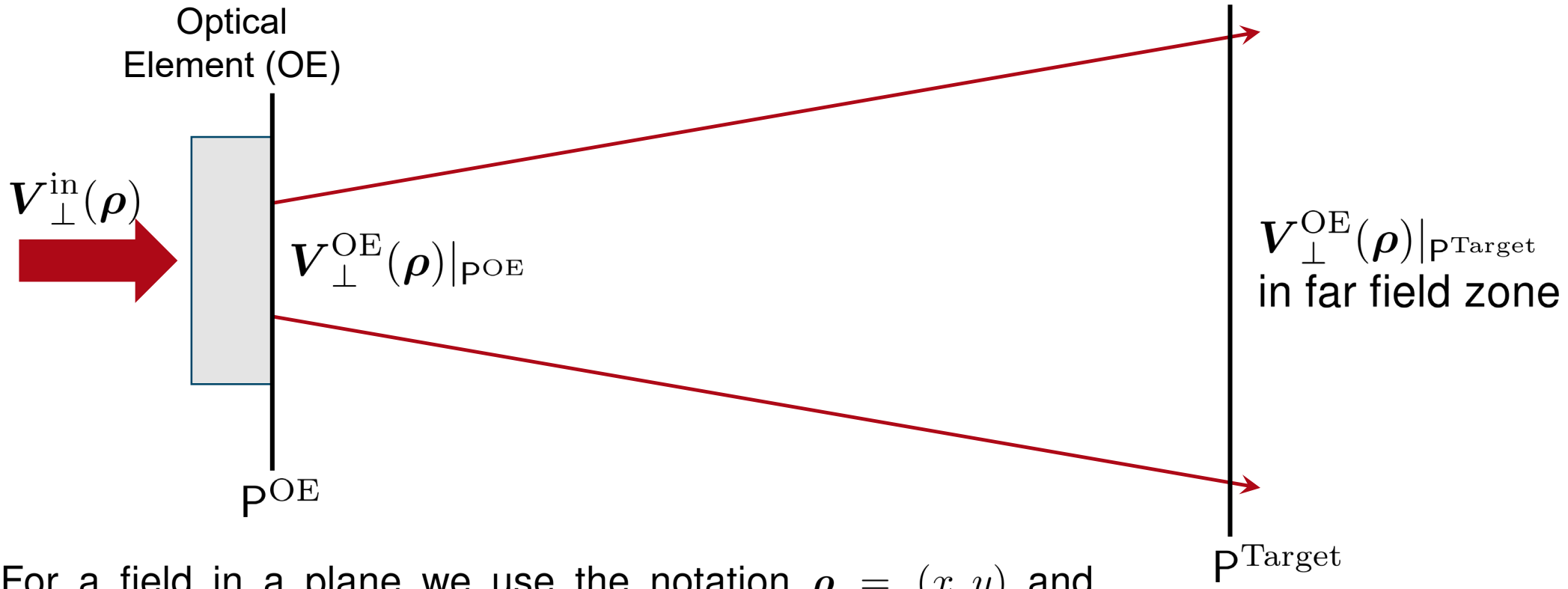
Photonics West 2019, February 6, San Francisco

How the design concepts of high-NA beam splitters & diffusers, as well as of beam shapers by freeform surfaces, are related

Liangxin Yang and Frank Wyrowski, University of Jena
Roberto Knoth, LightTrans International
Christian Hellmann, Wyrowski Photonics

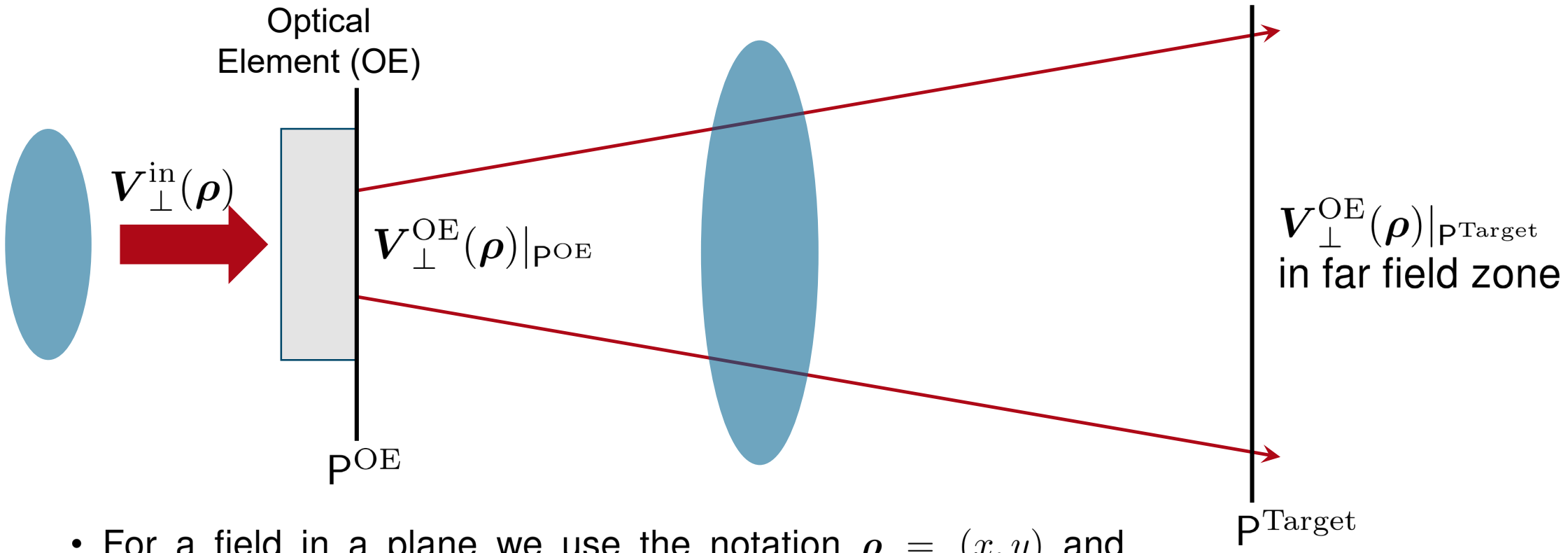
Physical-optics view on light shaping

Shaping the Far Field of an Incident Light Beam



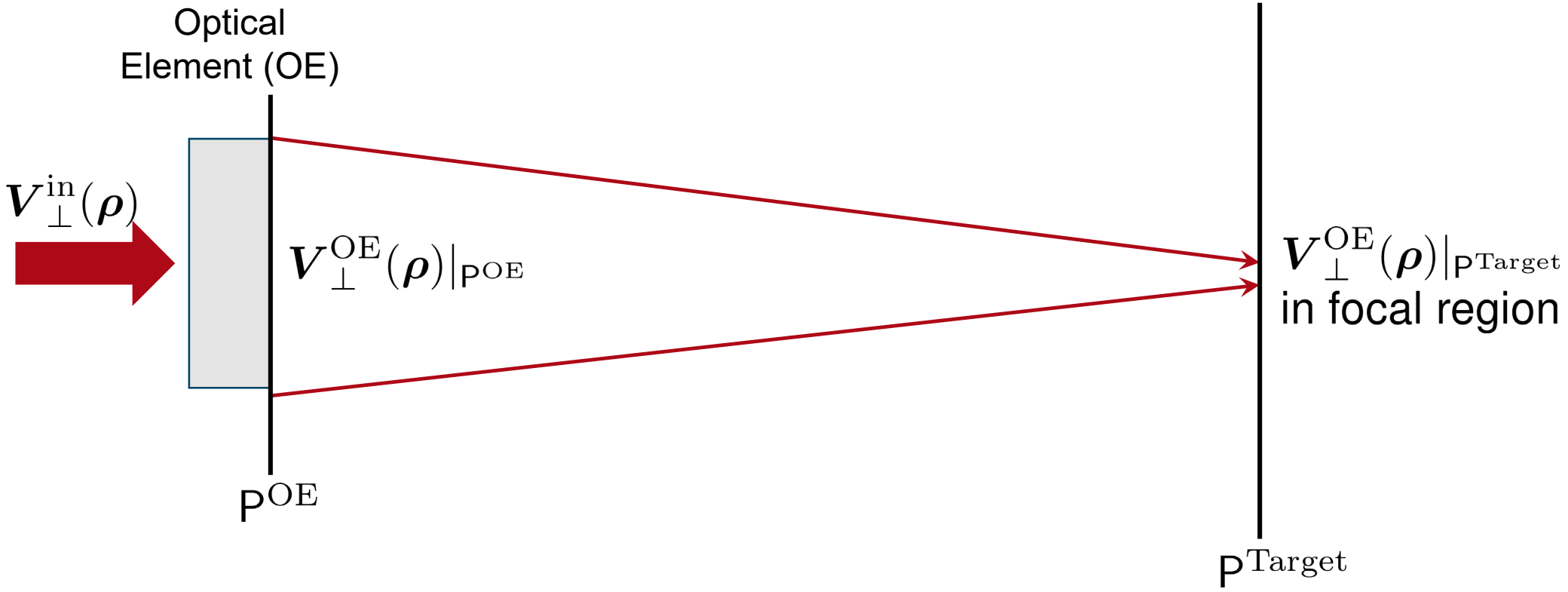
- For a field in a plane we use the notation $\boldsymbol{\rho} = (x, y)$ and $\boldsymbol{V}_{\perp}(\boldsymbol{\rho}) = (E_x(\boldsymbol{\rho}), E_y(\boldsymbol{\rho}))$.
- In k-domain we obtain $\tilde{\boldsymbol{V}}_{\perp}(\boldsymbol{\kappa}) = \boldsymbol{\mathcal{F}}_k \boldsymbol{V}_{\perp}(\boldsymbol{\rho})$ with $\boldsymbol{\kappa} = (k_x, k_y)$ and the Fourier transfrom operator $\boldsymbol{\mathcal{F}}_k$.

Shaping the Far Field of an Incident Light Beam

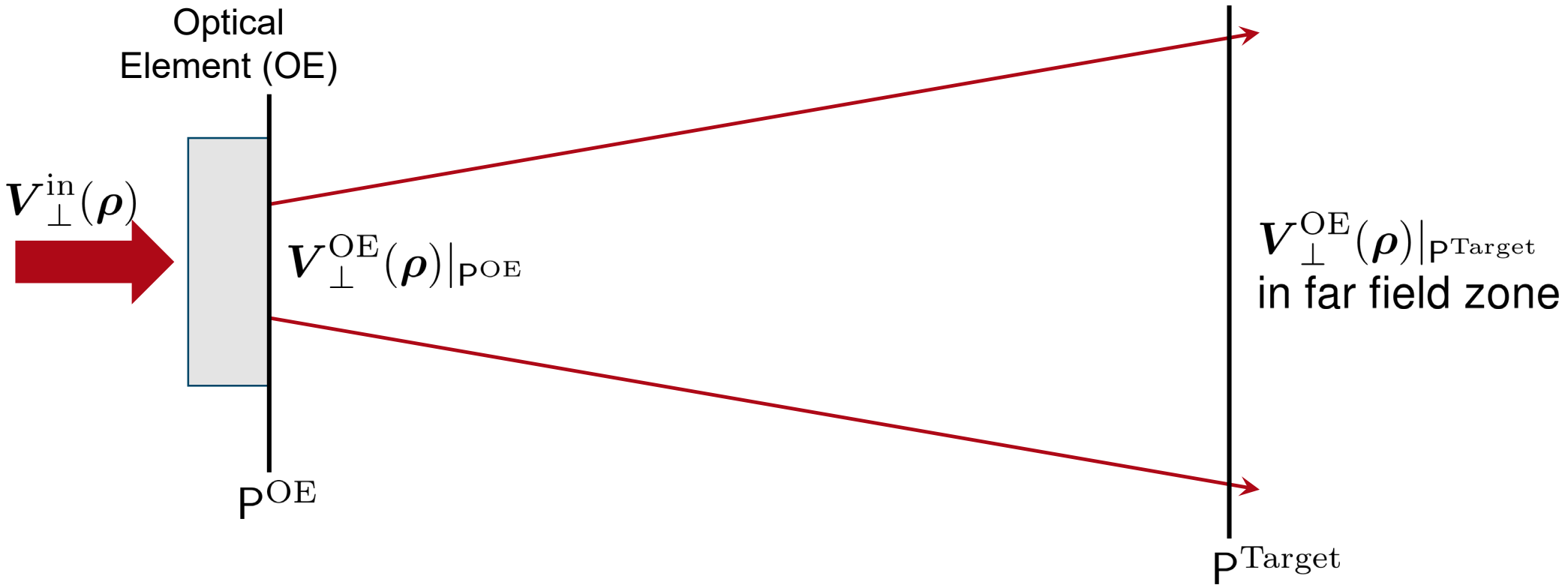


- For a field in a plane we use the notation $\boldsymbol{\rho} = (x, y)$ and $\boldsymbol{V}_{\perp}(\boldsymbol{\rho}) = (E_x(\boldsymbol{\rho}), E_y(\boldsymbol{\rho}))$.
- In k-domain we obtain $\tilde{\boldsymbol{V}}_{\perp}(\boldsymbol{\kappa}) = \boldsymbol{\mathcal{F}}_k \boldsymbol{V}_{\perp}(\boldsymbol{\rho})$ with $\boldsymbol{\kappa} = (k_x, k_y)$ and the Fourier transfrom operator $\boldsymbol{\mathcal{F}}_k$.

Shaping the Focal Region of an Incident Light Beam

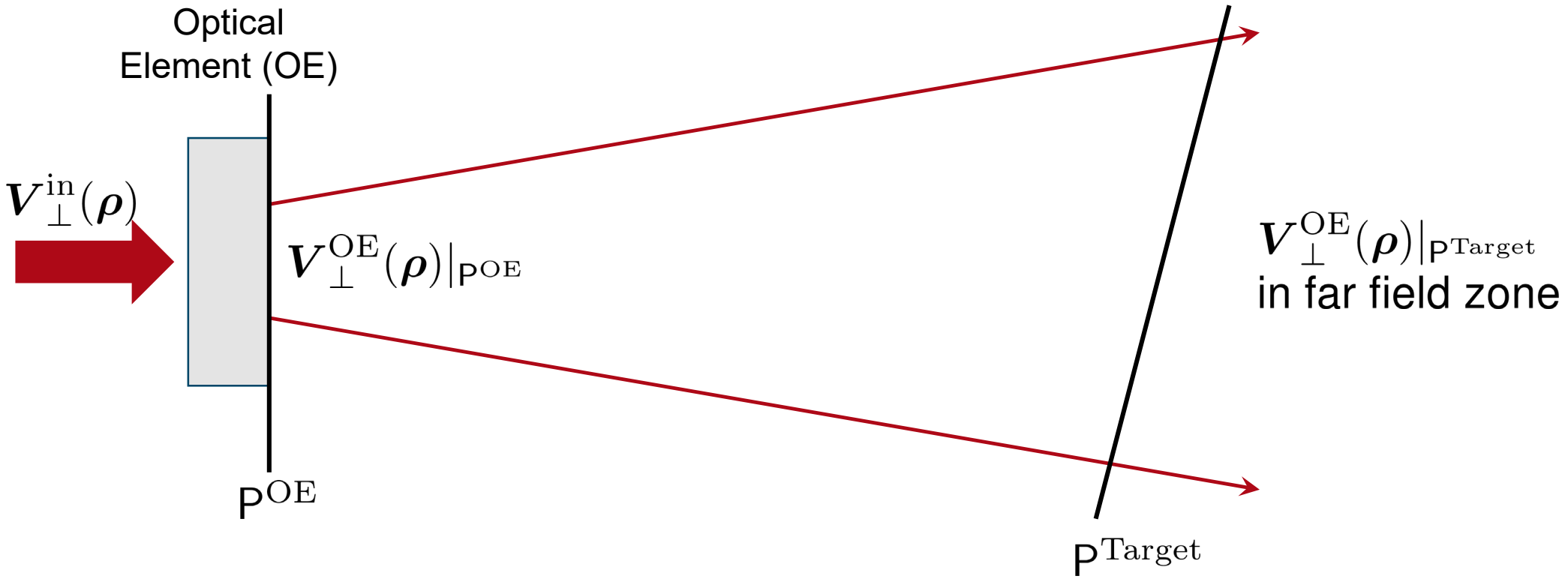


Shaping the Far Field of an Incident Light Beam

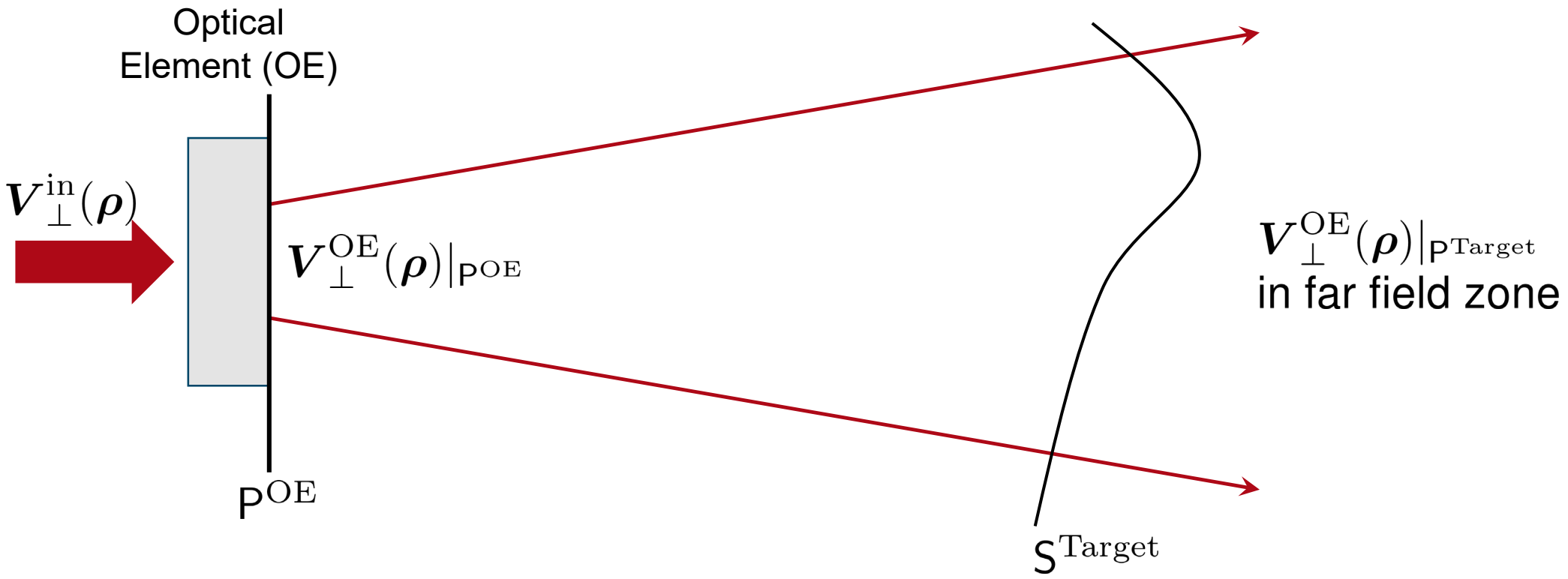


Design task: Shape the irradiance/illuminance (or other radiometric/photometric quantities) on the target plane. The connection of the field $\boldsymbol{V}_{\perp}^{\mathrm{OE}}|_{\mathsf{P}^{\mathrm{Target}}}$ and the radiometric/photometric quantities can be determined locally in any position $\boldsymbol{\rho}$.

