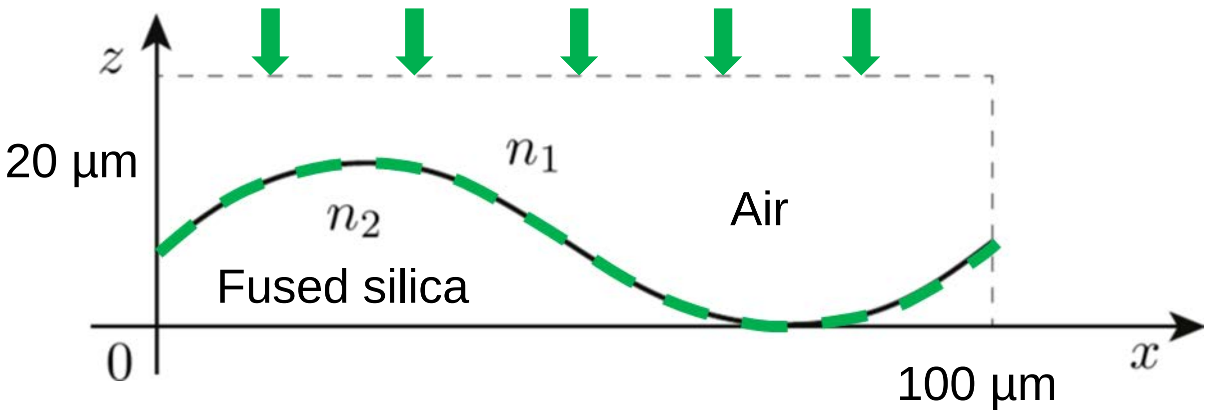
Field on interface: Rigoros



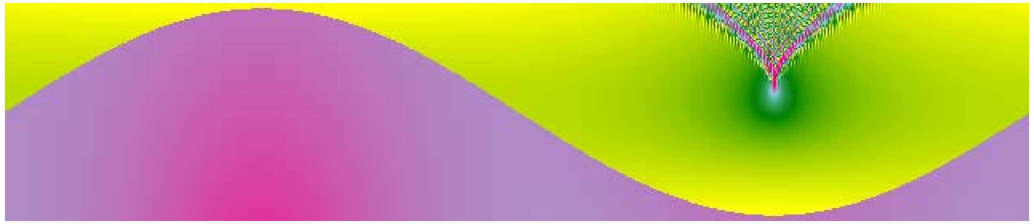
Field on interface: LPIA



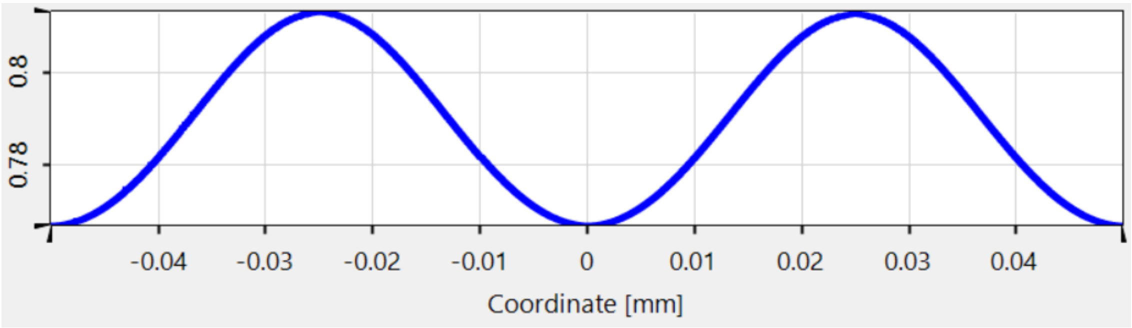
Simulations: Transmitted Field On Surface (TE polarized)



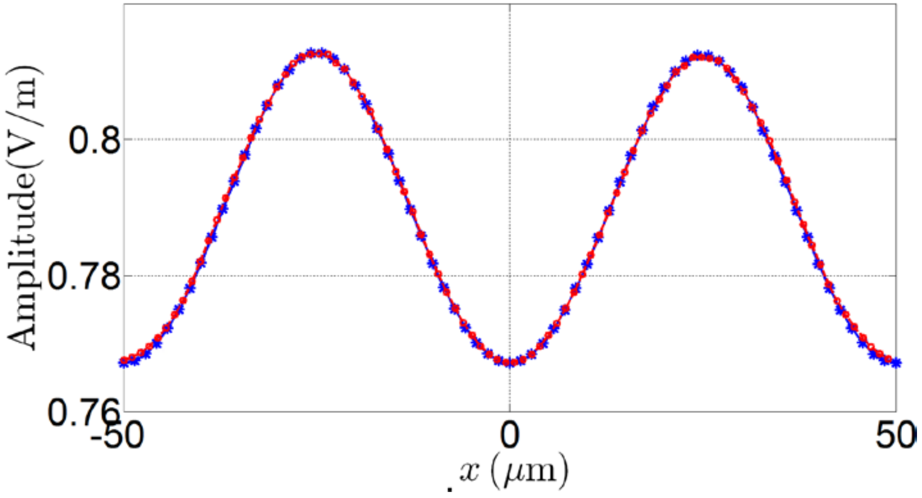
Field inside



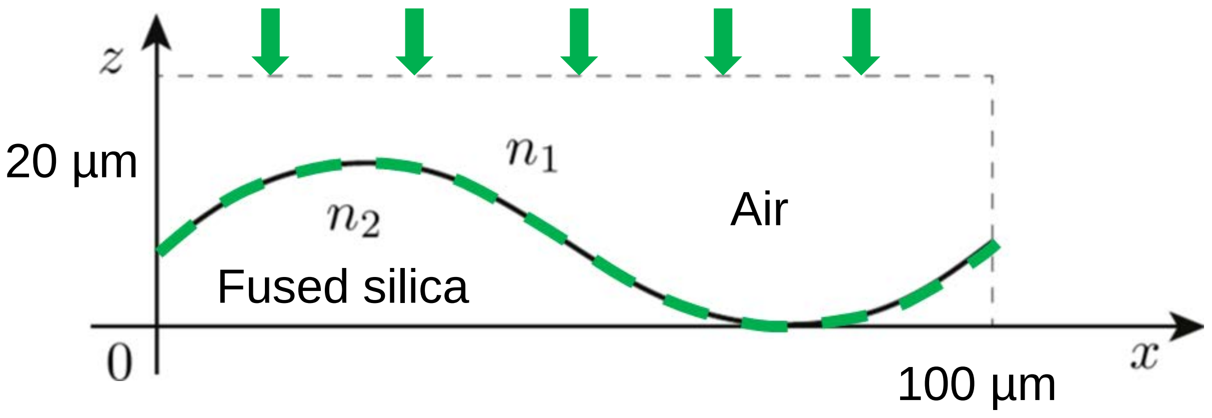
Field on interface: Rigoros



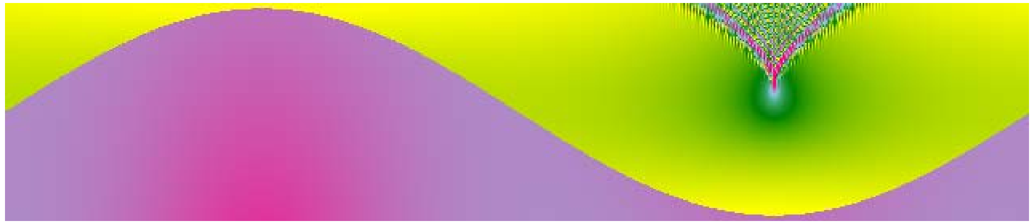
Field on interface: Comparison



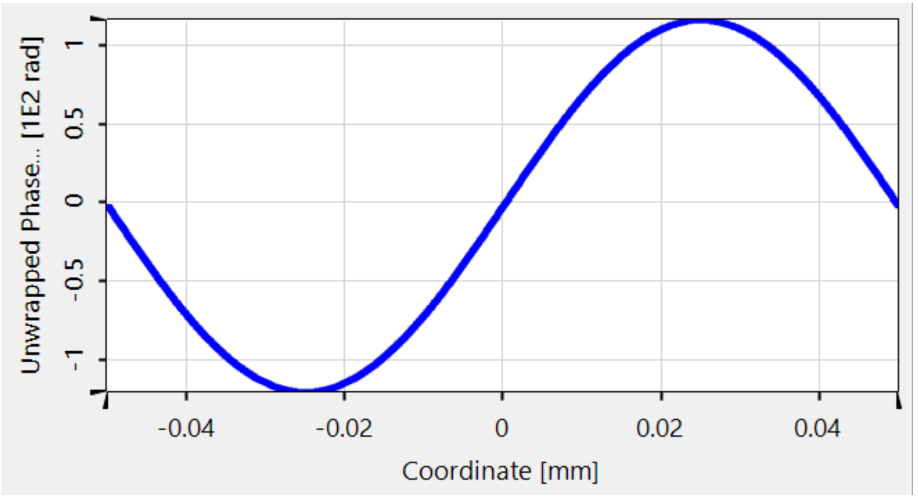
Simulations: Transmitted Phase On Surface (TE polarized)



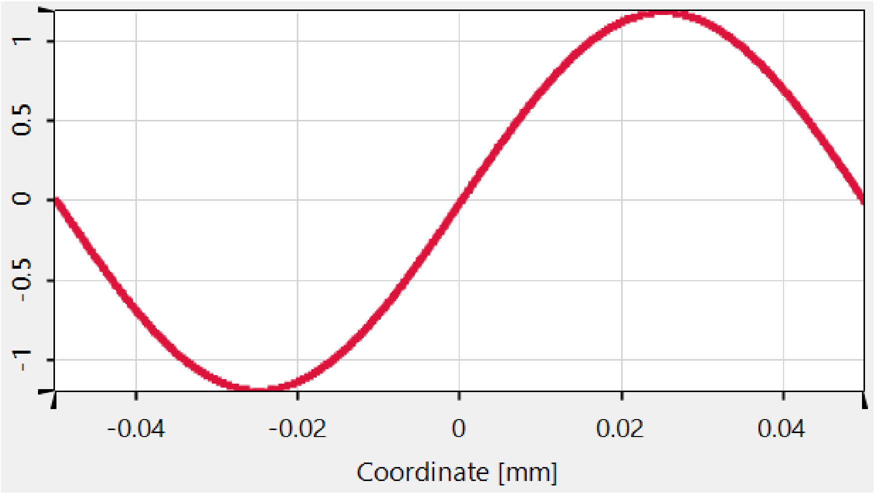
Field inside



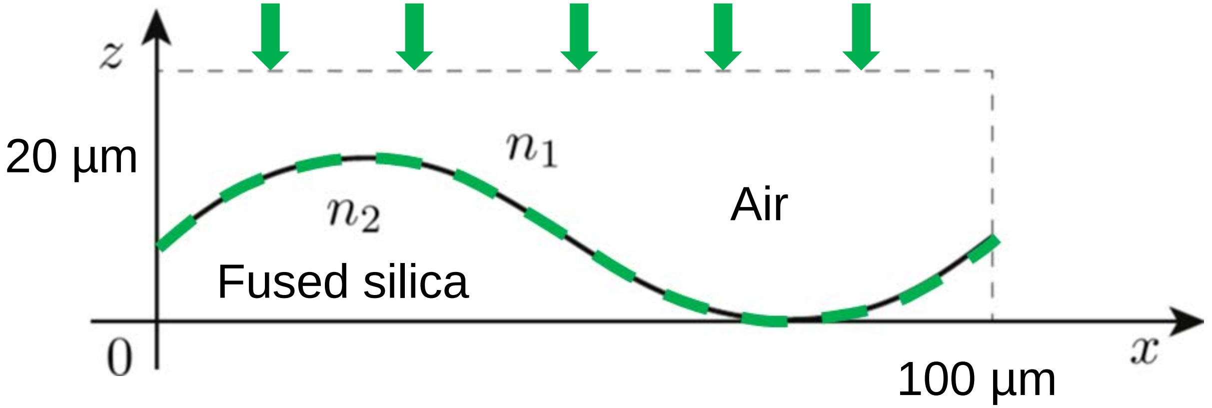
Phase on interface: Rigoros



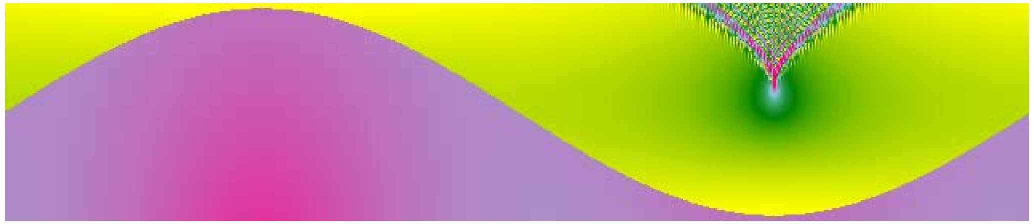
Phase on interface: LPIA



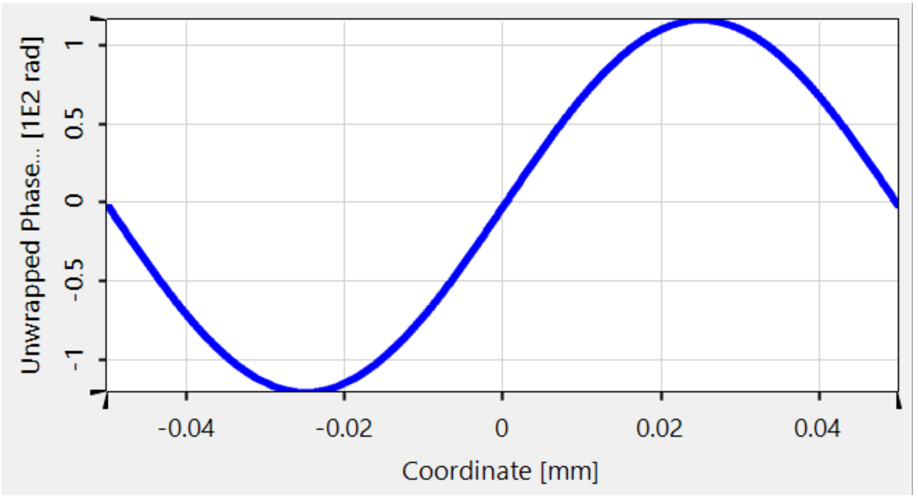
Simulations: Transmitted Phase On Surface (TE polarized)



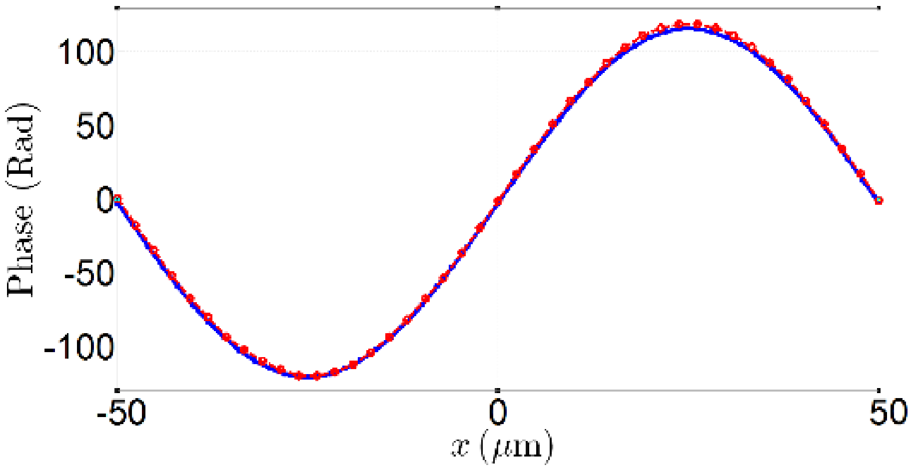
Field inside



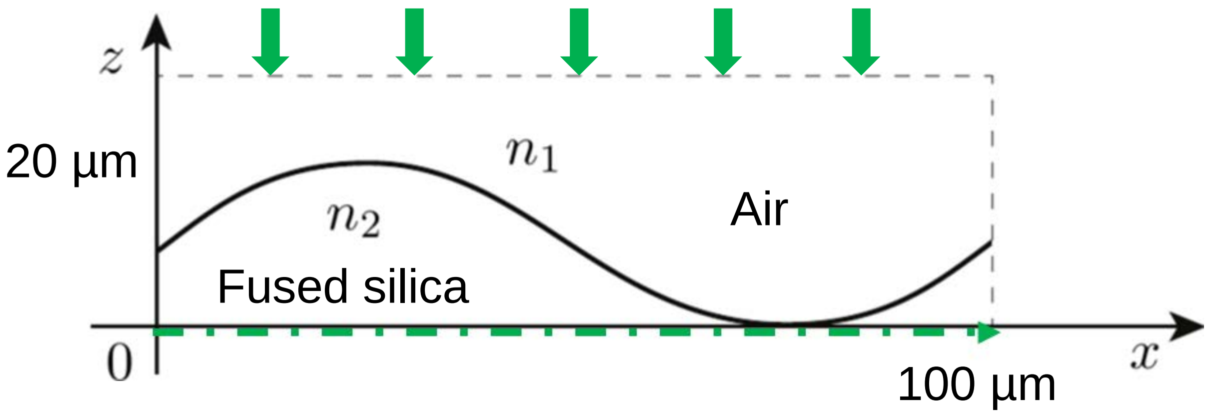
Phase on interface: Rigoros



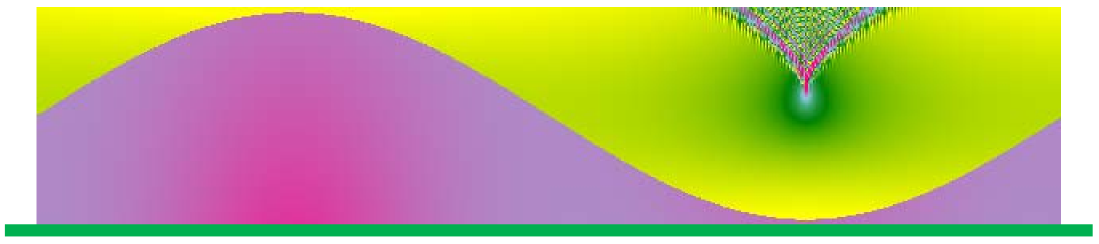
Phase on interface: Comparison



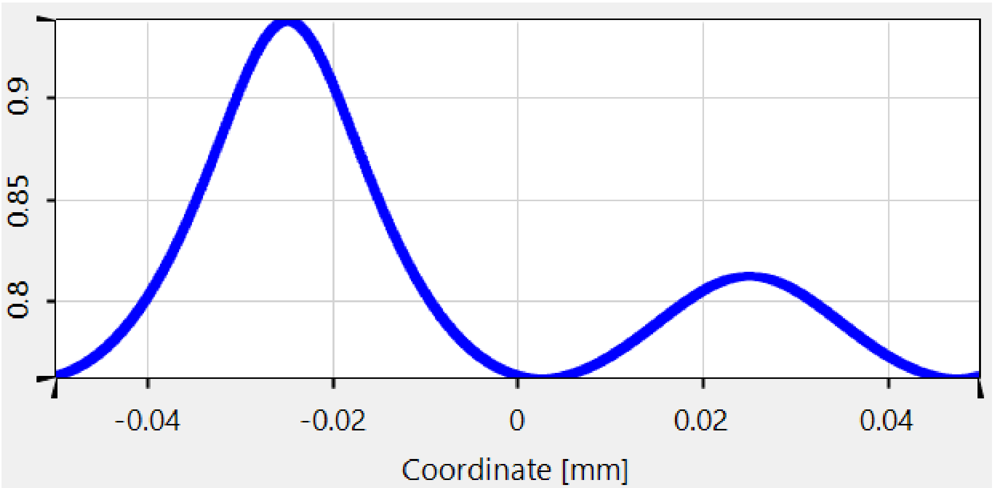
Simulations: Transmitted Field Propagated (TE polarized)



