Photonics West 2019, February 6, San Francisco

# Physical-optics modeling of diffractive and meta-lenses and their design

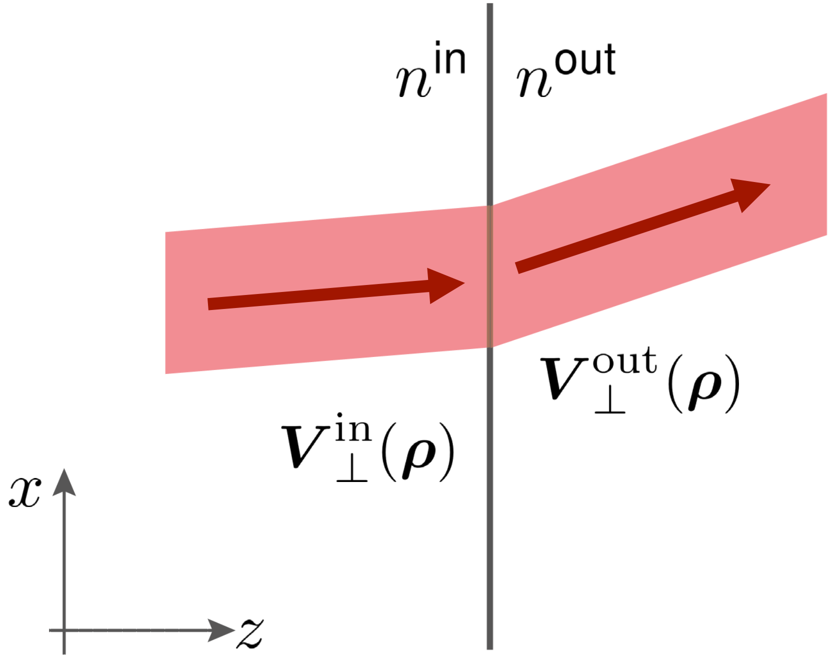
Liangxin Yang and Frank Wyrowski, University of Jena
Site Zhang, LightTrans International
Christian Hellmann, Wyrowski Photonics

## Emphasis of the common theoretical background for different types of microstructured layers

## Wavefront Surface Response Function

Unifying approach for dealing with different structure concepts in optical modeling and design

# Plane Wave Interaction with Plane Surface




- For a field in a plane we use the notation $\boldsymbol{\rho} = (x, y)$ and $\boldsymbol{V}_\perp(\boldsymbol{\rho}) = (E_x(\boldsymbol{\rho}), E_y(\boldsymbol{\rho}))$.
- A plane input field is given by

$$\boldsymbol{V}_\perp^{\text{in}}(\boldsymbol{\rho}) = \boldsymbol{U}_\perp^{\text{in}} \exp\left(\mathrm{i}\boldsymbol{\kappa}^{\text{in}}\boldsymbol{\rho}\right)$$

  with $\boldsymbol{\kappa} = (k_x, k_y)$.
- The transmitted plane output field is given by

$$\boldsymbol{V}_\perp^{\text{out}}(\boldsymbol{\rho}) = \left(\mathbf{B}(\boldsymbol{\kappa}^{\text{in}})\boldsymbol{U}_\perp^{\text{in}}\right) \exp\left(\mathrm{i}\boldsymbol{\kappa}^{\text{out}}\boldsymbol{\rho}\right)$$

  with the Fresnel effect at the surface is expressed by the $2 \times 2$ matrix $\mathbf{B}(\boldsymbol{\kappa}^{\text{in}})$.
- From the boundary conditions follows: $\boldsymbol{\kappa}^{\text{out}} \stackrel{!}{=} \boldsymbol{\kappa}^{\text{in}}$

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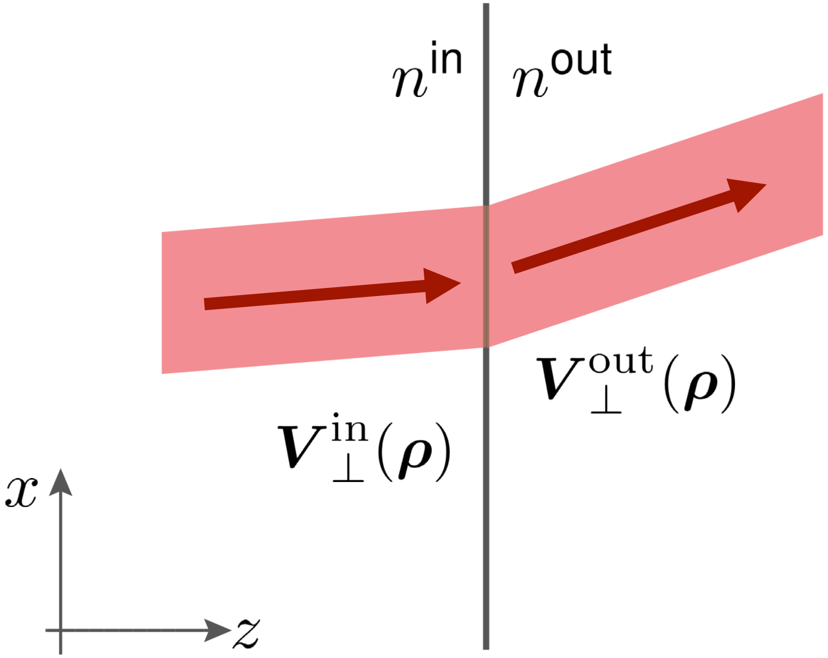
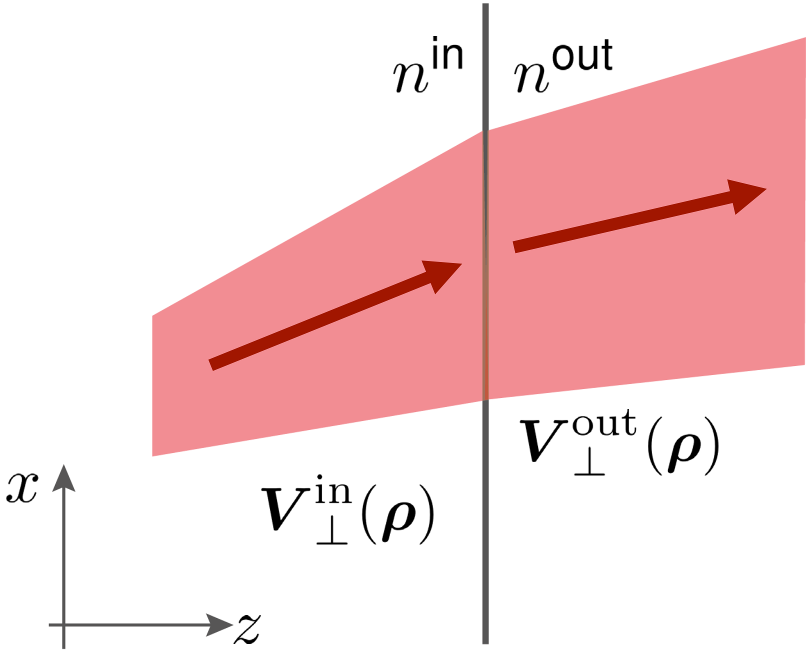