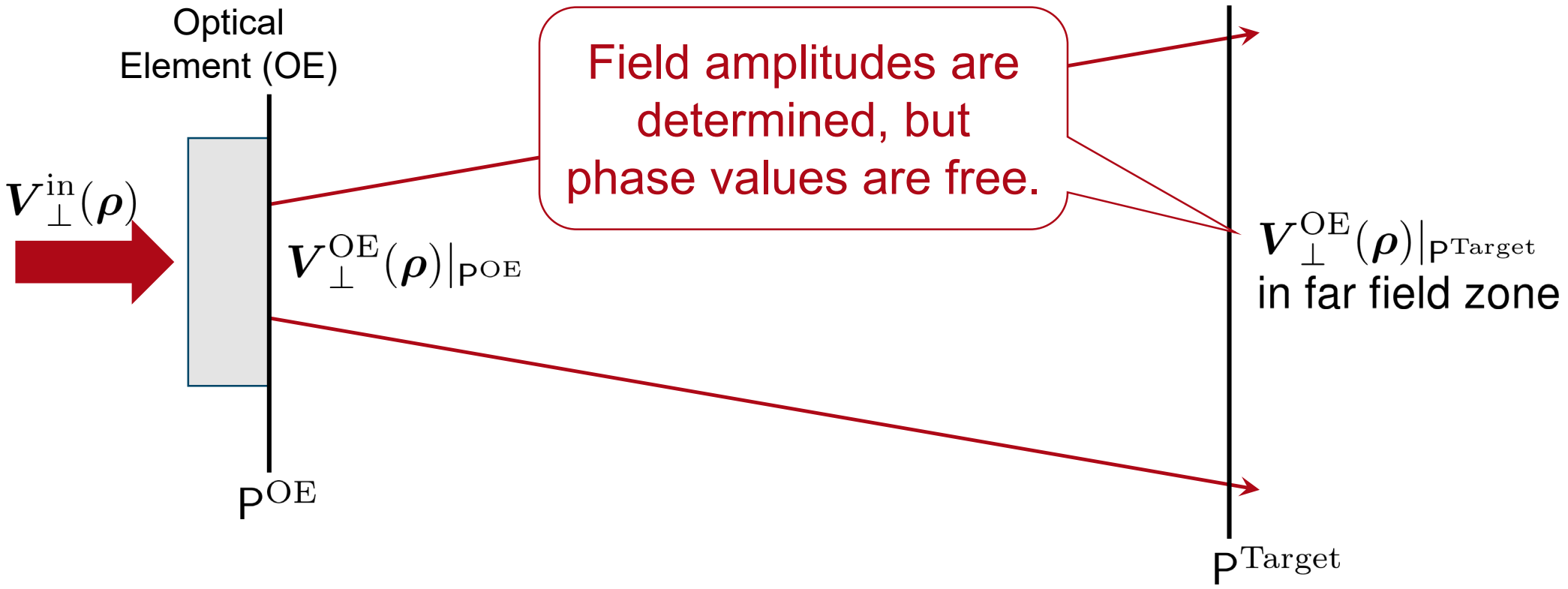
Shaping the Far Field of an Incident Light Beam



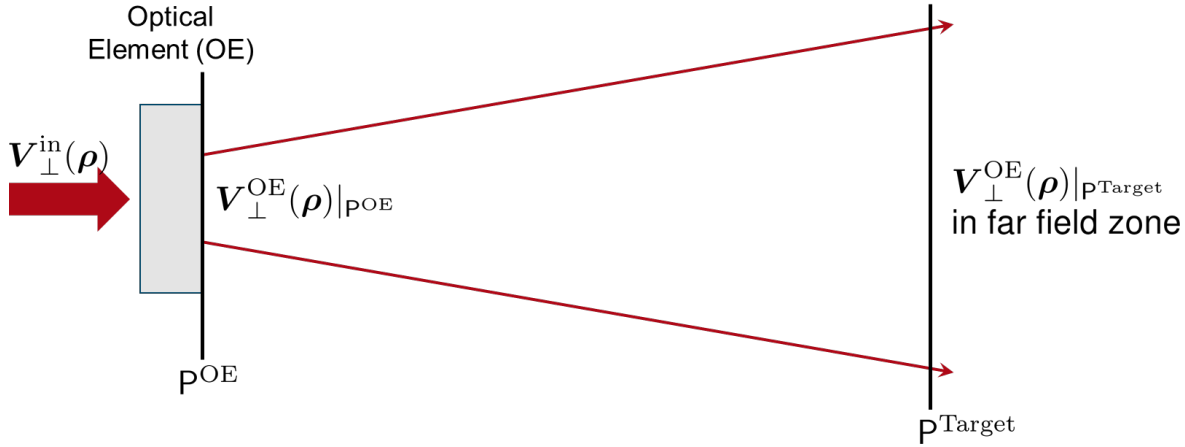
Shaping the Far Field of an Incident Light Beam

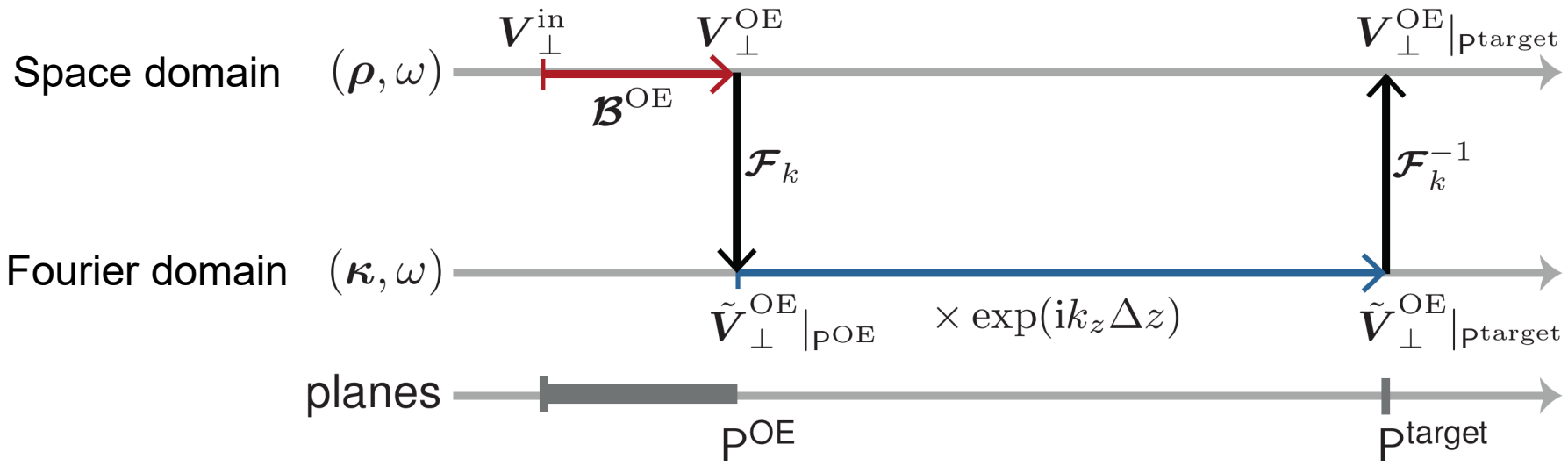


Shaping the Far Field of an Incident Light Beam



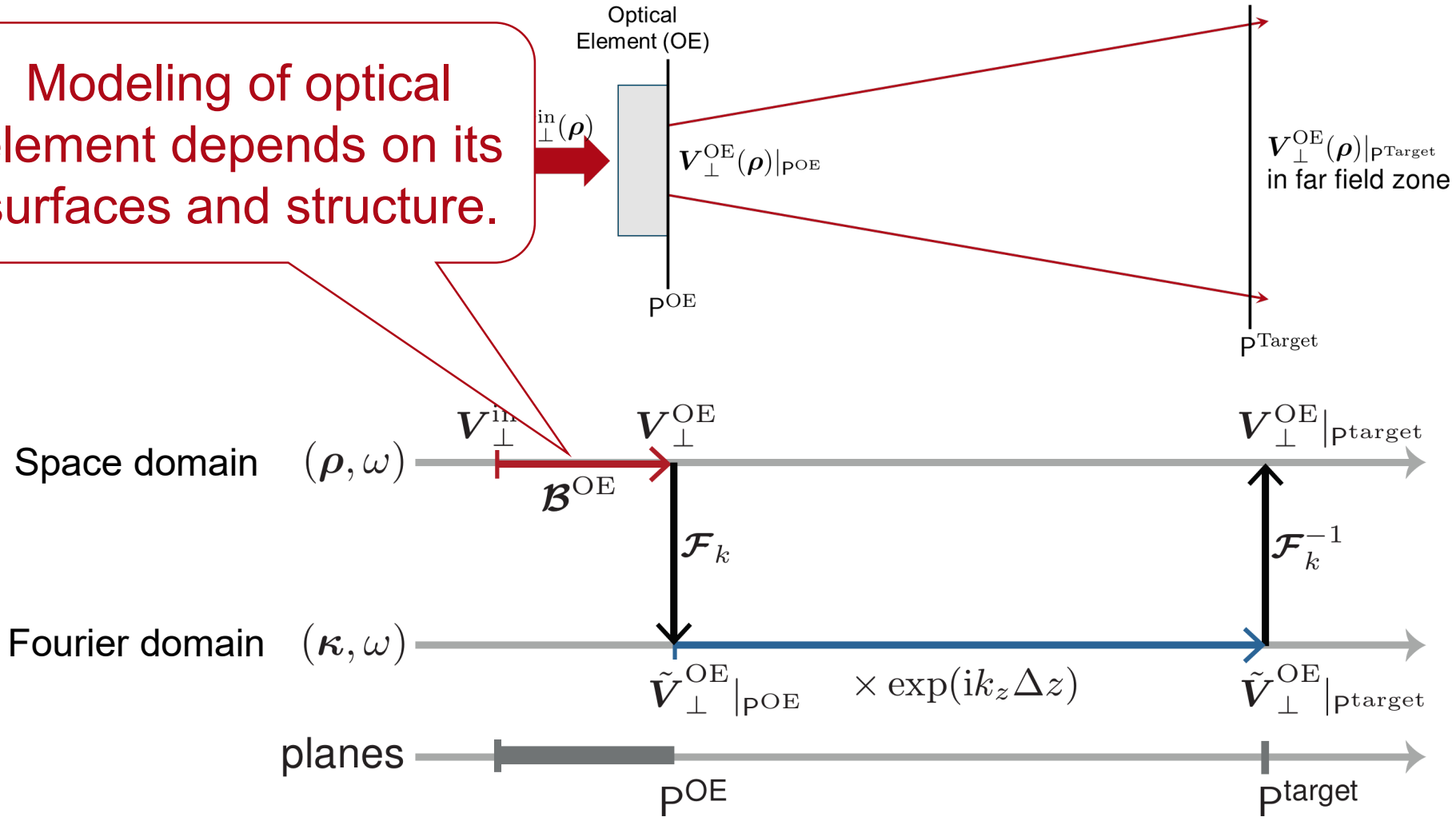
Mathematical Formulation in Field Tracing Diagram



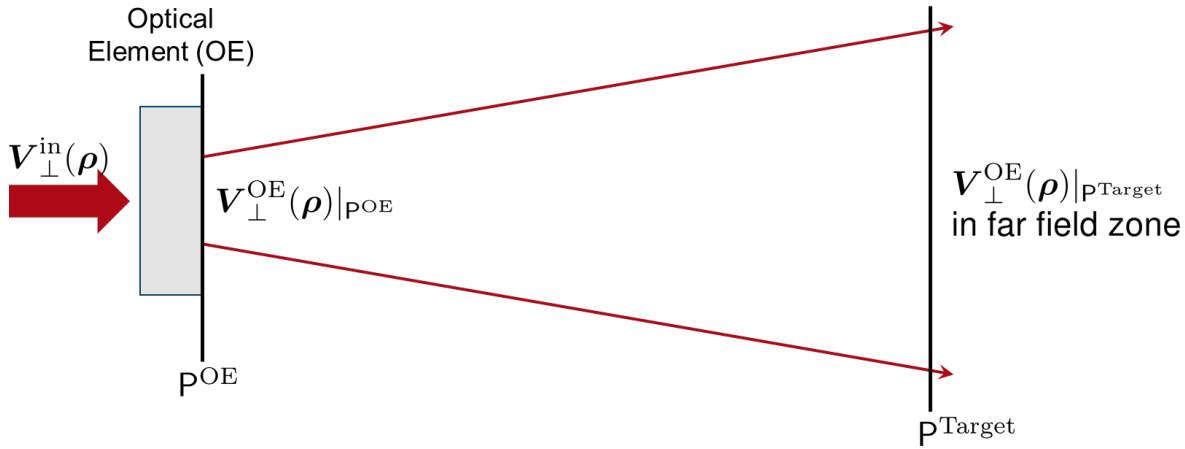


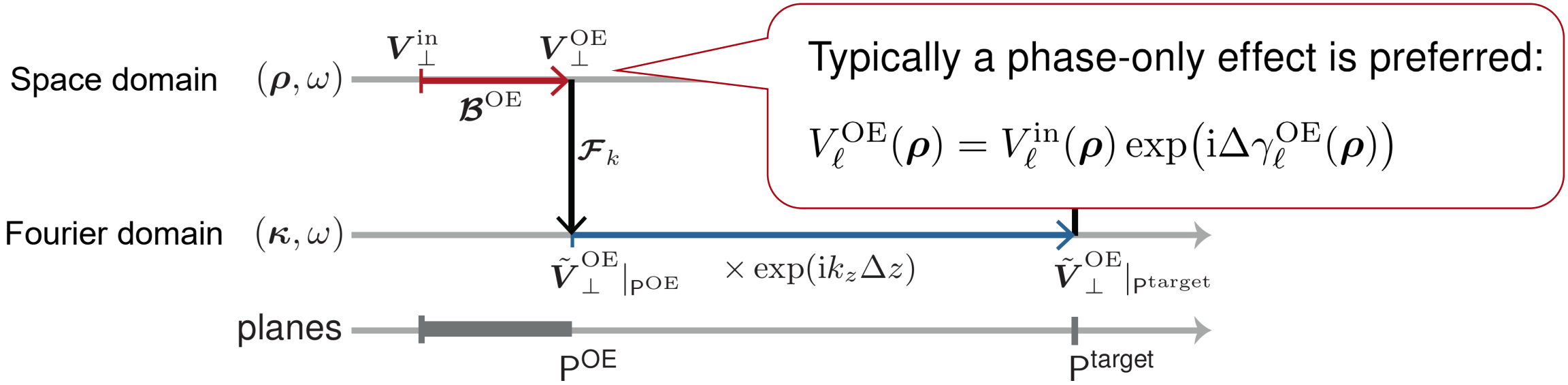
Mathematical Formulation in Field Tracing Diagram

Modeling of optical element depends on its surfaces and structure.



Mathematical Formulation in Field Tracing Diagram

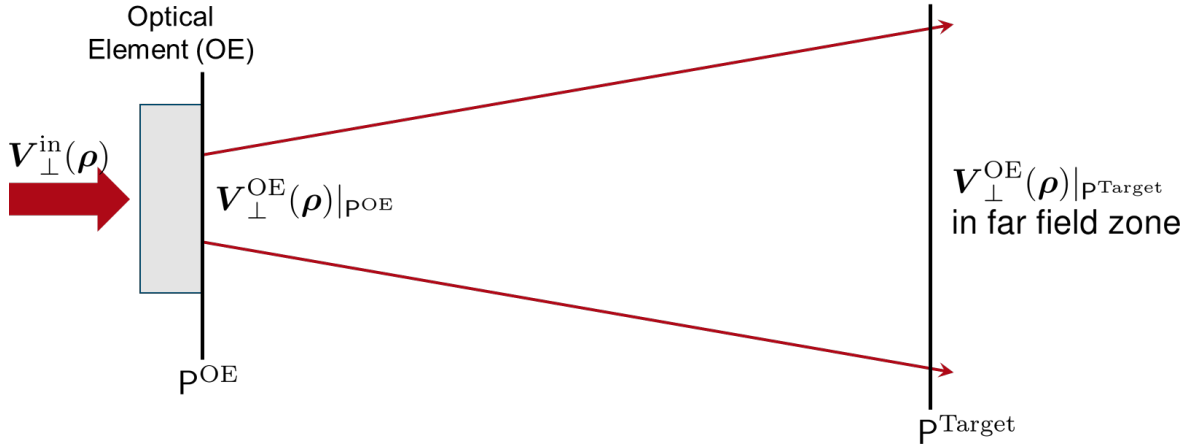


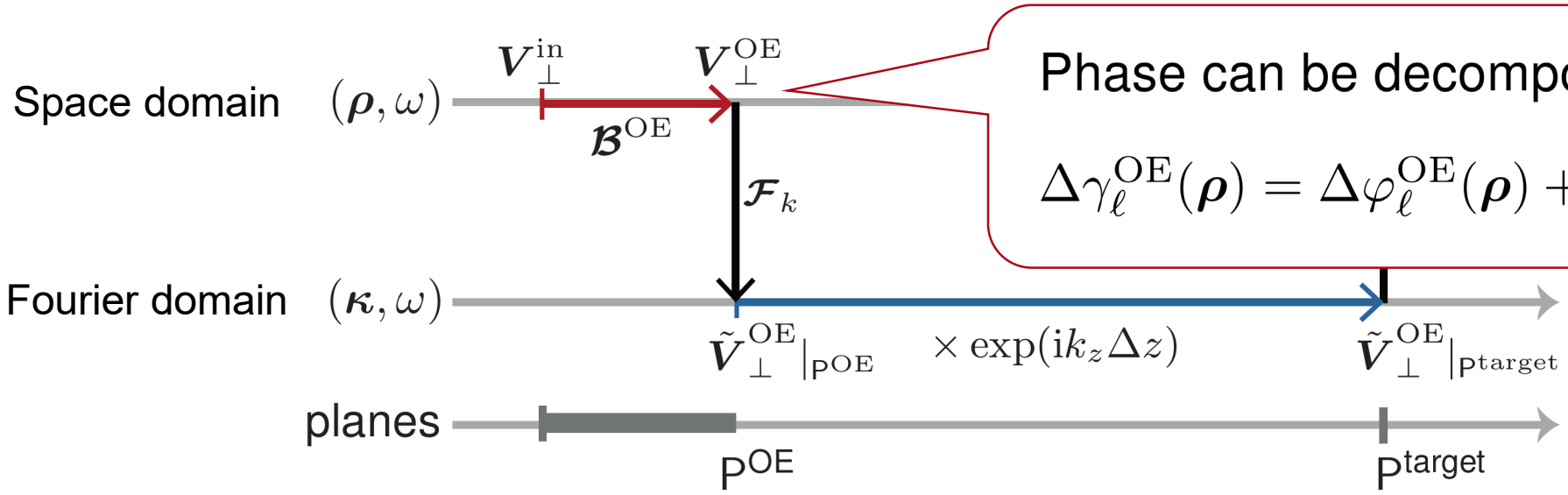


Typically a phase-only effect is preferred:

$$V_\ell^{\mathrm{OE}}(\boldsymbol{\rho}) = V_\ell^{\mathrm{in}}(\boldsymbol{\rho})\exp\left(\mathrm{i}\Delta\gamma_\ell^{\mathrm{OE}}(\boldsymbol{\rho})\right)$$

Mathematical Formulation in Field Tracing Diagram

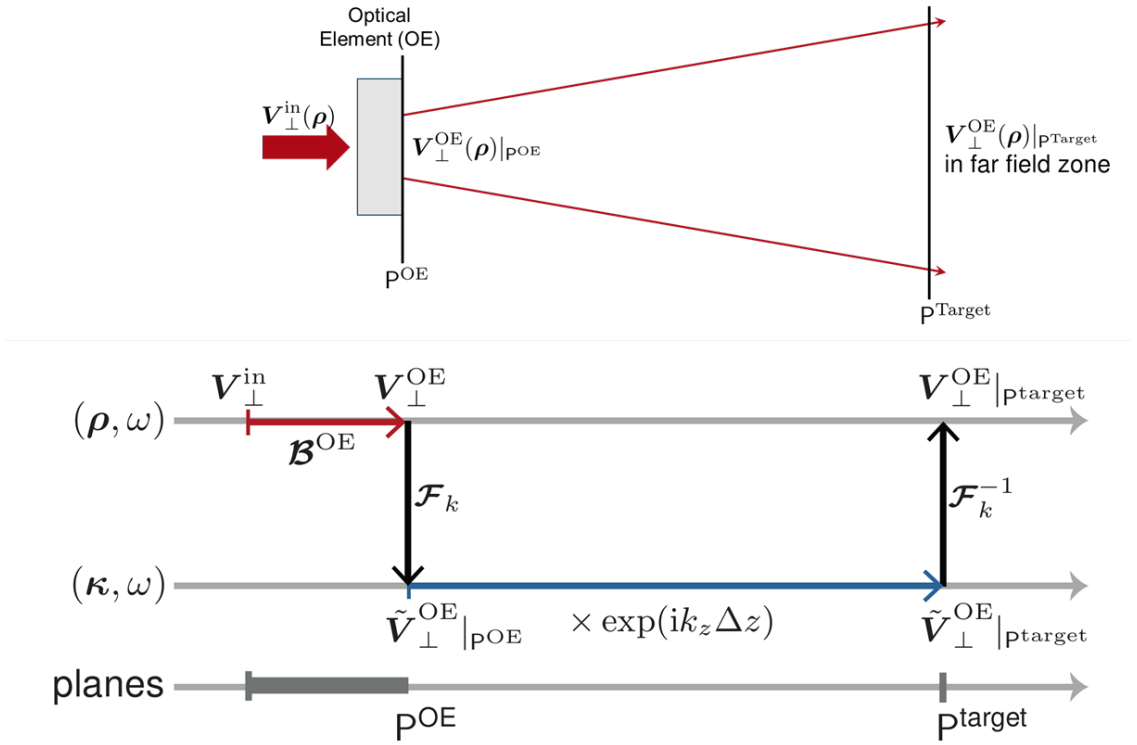




Phase can be decomposed into two parts:

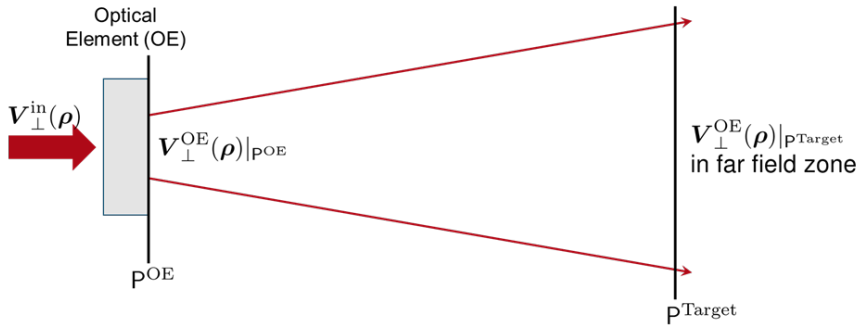
$$\Delta\gamma_\ell^{\mathrm{OE}}(\boldsymbol{\rho}) = \Delta\varphi_\ell^{\mathrm{OE}}(\boldsymbol{\rho}) + \Delta\psi^{\mathrm{OE}}(\boldsymbol{\rho})$$

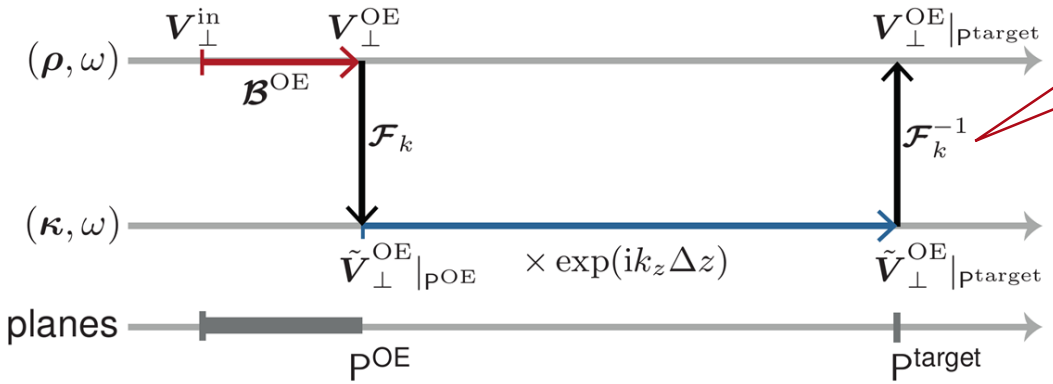
Systematic Understanding of Shaping Concepts



- Light shaping is done by freeform surfaces, DOE's, beam splitters, diffusers, lens arrays, and other types of optical elements.
- All follow the same physical optics modeling description (left), though some of them are discussed in ray optics and some in diffractive optics.
- How can that be understood mathematically?
- What are the consequences for design strategies?