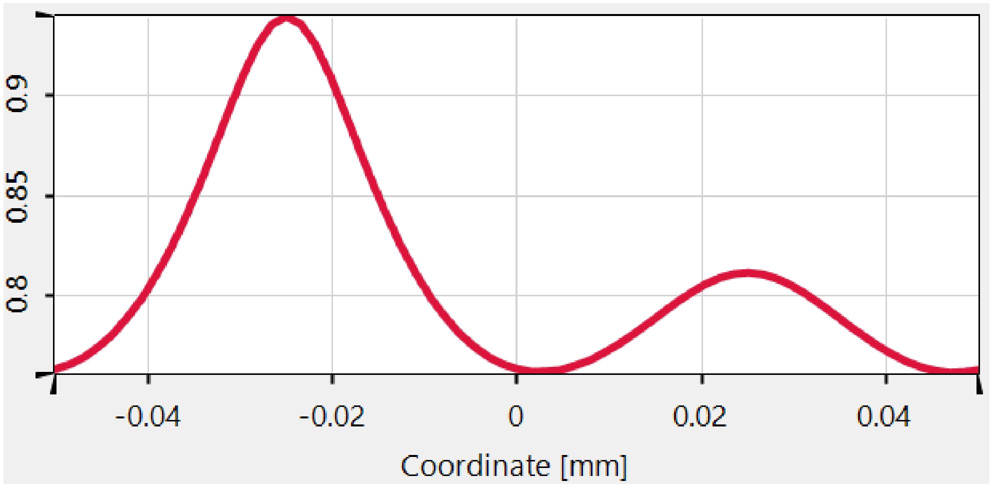
Field inside



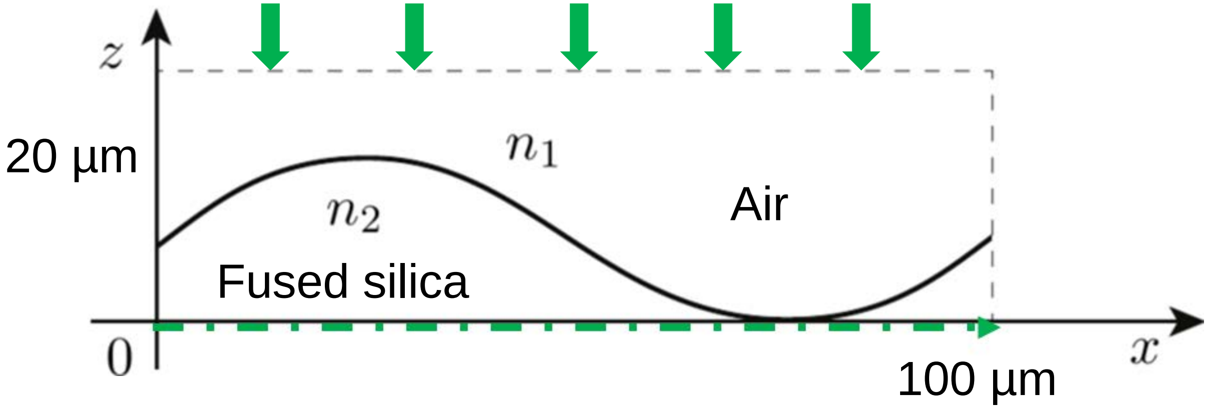
Field propagated: Rigoros



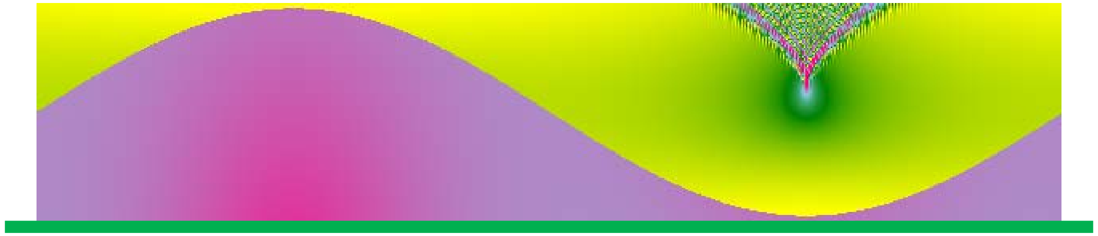
Field propagated: LPIA + Propagation



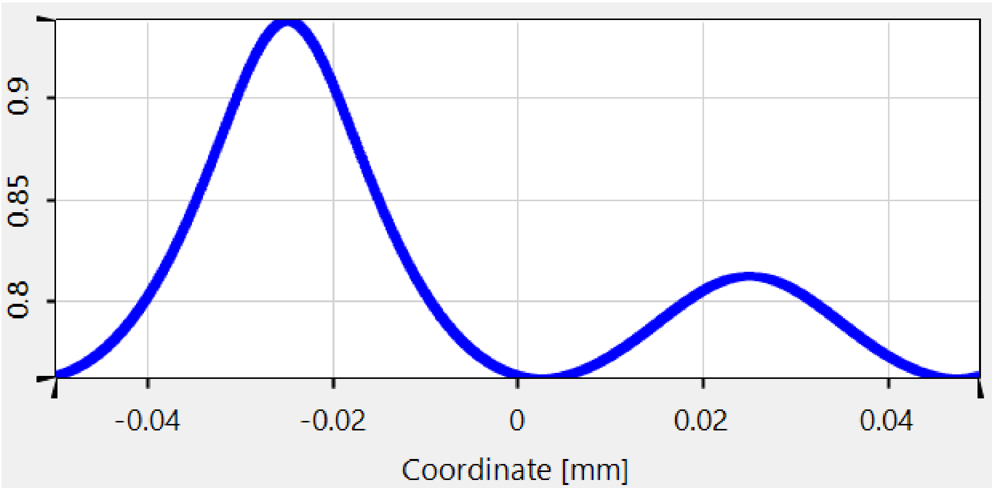
Simulations: Transmitted Field Propagated (TE polarized)



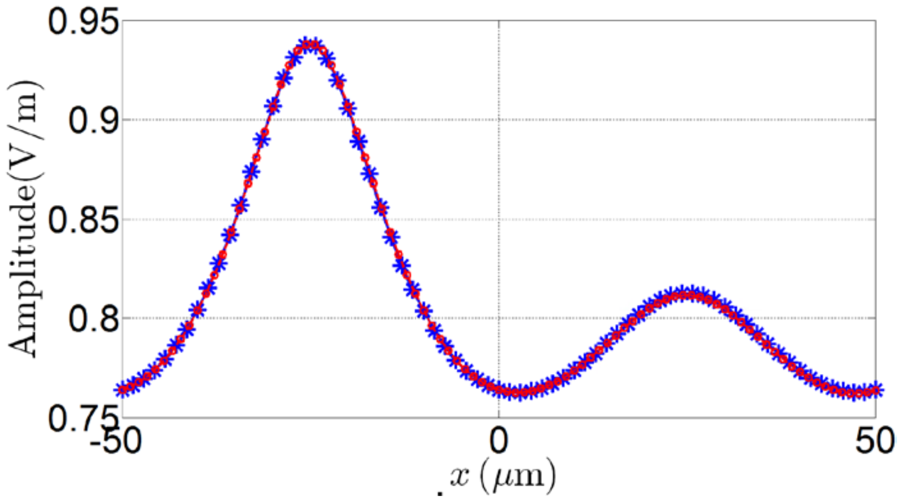
Field inside



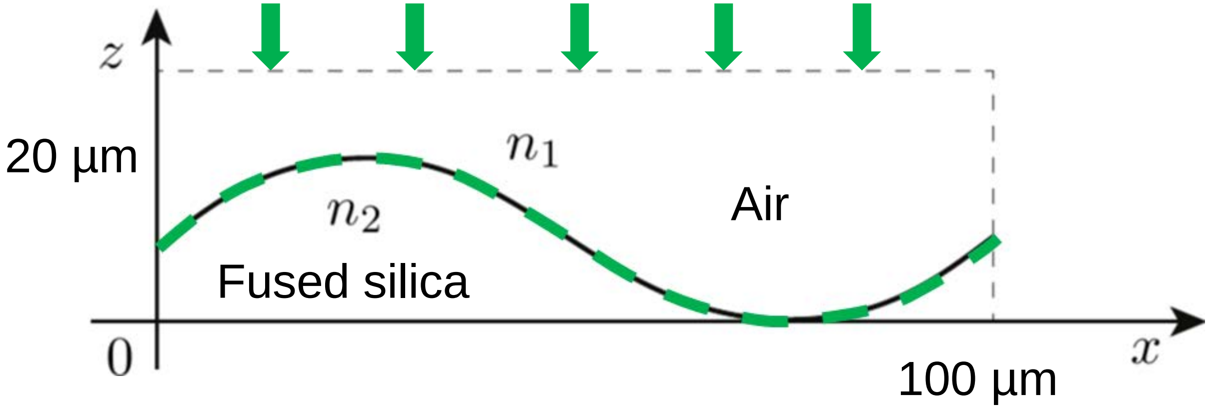
Field propagated: Rigoros



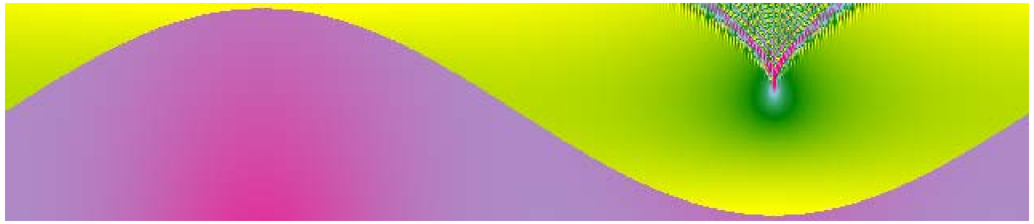
Field propagated: Comparison



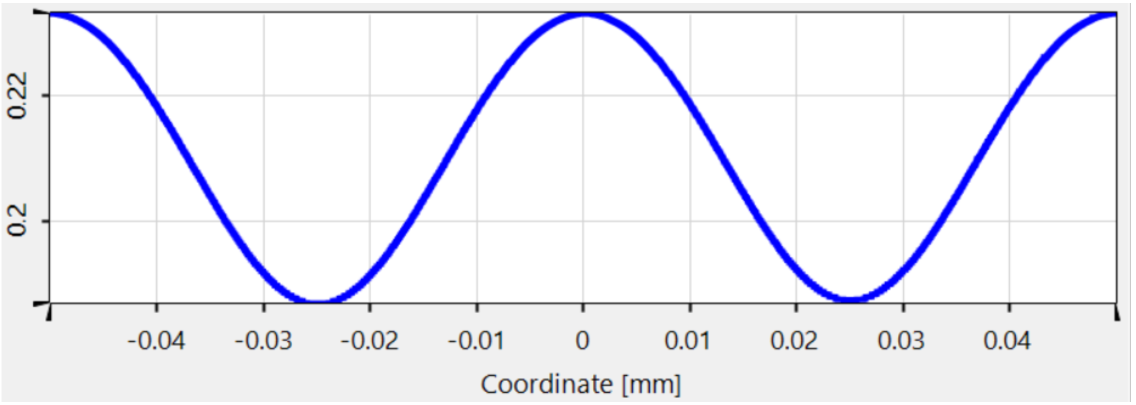
Simulations: Reflected Field On Surface (TE polarized)



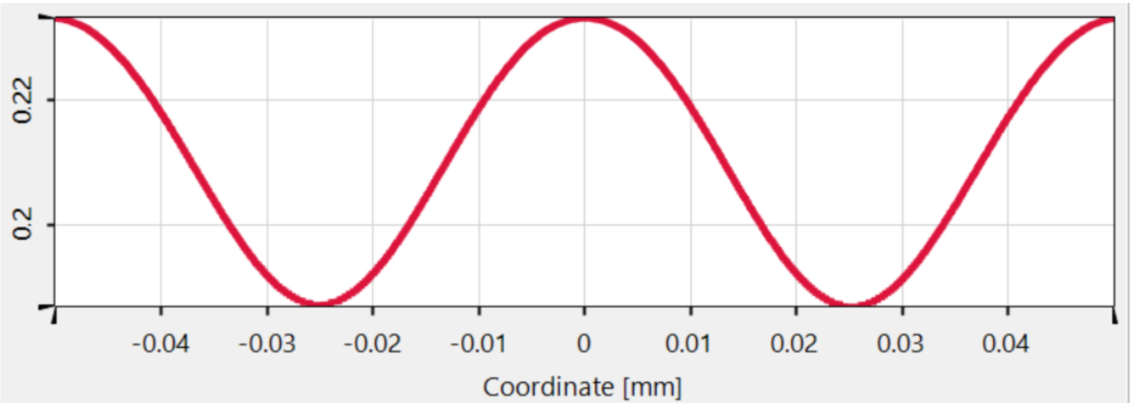
Field inside



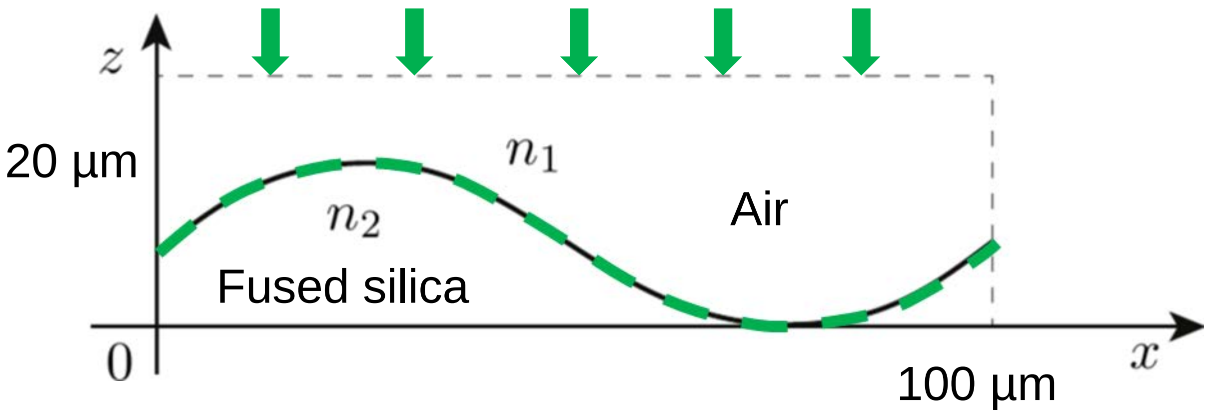
Field on interface: Rigoros



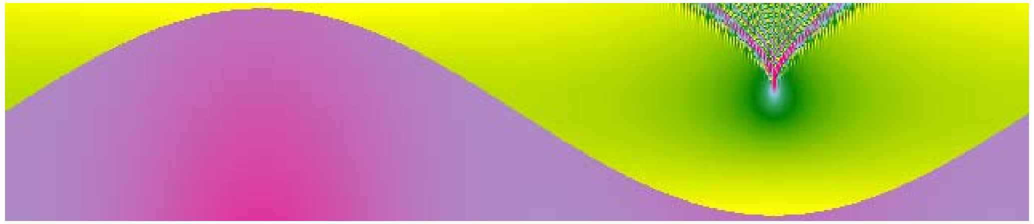
Field on interface: LPIA



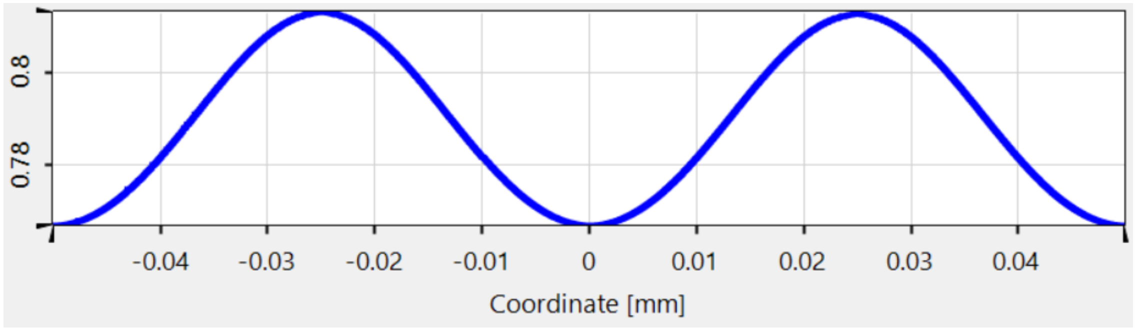
Simulations: Reflected Field On Surface (TE polarized)