- From the boundary conditions follows: $\boldsymbol{\kappa}^{\text{out}} \stackrel{!}{=} \boldsymbol{\kappa}^{\text{in}}$
- With $\boldsymbol{\kappa} = k_0 n \hat{\boldsymbol{s}}_{\perp} = k_0 n(\sin\theta\cos\phi, \sin\theta\cos\phi)$ and $\phi = 0$ the law of refraction follows: $n^{\text{in}} \sin\theta^{\text{in}} = n^{\text{out}} \sin\theta^{\text{out}}$.
- For plane wave fields the wavefront phase is given by $\psi(\boldsymbol{\rho}) = \boldsymbol{\kappa} \cdot \boldsymbol{\rho}$.
- With $\boldsymbol{\kappa}^{\text{out}} \stackrel{!}{=} \boldsymbol{\kappa}^{\text{in}}$ we conclude $\psi^{\text{out}}(\boldsymbol{\rho}) = \psi^{\text{in}}(\boldsymbol{\rho})$.
- With $\nabla_{\perp} = \left\{\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right\}$ we find for plane wave fields:

$$\boldsymbol{\kappa} = \nabla_{\perp}\psi(\boldsymbol{\rho})$$

- Then $\boldsymbol{\kappa}^{\text{out}} \stackrel{!}{=} \boldsymbol{\kappa}^{\text{in}}$, that is the law of refraction, can be expressed by $\nabla_{\perp}\psi^{\text{in}}(\boldsymbol{\rho}) = \nabla_{\perp}\psi^{\text{out}}(\boldsymbol{\rho})$.

# General Field Interaction with Plane Surface




- Consider a general input field $\boldsymbol{V}_\perp^{\text{in}}(\boldsymbol{\rho}) = \boldsymbol{U}_\perp^{\text{in}}(\boldsymbol{\rho}) \exp\big(\mathrm{i}\psi^{\text{in}}(\boldsymbol{\rho})\big)$.
- In general the effect of the plane surface on the input field can be expressed by the operator equation $\boldsymbol{V}_\perp^{\text{out}}(\boldsymbol{\rho}) = \boldsymbol{\mathcal{B}}\boldsymbol{V}_\perp^{\text{in}}(\boldsymbol{\rho})$.
- Next we assume that we are allowed to apply the plane wave results locally. In physical optics that means the fields are in its homeomorphic field zone (HFZ).
- In the homeomorphic field zone we can express the effect of the plane surface on the input field locally:

$$\boldsymbol{U}_\perp^{\text{out}}(\boldsymbol{\rho}) \exp\big(\mathrm{i}\psi^{\text{out}}(\boldsymbol{\rho})\big) = \mathbf{B}(\boldsymbol{\rho}) \left\{\boldsymbol{U}_\perp^{\text{in}}(\boldsymbol{\rho}) \exp\big(\mathrm{i}\psi^{\text{in}}(\boldsymbol{\rho})\big)\right\}$$

- The local use of the law of refraction can be expressed by:

$$\nabla_\perp \psi^{\text{in}}(\boldsymbol{\rho}) = \nabla_\perp \psi^{\text{out}}(\boldsymbol{\rho})$$

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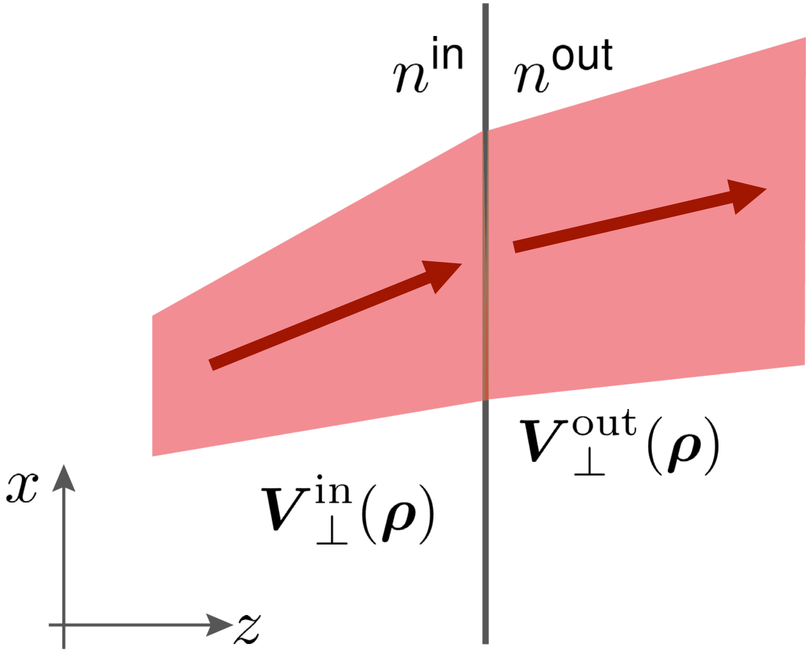


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- The local use of the law of refraction is expressed by:

$$\nabla_{\perp}\psi^{\text{in}}(\boldsymbol{\rho}) = \nabla_{\perp}\psi^{\text{out}}(\boldsymbol{\rho})$$

- In conclusion the effect on the phase is described by the local Fresnel effects:

$$\boldsymbol{U}_{\perp}^{\text{out}}(\boldsymbol{\rho})\exp\big(\mathrm{i}\psi^{\text{in}}(\boldsymbol{\rho})\big) = \left\{\mathbf{B}(\boldsymbol{\rho};\psi^{\text{in}})\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})\right\}\exp\big(\mathrm{i}\psi^{\text{in}}(\boldsymbol{\rho})\big)$$

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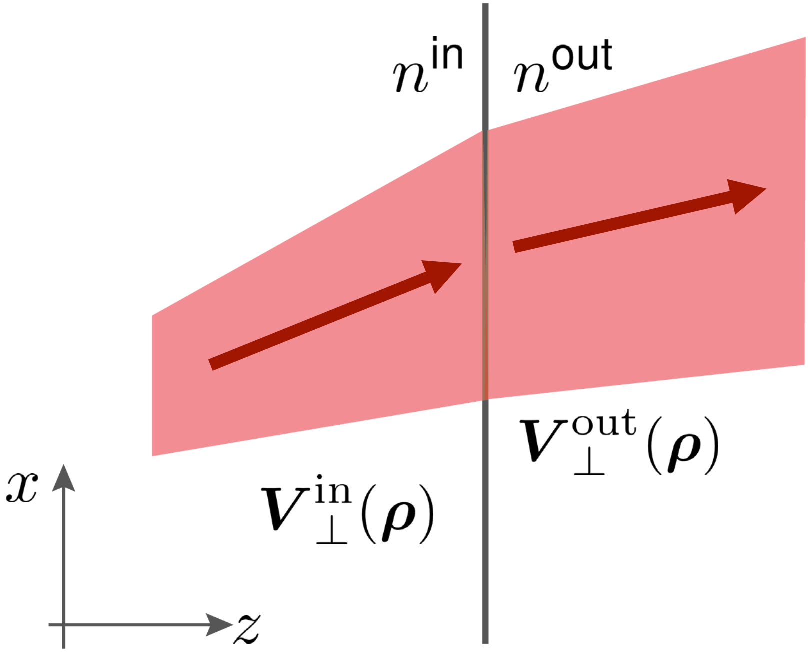
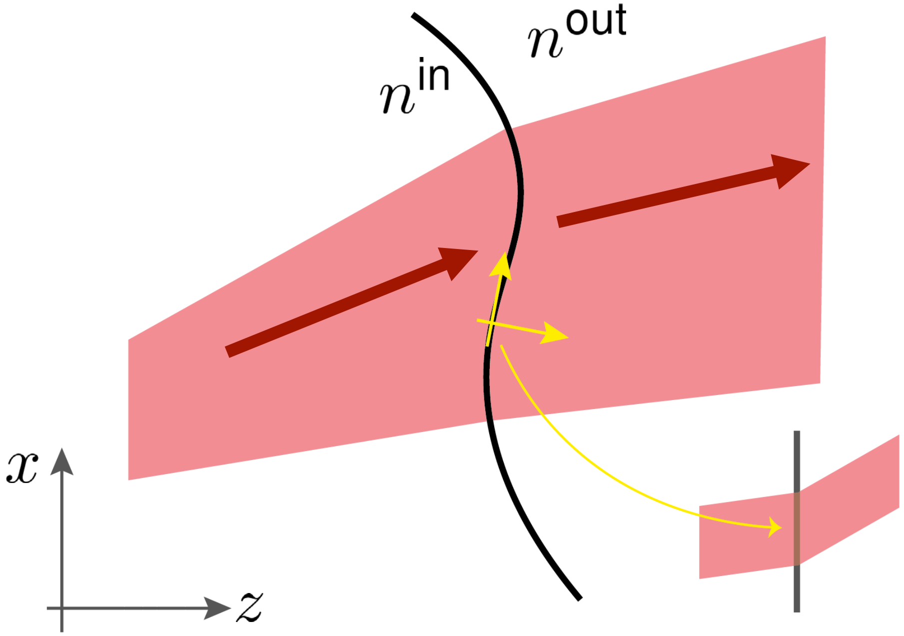