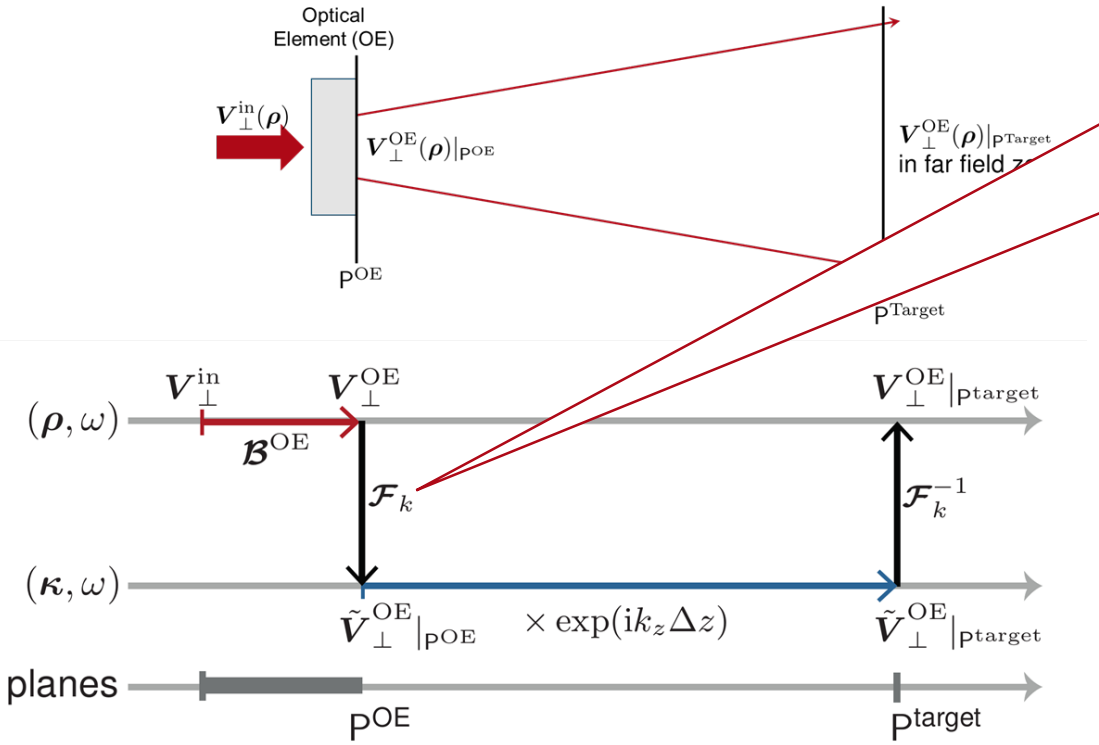
Systematic Understanding of Shaping Concepts





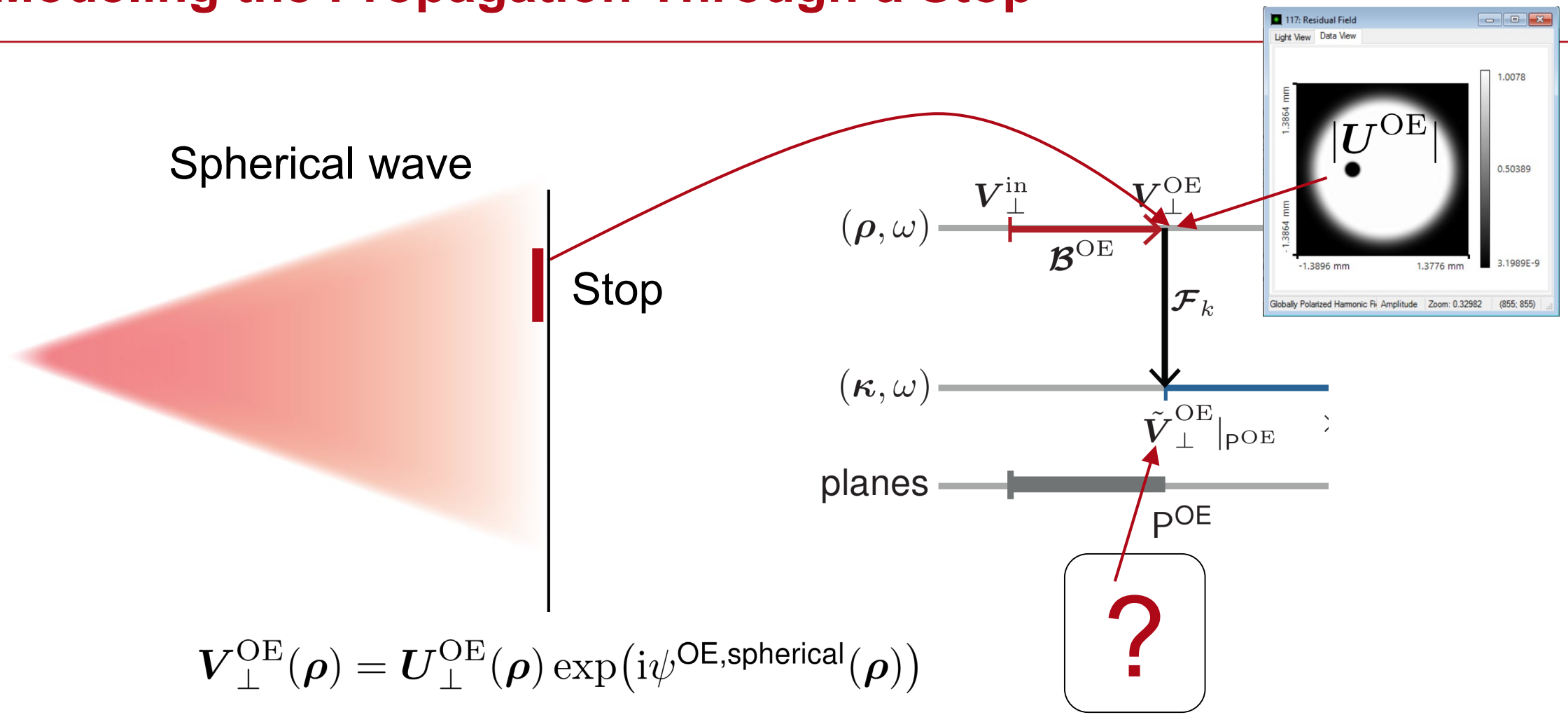
The key for an answer lies in the character of the Fourier transform!

Systematic Understanding of Shaping Concepts



The key for an answer lies in the character of the Fourier transform!

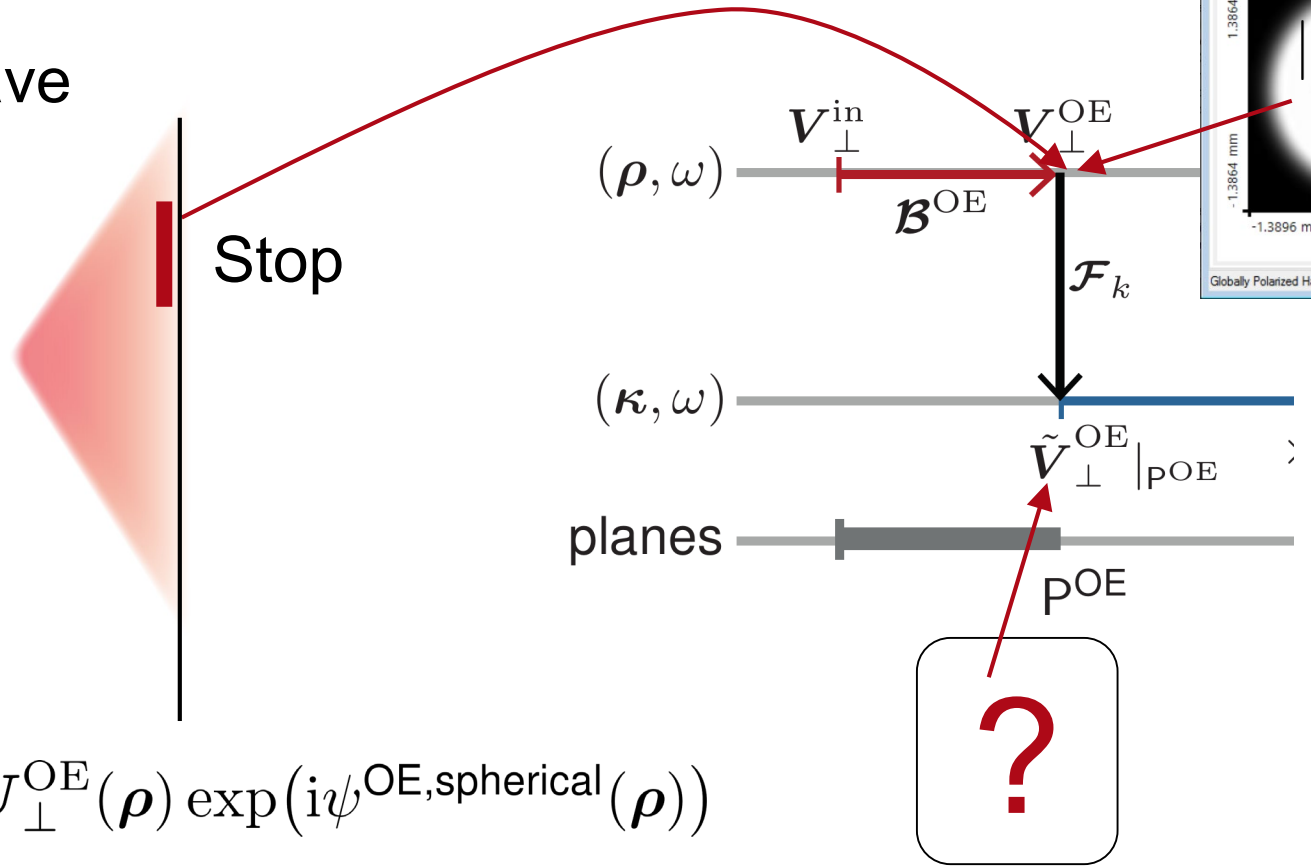
Modeling the Propagation Through a Stop

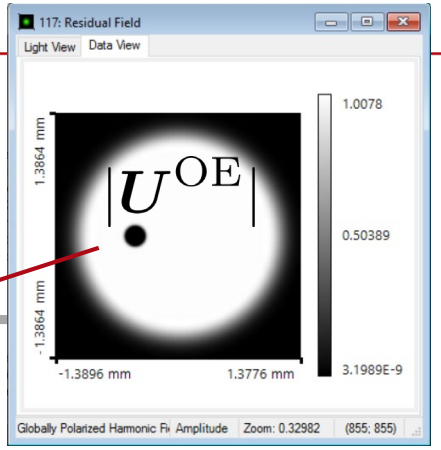


$$\boldsymbol{V}_\perp^{\mathrm{OE}}(\boldsymbol{\rho}) = \boldsymbol{U}_\perp^{\mathrm{OE}}(\boldsymbol{\rho}) \exp\left(\mathrm{i}\psi^{\mathsf{OE,spherical}}(\boldsymbol{\rho})\right)$$

Modeling the Propagation Through a Stop

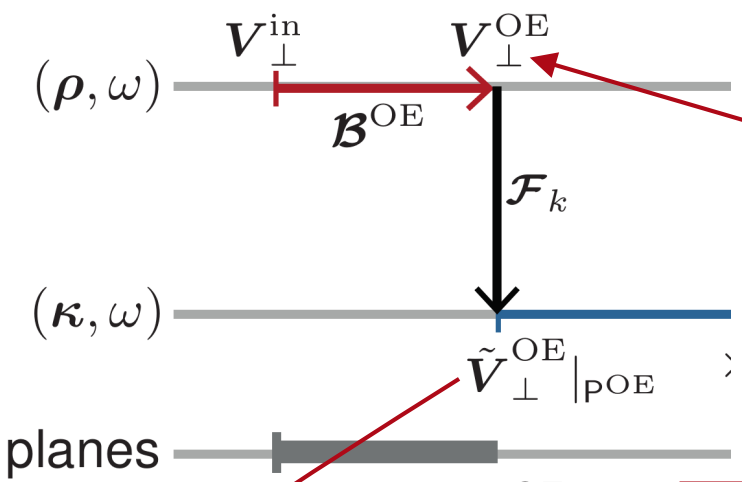
Spherical wave

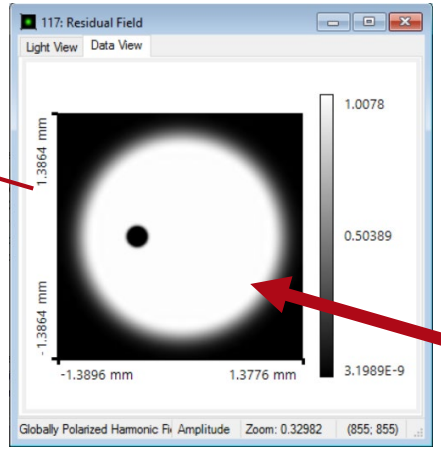




$$\boldsymbol{V}_\perp^{\text{OE}}(\boldsymbol{\rho}) = \boldsymbol{U}_\perp^{\text{OE}}(\boldsymbol{\rho}) \exp\left(\mathrm{i}\psi^{\text{OE,spherical}}(\boldsymbol{\rho})\right)$$

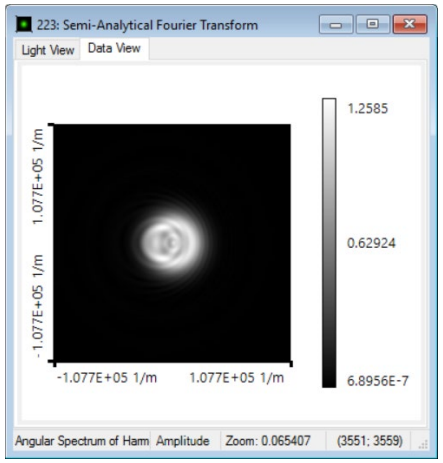
Results of Fourier Transform

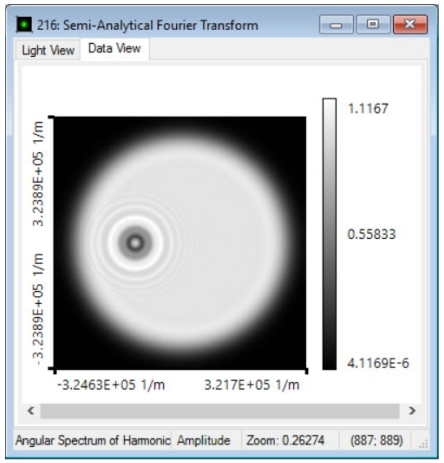


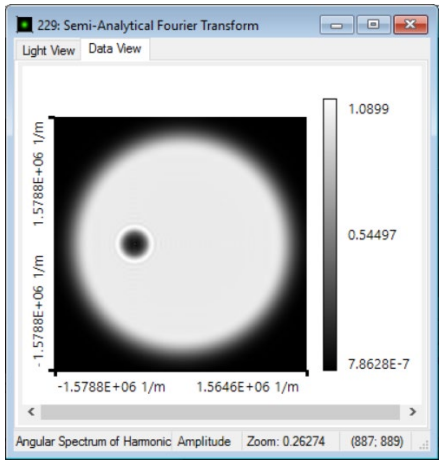


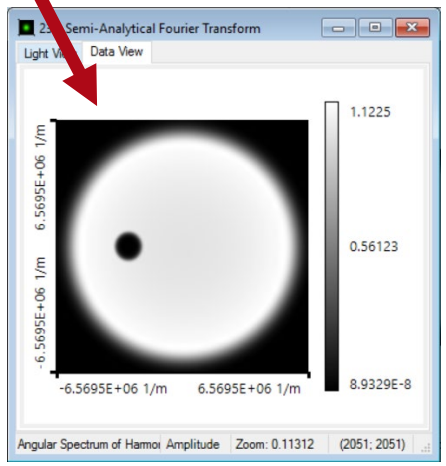
For strong wavefront phase Fourier transform performs 1:1 mapping!

Increasing NA

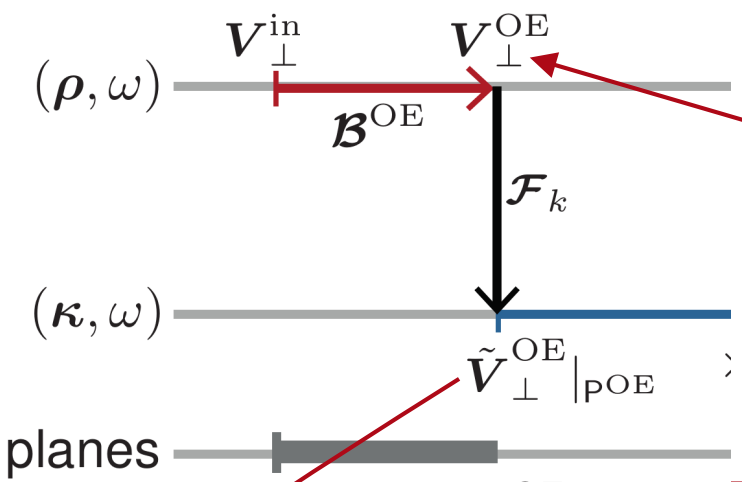


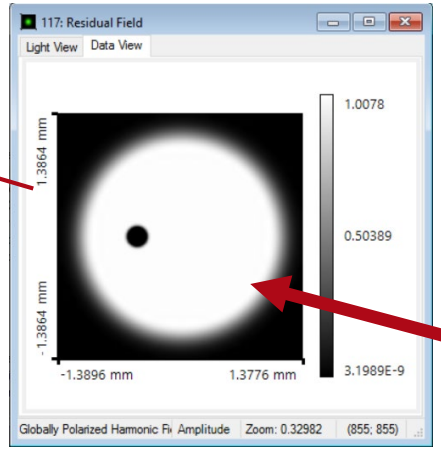






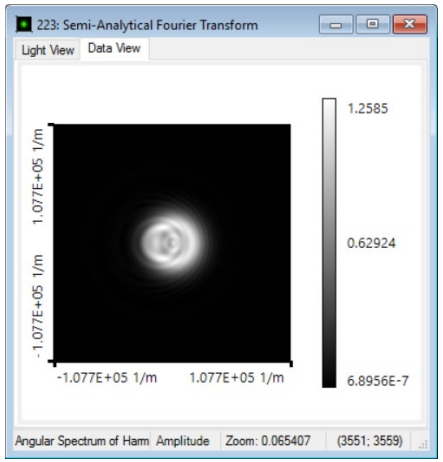
Results of Fourier Transform

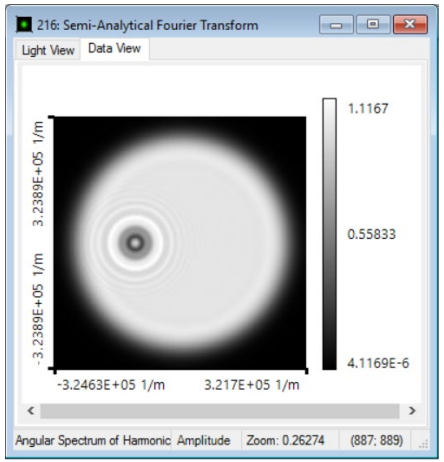


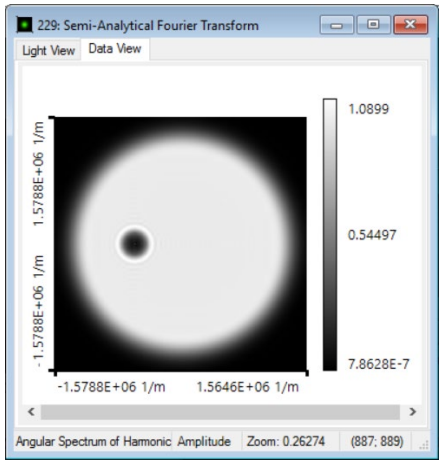


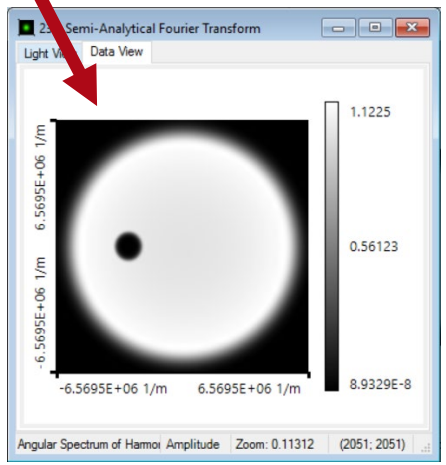
For strong wavefront phase we obtain homeomorphism between both domains.