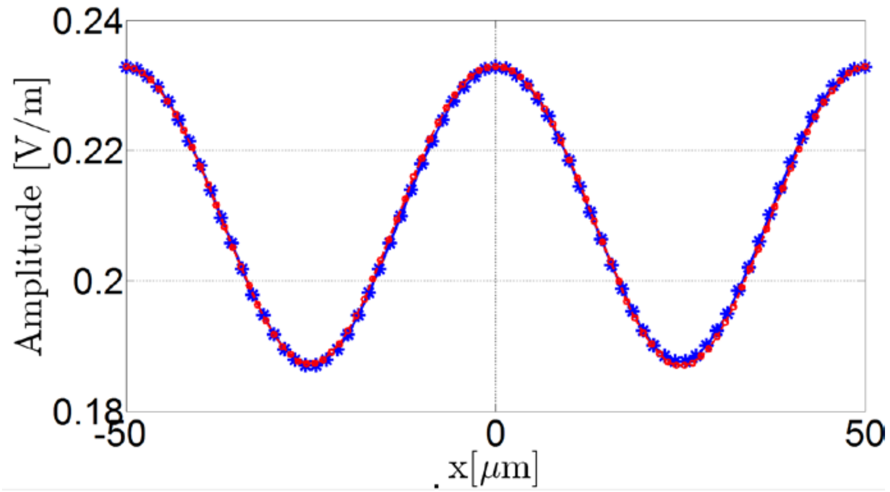
Field inside



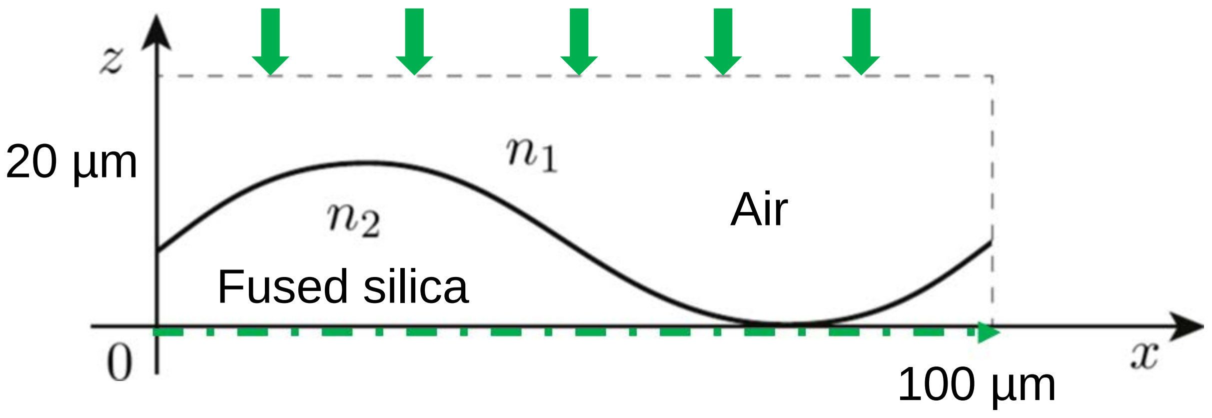
Field on interface: Rigoros



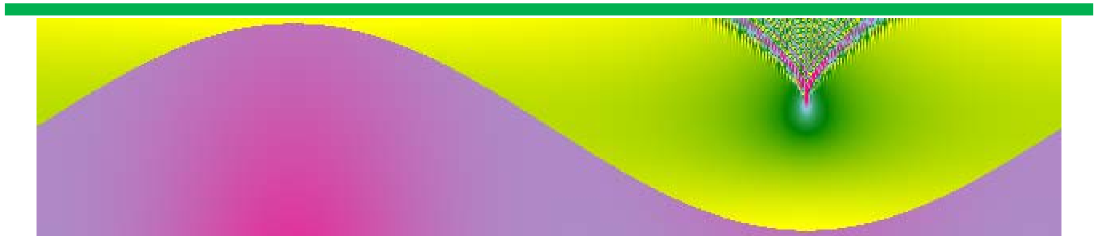
Field on interface: Comparison



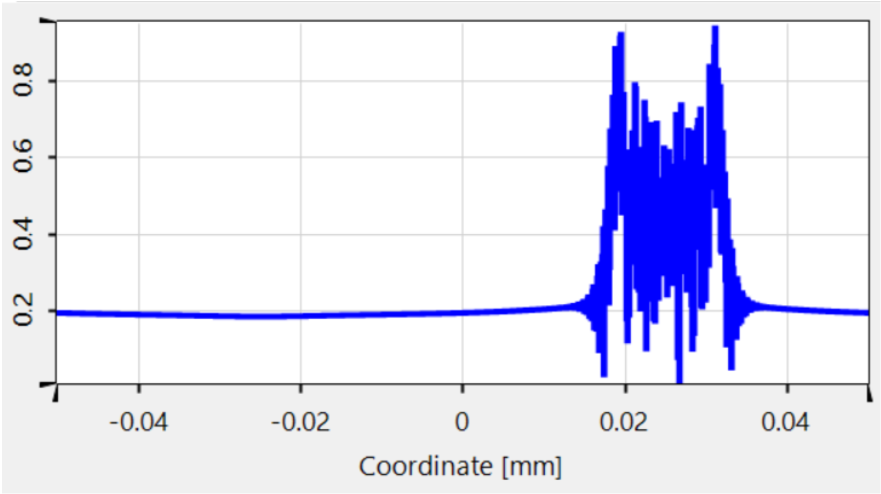
Simulations: Reflected Field Propagated (TE polarized)



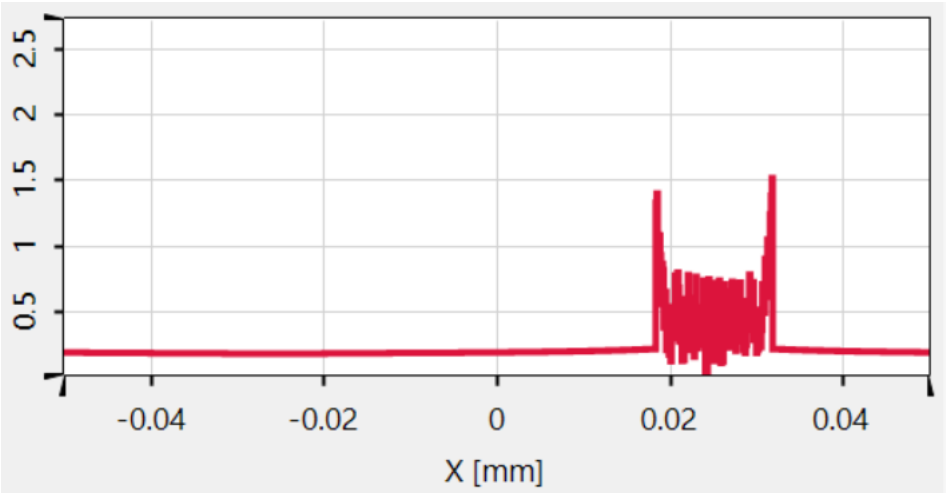
Field inside



Field propagated: Rigoros

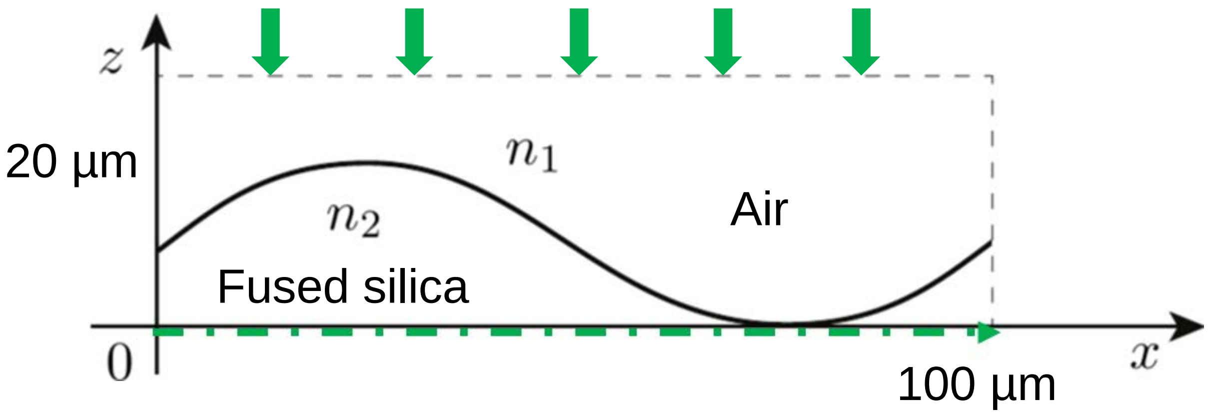


Field propagated: LPIA + Propagation

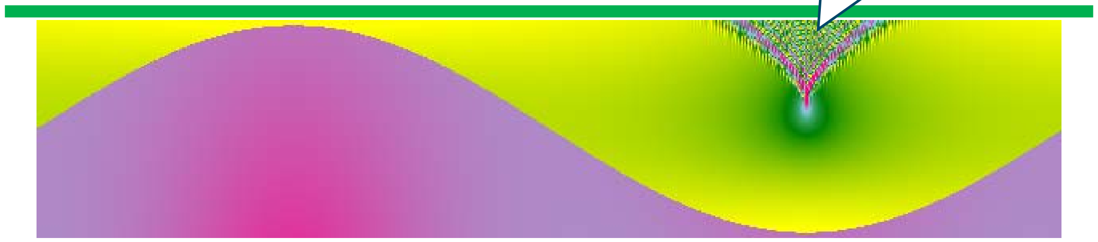


Simulations: Reflected Field Propagated (TE p

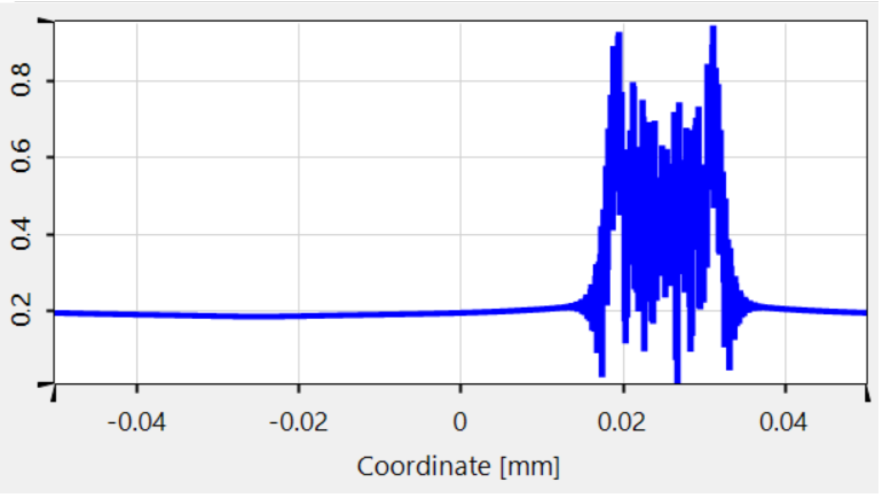
Propagation problem only! Can be solved.



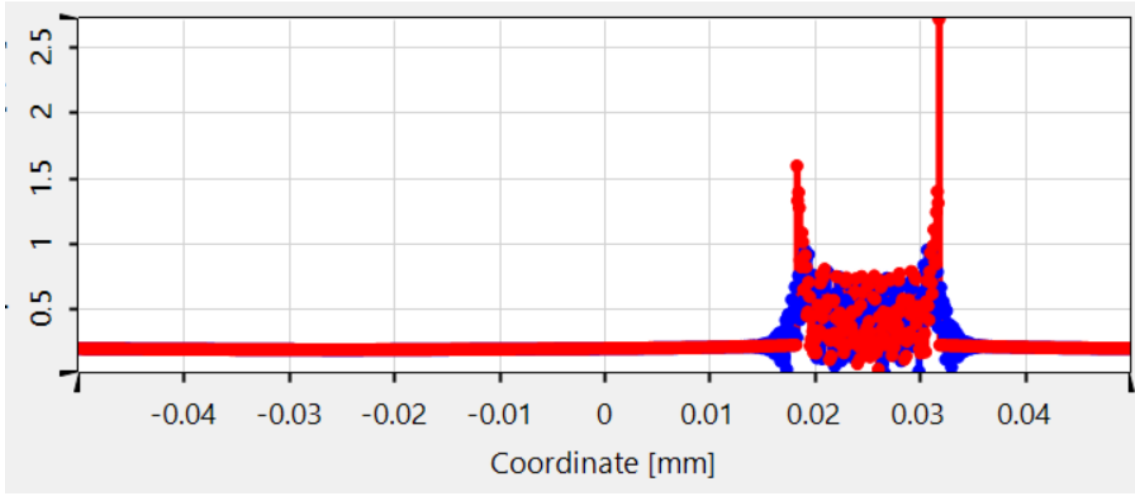
Field inside



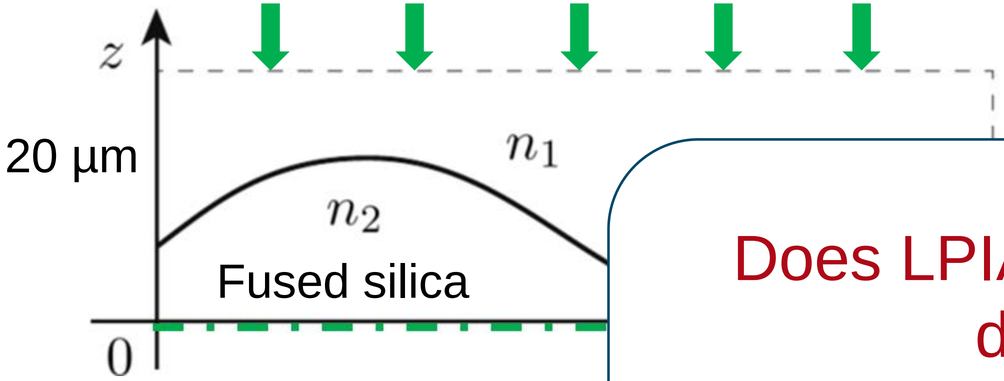
Field propagated: Rigoros



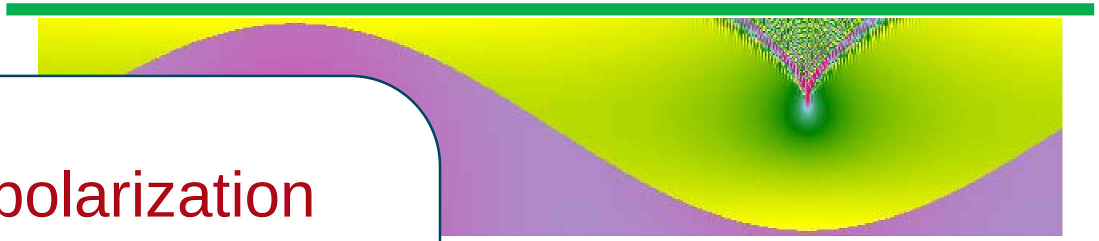
Field propagated: Comparison



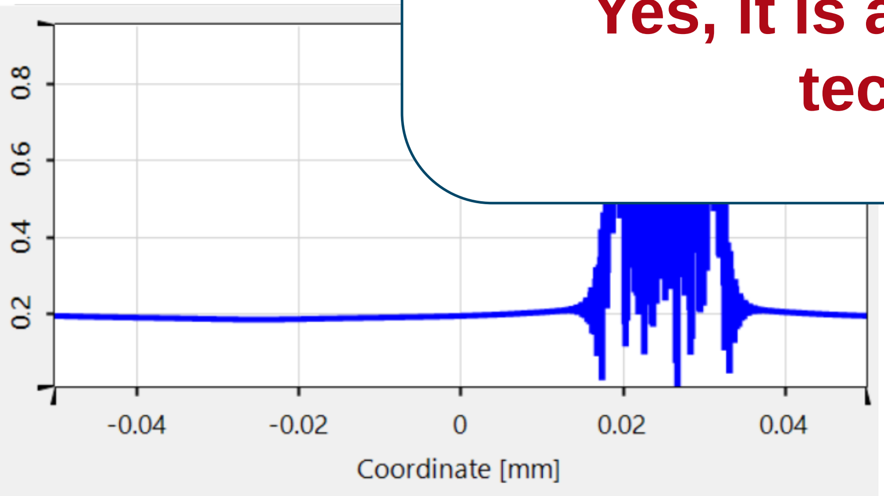
Simulations: Reflected Field Propagated (TE polarized)



Field inside



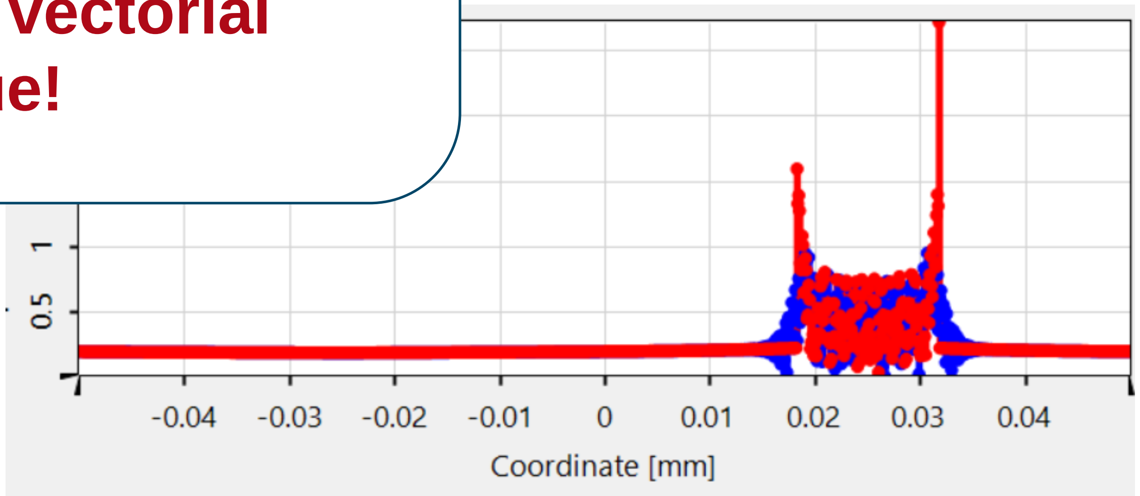
Field propagated: Ri



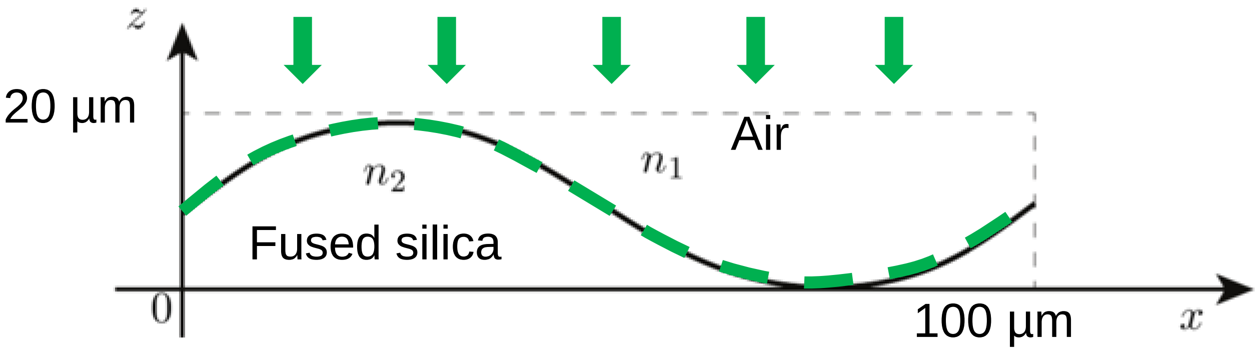
Does LPIA include polarization dependency?

Yes, it is a fully vectorial technique!