- Intermediate result: Plane surfaces do not change the wavefront phase of the incident field.

# General Field Interaction with Curved Surface




- Intermediate result: Plane surfaces do not change the wavefront phase of the incident field.
- For curved surfaces the modeling can be still applied locally. Then the wavefront phase can be shaped by the shape of the surface:

$$\psi^{\text{out}}(\boldsymbol{\rho}) \neq \psi^{\text{in}}(\boldsymbol{\rho})$$

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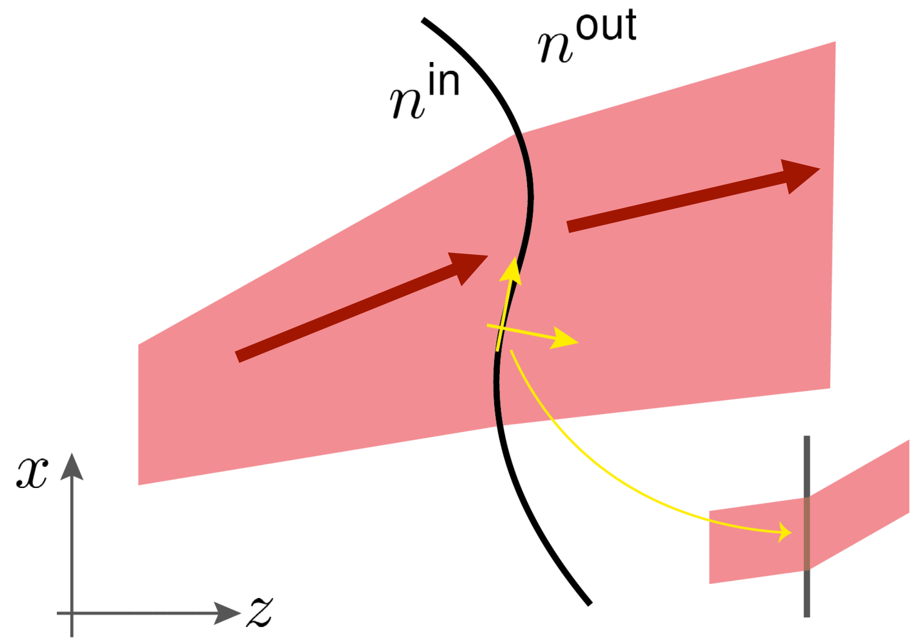


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- For curved surfaces the modeling can be still applied locally. Then the wavefront phase can be shaped by the shape of the surface:

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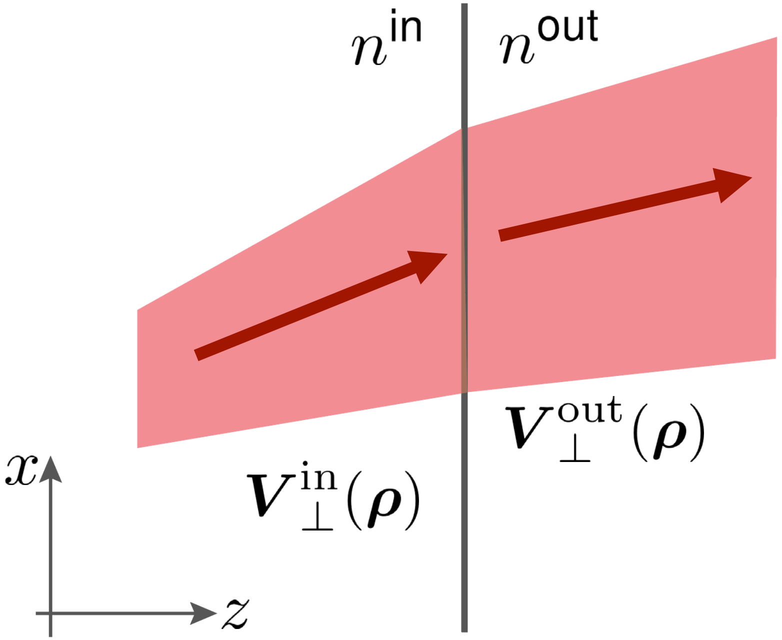
- In what follows a change of the wavefront phase should be enforced by assuming a Wavefront Response Function $\Delta\psi(\boldsymbol{\rho})$ instead of (in addition to) a curved surface.

# Wavefront Surface Response (WSR)




- For plane surfaces we found $\nabla_{\perp}\psi^{\mathsf{in}}(\boldsymbol{\rho}) = \nabla_{\perp}\psi^{\mathsf{out}}(\boldsymbol{\rho})$ and in conclusion

$$\boldsymbol{U}_{\perp}^{\mathrm{out}}(\boldsymbol{\rho})\exp\bigl(\mathrm{i}\psi^{\mathsf{in}}(\boldsymbol{\rho})\bigr) = \left\{\mathbf{B}(\boldsymbol{\rho};\psi^{\mathsf{in}})\boldsymbol{U}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho})\right\}\exp\bigl(\mathrm{i}\psi^{\mathsf{in}}(\boldsymbol{\rho})\bigr)\,.$$