Increasing NA

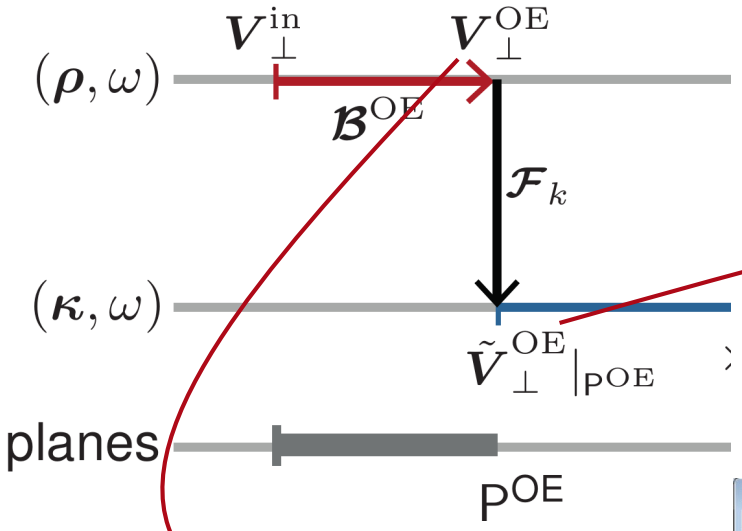


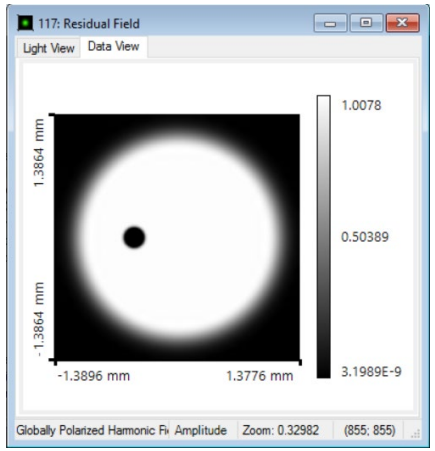


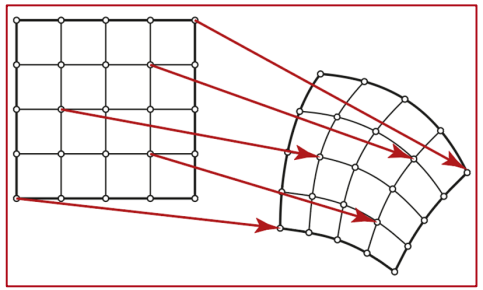


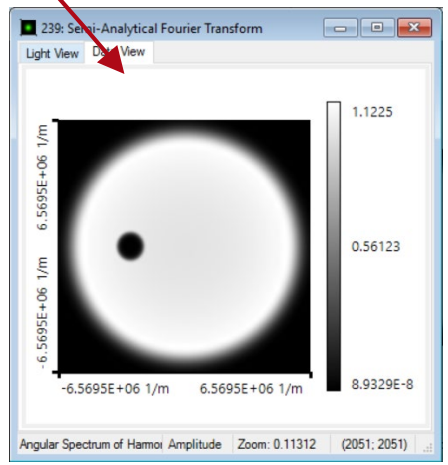


Homeomorphic Fourier Transform

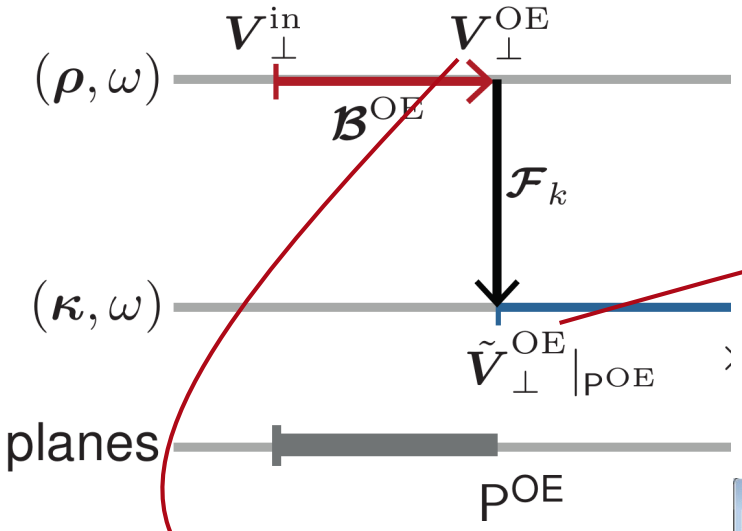




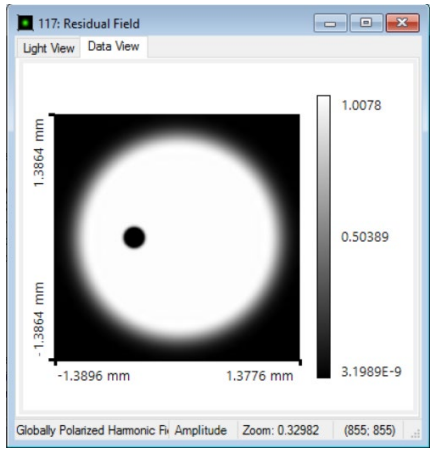


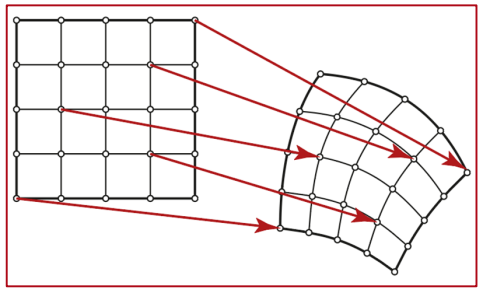


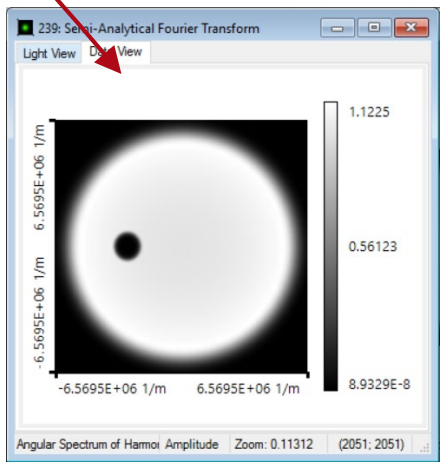
Homeomorphic Fourier Transform



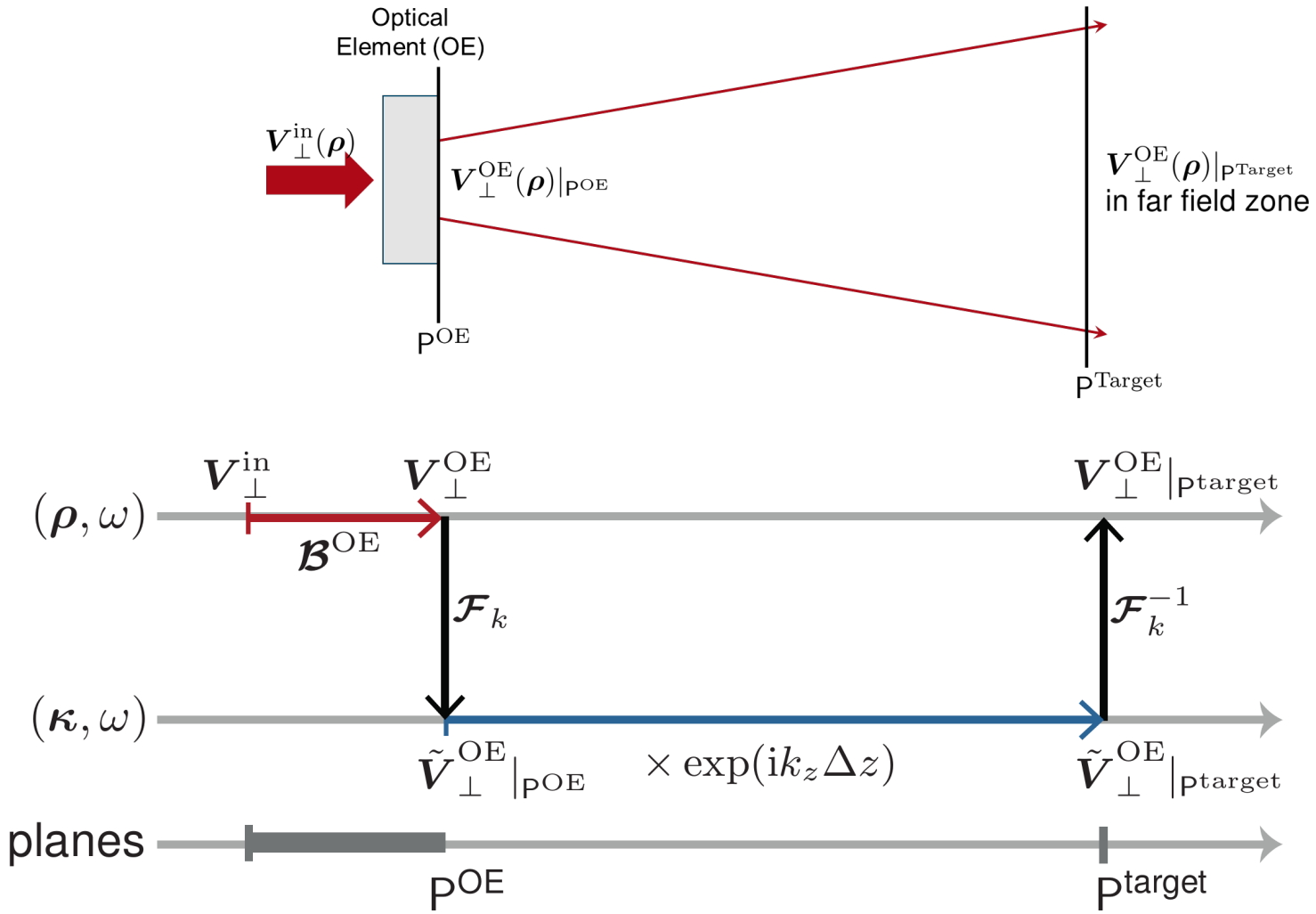
Fields which can be accurately transformed into the *k*-domain by the homeomorphic Fourier transform are situated in its homeomorphic zone (HFZ).



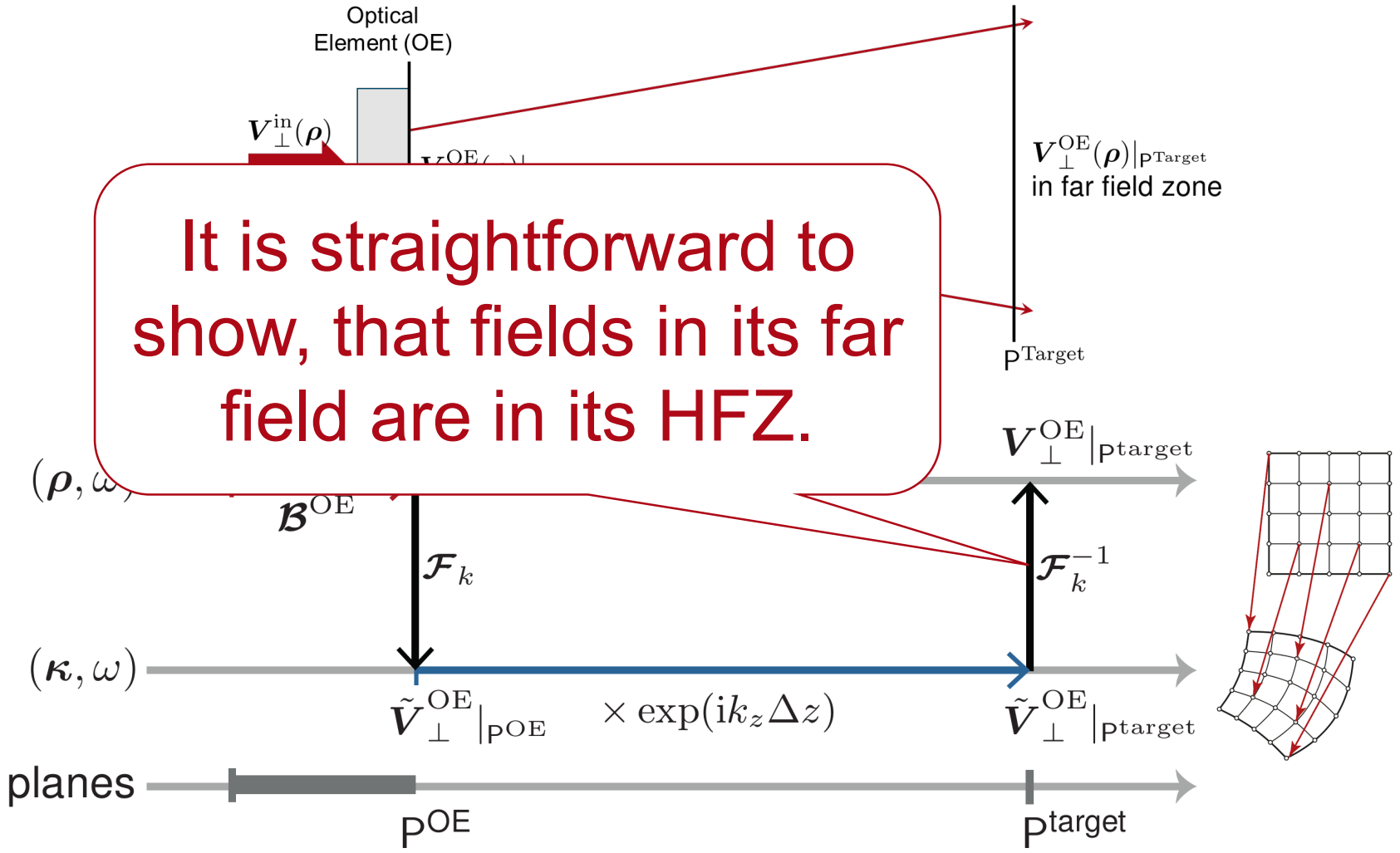




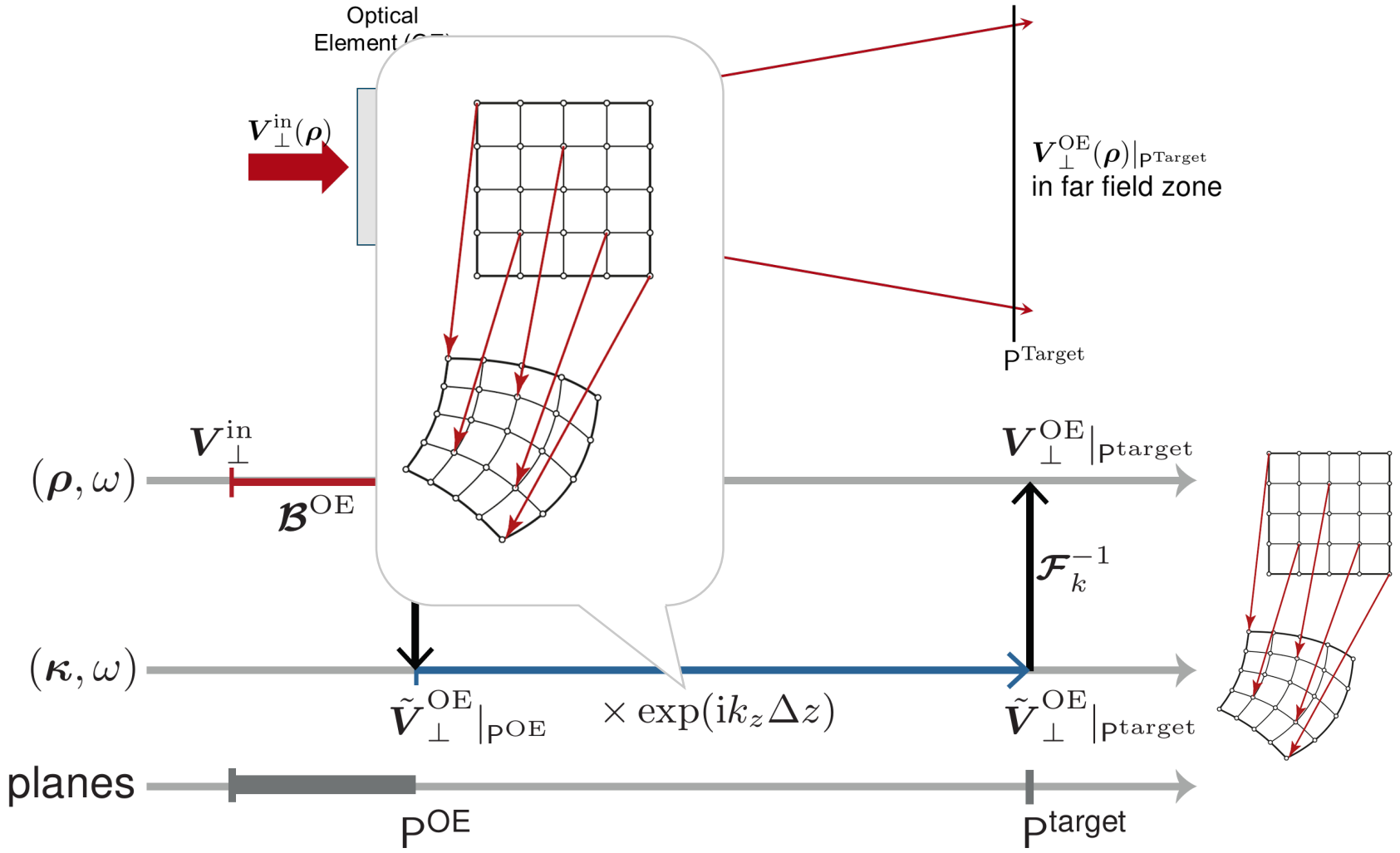
Mathematical Formulation in Field Tracing Diagram



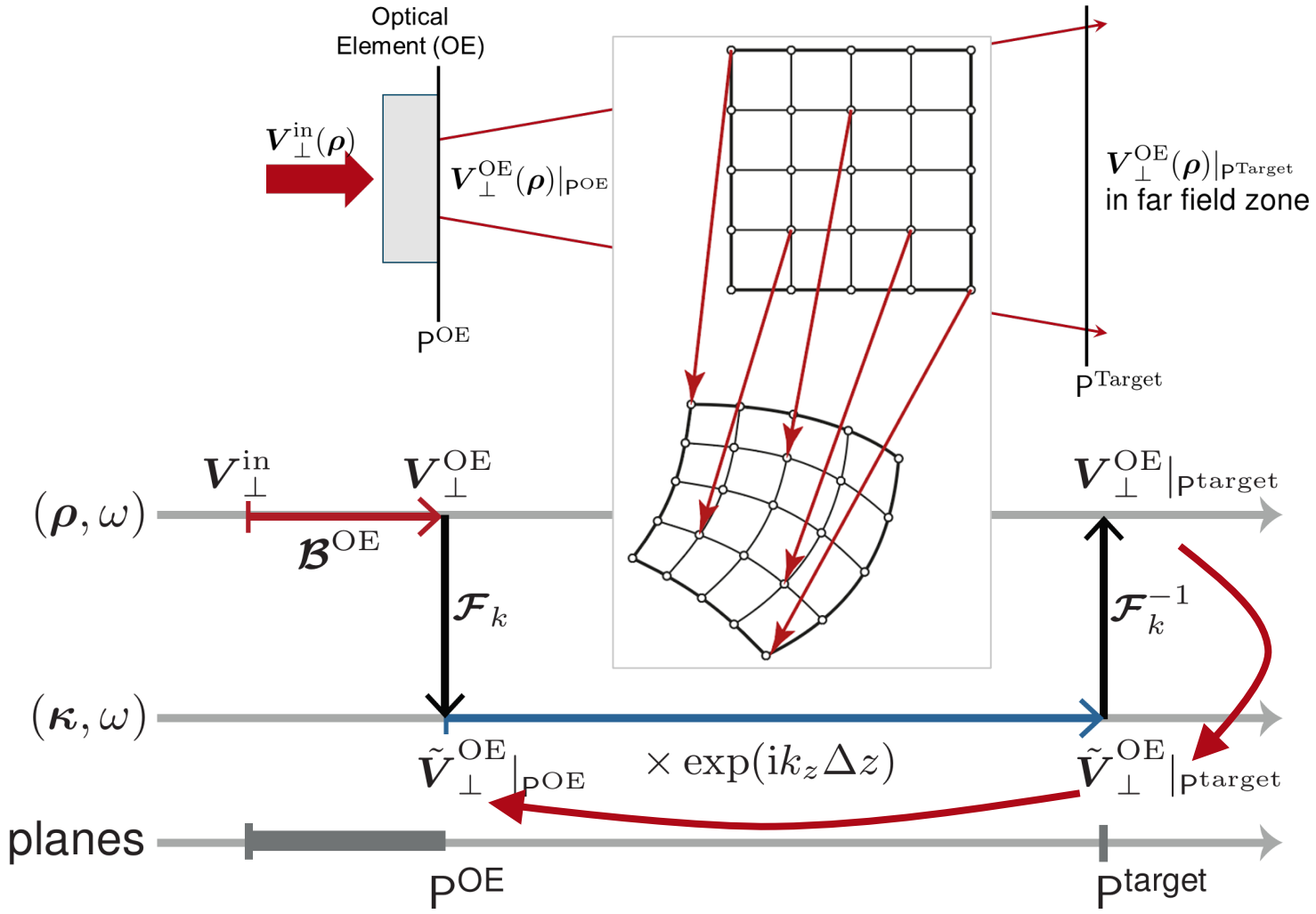
Light Shaping and Homeomorphic Operations