: Comparison

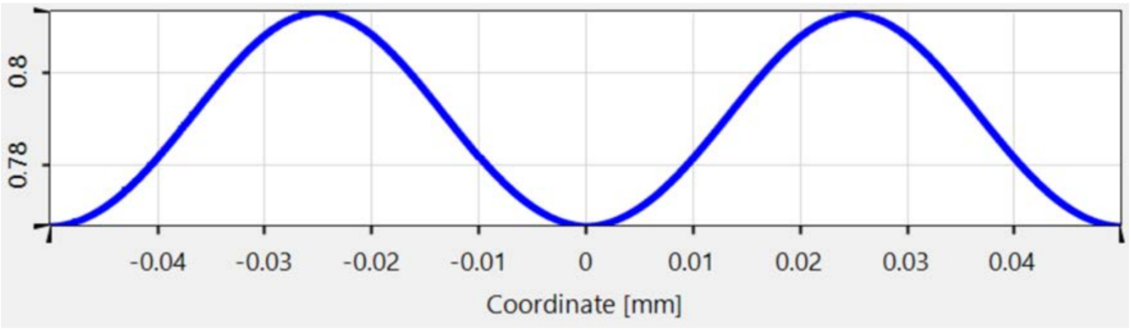


Simulations: Transmitted Field On Surface (TM polarized)

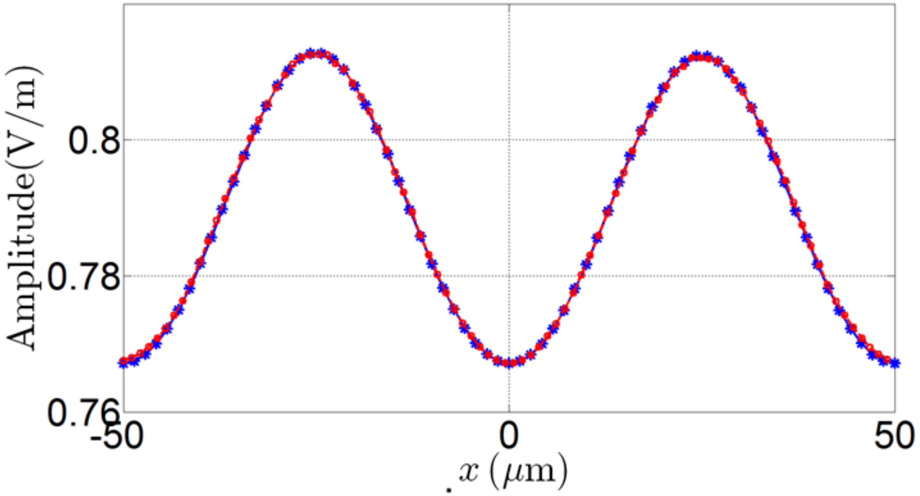


Field inside

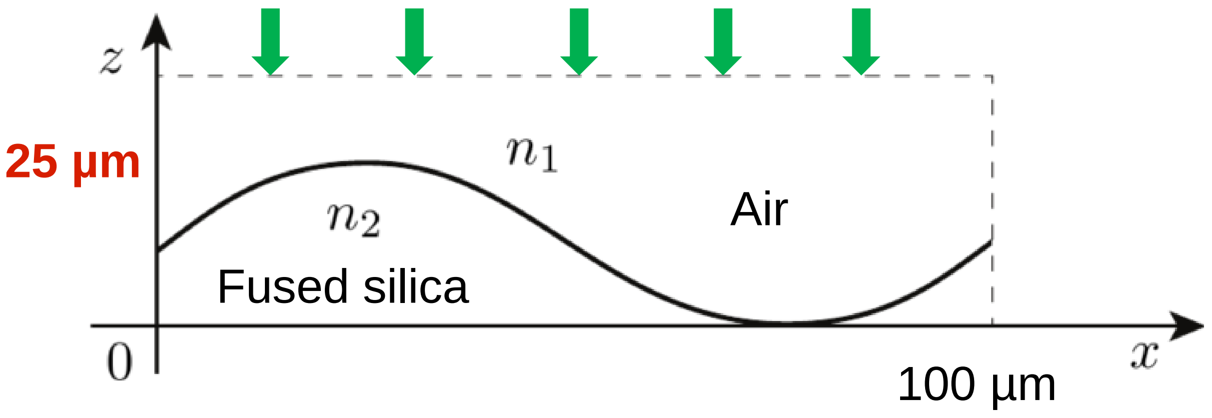
Field on interface: Rigorous

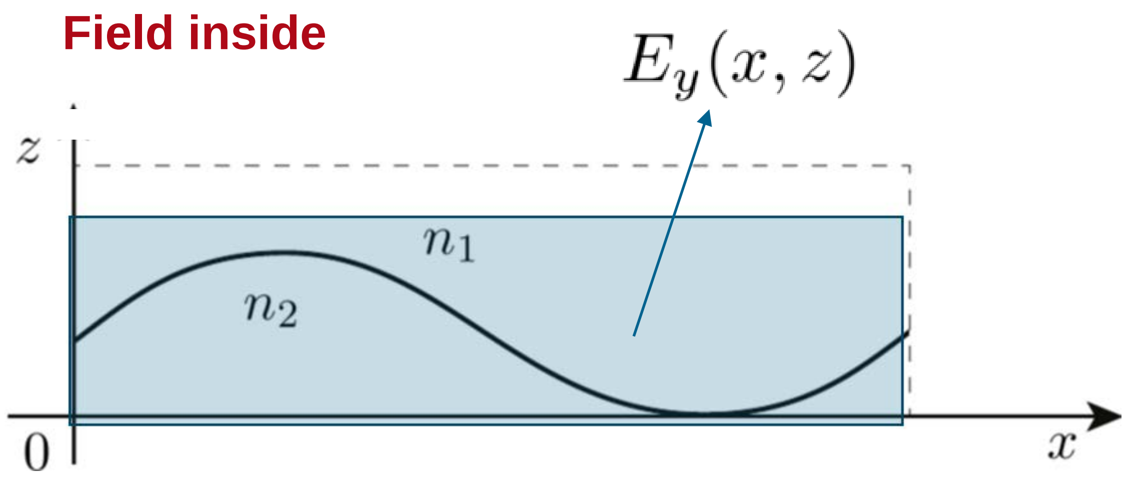


Field on interface: Comparison

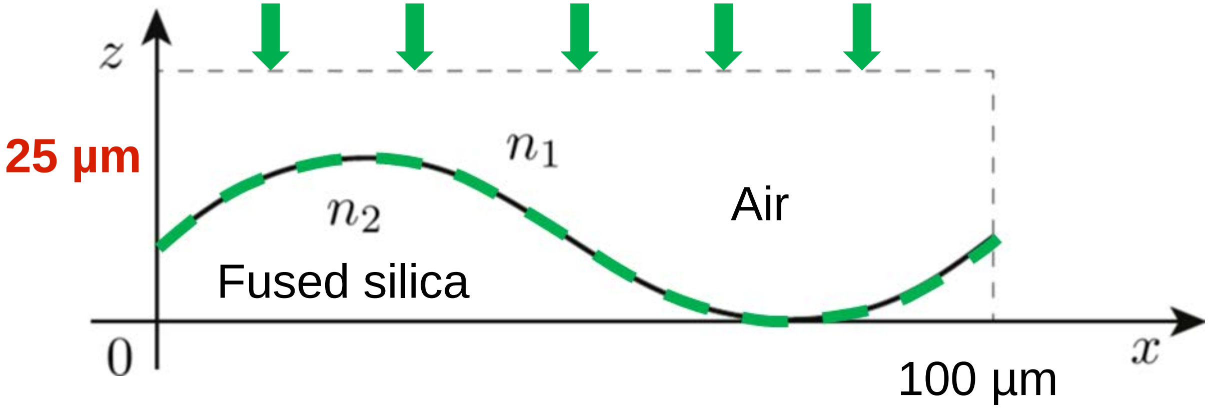


Simulations: Specifications Changed

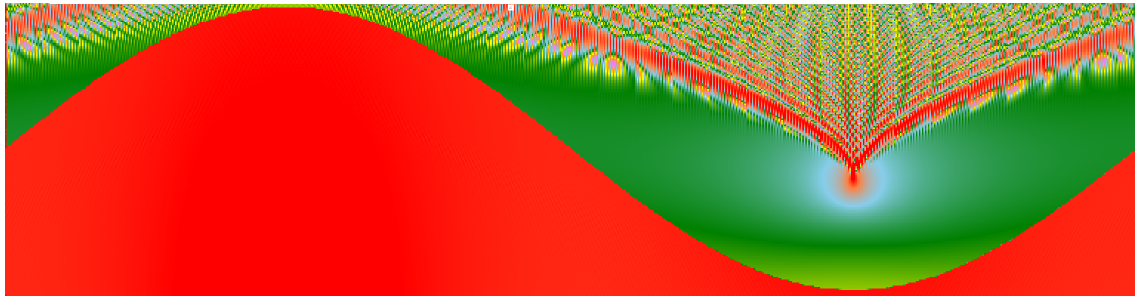




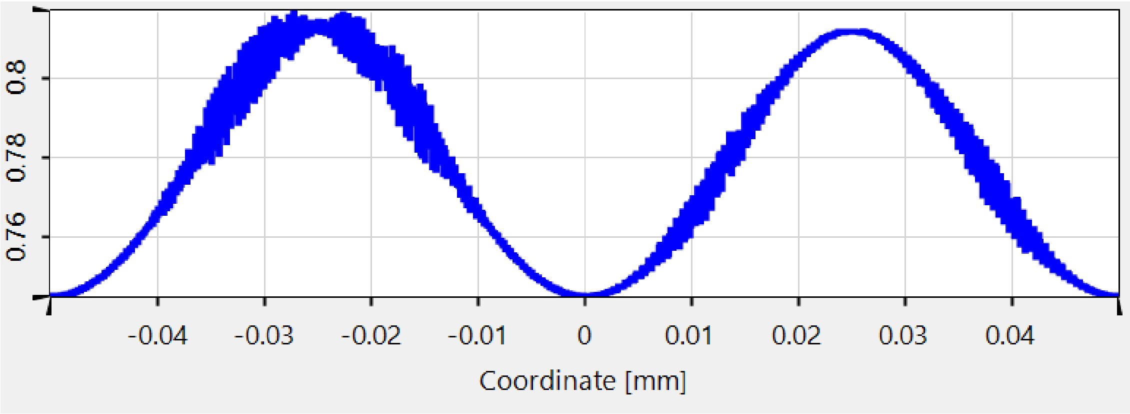
Simulations: Transmitted Field On Surface



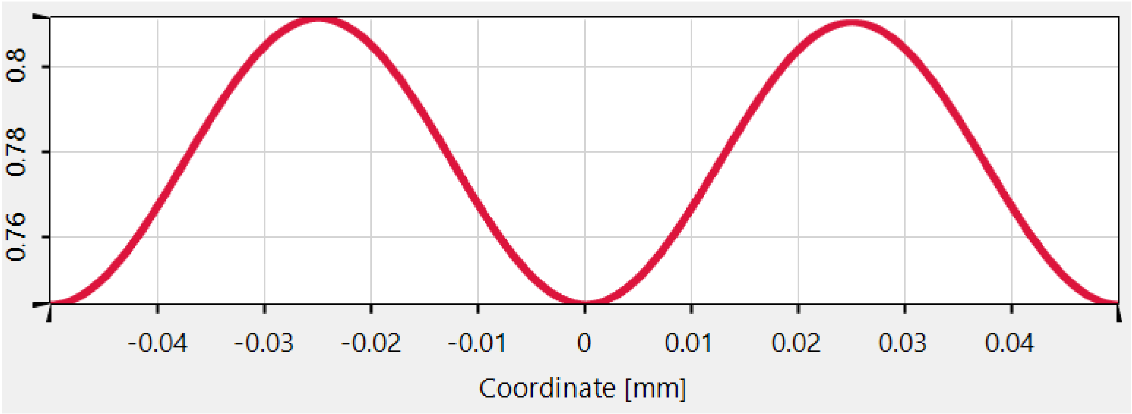
Field inside



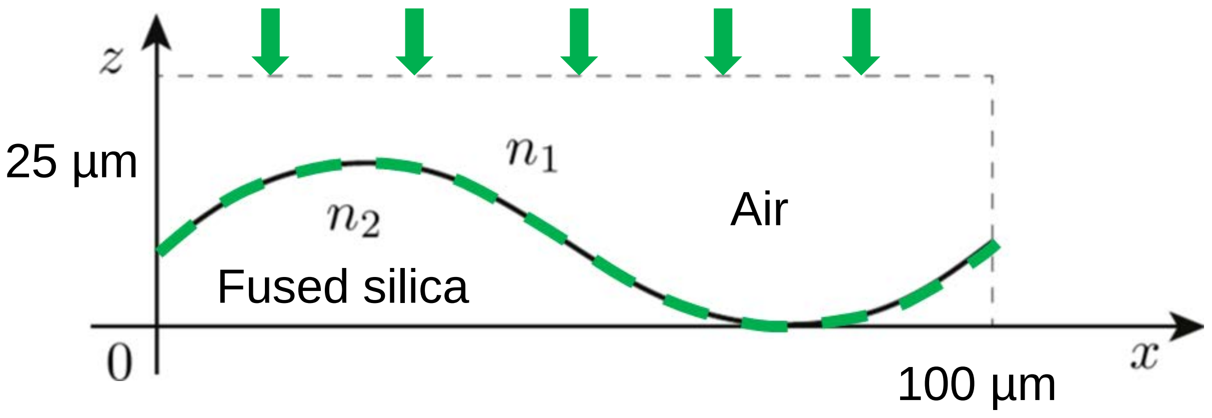
Field on interface: Rigoros



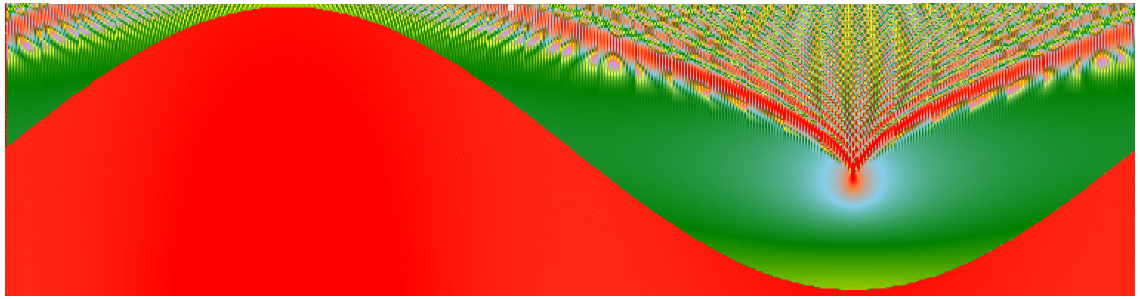
Field on interface: LPIA



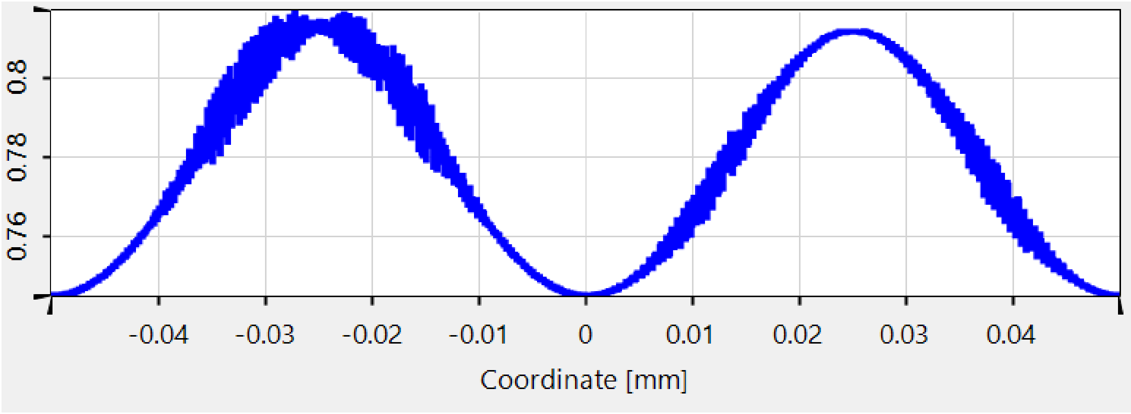
Simulations: Transmitted Field On Surface



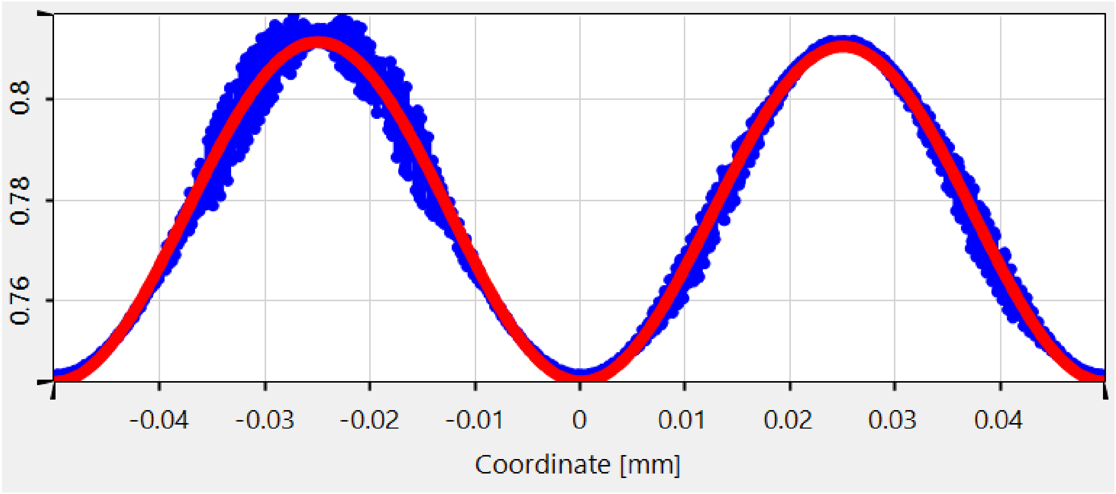
Field inside



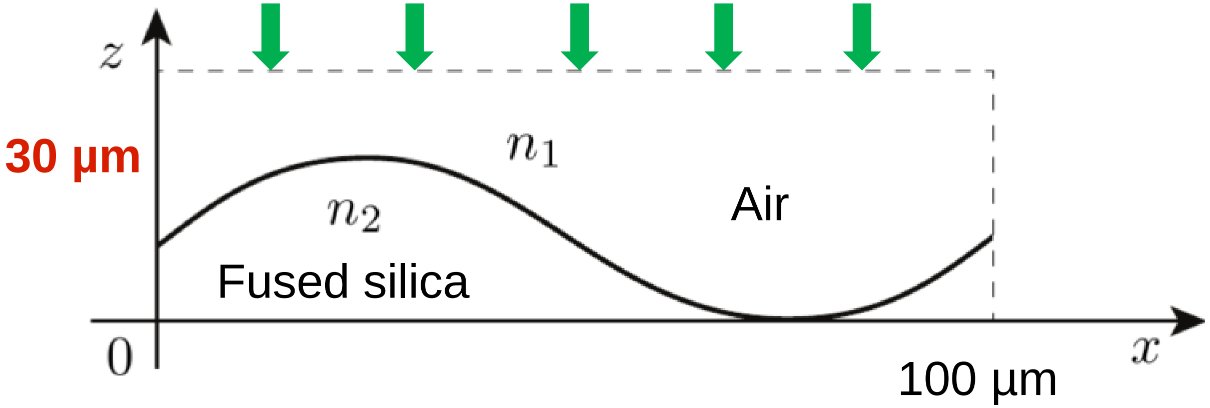
Field on interface: Rigoros



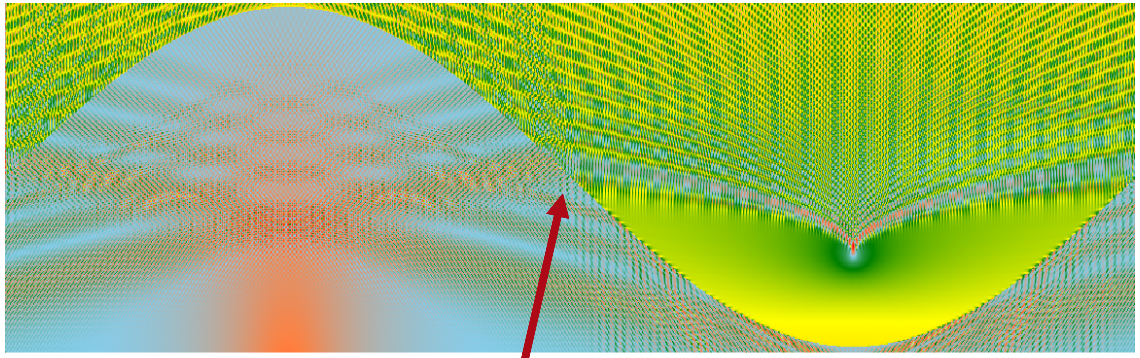
Field on interface: Comparison



Simulations: Specifications Changed

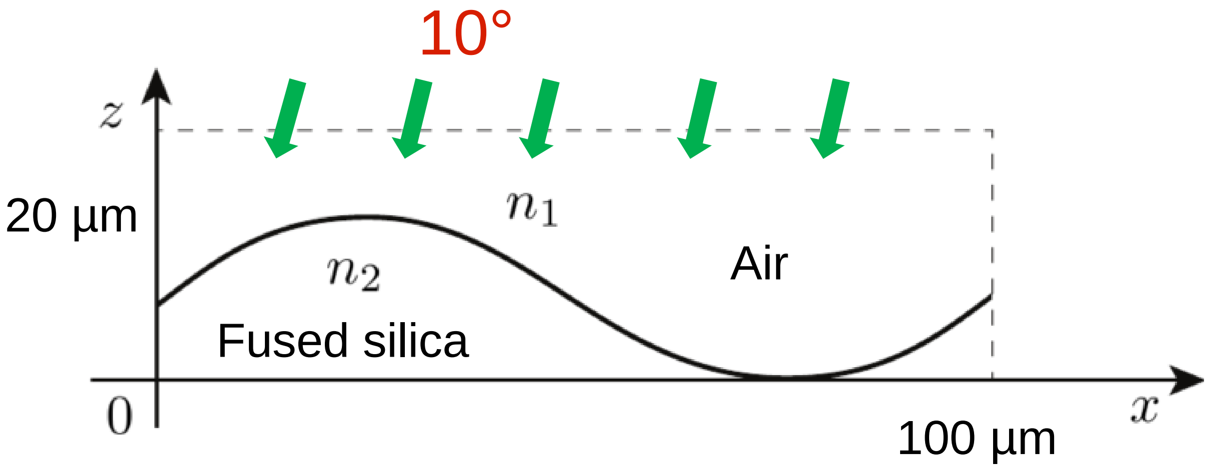


Field inside

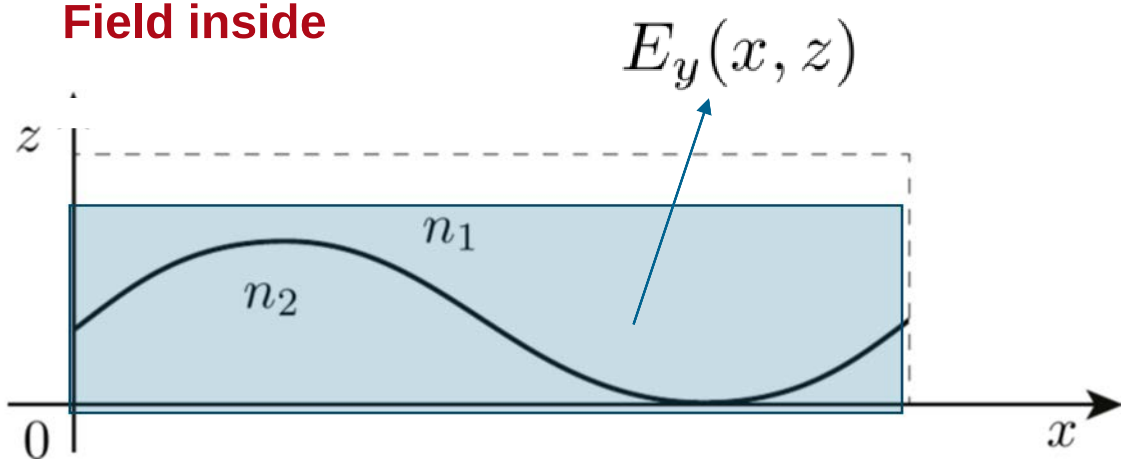


- Multiple scattering at surface is not taken into account by LPIA
- Observed for very high local NA
- LPIA algorithm can easily detect it!