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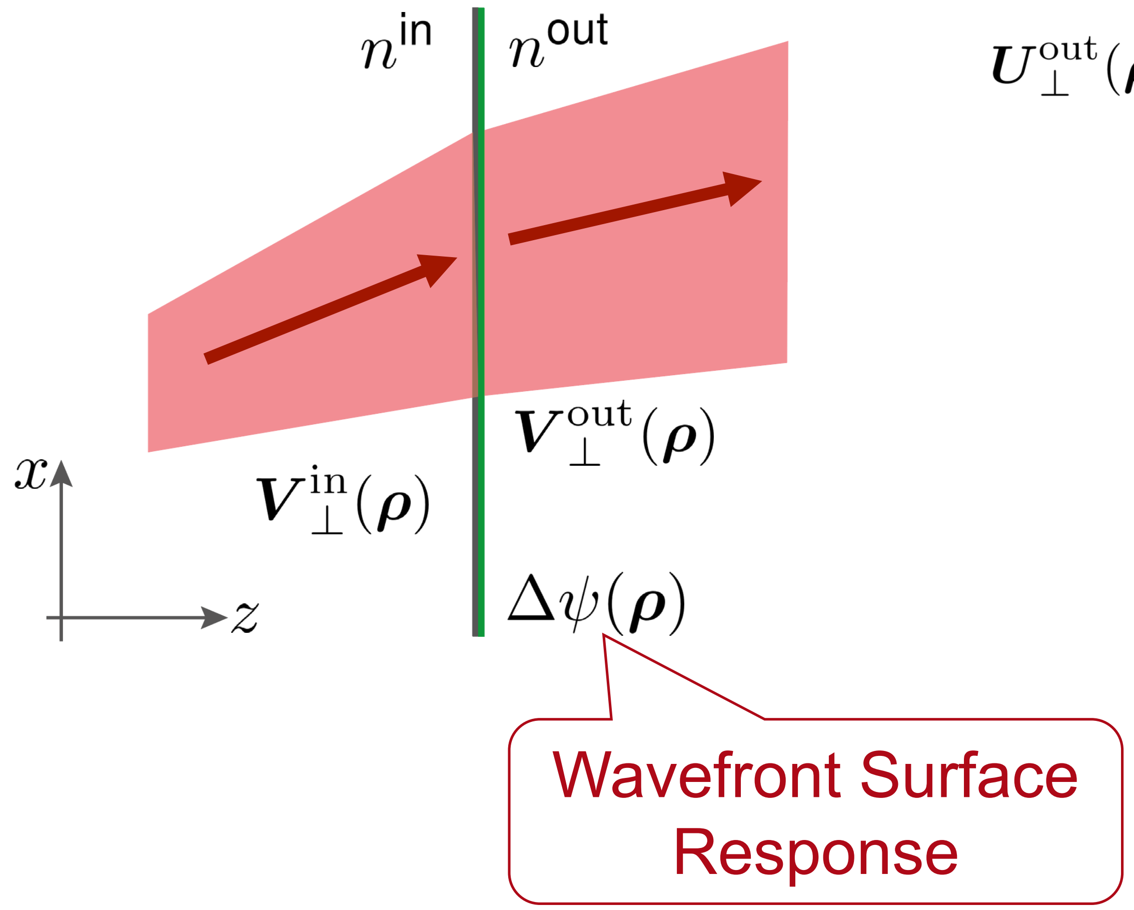


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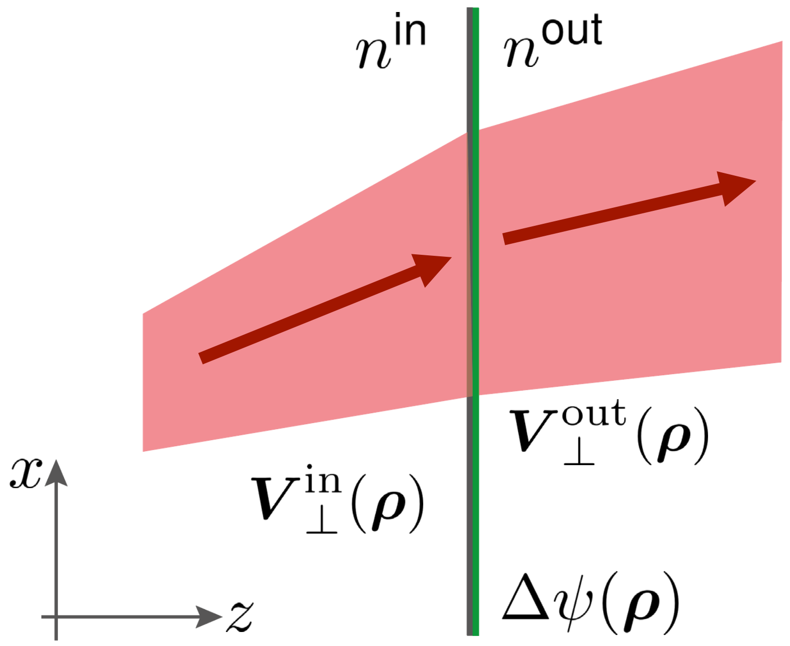


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- By introducing the wavefront surface response we assume an effect at the surface of the form

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with

$$\psi^{\mathrm{out}}(\boldsymbol{\rho}) = \psi^{\mathrm{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})$$

and thus

$$\nabla_{\perp}\psi^{\mathrm{out}}(\boldsymbol{\rho}) = \nabla_{\perp}\psi^{\mathrm{in}}(\boldsymbol{\rho}) + \nabla_{\perp}\left(\Delta\psi(\boldsymbol{\rho})\right).$$

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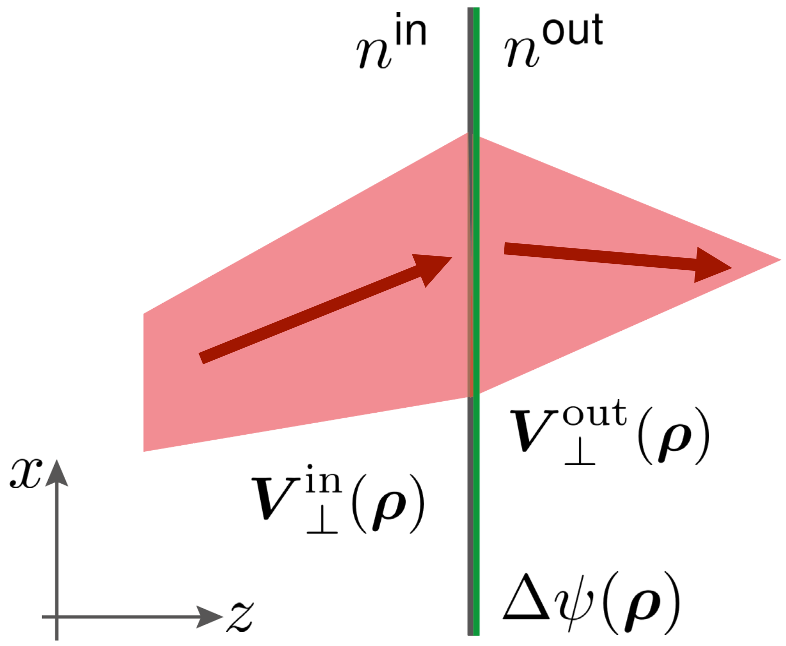


- For plane surfaces we found $\nabla_\perp \psi^{\text{in}}(\boldsymbol{\rho}) = \nabla_\perp \psi^{\text{out}}(\boldsymbol{\rho})$ and in conclusion

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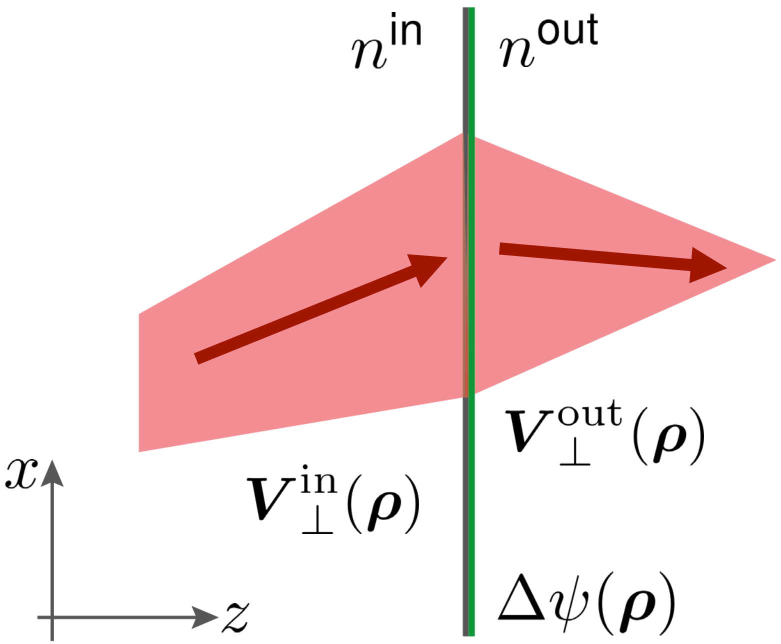