Light Shaping and Homeomorphic Operations

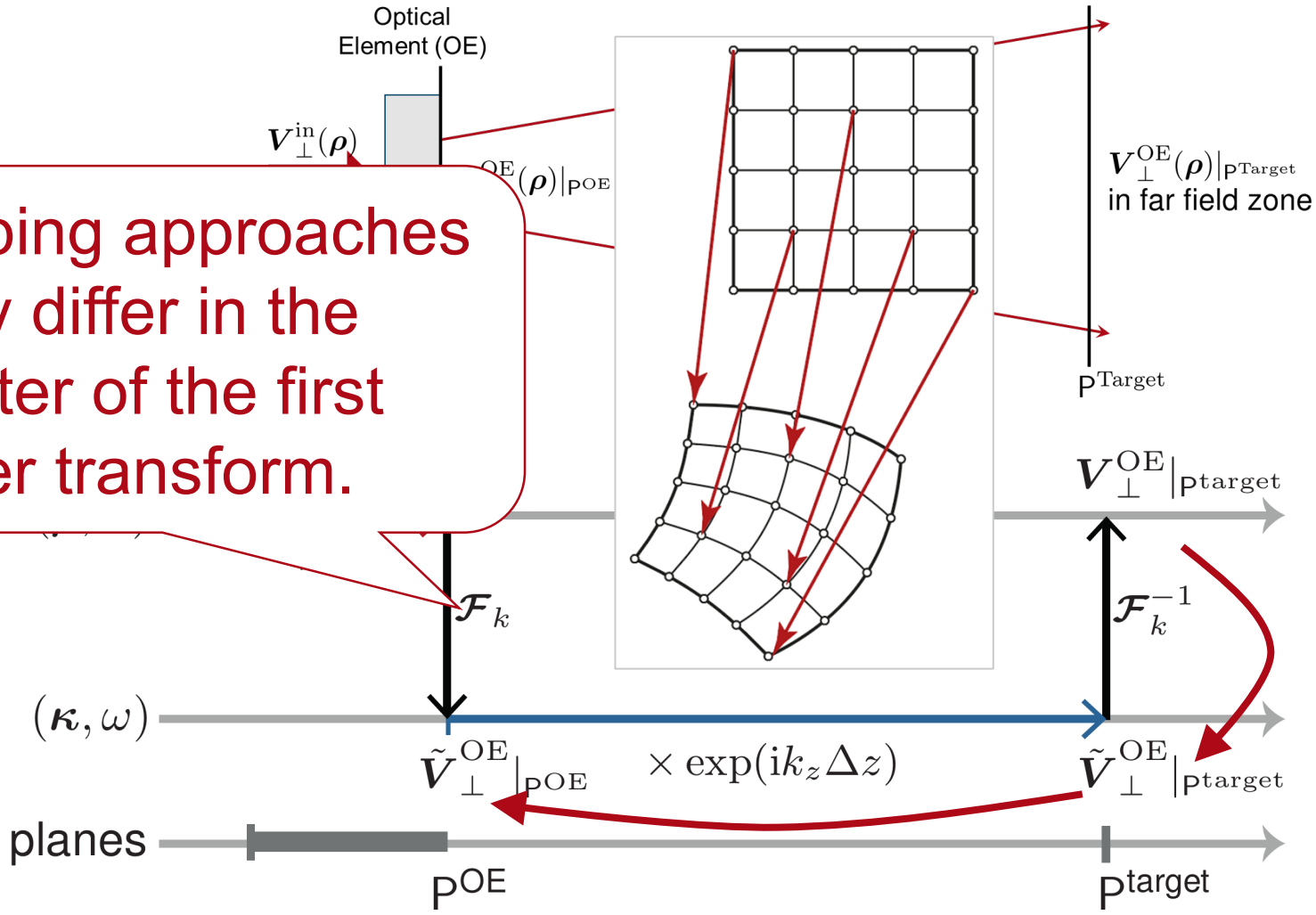


Light Shaping and Homeomorphic Operations

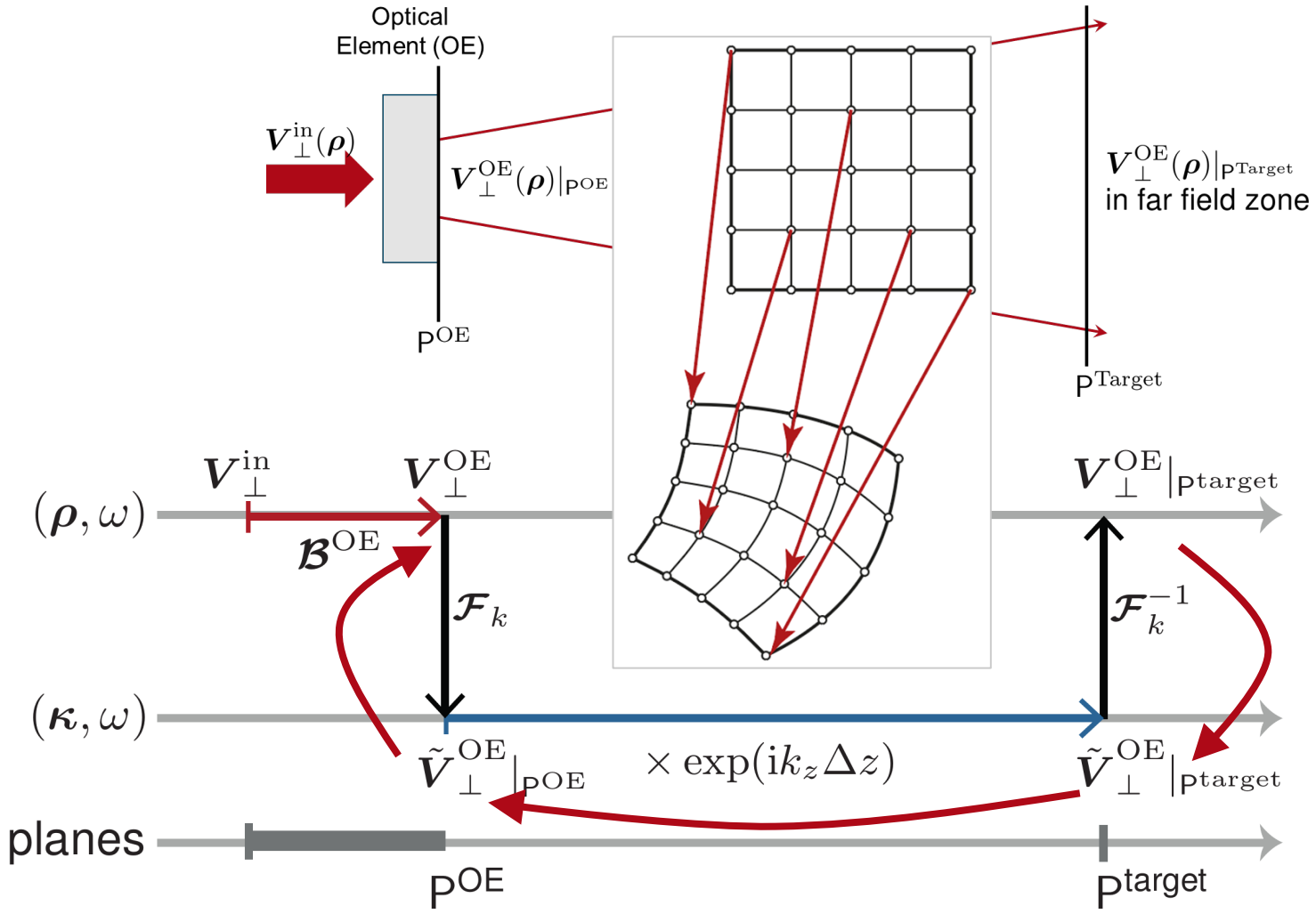


Light Shaping and Homeomorphic Operations

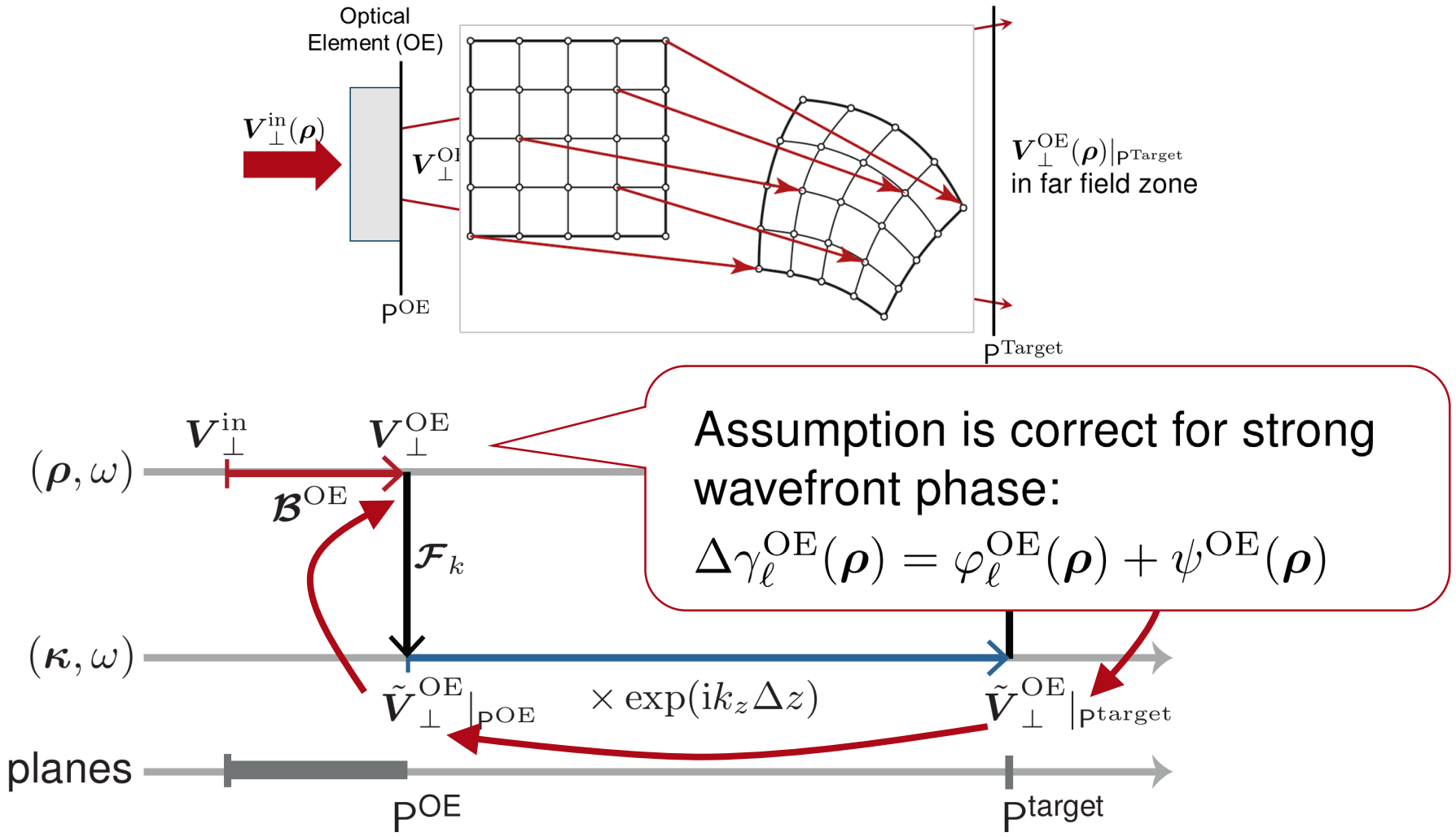
Light shaping approaches mainly differ in the character of the first Fourier transform.



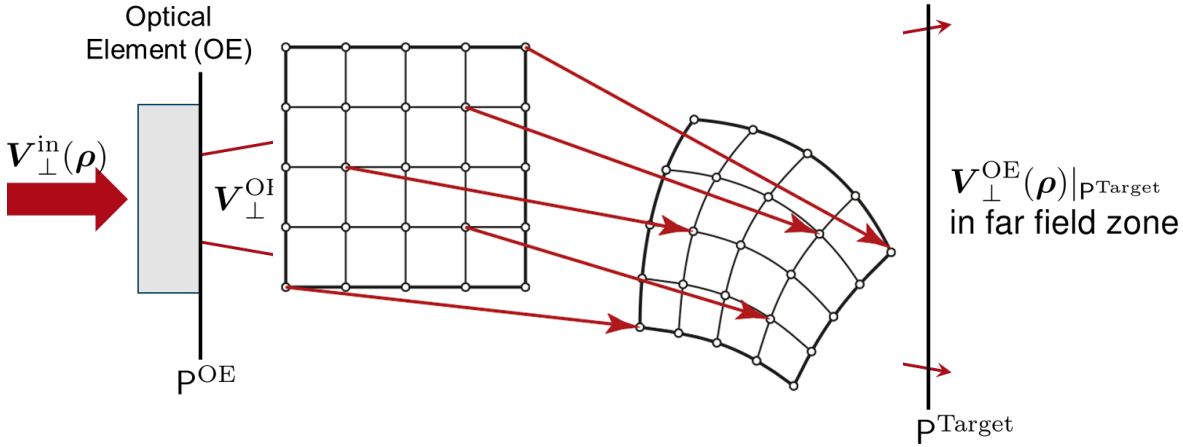
Light Shaping by Fully Homeomorphic Operations



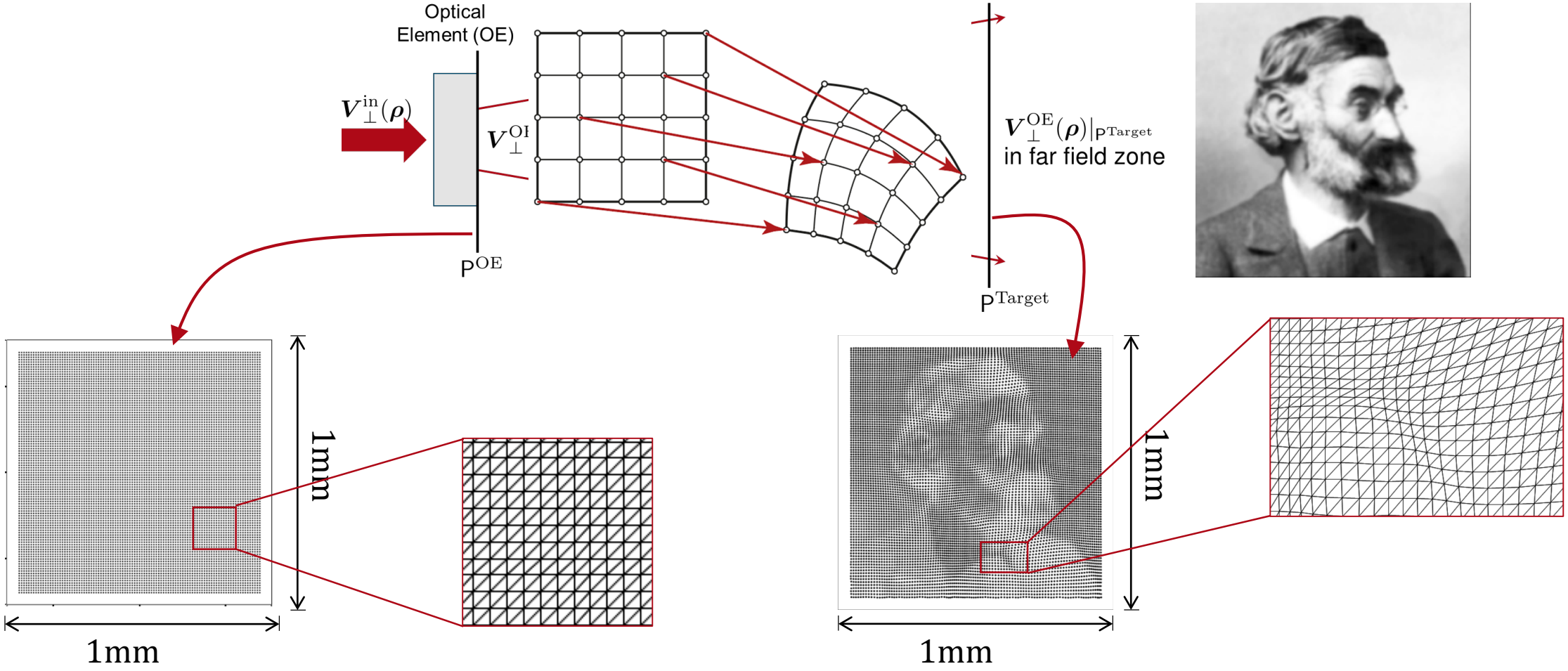
Light Shaping by Fully Homeomorphic Operations



Freeform Design for Light Shaping (Homeomorphism)

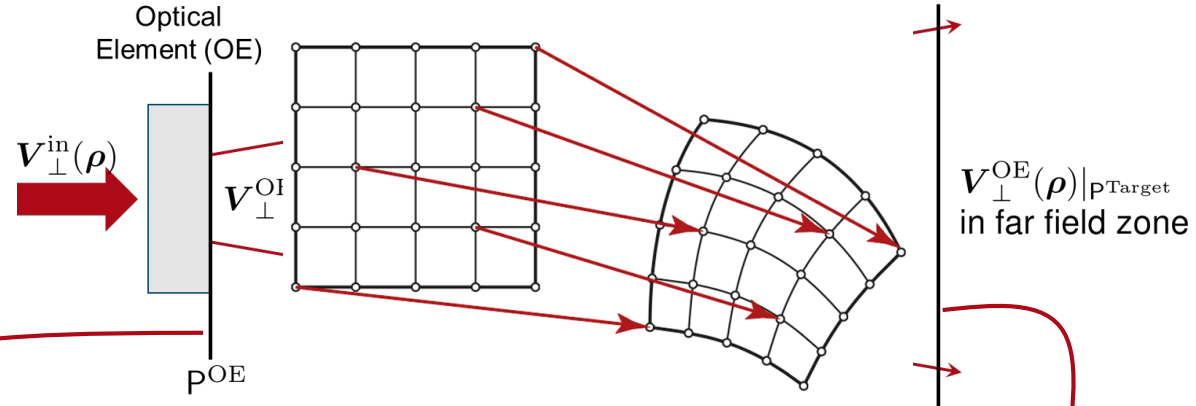


Freeform Design for Light Shaping (Homeomorphism)

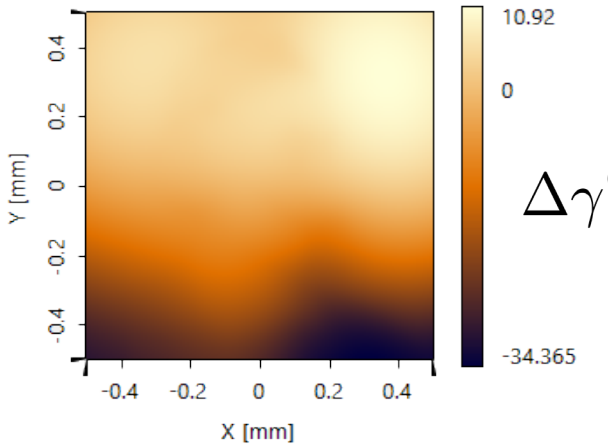


Prins, C, et al. A Least-Squares Method for Optimal Transport Using the Monge--Ampere Equation *SIAM Journal on Scientific Computing,* **2015**, *37*, B937-B961

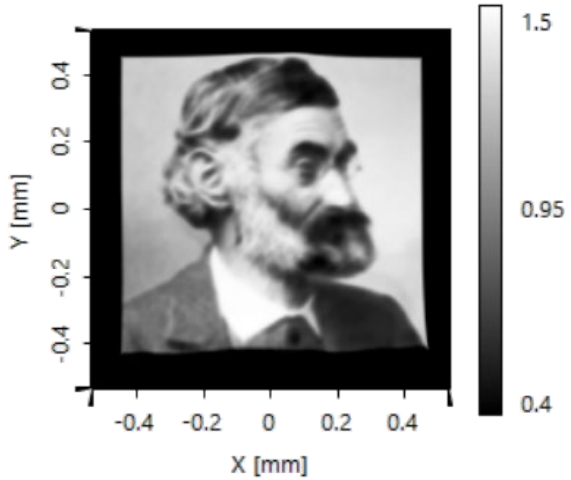
Freeform Design for Light Shaping (Homeomorphism)



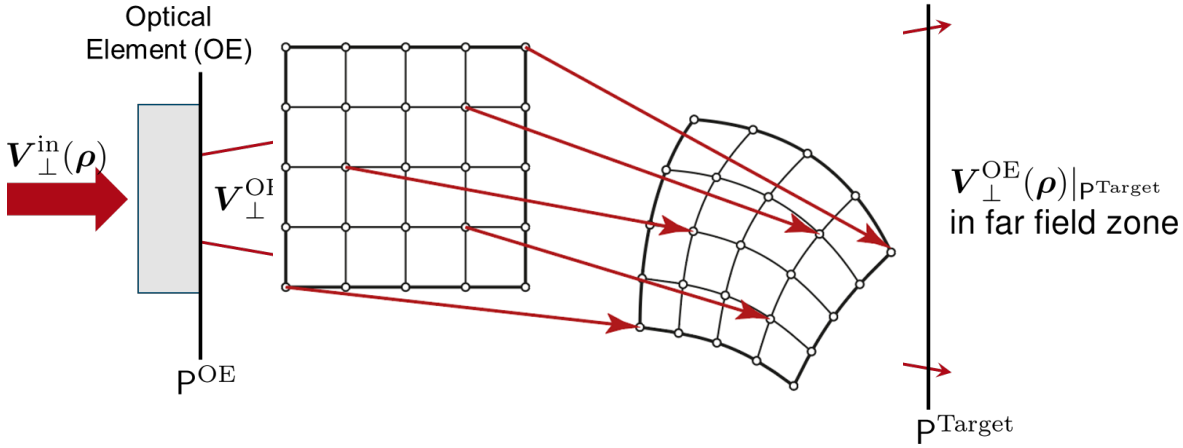
phase function (rad)