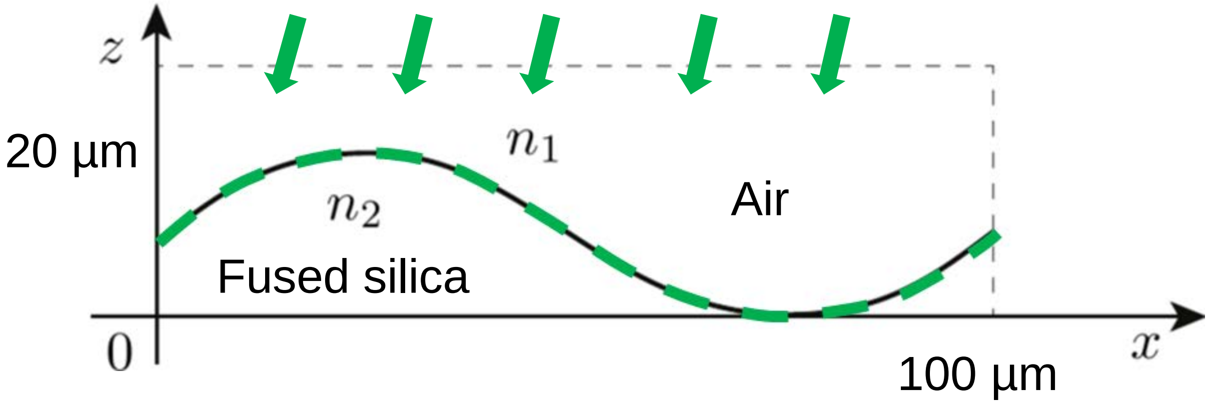
Simulations: Specifications Changed



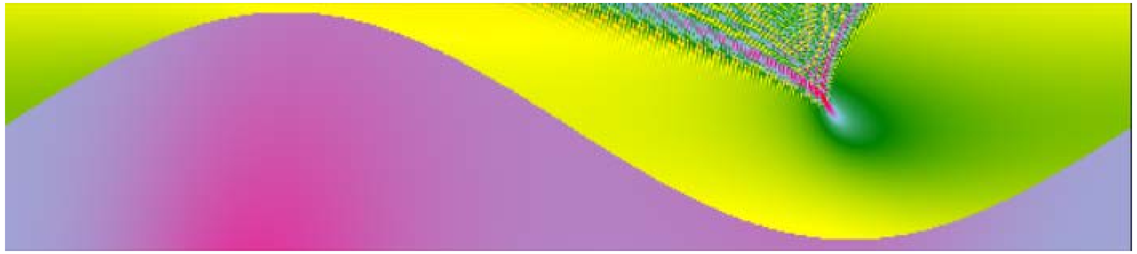
Field inside



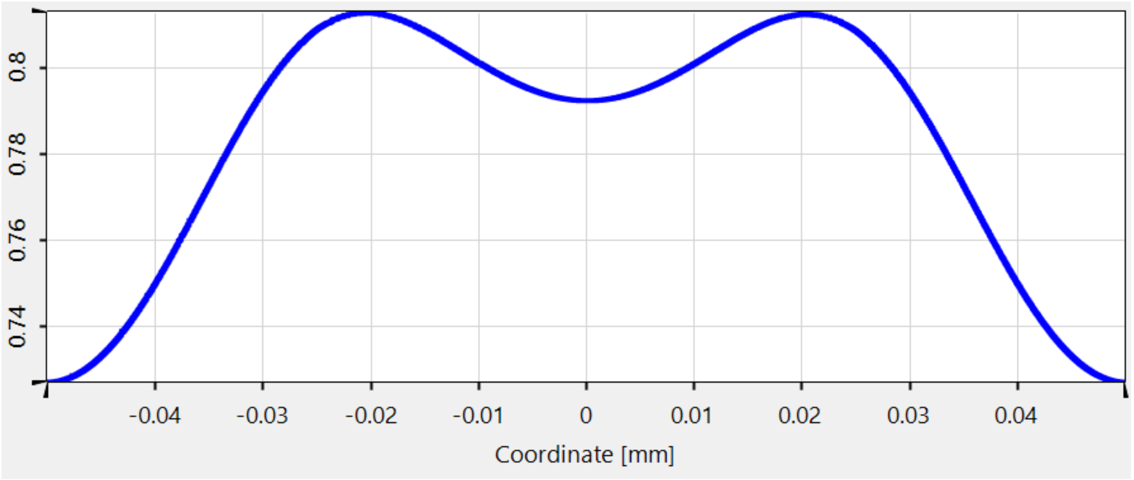
Simulations: Transmitted Field On Surface



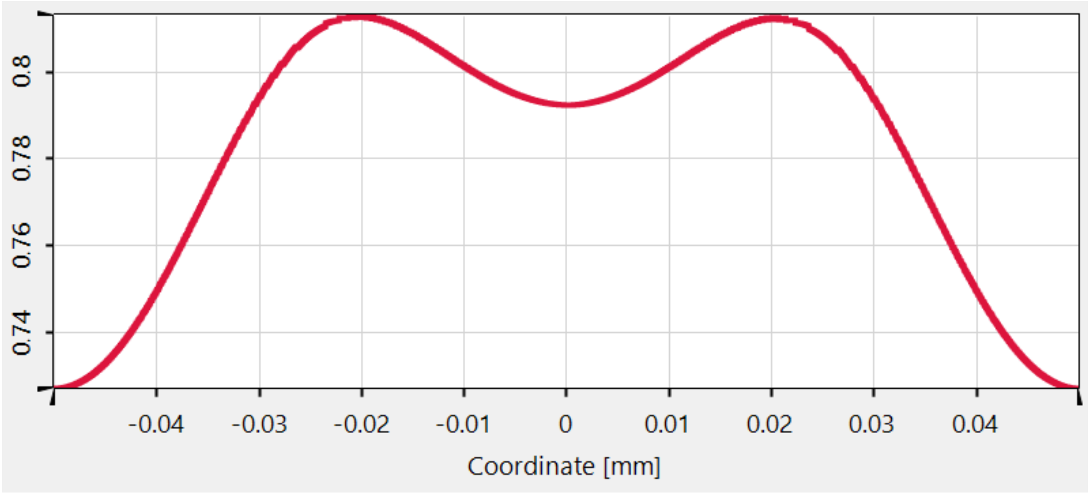
Field inside



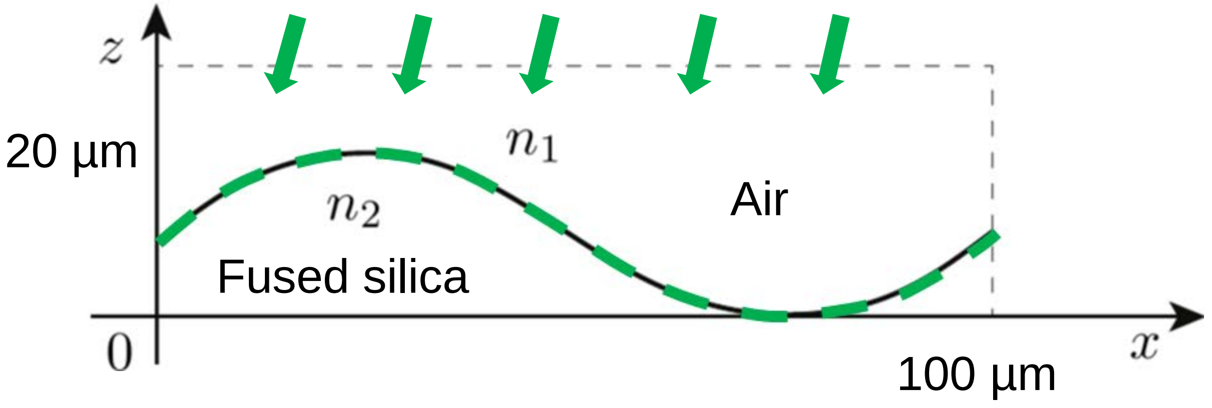
Field on interface: Rigoros



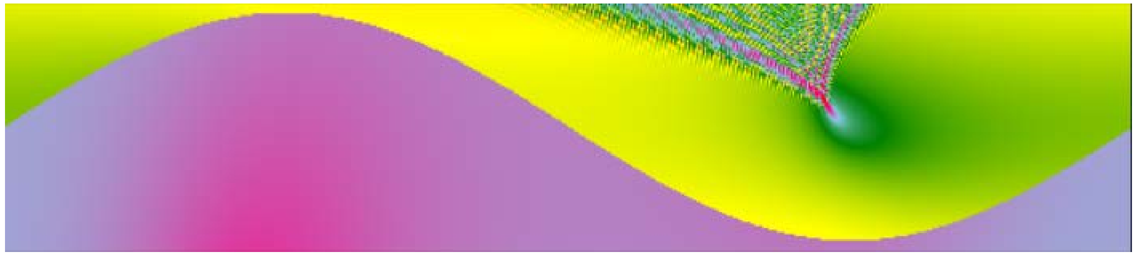
Field on interface: LPIA



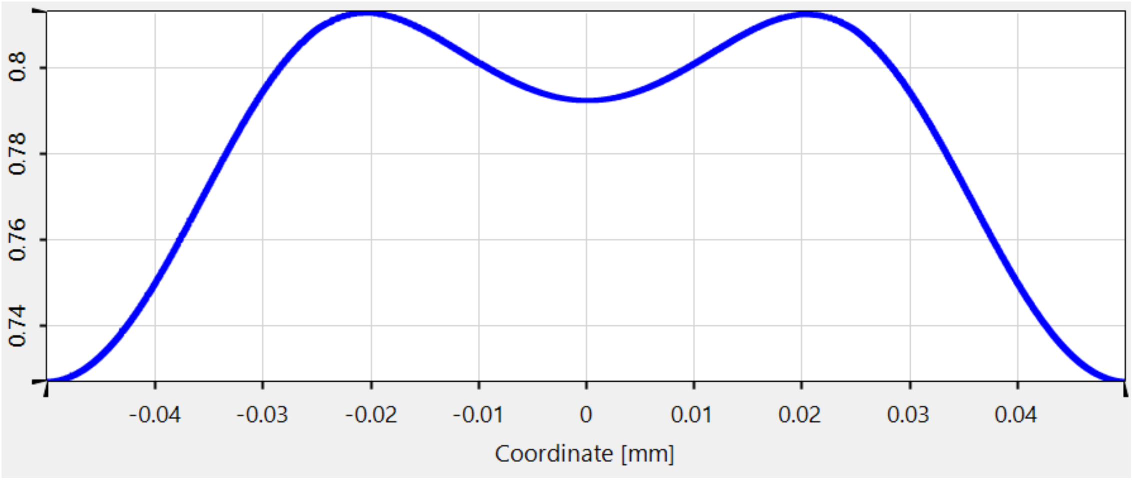
Simulations: Transmitted Field On Surface