- Because of the local plane wave assumption (homeomorphic zone)

$$\nabla_\perp \psi^{\text{out}}(\boldsymbol{\rho}) = \nabla_\perp \psi^{\text{in}}(\boldsymbol{\rho}) + \nabla_\perp \left(\Delta\psi(\boldsymbol{\rho})\right)$$

leads to

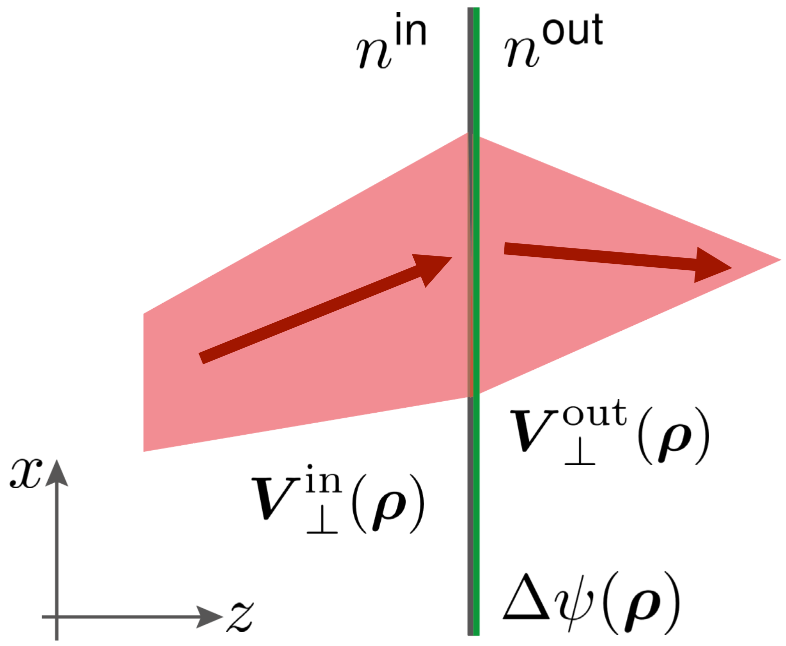
$$\boldsymbol{\kappa}^{\text{out}}(\boldsymbol{\rho}) = \boldsymbol{\kappa}^{\text{in}}(\boldsymbol{\rho}) + \boldsymbol{K}(\boldsymbol{\rho})$$

with $\boldsymbol{K}(\boldsymbol{\rho}) \overset{\text{def}}{=} \nabla_\perp \left(\Delta\psi(\boldsymbol{\rho})\right)$.

- That gives a direct access to ray tracing with $\boldsymbol{\kappa}(\boldsymbol{\rho}) = k_0 n \hat{\boldsymbol{s}}_\perp(\boldsymbol{\rho})$ via

$$n^{\text{out}} \hat{\boldsymbol{s}}_\perp^{\text{out}}(\boldsymbol{\rho}) = n^{\text{in}} \hat{\boldsymbol{s}}_\perp^{\text{in}}(\boldsymbol{\rho}) + \boldsymbol{K}(\boldsymbol{\rho})/k_0.$$

# How to Realize a Desired Wavefront Surface Response (WSR)?




- From a physical-optics point of view the question arises, if there exists any manipulation of the structure of the surface, which provides an effect of the form:

$$\boldsymbol{U}_{\perp}^{\text{out}}(\boldsymbol{\rho})\exp\big(\mathrm{i}\psi^{\text{in}}(\boldsymbol{\rho})+\Delta\psi(\boldsymbol{\rho})\big)=\big\{\mathbf{B}(\boldsymbol{\rho};\psi^{\text{in}})\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})\big\}\exp\big(\mathrm{i}\psi^{\text{in}}(\boldsymbol{\rho})\big)$$

- A detailed answer can only be given for a specific surface structure.
- By introducing microstructured layers onto the surface a wavefront surface response can be implemented:
  - Graded-index layer
  - Volume hologram layer
  - Diffractive layer
  - Metamaterial layer

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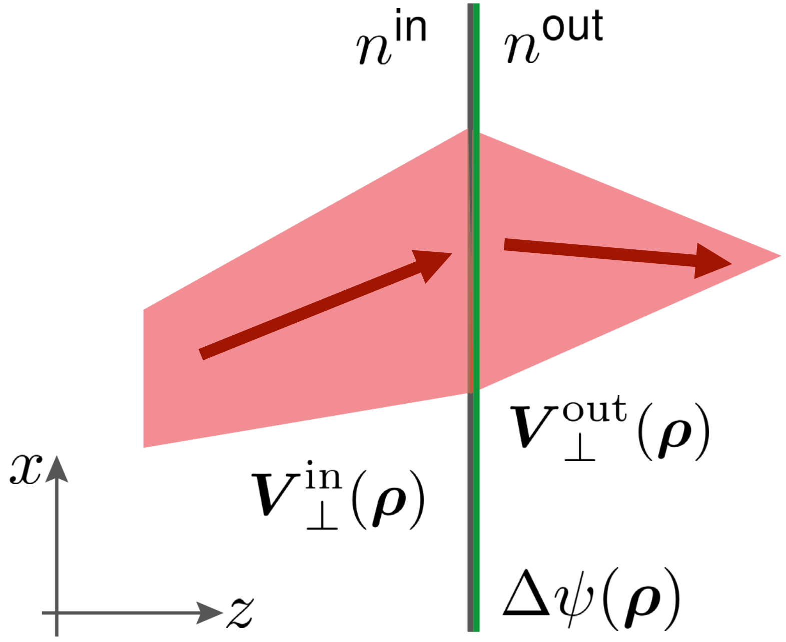


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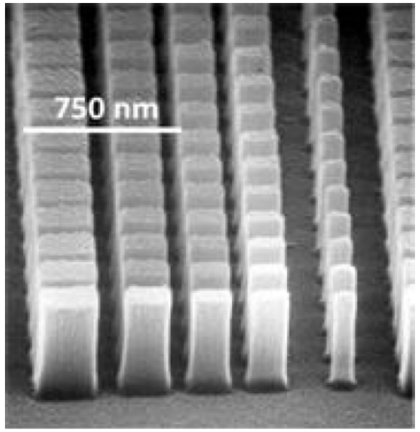
## Metasurfaces

Realization of wavefront surface responses by nanostructured layers

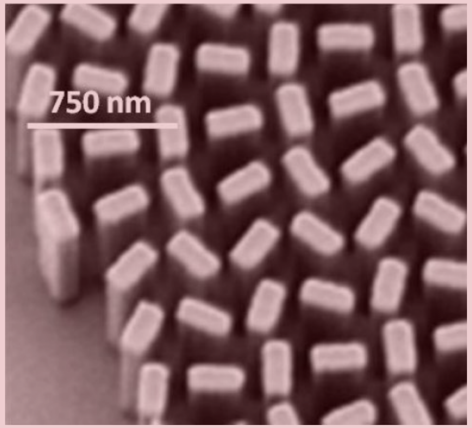
# Physical Effects for Realizing Metasurfaces