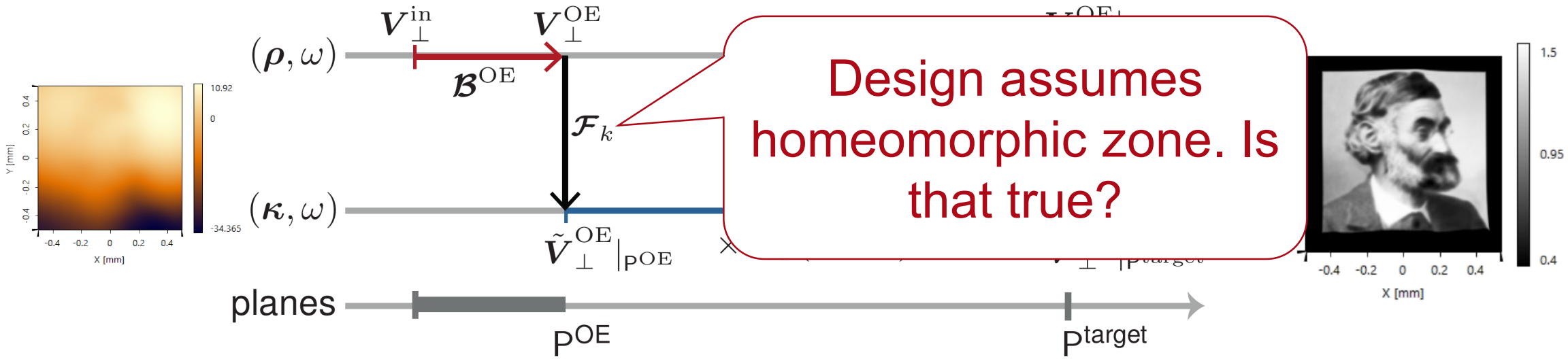


$\Delta\gamma^{\mathrm{OE}}(\rho)$



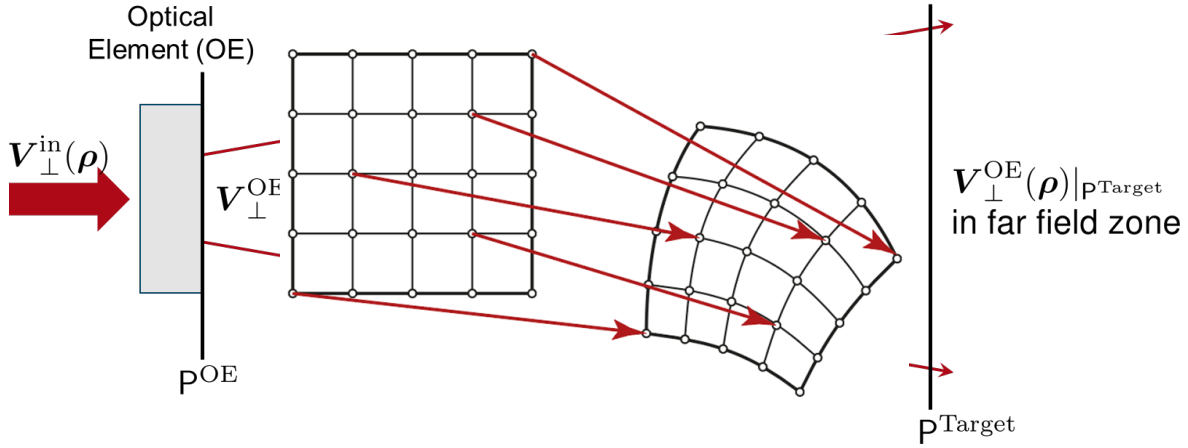
Freeform Design for Light Shaping

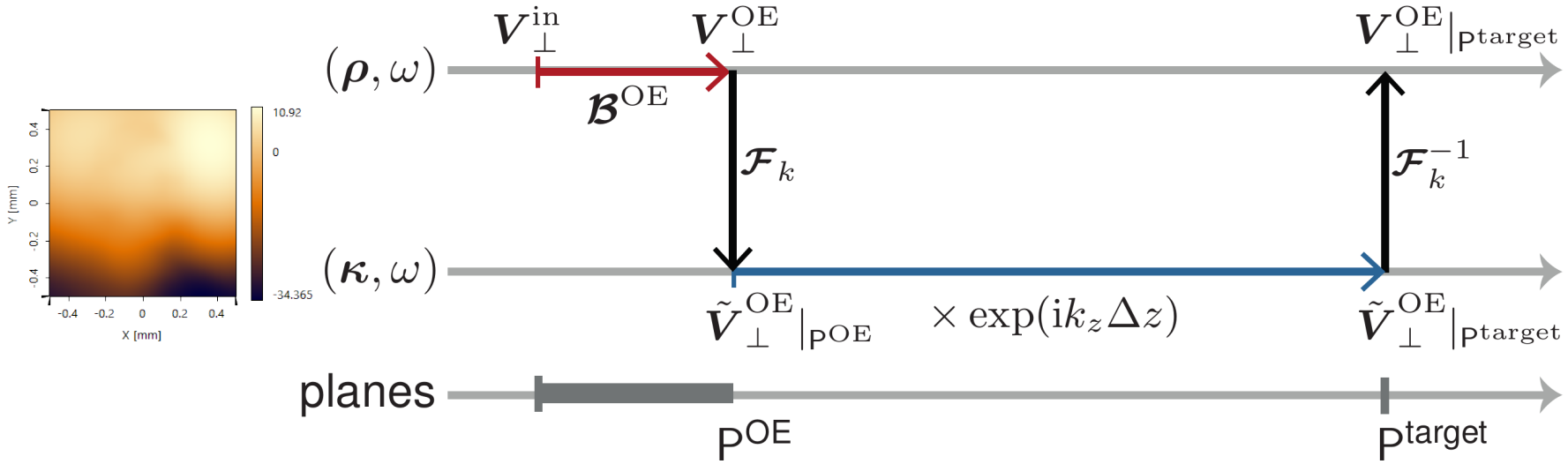


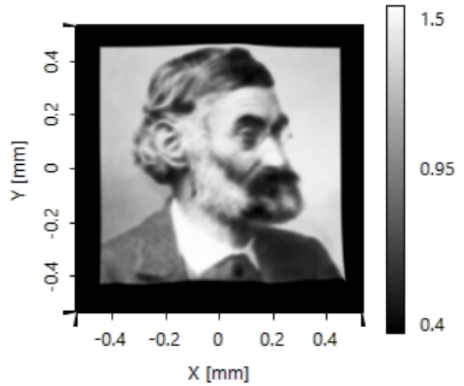


Design assumes homeomorphic zone. Is that true?

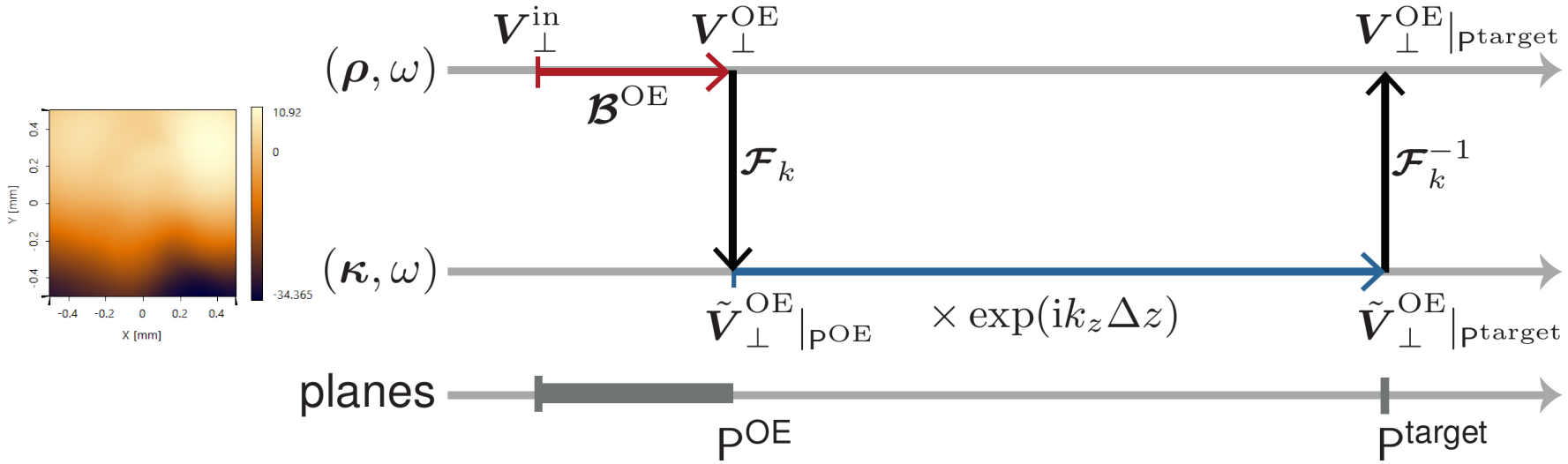
Freeform Design for Light Shaping

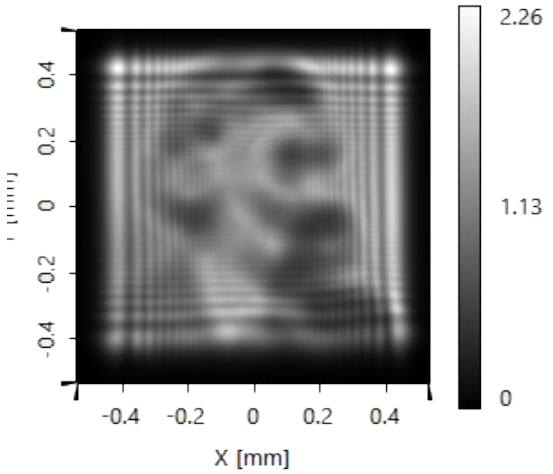




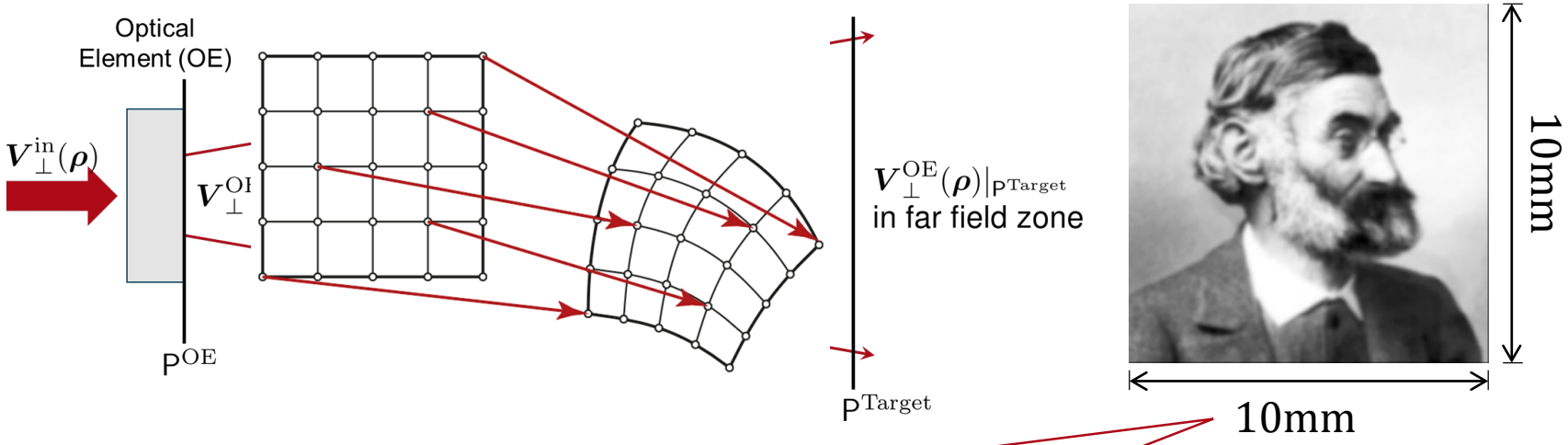


Freeform Design for Light Shaping



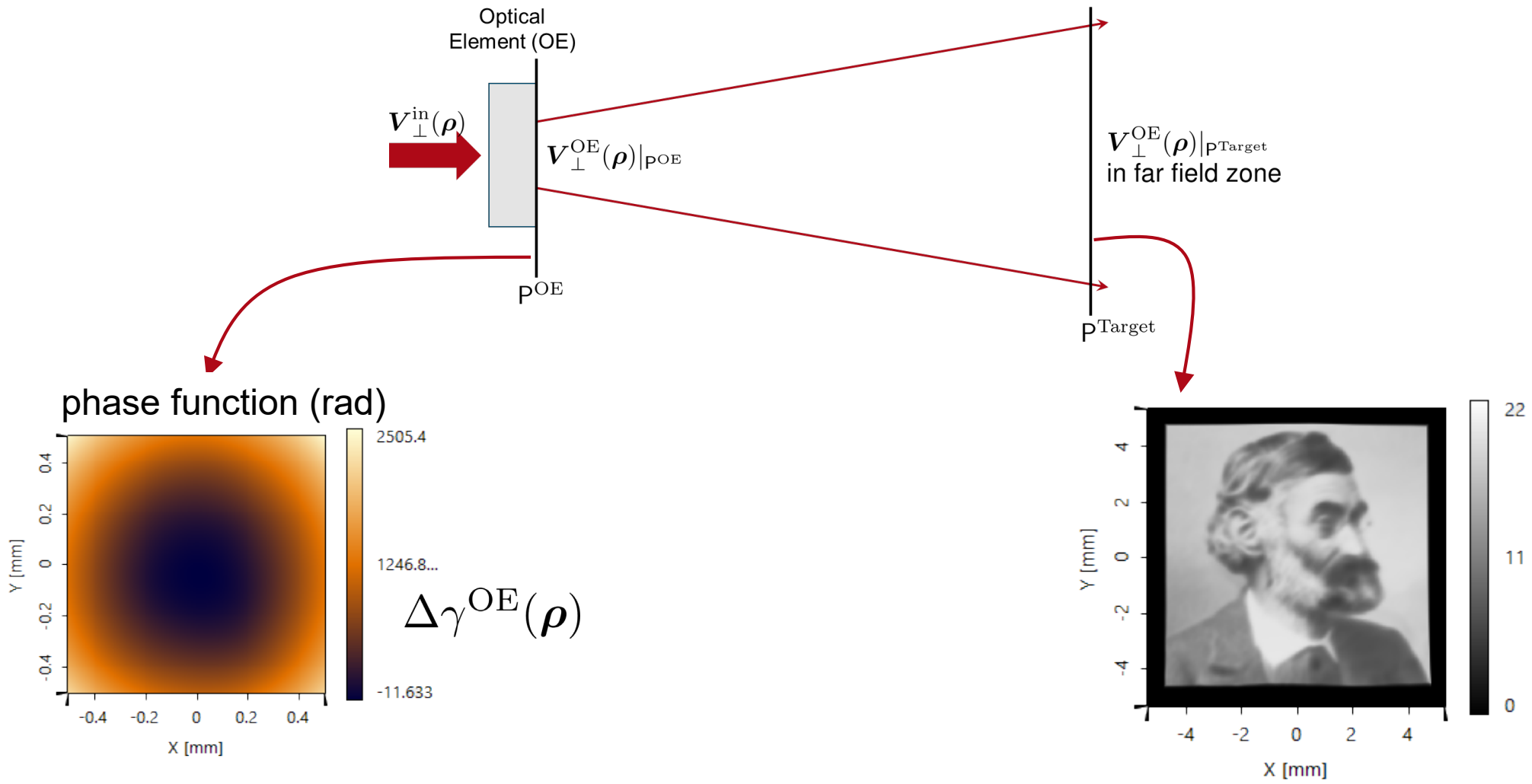


Freeform Design for Light Shaping: High Divergence



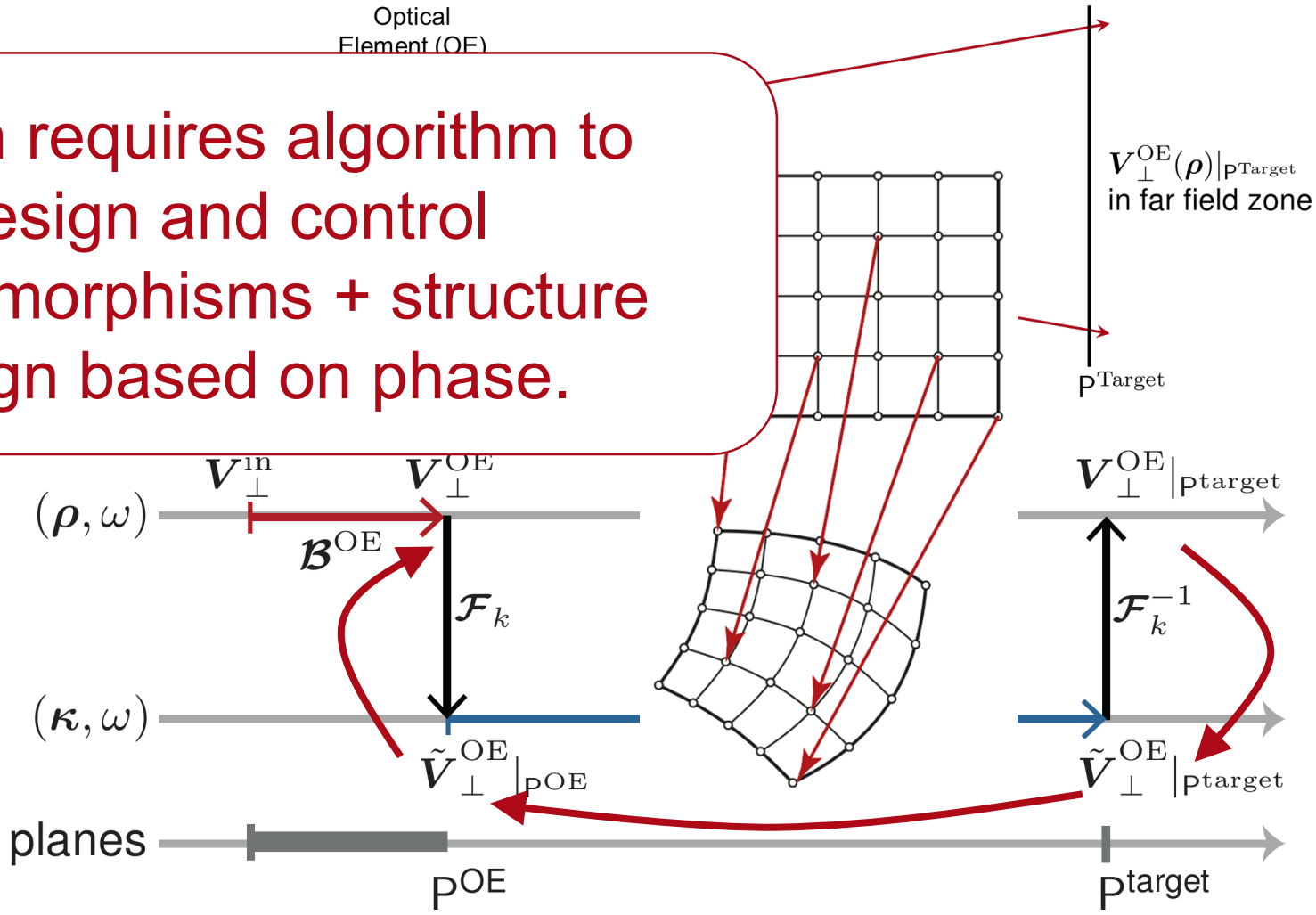
Increase size of pattern from 1 mm to 10 mm!

Freeform Design for Light Shaping: High Divergence

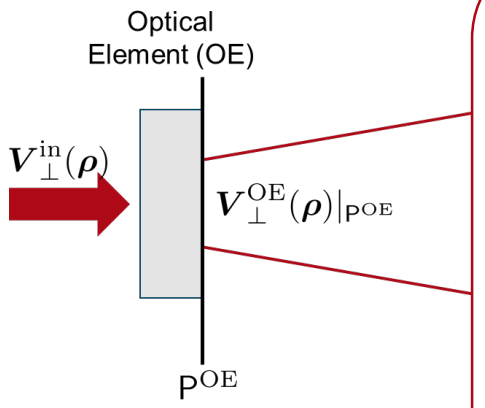


Light Shaping by Fully Homeomorphic Operations

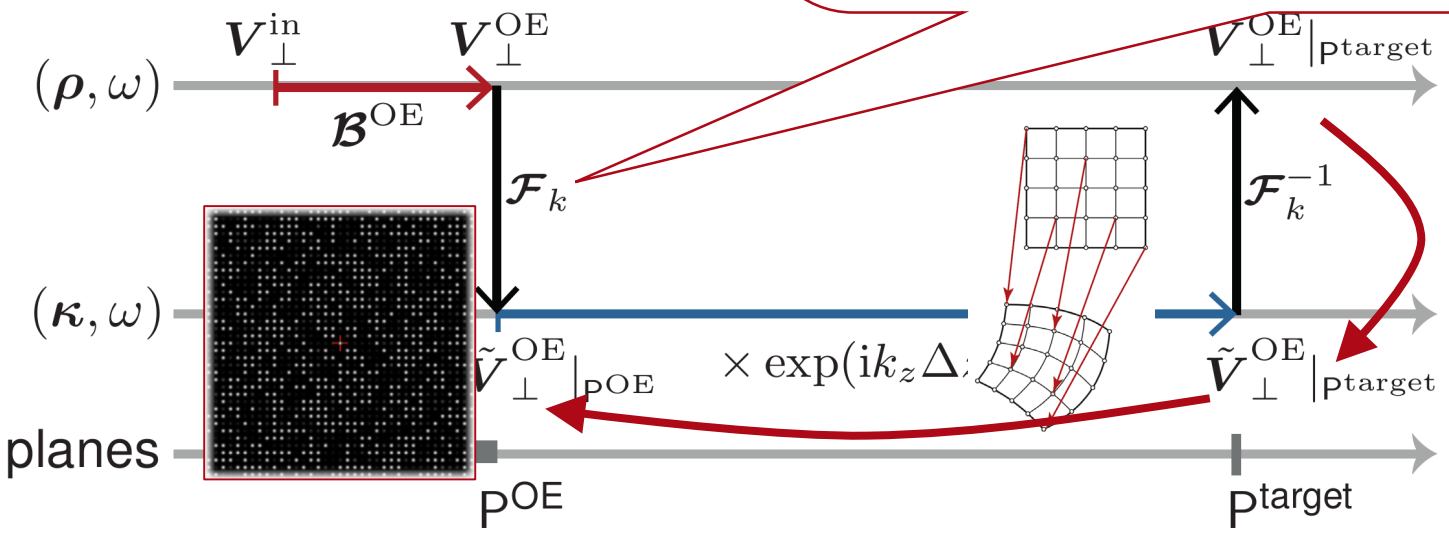
Design requires algorithm to design and control homeomorphisms + structure design based on phase.