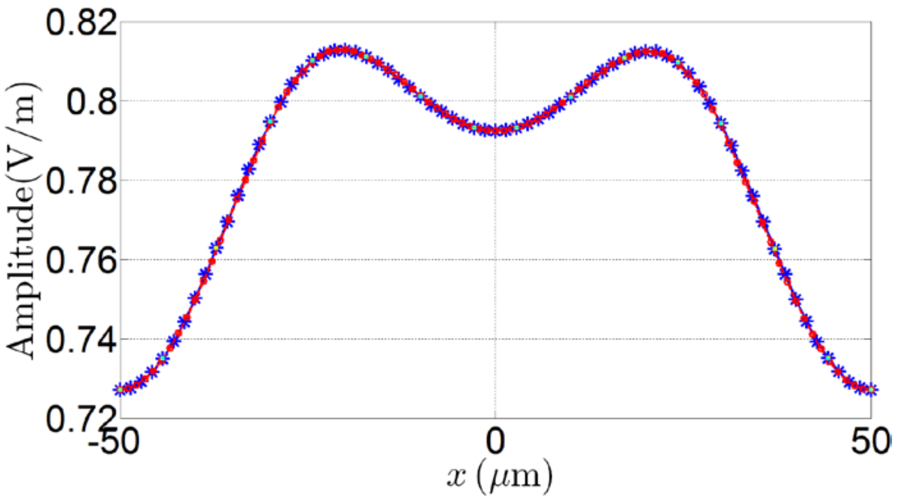
Field inside



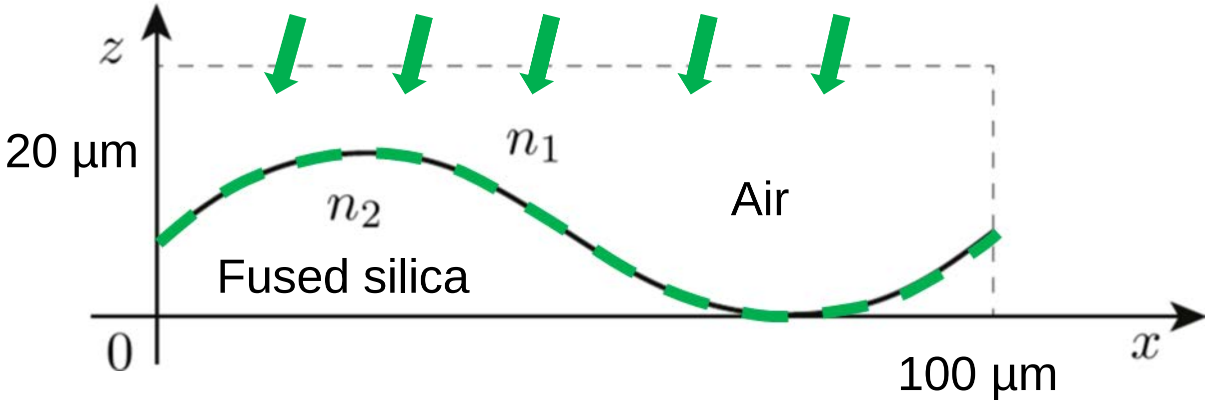
Field on interface: Rigoros



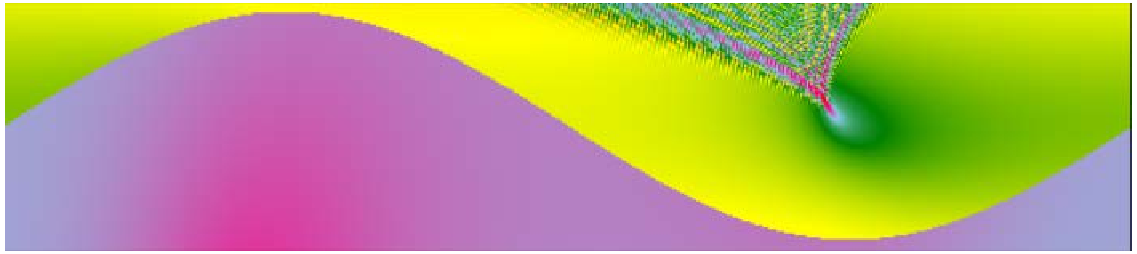
Field on interface: Comparison



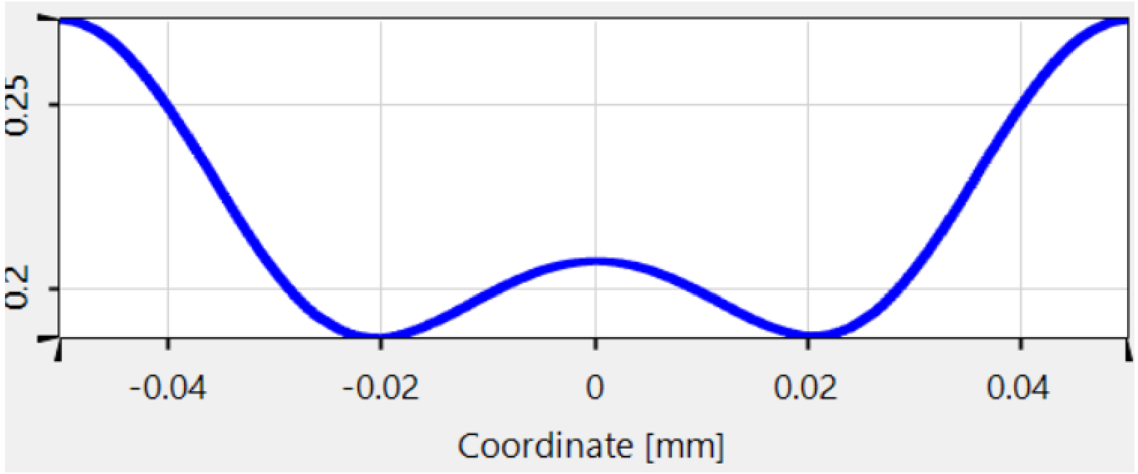
Simulations: Reflected Field On Surface



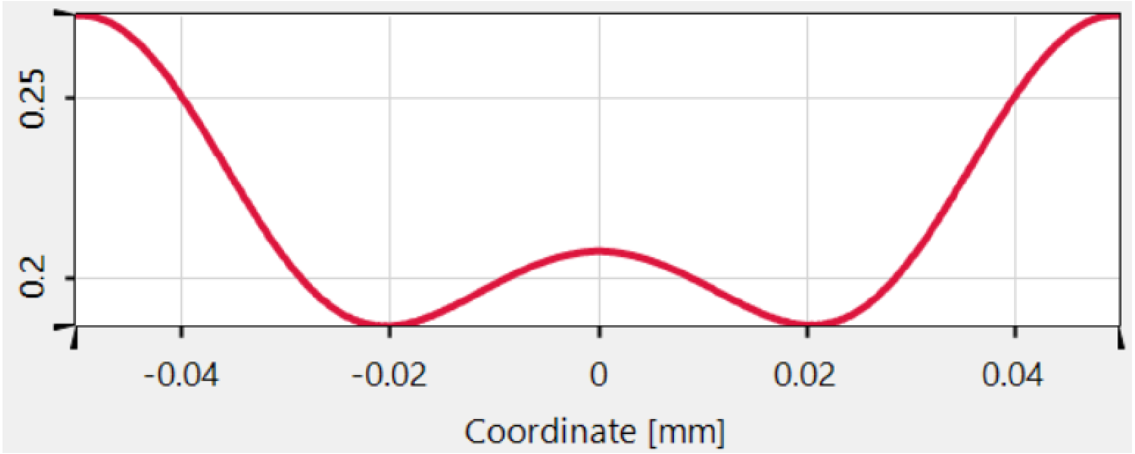
Field inside



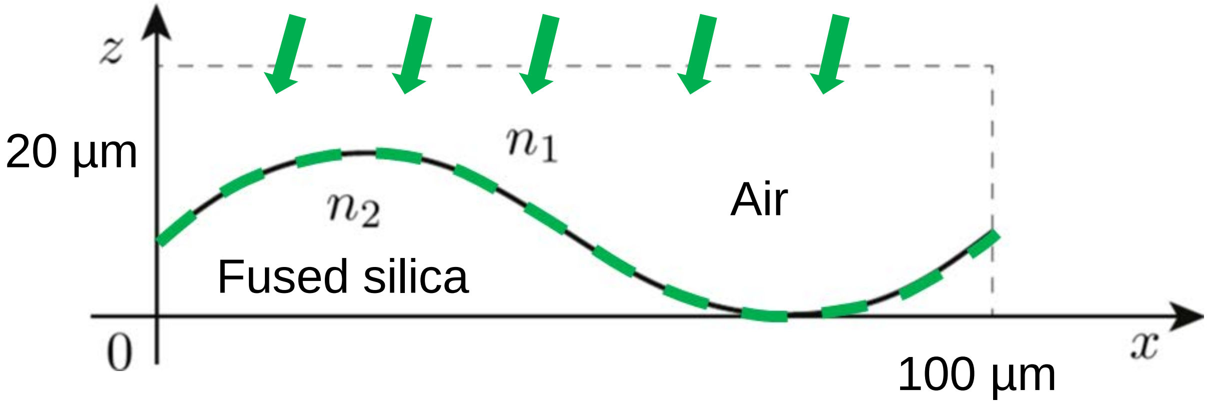
Field on interface: Rigoros



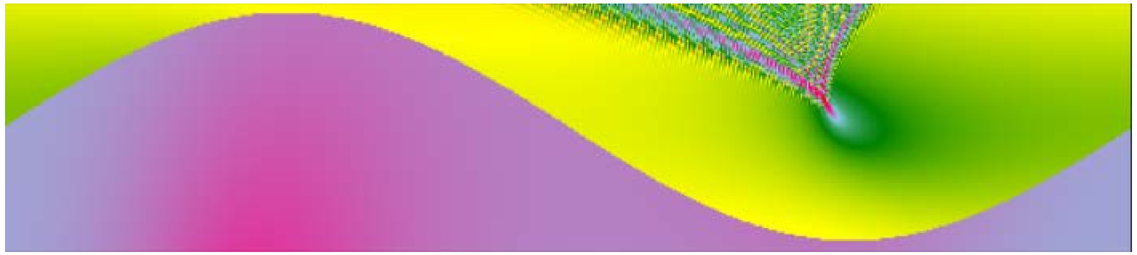
Field on interface: LPIA



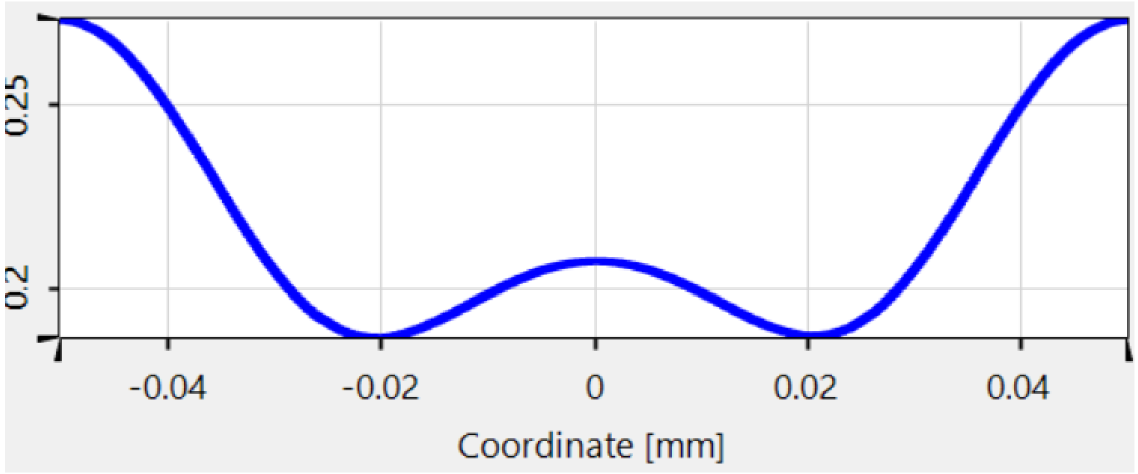
Simulations: Reflected Field On Surface



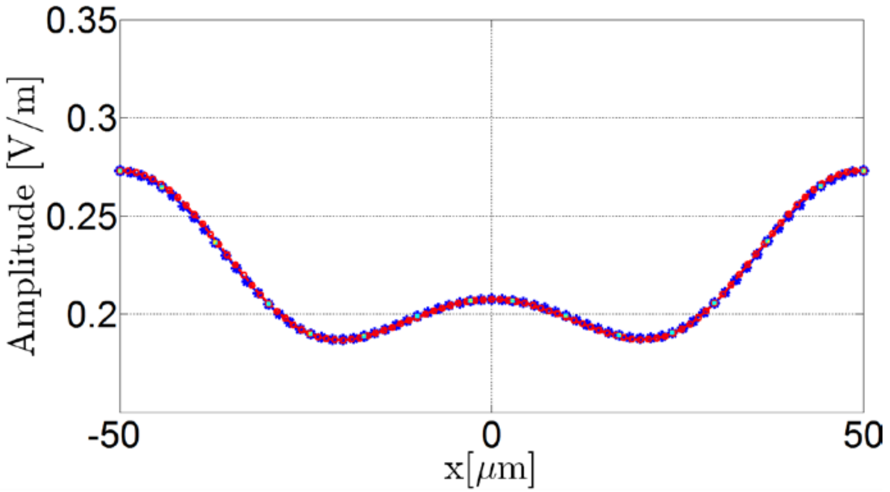
Field inside



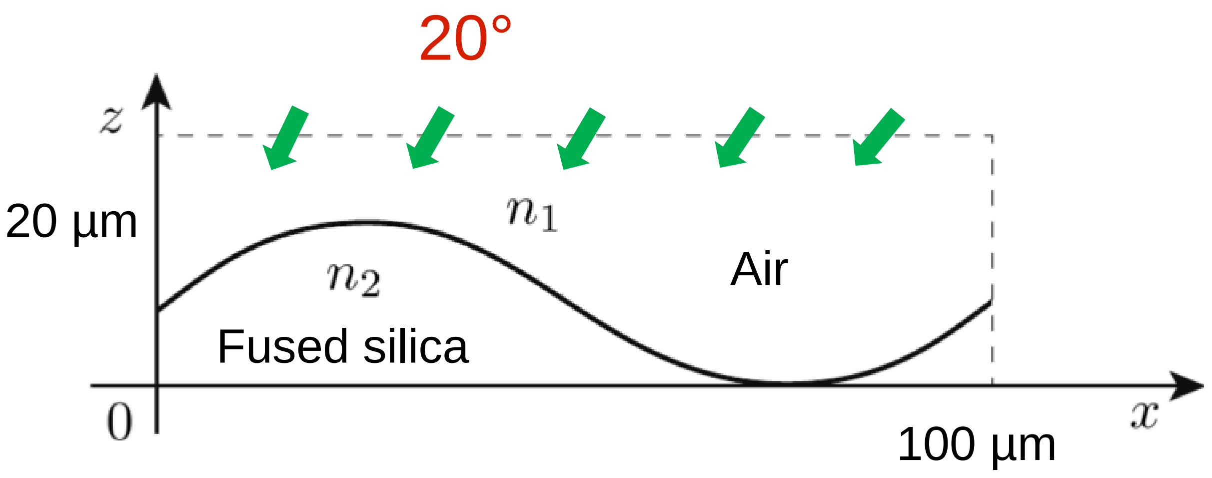
Field on interface: Rigoros



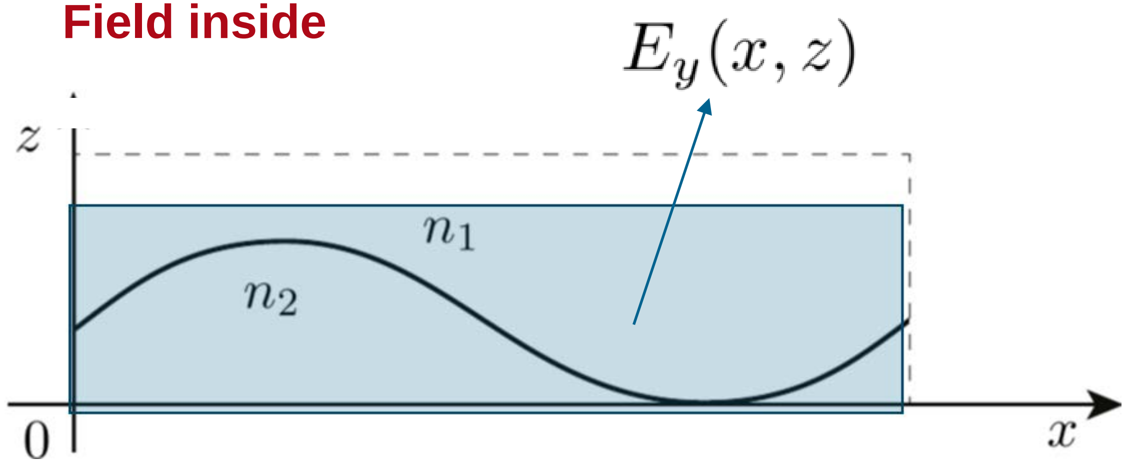
Field on interface: Comparison



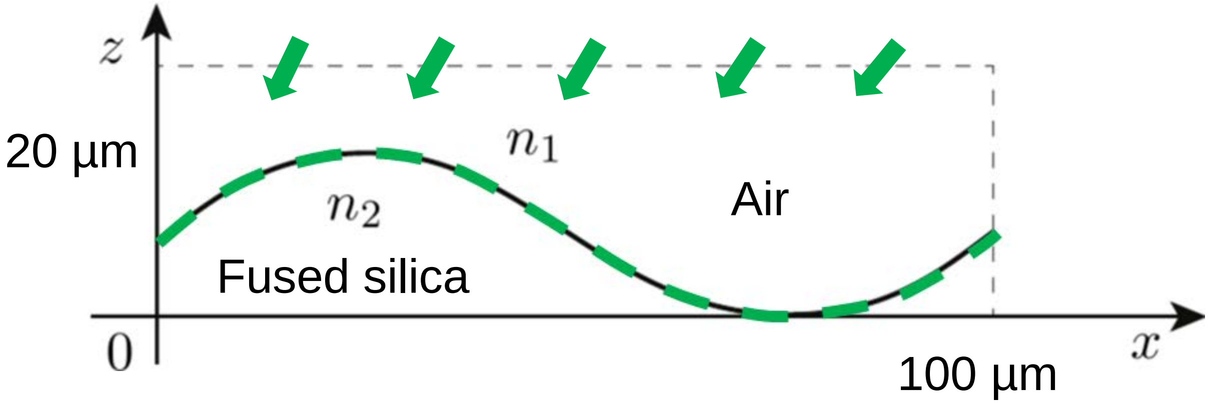
Simulations: Specifications Changed



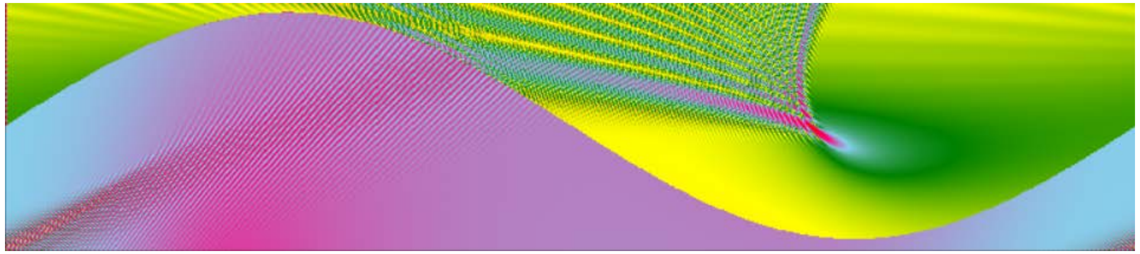
Field inside



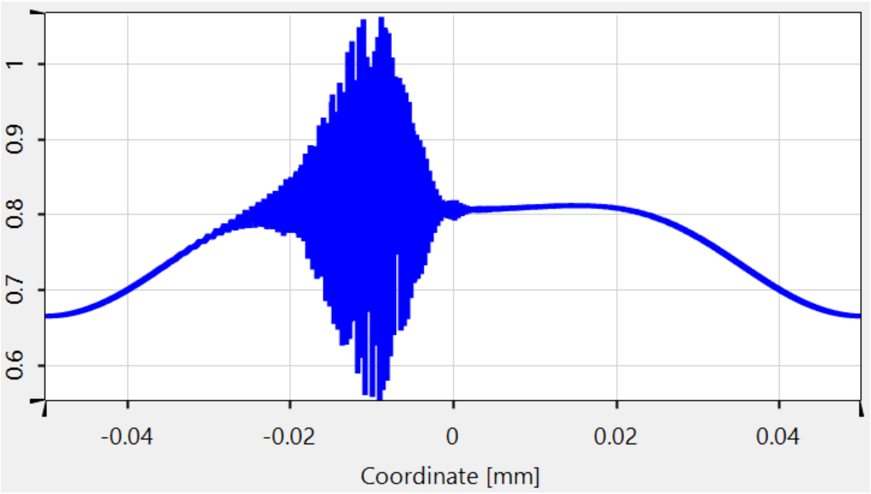
Simulations: Transmitted Field On Surface



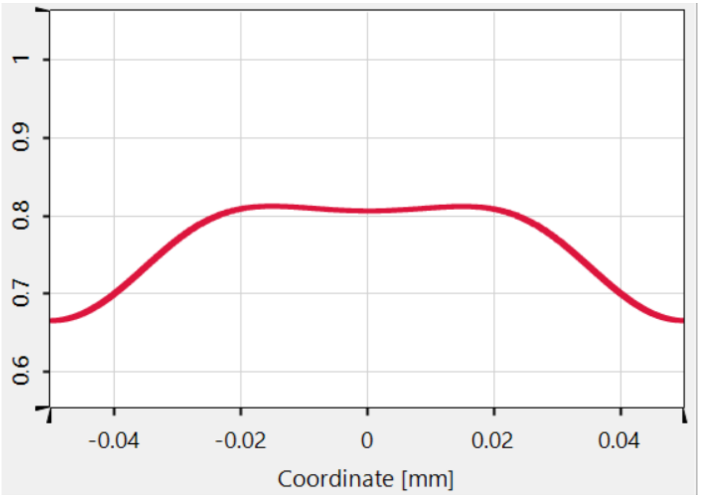
Field inside



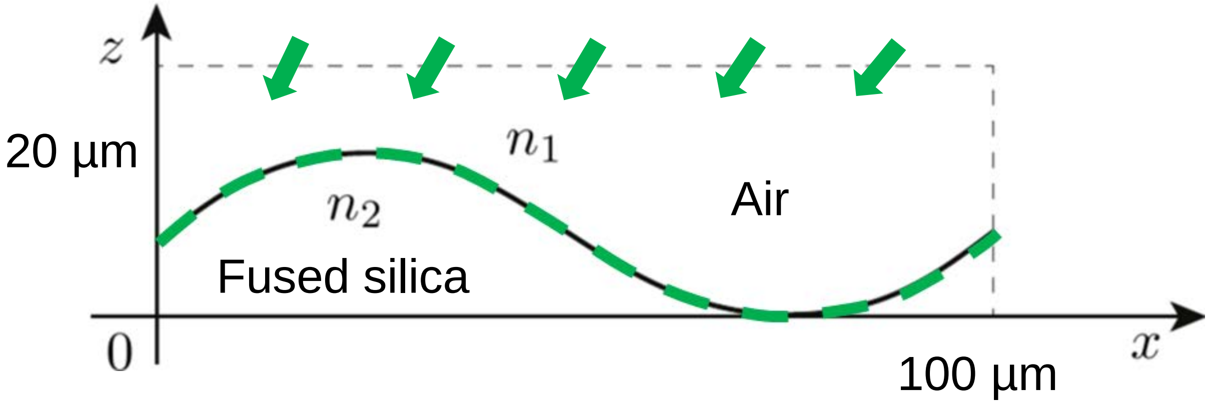
Field on interface: Rigoros



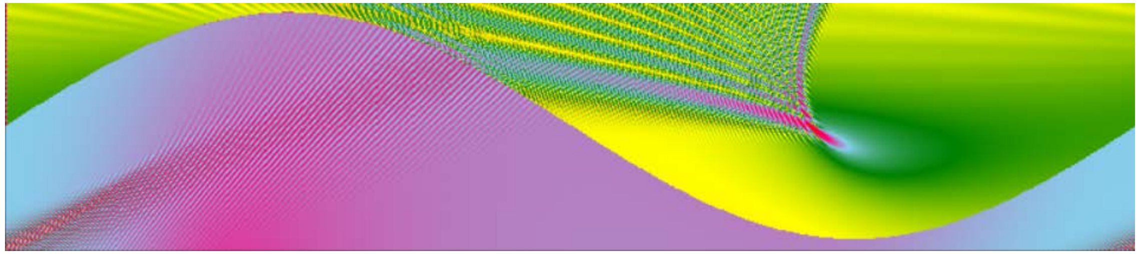
Field on interface: LPIA



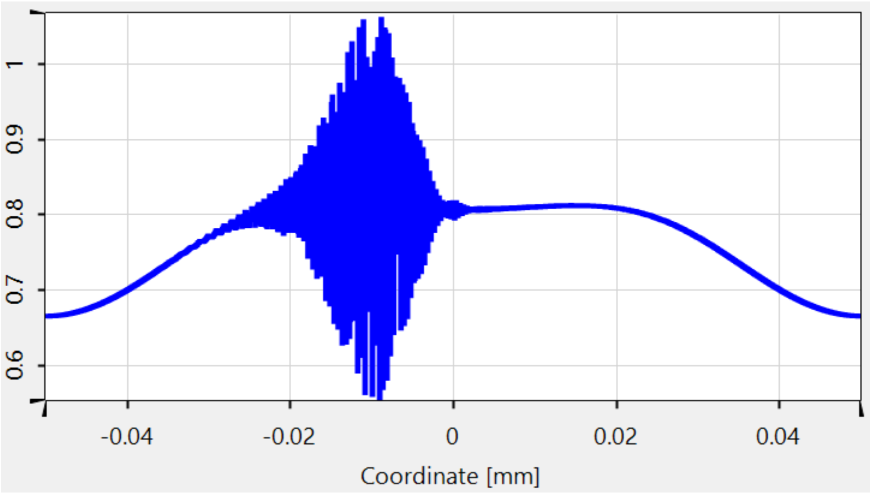
Simulations: Transmitted Field On Surface