- Propagation phase delay
  - Centrosymmetric (polarization insensitive)



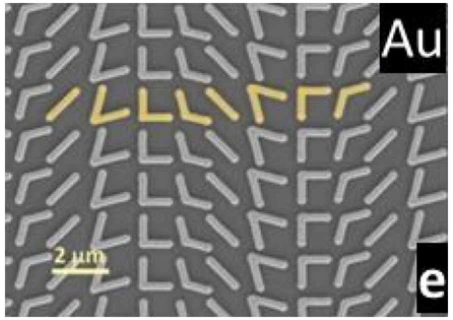

P. Lalanne *et al.*, J. Opt. Soc. Am. A **16**, 1143-1156 (1999).

  - Rotationally asymmetric (form birefringence)

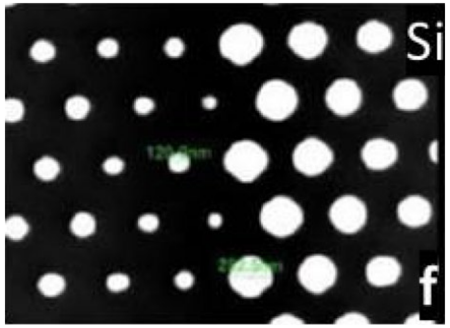



M. Khorasaninejad *et al.*, Science **352**, 1190-1194 (2016).

- Resonance phase delay




N. Yu *et al.*, Science **334**, 333–337 (2011).




Y. F. Yu *et al.*, Laser Photonics Rev. **9**, 412-418 (2015).

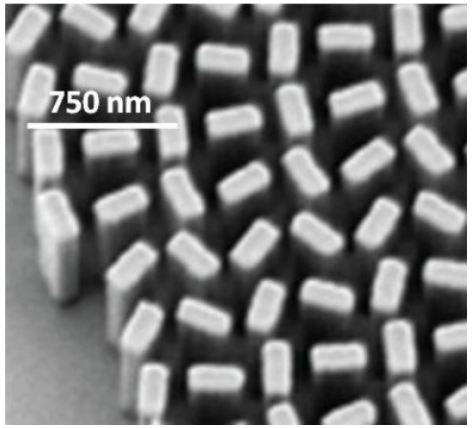
# Physical Optics Modeling: Metasurface Layer

- In general a real surface layer structure does not just realize the desired wavefront response but additional effects and fields:

$$\boldsymbol{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \left\{\mathbf{B}(\boldsymbol{\rho}; \psi^{\mathsf{in}})\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})\right\} \exp\left(\mathrm{i}\psi^{\mathsf{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})\right) + \boldsymbol{V}_{\perp}^{\text{res}}(\boldsymbol{\rho})$$

- For nanofin-based metalayers the typical result can be written as:

$$\begin{aligned}\boldsymbol{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = &\left\{\mathbf{B}^{+}(\boldsymbol{\rho}; \psi^{\mathsf{in}})\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})\right\} \exp\left(\mathrm{i}\psi^{\mathsf{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})\right) \\ &+ \left\{\mathbf{B}^{-}(\boldsymbol{\rho}; \psi^{\mathsf{in}})\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})\right\} \exp\left(\mathrm{i}\psi^{\mathsf{in}}(\boldsymbol{\rho}) - \Delta\psi(\boldsymbol{\rho})\right)\end{aligned}$$

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Calculation by Fourier modal method (FMM), a.k.a. RCWA

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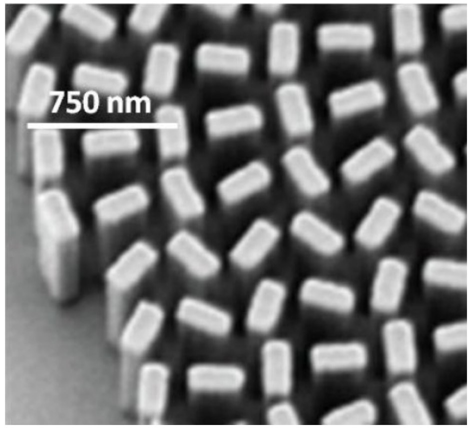


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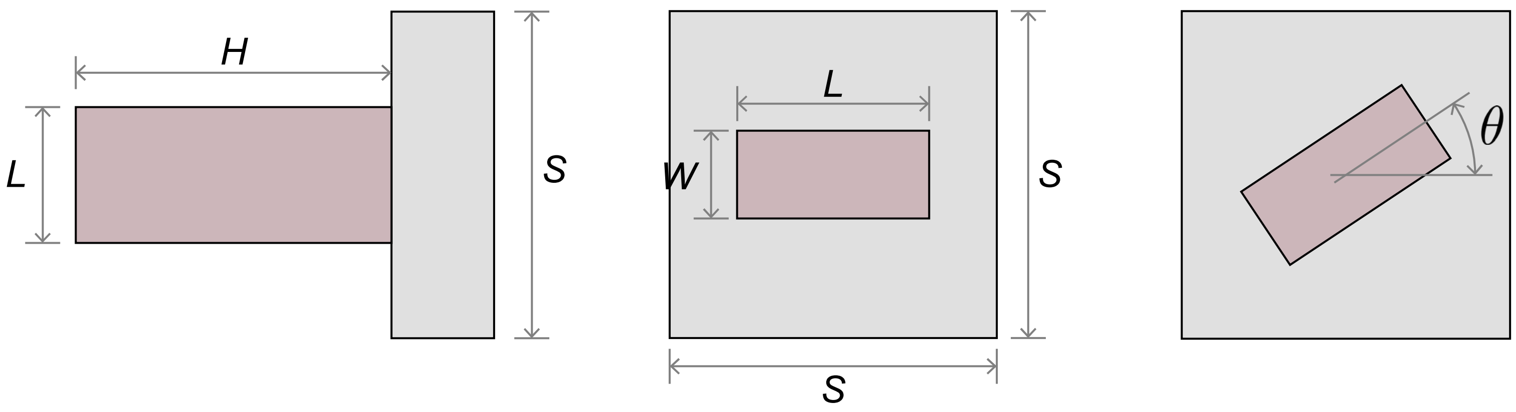
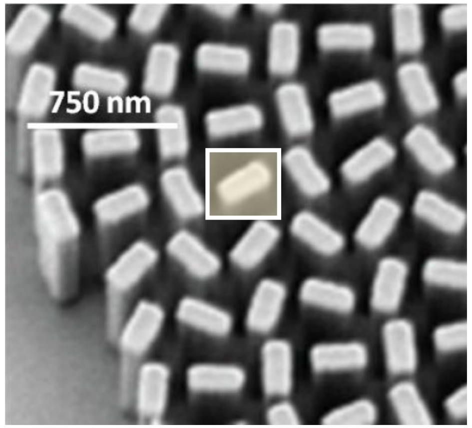
# Metasurface Building Block




- Building blocks over the whole surface are same, but with different rotation angle $\theta(\boldsymbol{\rho})$. That can be expressed as

$$\boldsymbol{V}_{\perp}^{\mathrm{out}}(\boldsymbol{\rho}) = \begin{pmatrix} \cos\theta(\boldsymbol{\rho}) & -\sin\theta(\boldsymbol{\rho}) \\ \sin\theta(\boldsymbol{\rho}) & \cos\theta(\boldsymbol{\rho}) \end{pmatrix} \boxed{\begin{pmatrix} b_{xx} & b_{xy} \\ b_{yx} & b_{yy} \end{pmatrix}} \begin{pmatrix} \cos\theta(\boldsymbol{\rho}) & \sin\theta(\boldsymbol{\rho}) \\ -\sin\theta(\boldsymbol{\rho}) & \cos\theta(\boldsymbol{\rho}) \end{pmatrix} \boldsymbol{V}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho})$$