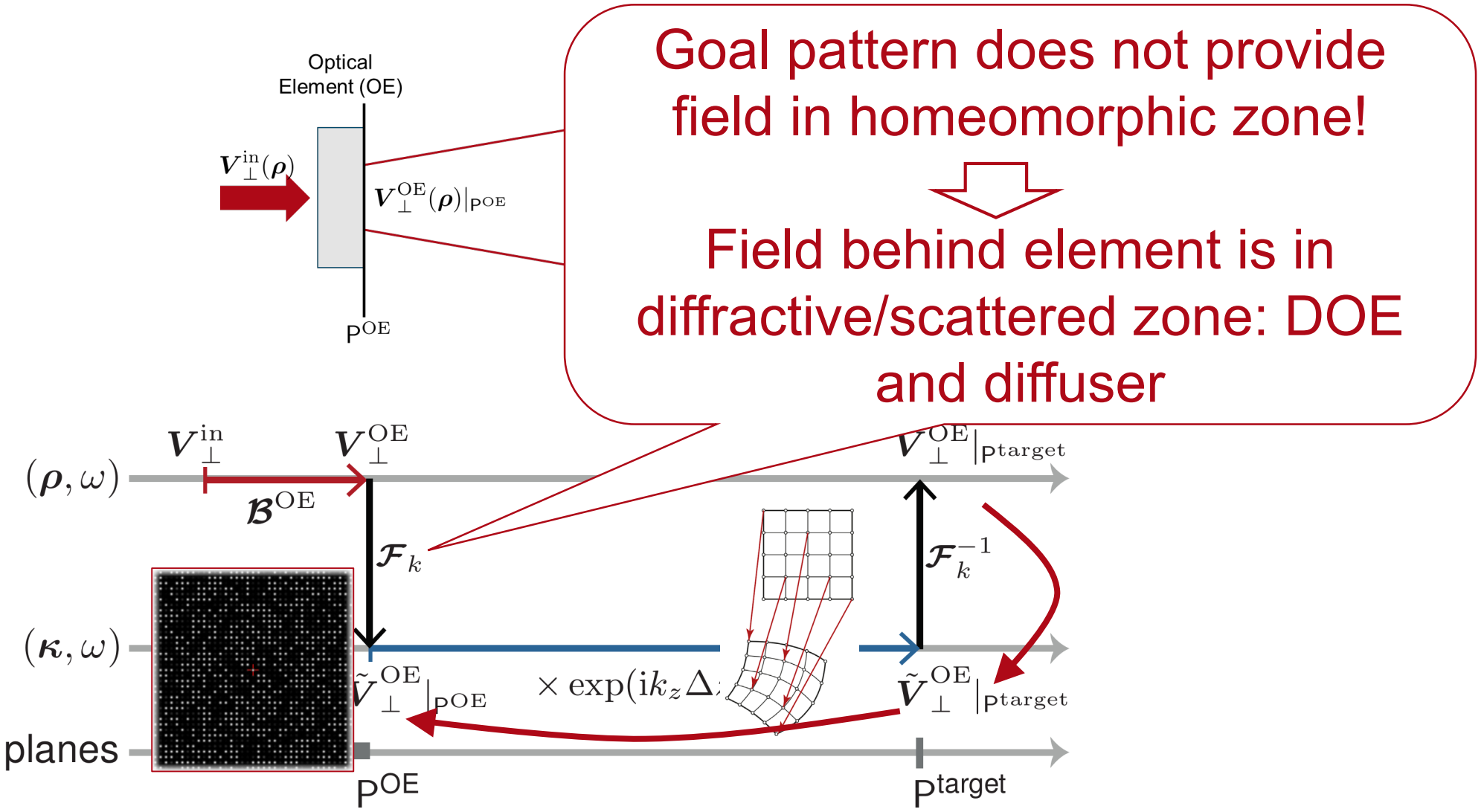
Light Shaping by Diffractive Optics



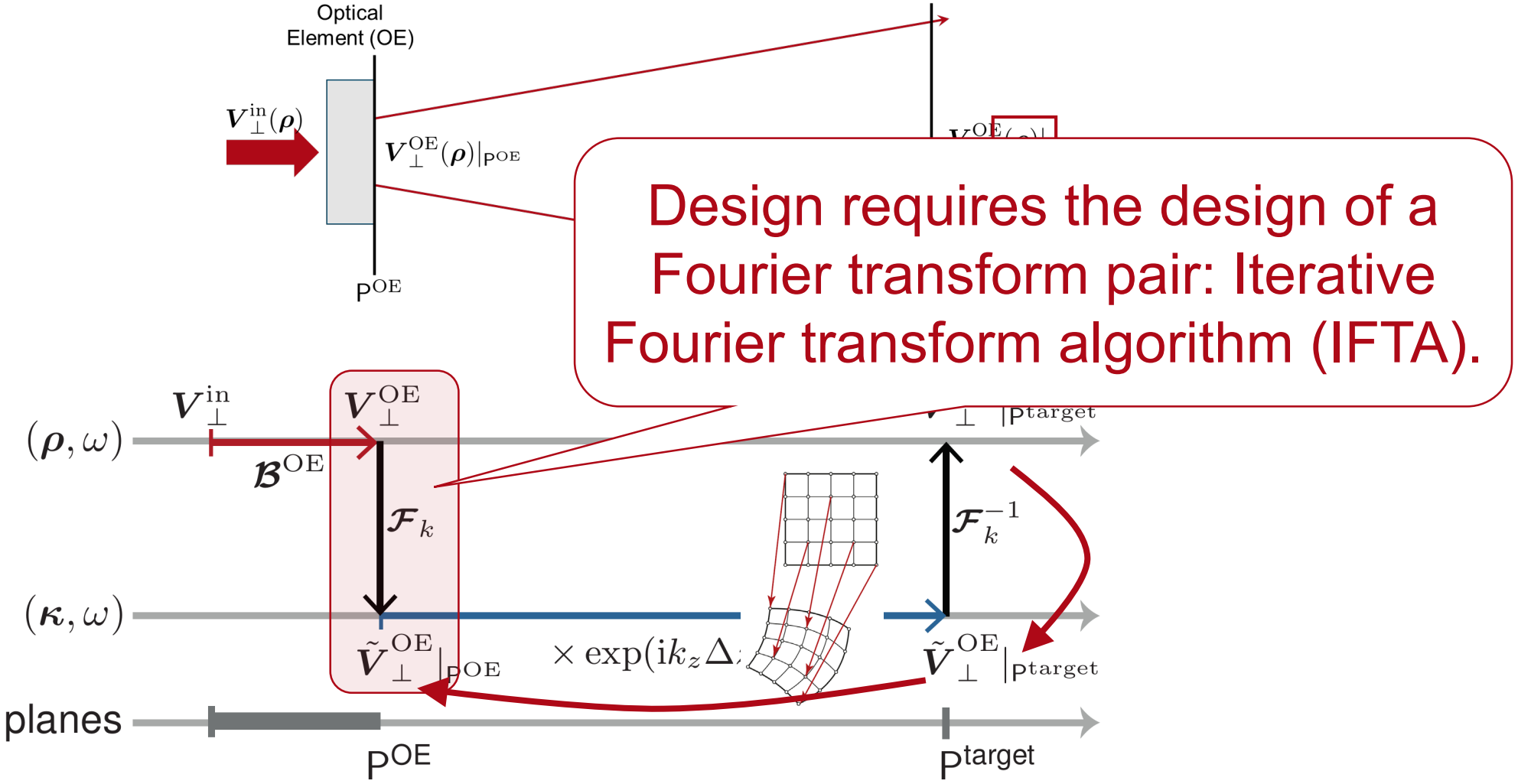
Goal pattern does not provide field in homeomorphic zone!