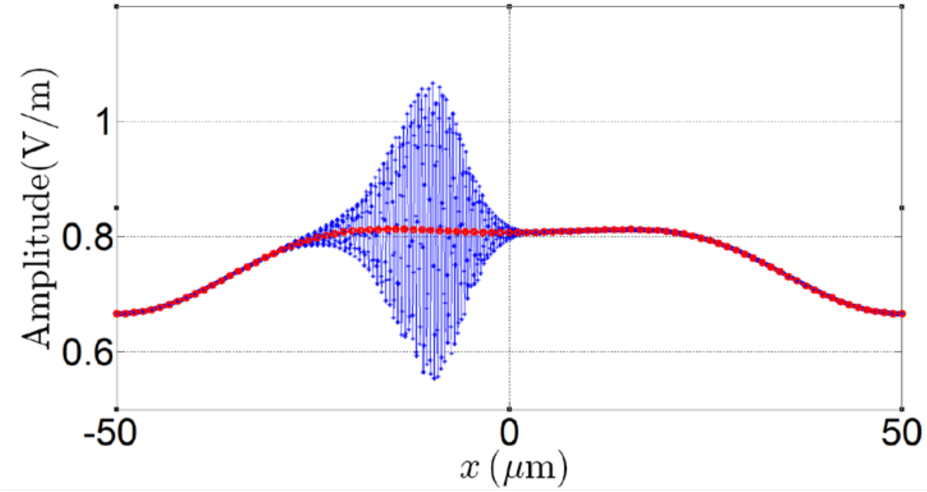
Field inside



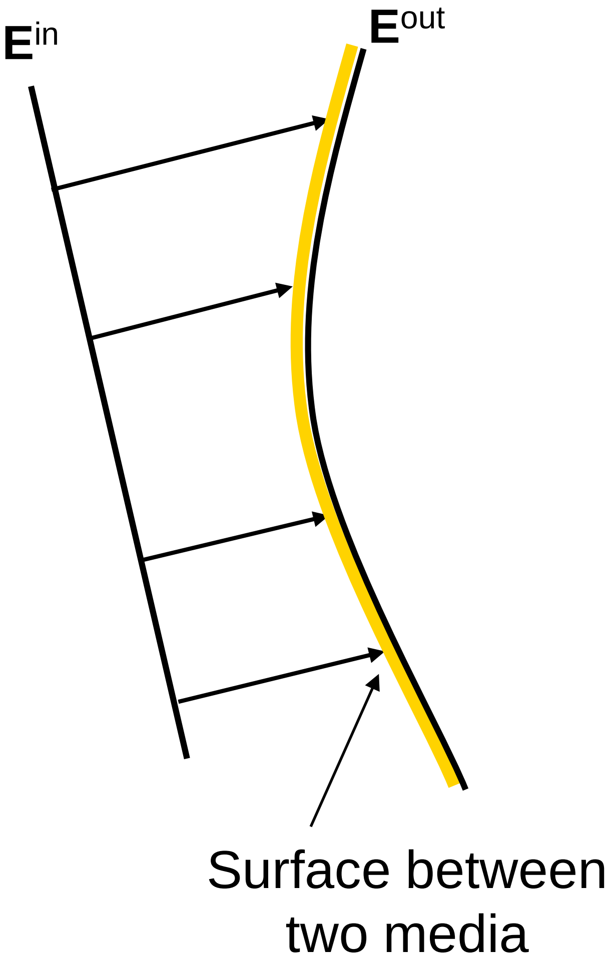
Field on interface: Rigoros



Field on interface: Comparison



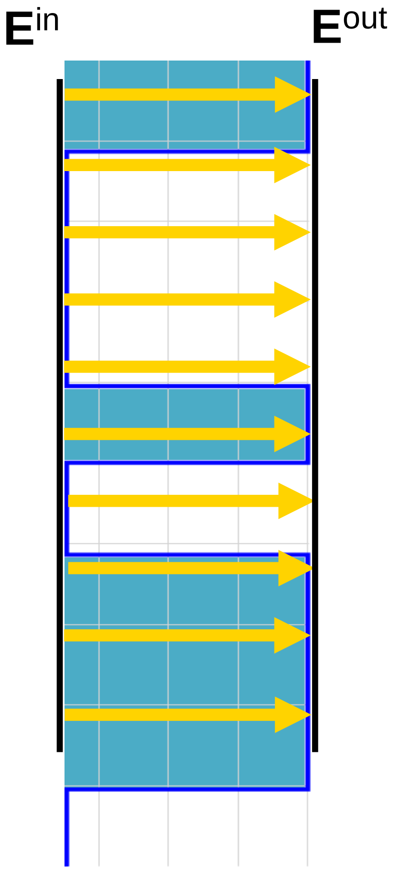
Local Plane Interface Approximation (LPIA)



- Very reasonable technique for smooth surfaces.
- Multiple scattering at surface limits validity of LPIA.
- Can be easily detected by LPIA algorithm!
- No limitation of tickness but of local NA.
- **Remark: Application of LPIA in geometric zones leads to fast B operators!**

$$\tilde{V}^{\text{out}}(k_x,k_y) = \int_{K^2} \tilde{B}(k_x,k_y,k'_x,k'_y)\,\tilde{V}^{\text{in}}(k'_x,k'_y)\,\mathrm{d}k'_x\,\mathrm{d}k'_y$$

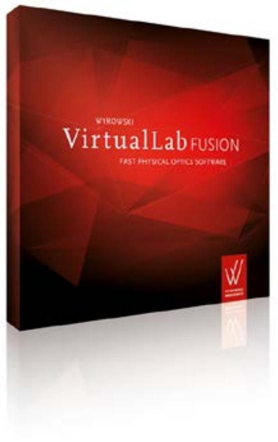
Discrete Height Steps: LPIA Identical with TEA



Structure
Height Profile

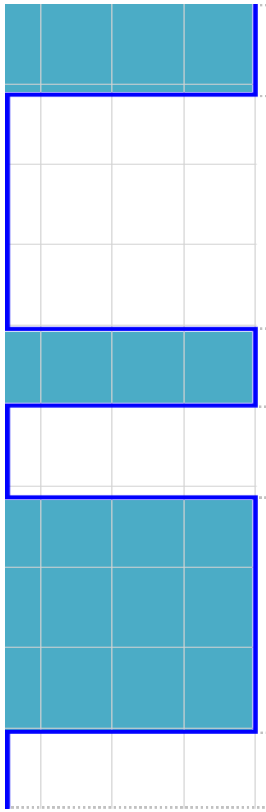
- For discrete height steps LPIA is identical with TEA.
- Diffraction/Scattering at edges WITHIN structure not taken into account
- Multiple scattering in structure not included.
- Vaildity: For thin profiles only (wavelength)
- Otherwise other methods, e.g. split-step (BPM, WPM, …)

$$\tilde{V}^{\mathrm{out}}(k_x,k_y) = \int_{K^2} \tilde{B}(k_x,k_y,k_x',k_y')\,\tilde{V}^{\mathrm{in}}(k_x',k_y')\,\mathrm{d}k_x'\,\mathrm{d}k_y'$$