… in the “eigen” coordinate system of each nanofin

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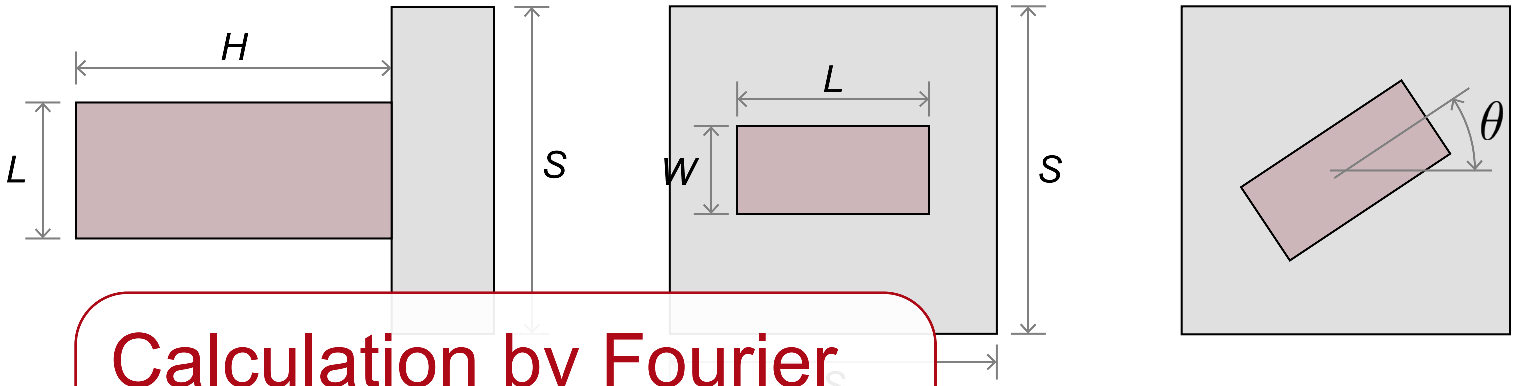


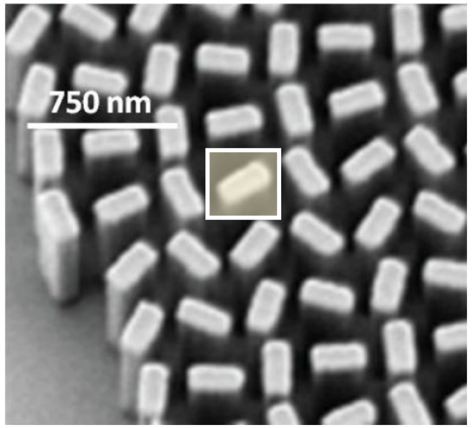


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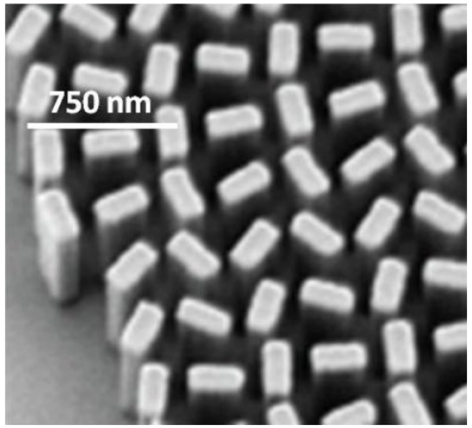
- In depth investigation reveals, that $+\Delta\psi$ occurs for R-circularly polarized input fields and the conjugate phase for L-circularly polarized input.

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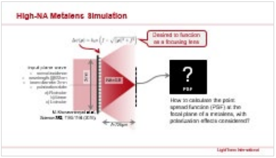
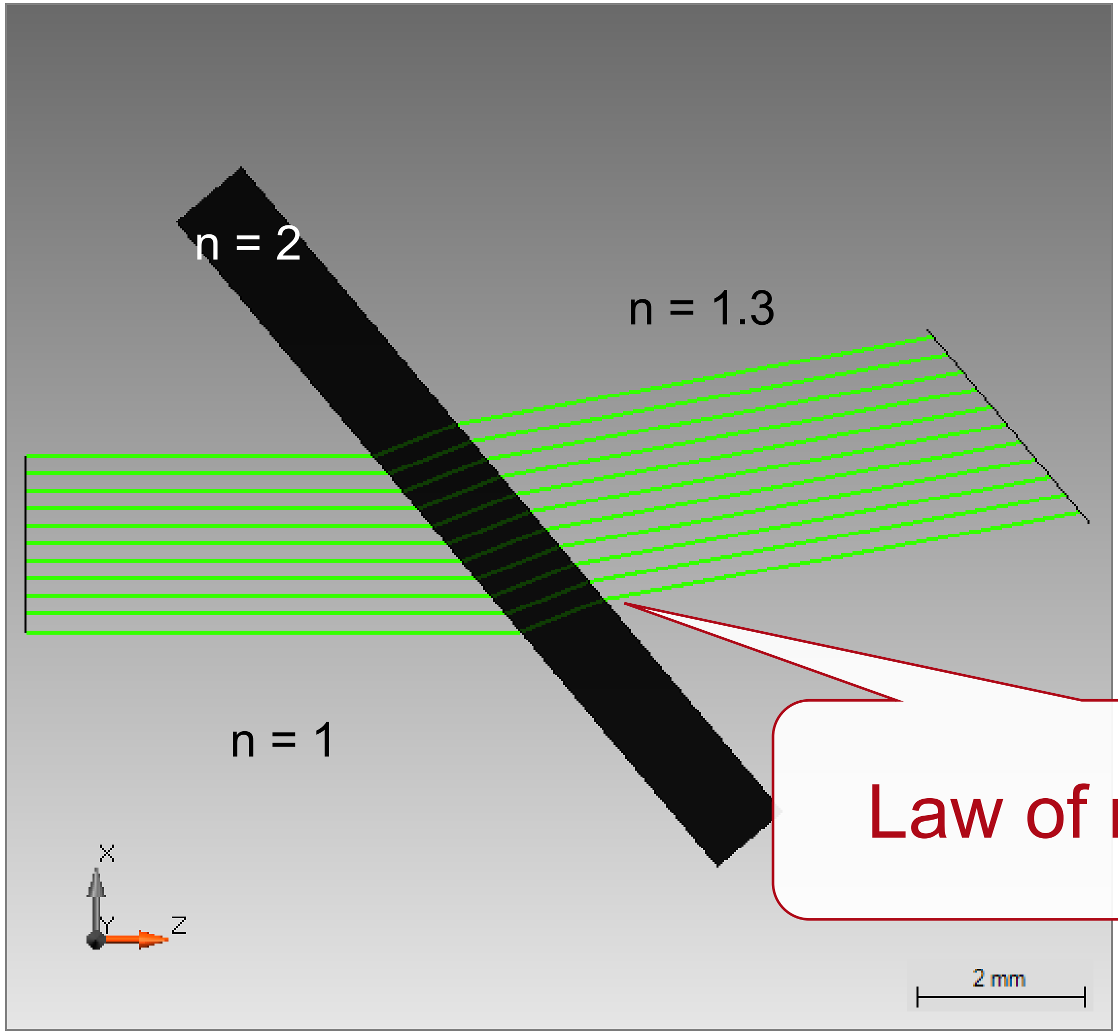