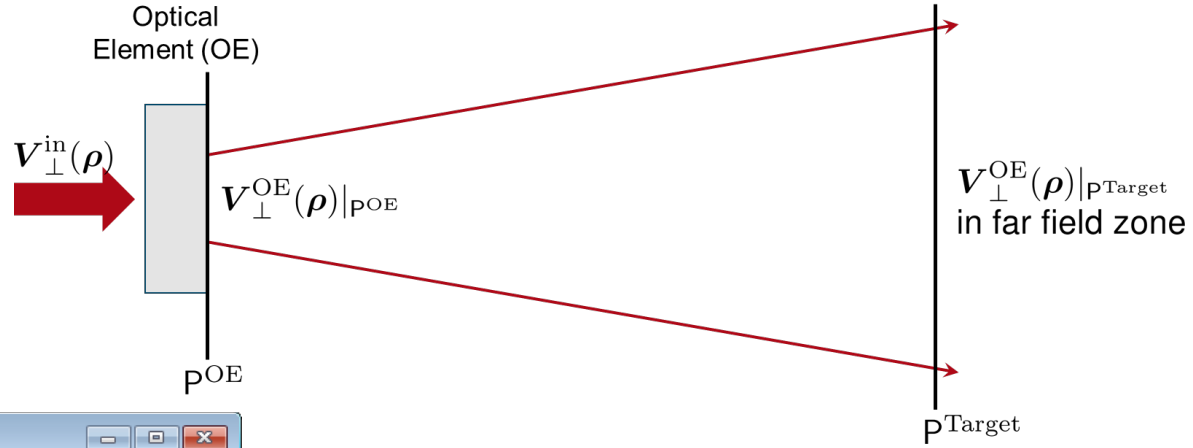
Light Shaping by Diffractive Optics

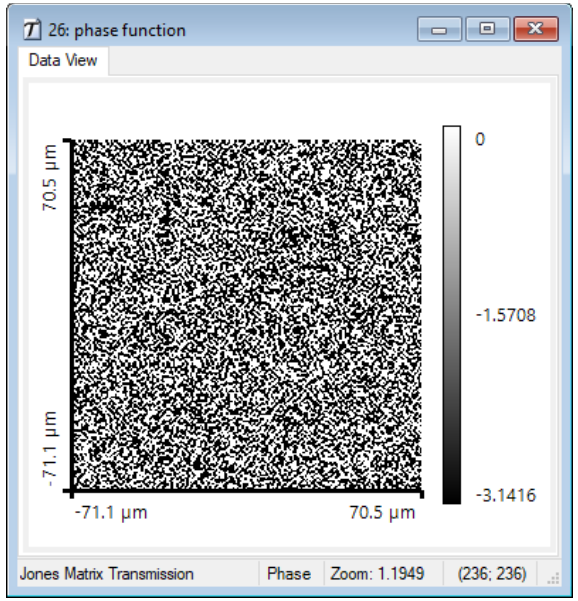


Light Shaping by Diffractive Optics

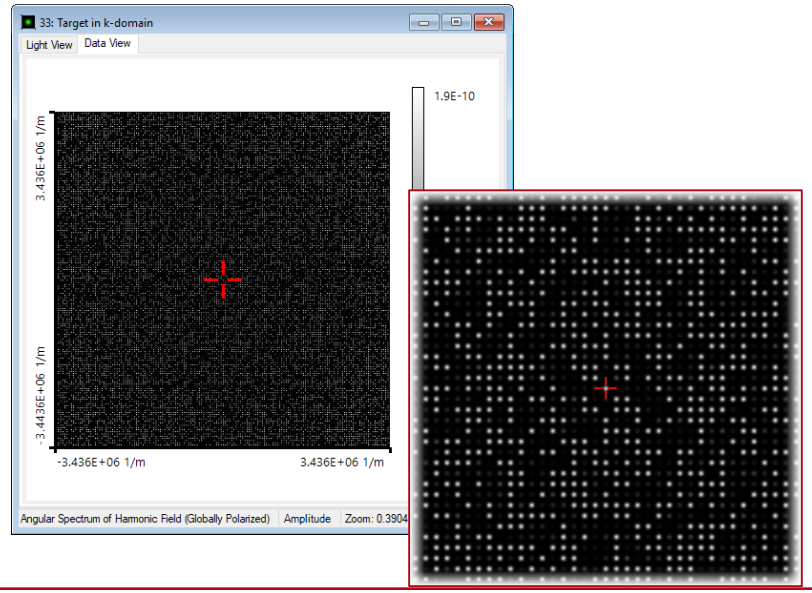


Point Cloud Generation

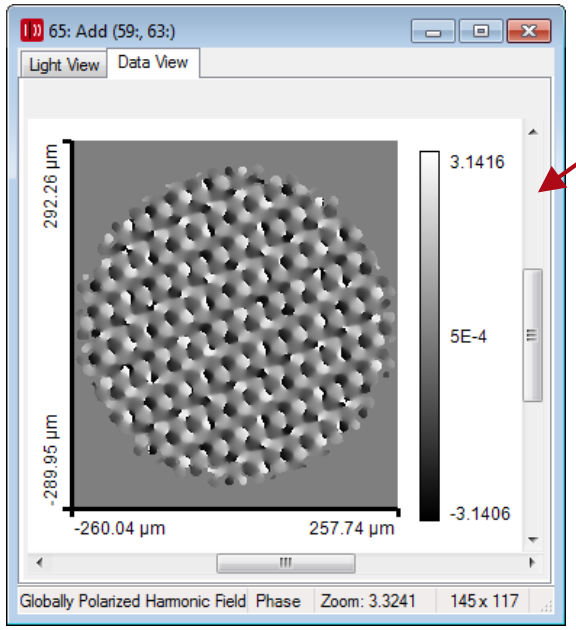


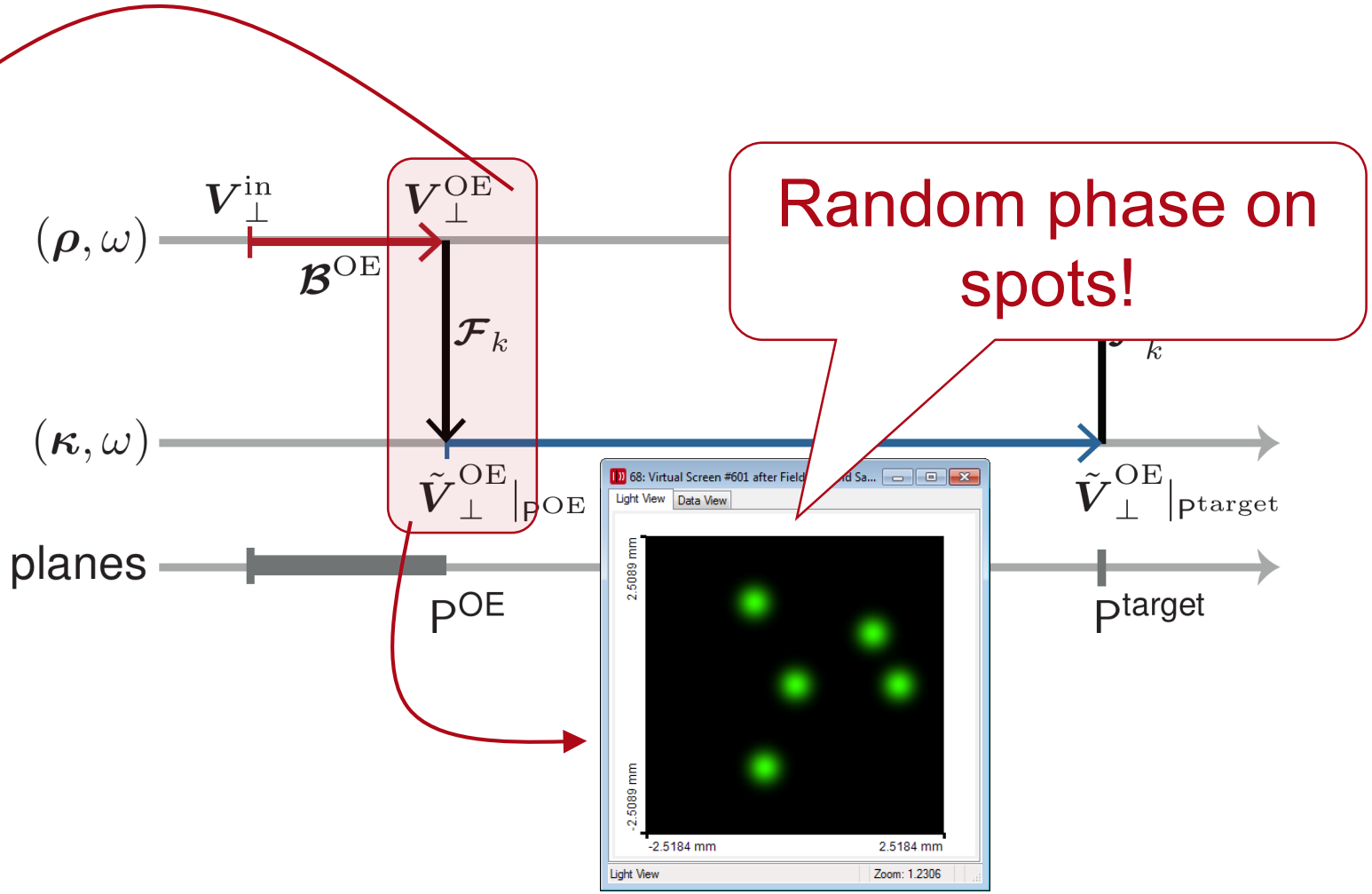


$\Delta\gamma^{\mathrm{OE}}(\rho)$

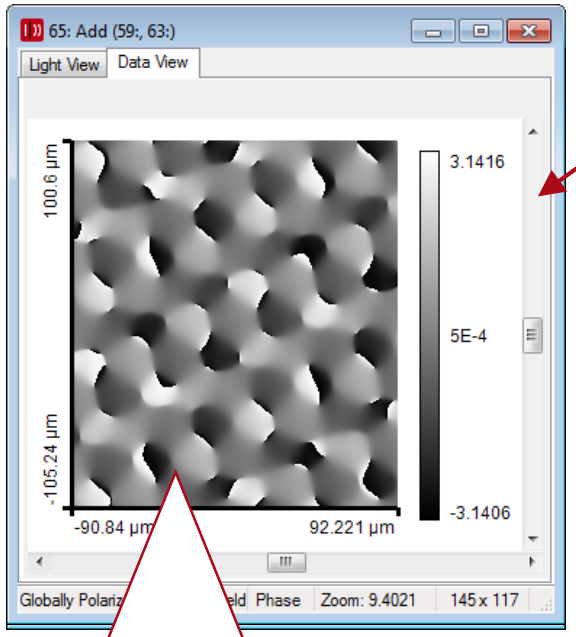


Light Shaping by Diffractive Optics

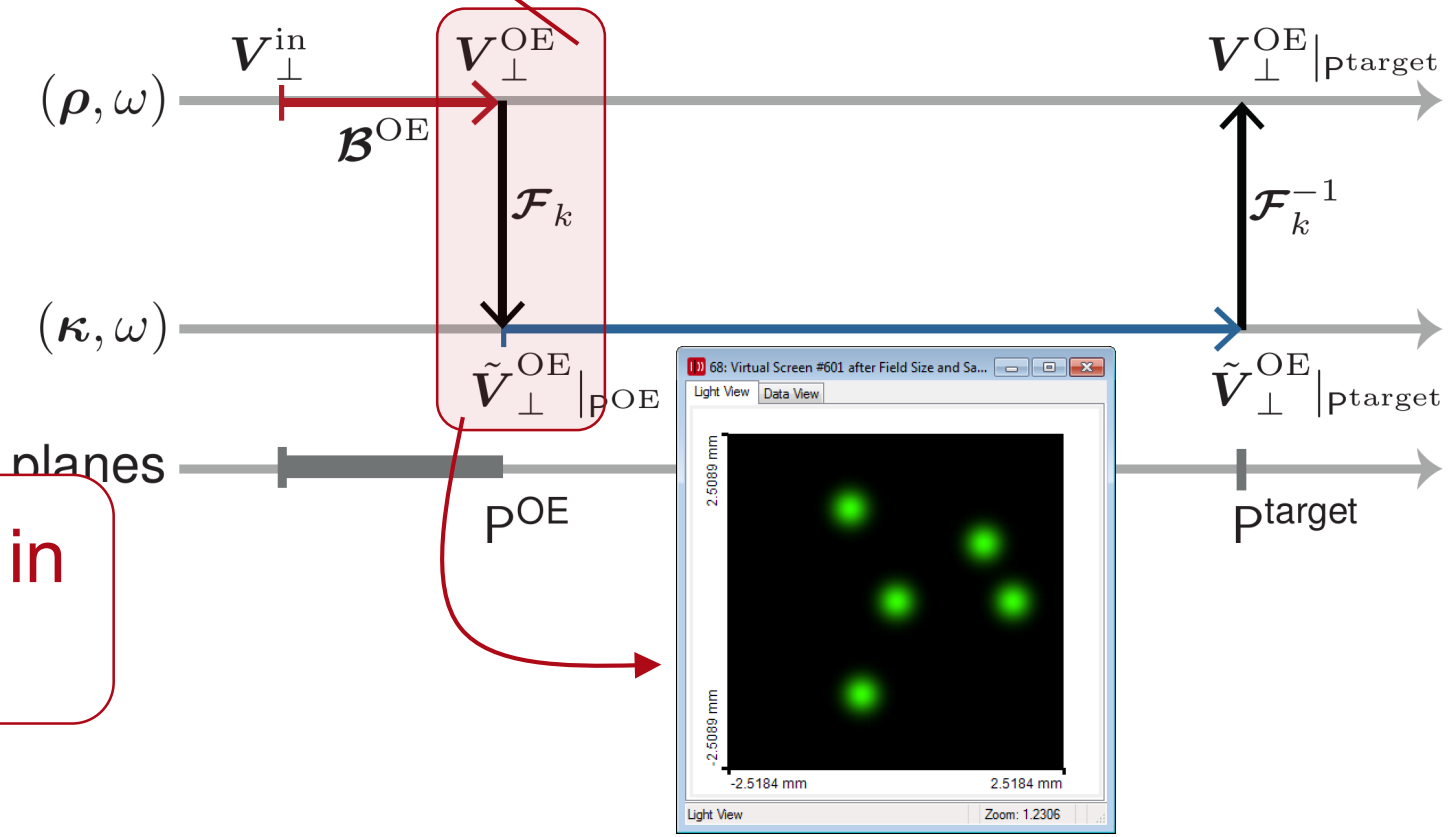




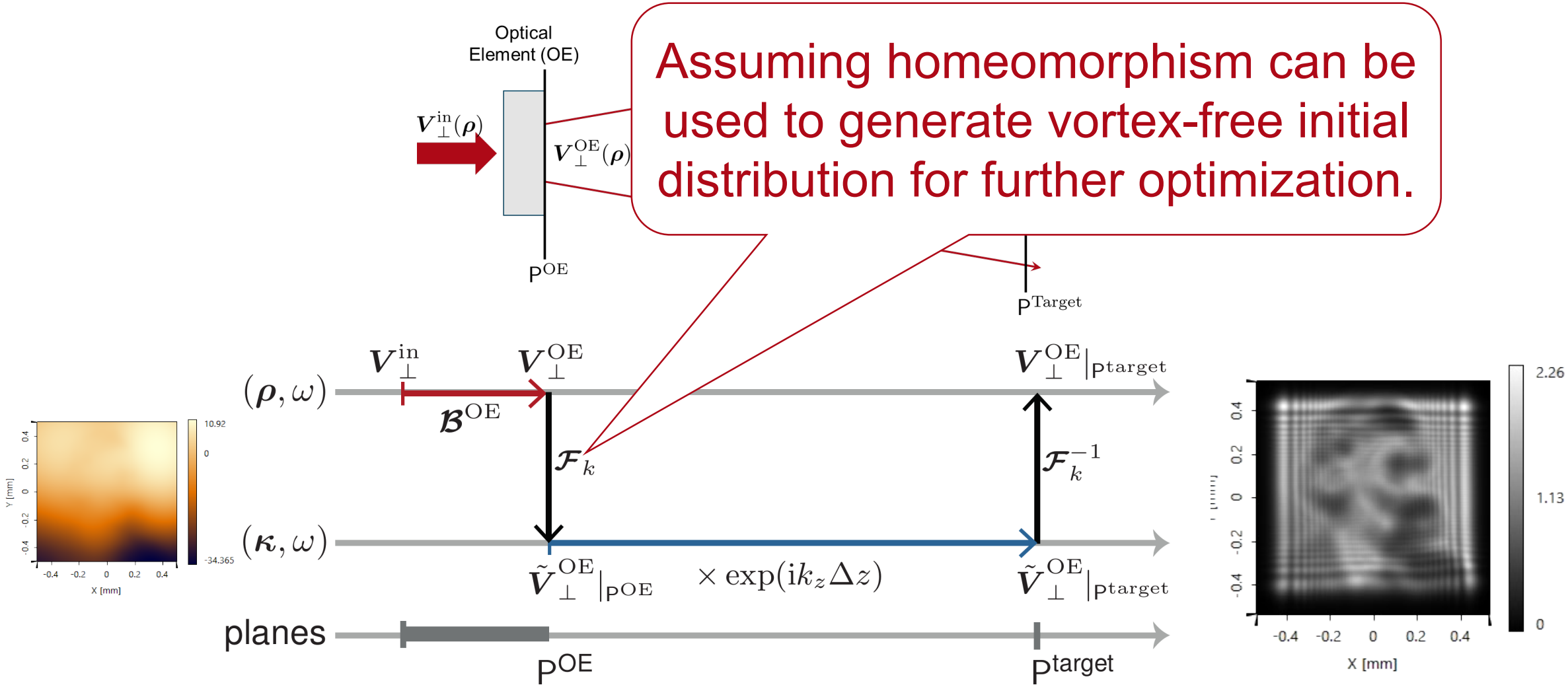
Light Shaping by Diffractive Optics



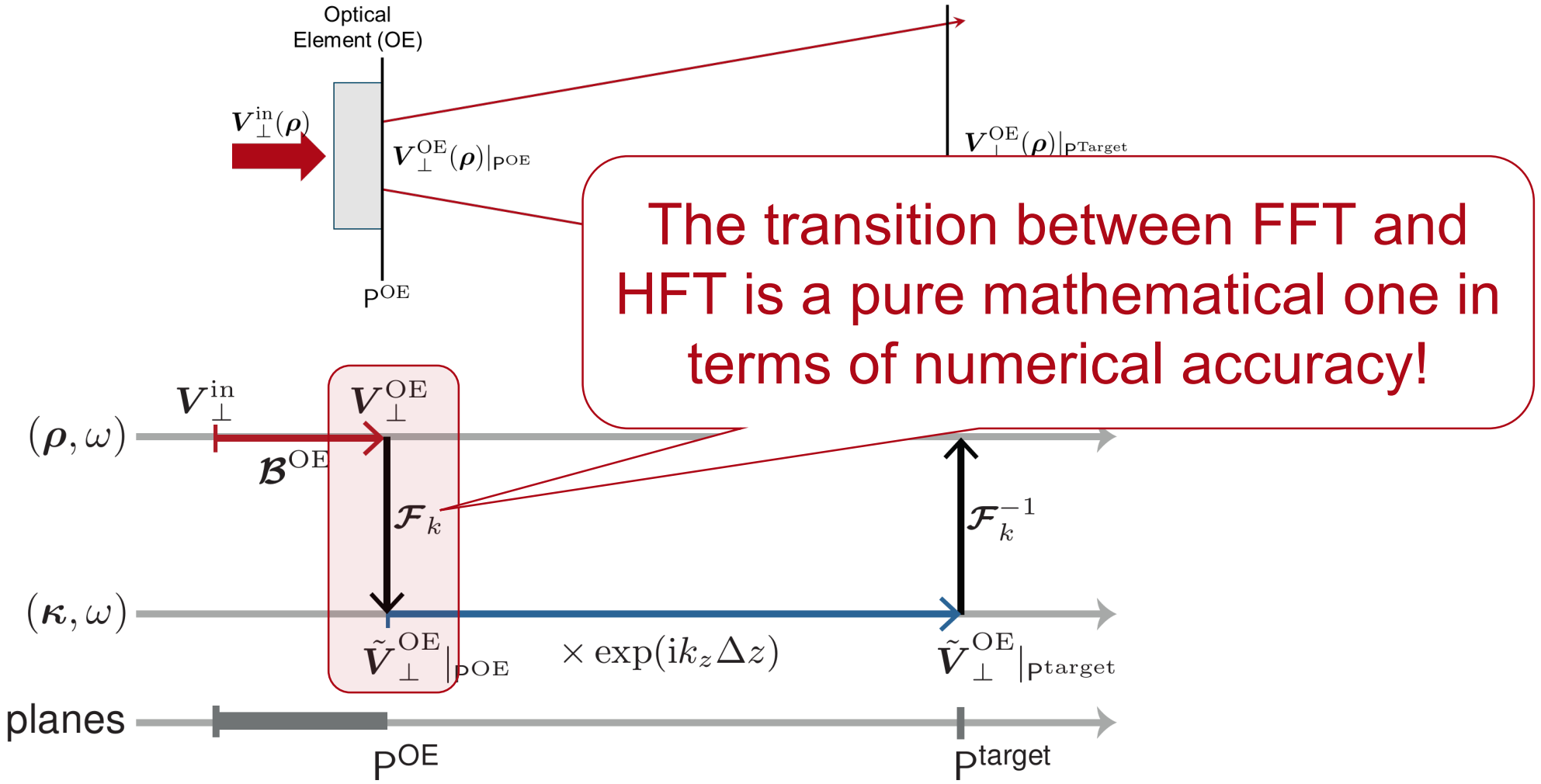
Stagnation of IFTA in vortexes!



Freeform Design for Light Shaping



Initialization of IFTA by Homeomorphism

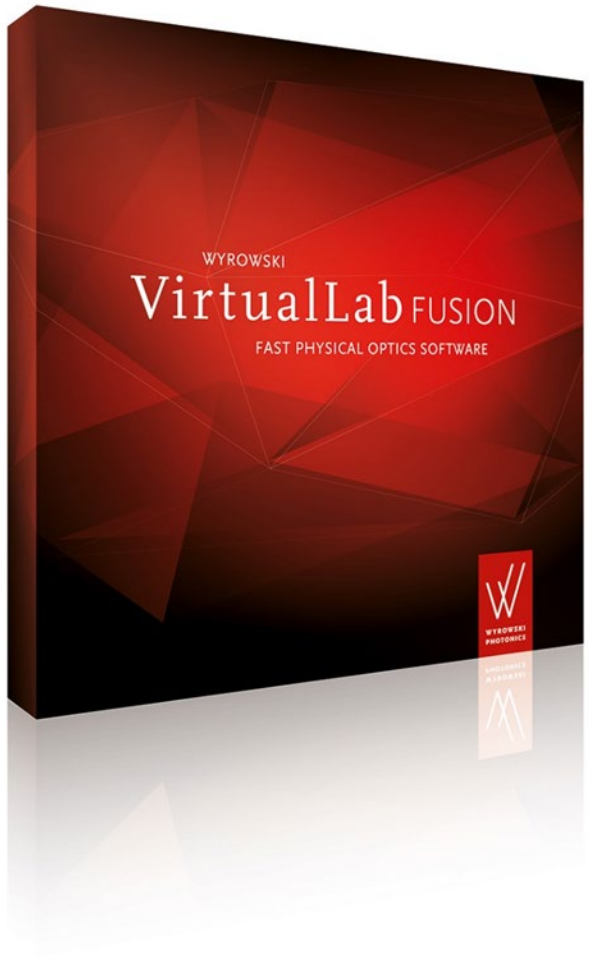


Physical-optics view on light shaping

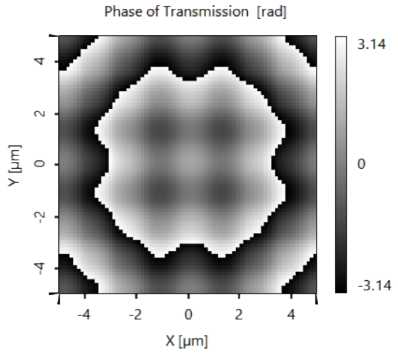
… enables a deep understanding of light shaping ranging from pure ray optics to diffractive optics. The transition between both can be mastered with one unifying theory!

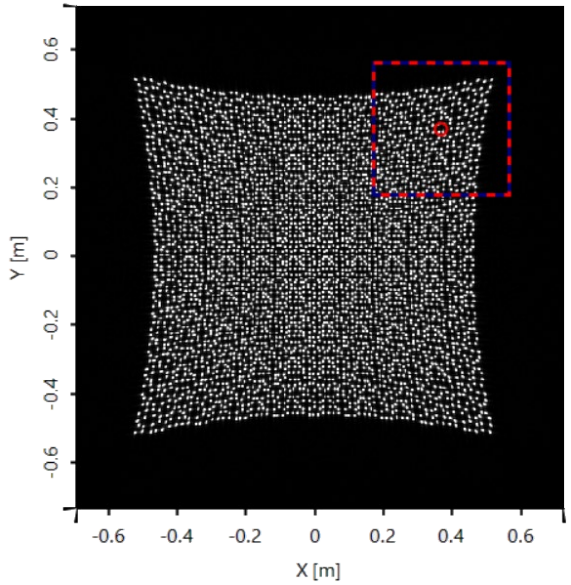
VirtualLab Light Shaping Solutions

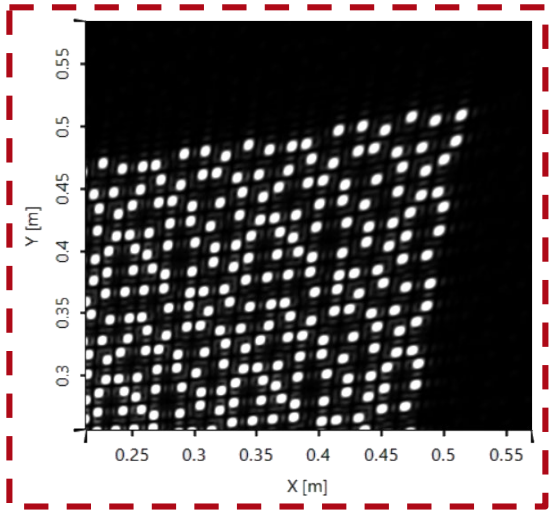
- We prepare a new VirtualLab product for Light Shaping to be released in 2019.
- It will be based on our developed theoretical understanding and provide the tremendous benefits to the illumination and lighting experts and the photonics community in general.

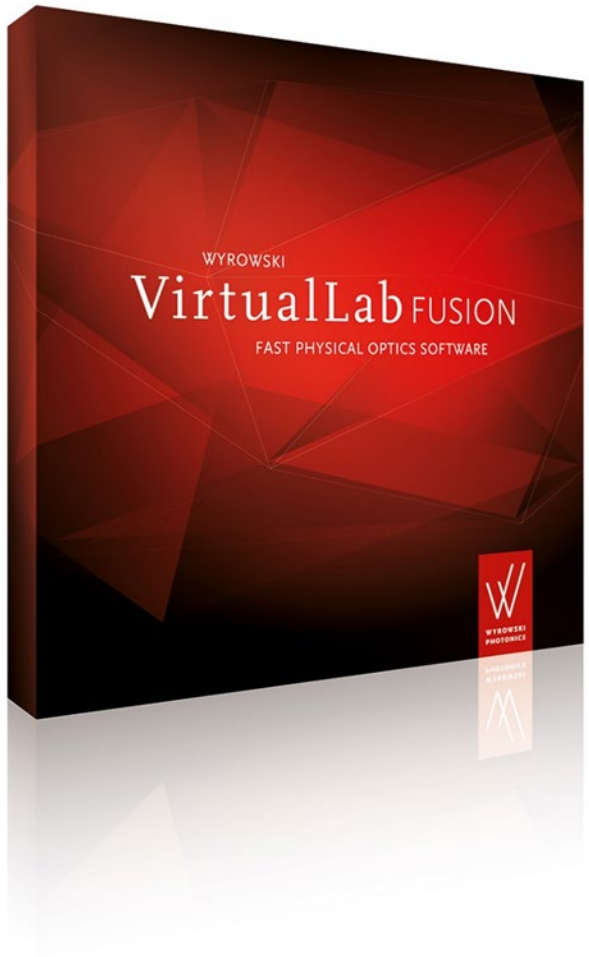


VirtualLab Illumination Solutions: Point Cloud

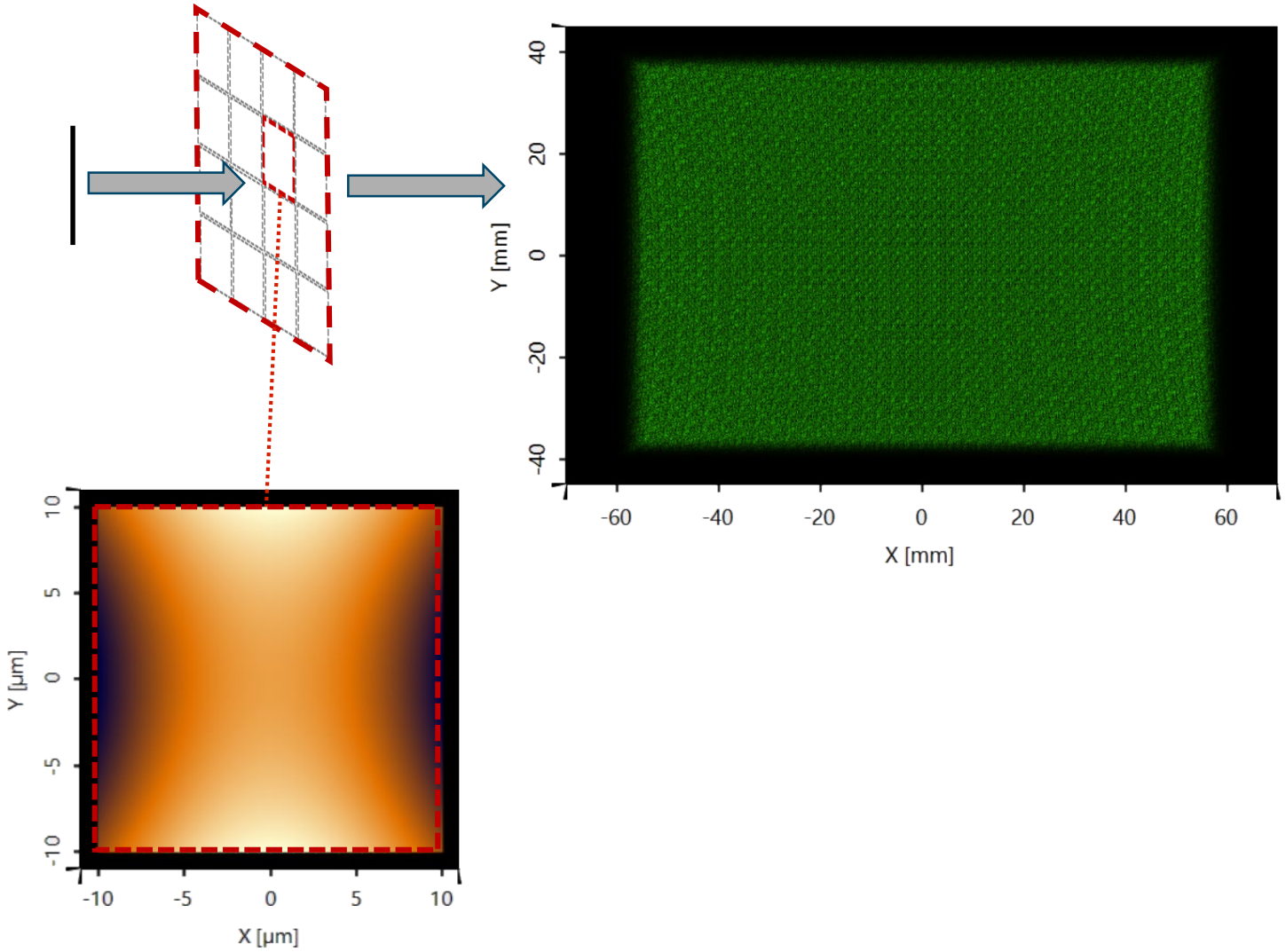


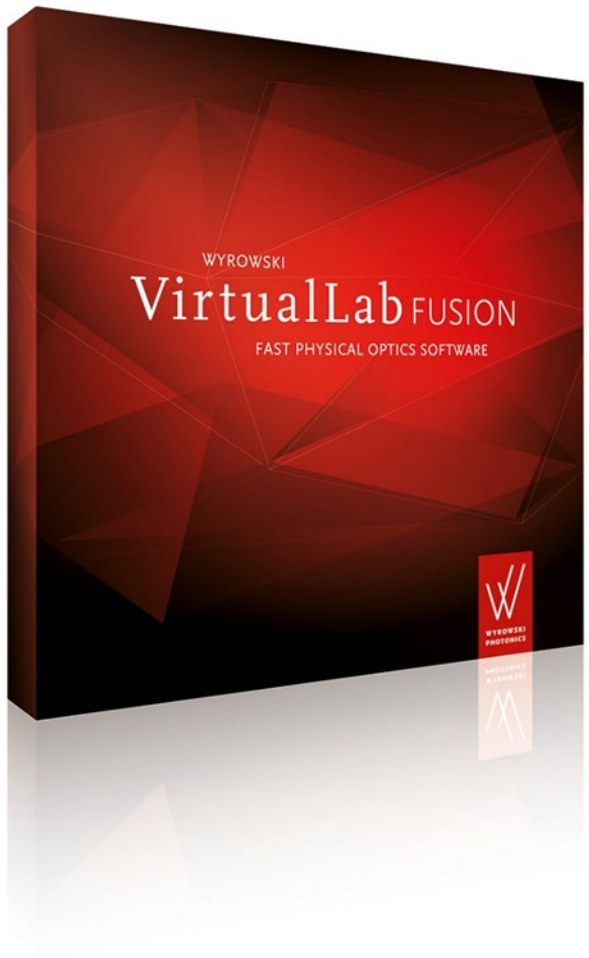






VirtualLab Illumination Solutions: Diffuser





VirtualLab Light Shaping Solutions

- We prepare a new VirtualLab product for Light Shaping to be released in 2019.
- It will be based on our developed theoretical understanding and provide the tremendous benefits to the illumination and lighting experts and the photonics community in general.