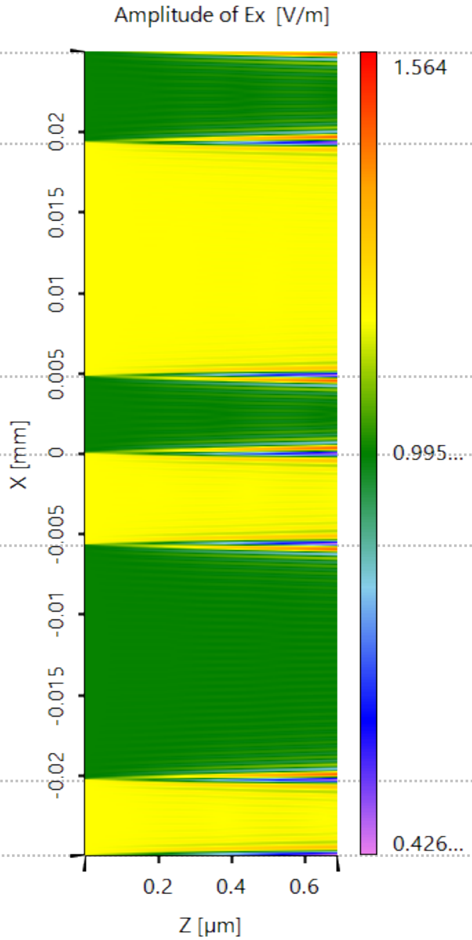


Period = 50µm; Smallest Feature = 4.7410µm

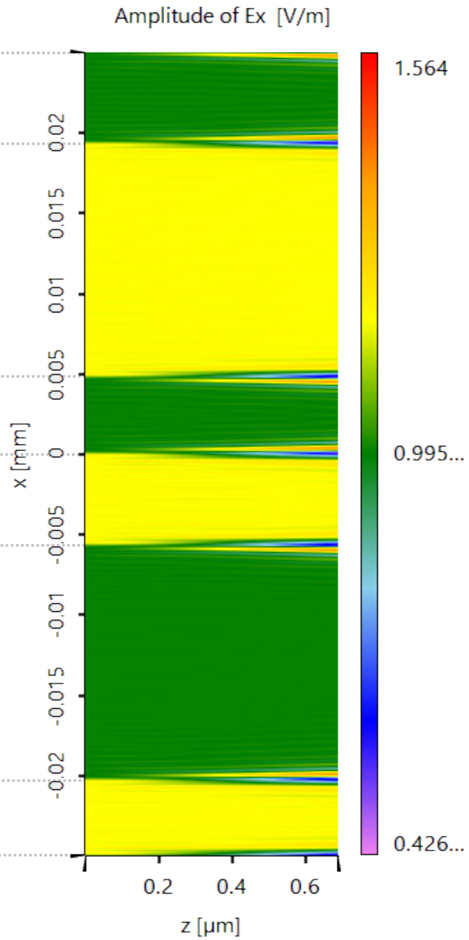
Structure Height Profile



Split Step

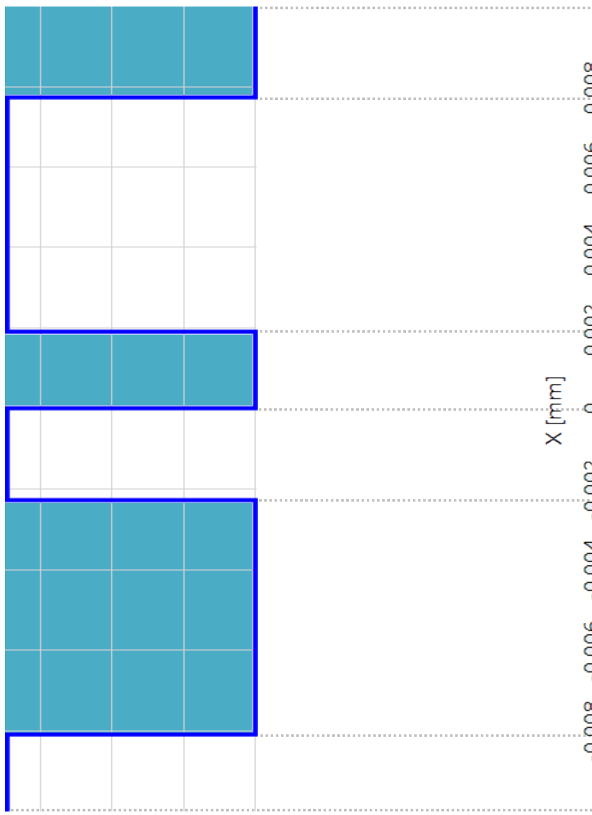


FMM

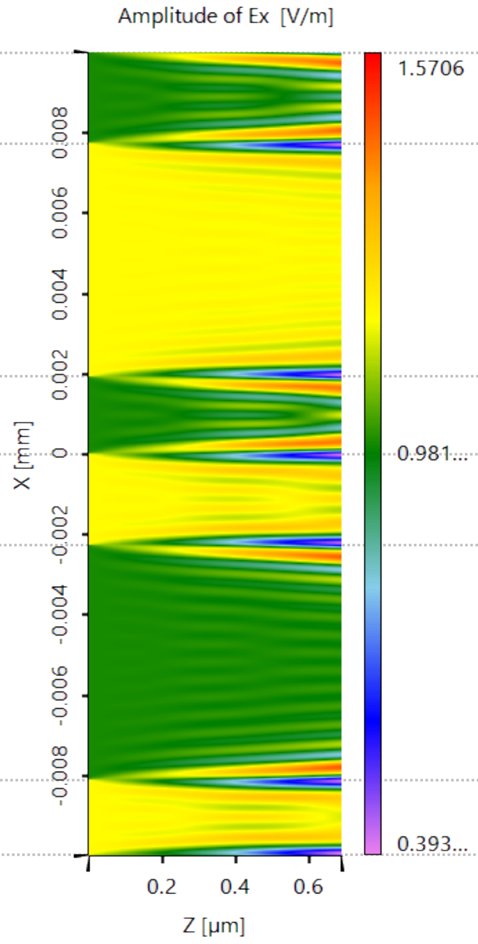


Period = 20µm; Smallest Feature = 1.8964µm

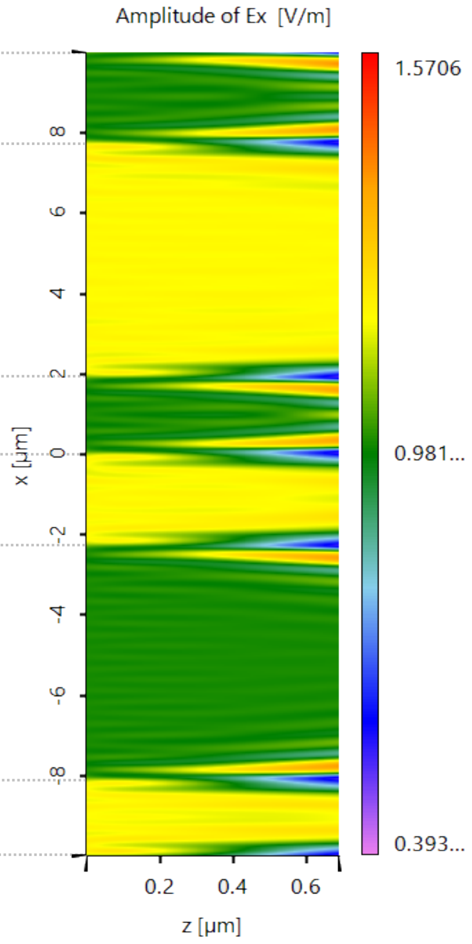
Structure Height Profile



Split Step

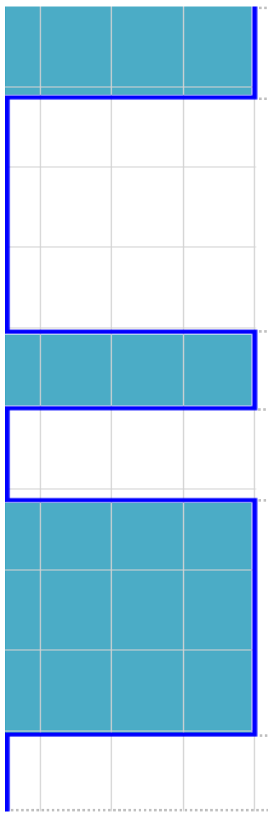


FMM

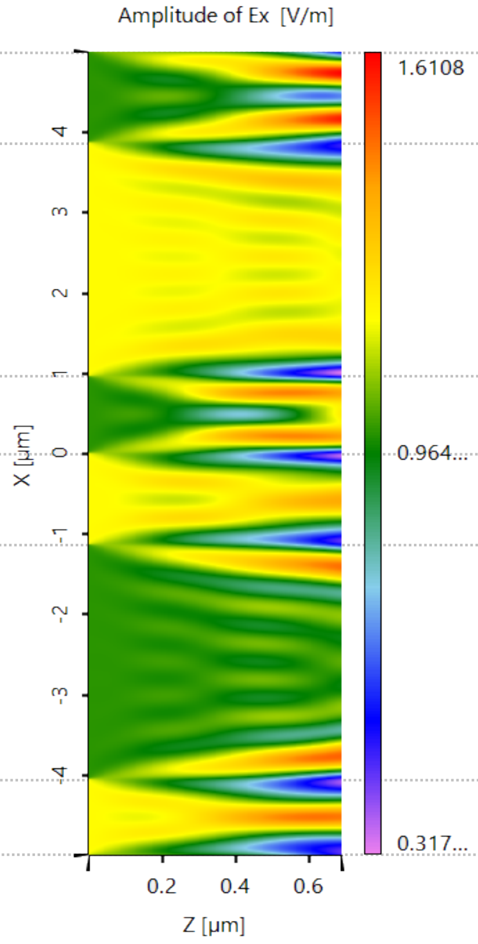


Period = 10µm; Smallest Feature = 0.9482µm

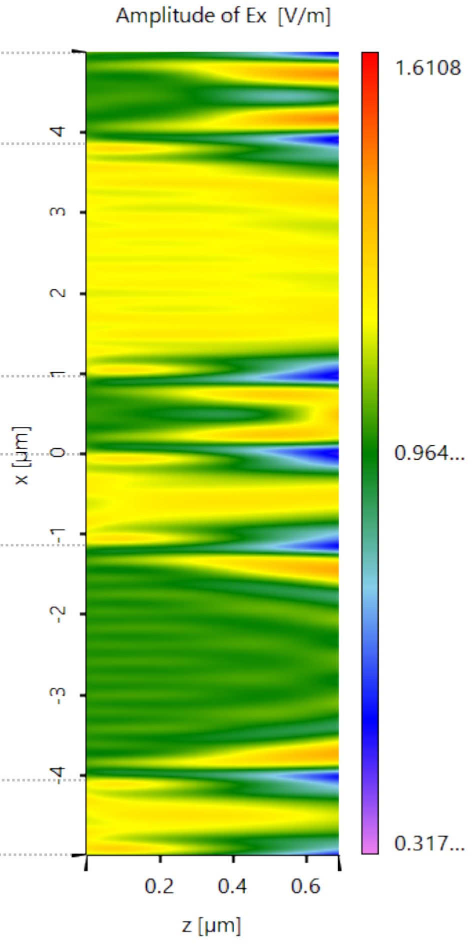
Structure Height Profile



Split Step

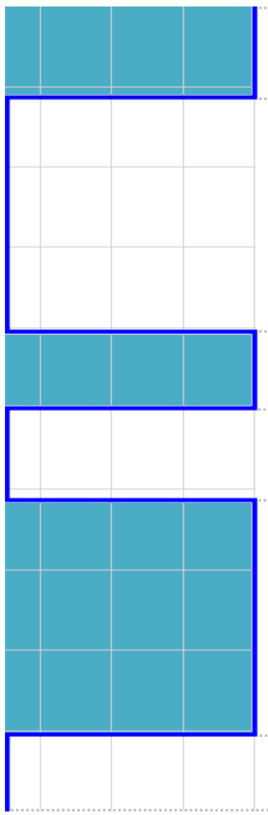


FMM

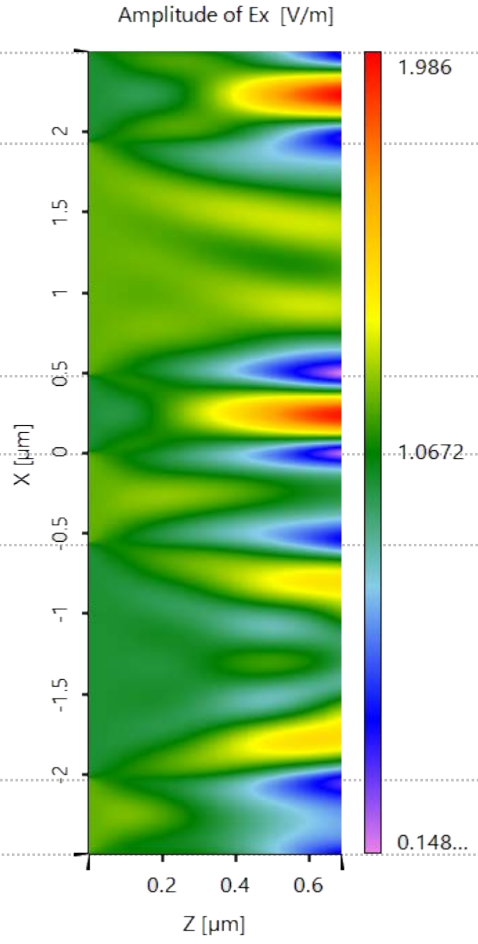


Period = 5µm; Smallest Feature = 0.4741µm

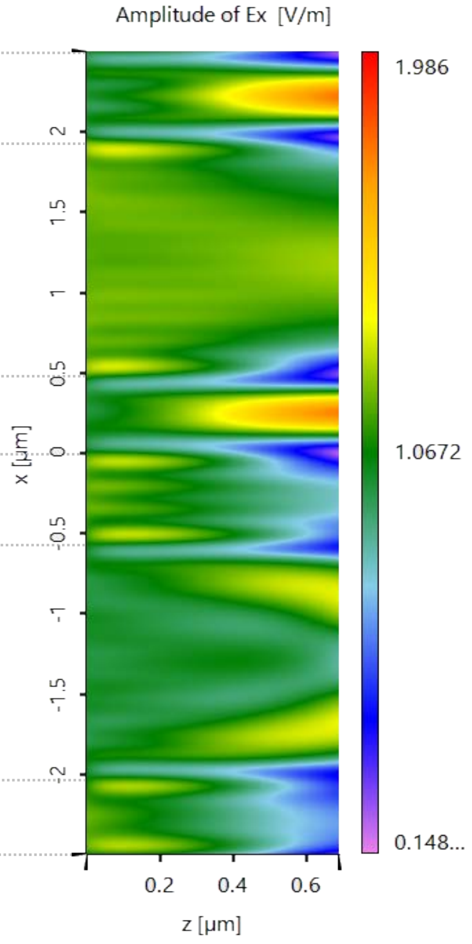
Structure Height Profile



Split Step

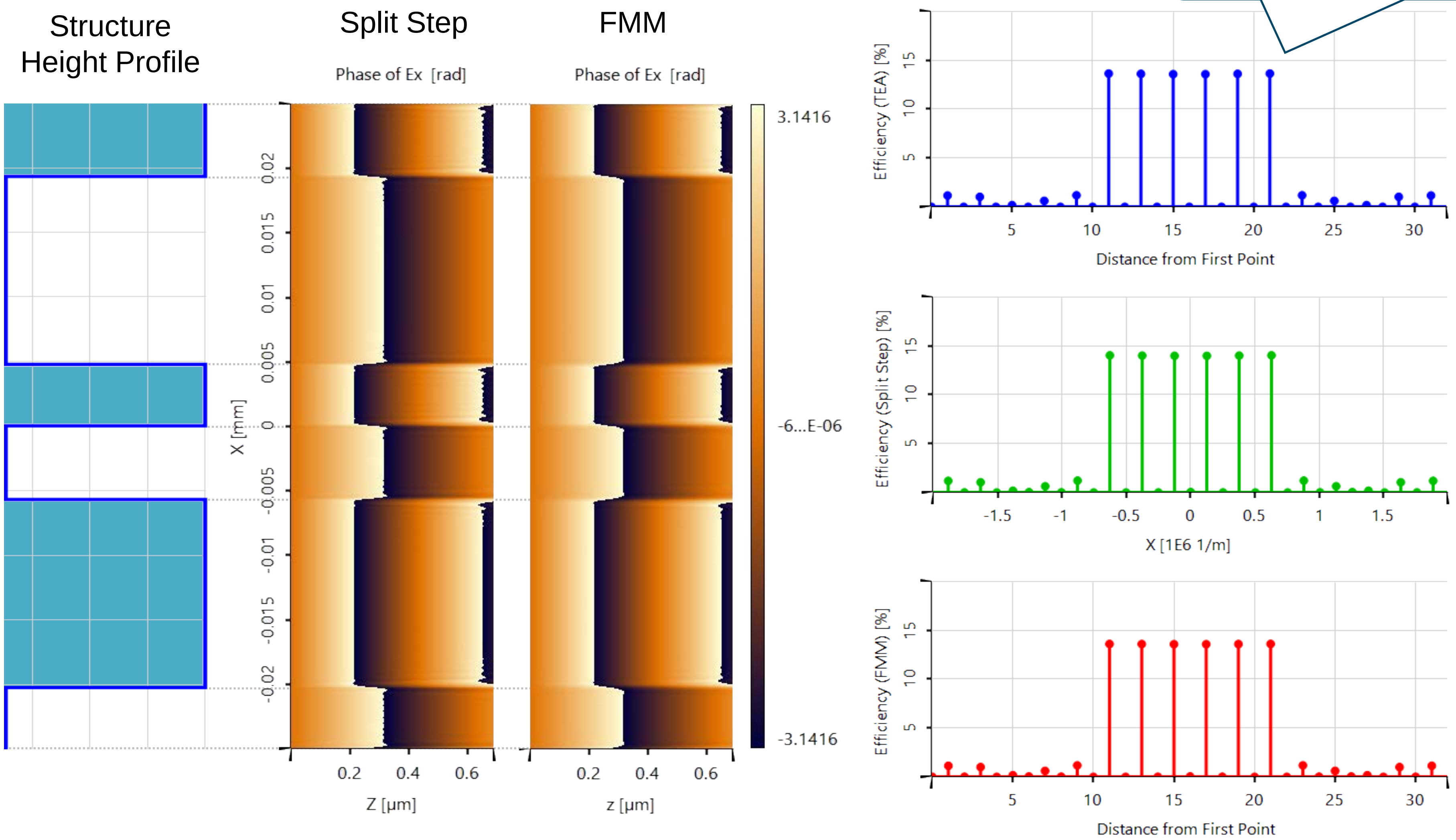


FMM

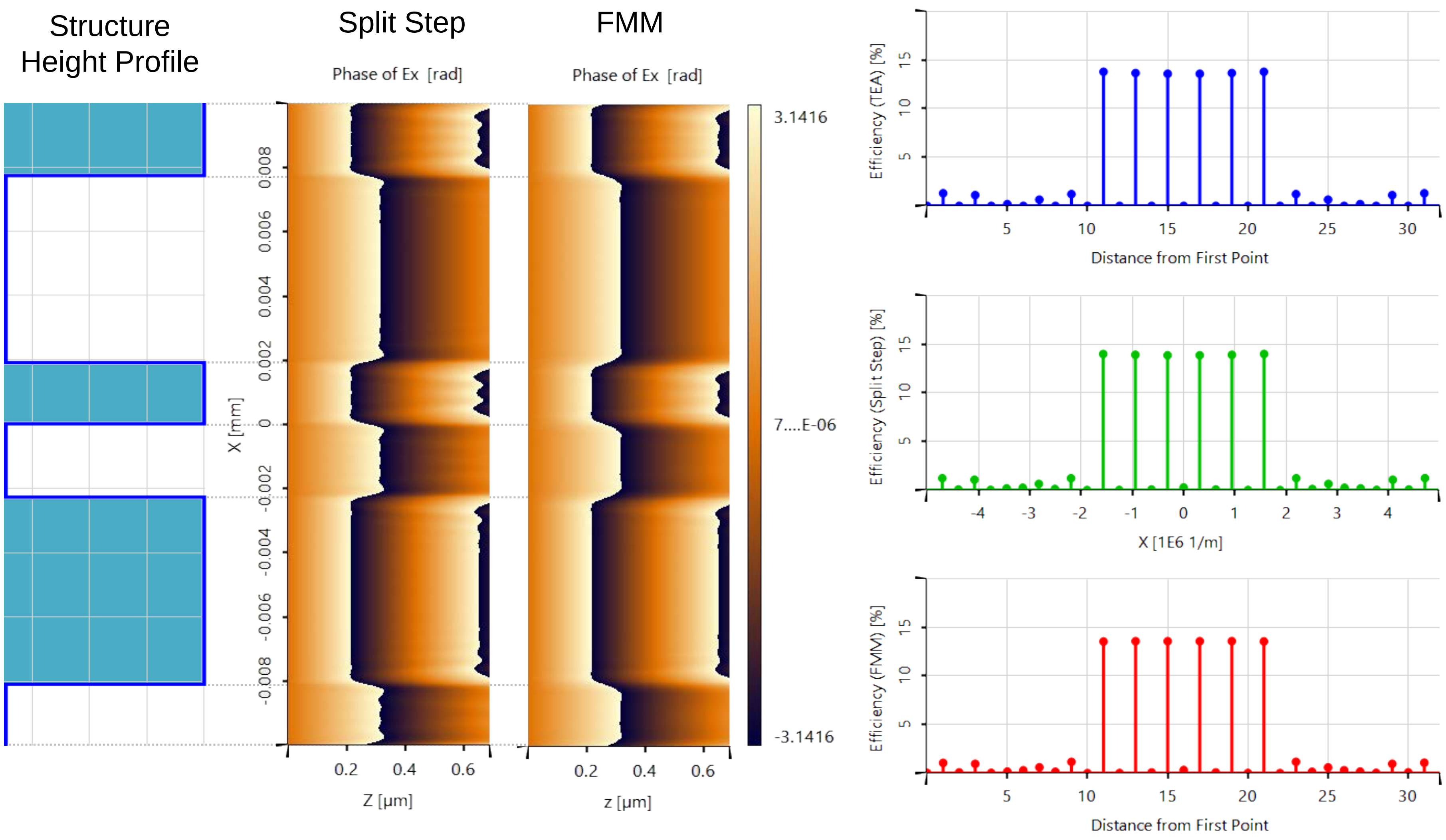


Period = 50µm; Smallest Feature = 4.7410µm

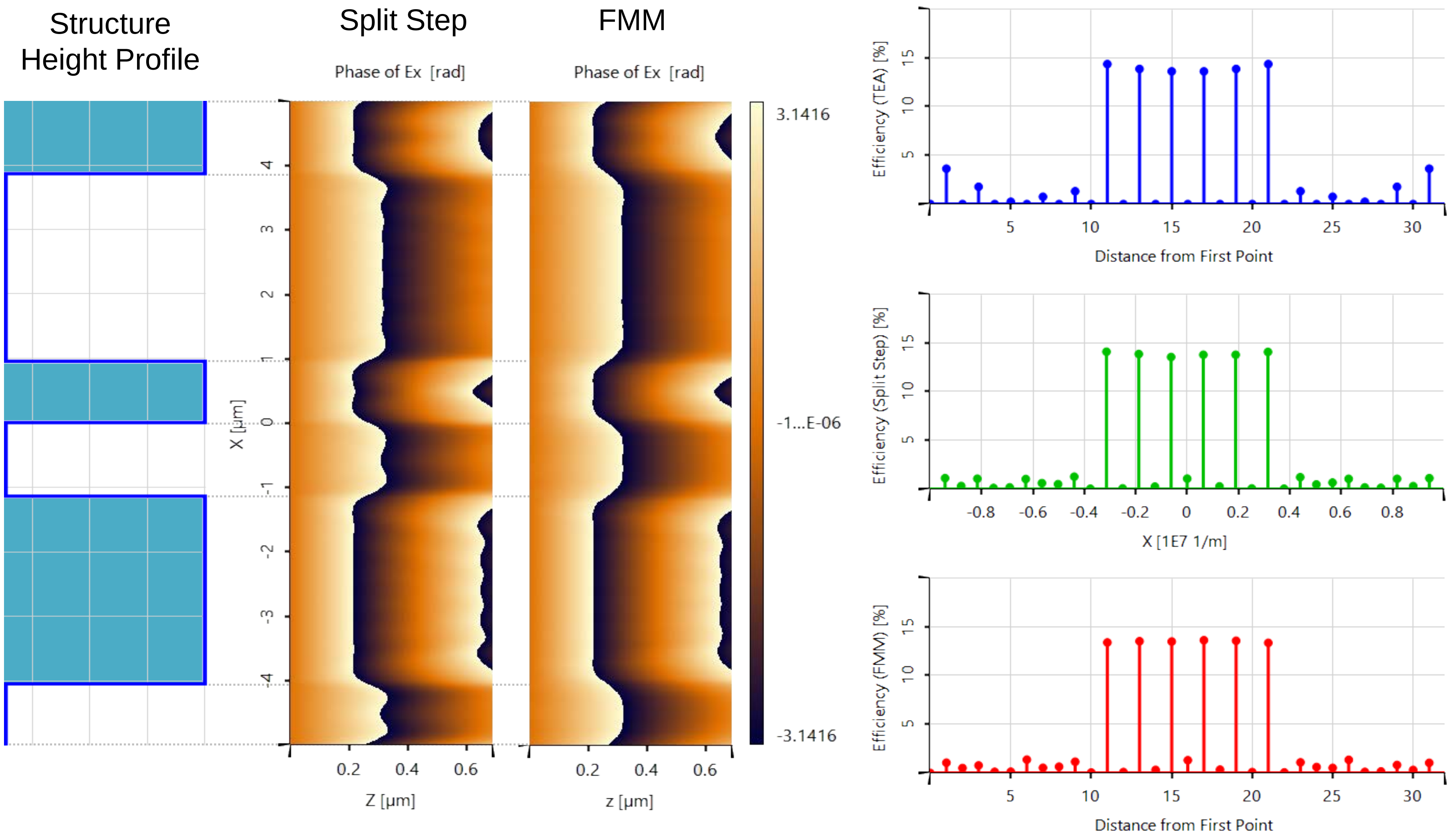
Diffraction efficiencies



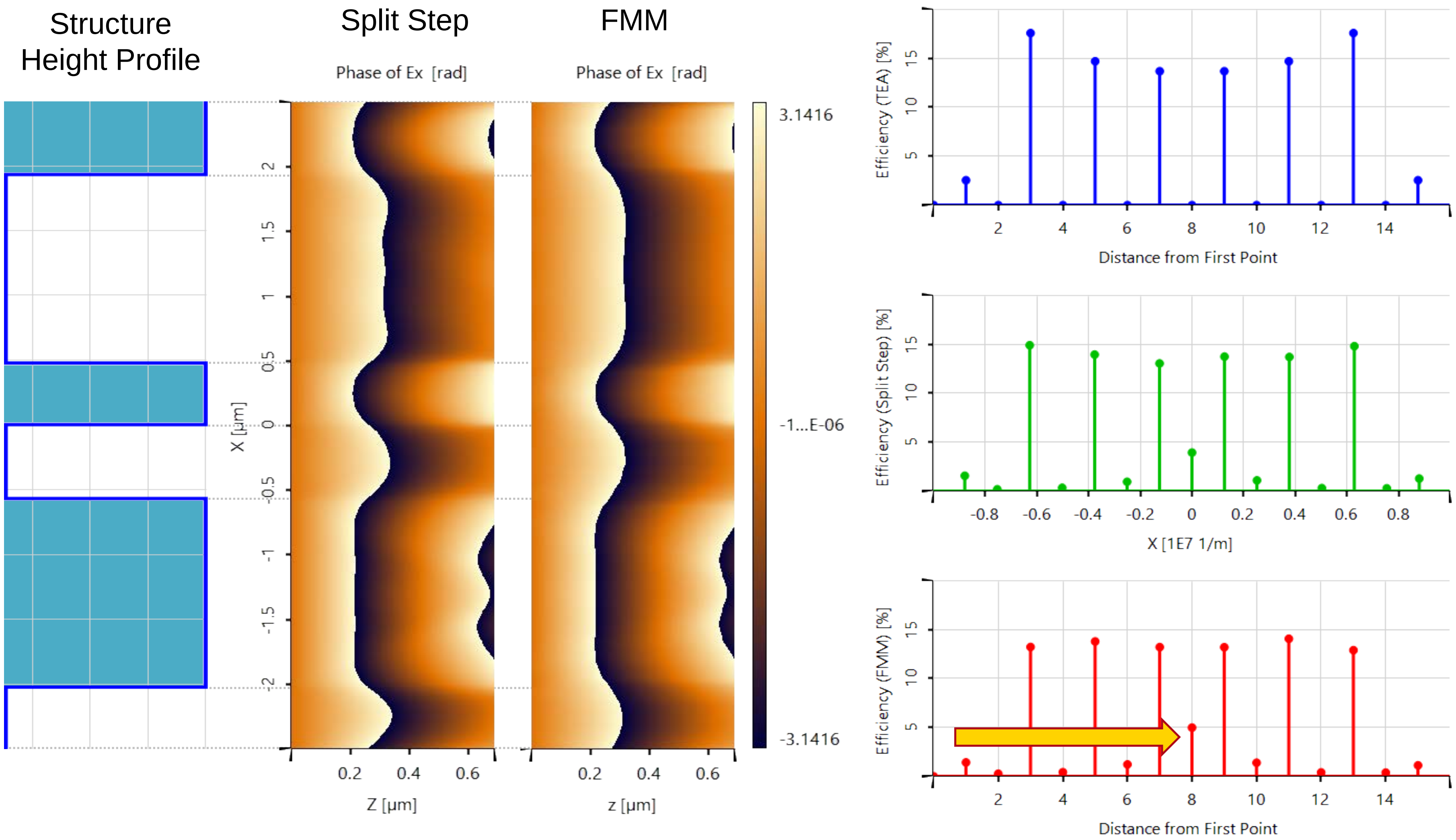
Period = 20µm; Smallest Feature = 1.8964µm



Period = 10µm; Smallest Feature = 0.9482µm



Period = 5µm; Smallest Feature = 0.4741µm



Summary

- Introduction of a geometric branch of physical optics with the help of the geometric Fourier transform.
- Fast physical optics enabled by non-sequential coupling of Maxwell solvers, which are mathematically expressed by B-operators.
- Generalization of BSDF concept in ray tracing.
- B-operators for curved surfaces by LPIA justified and limitation of validity can be detected by algorithm.
- B operator for thin stepped height profiles: TEA
- Thicker stepped profiles must be modeled by split-step type techniques or rigorous approaches.

- Simulations done with VirtualLab Fusion software

- FEM with JCMSuite