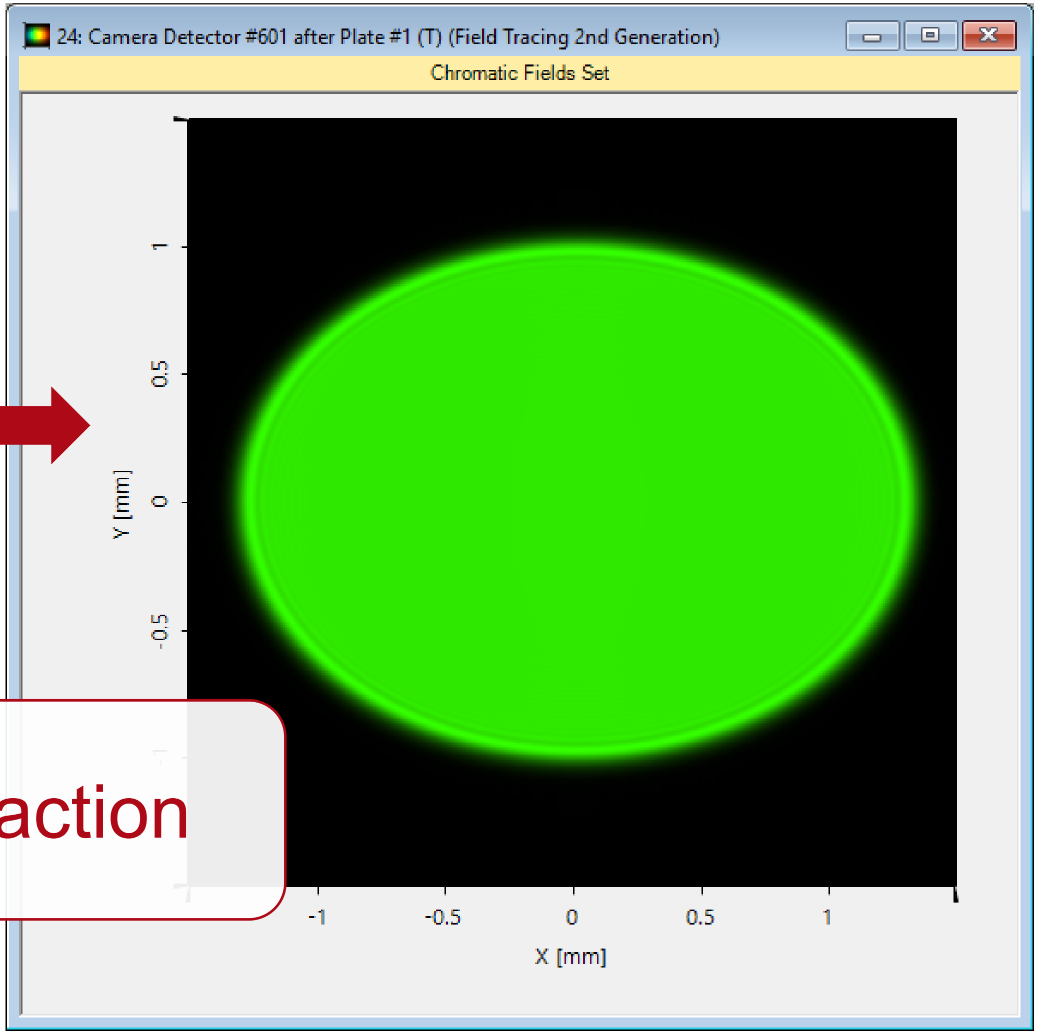
## Metasurface layer

Linear Wavefront Response Function

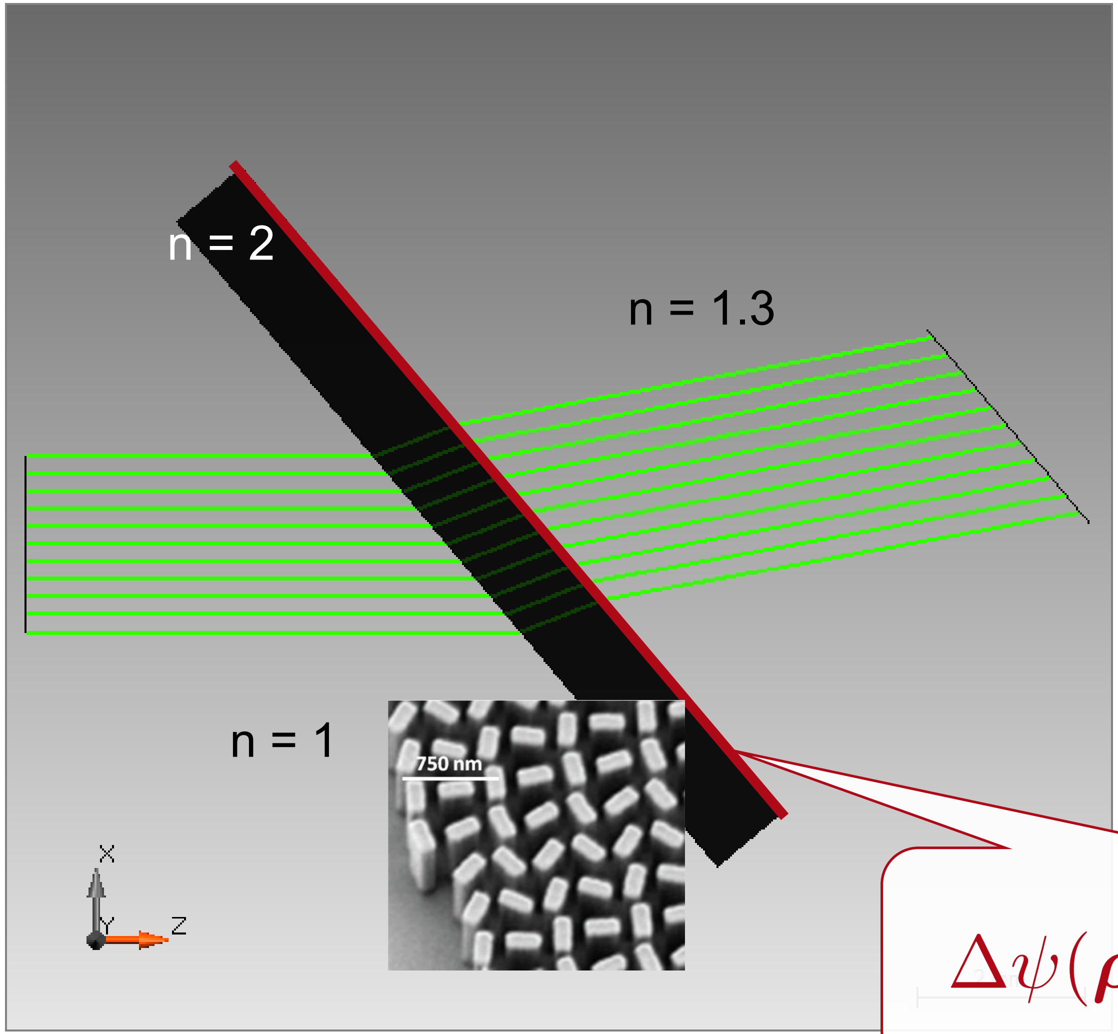
# Plane Wave through Plate: Ray and Field Tracing

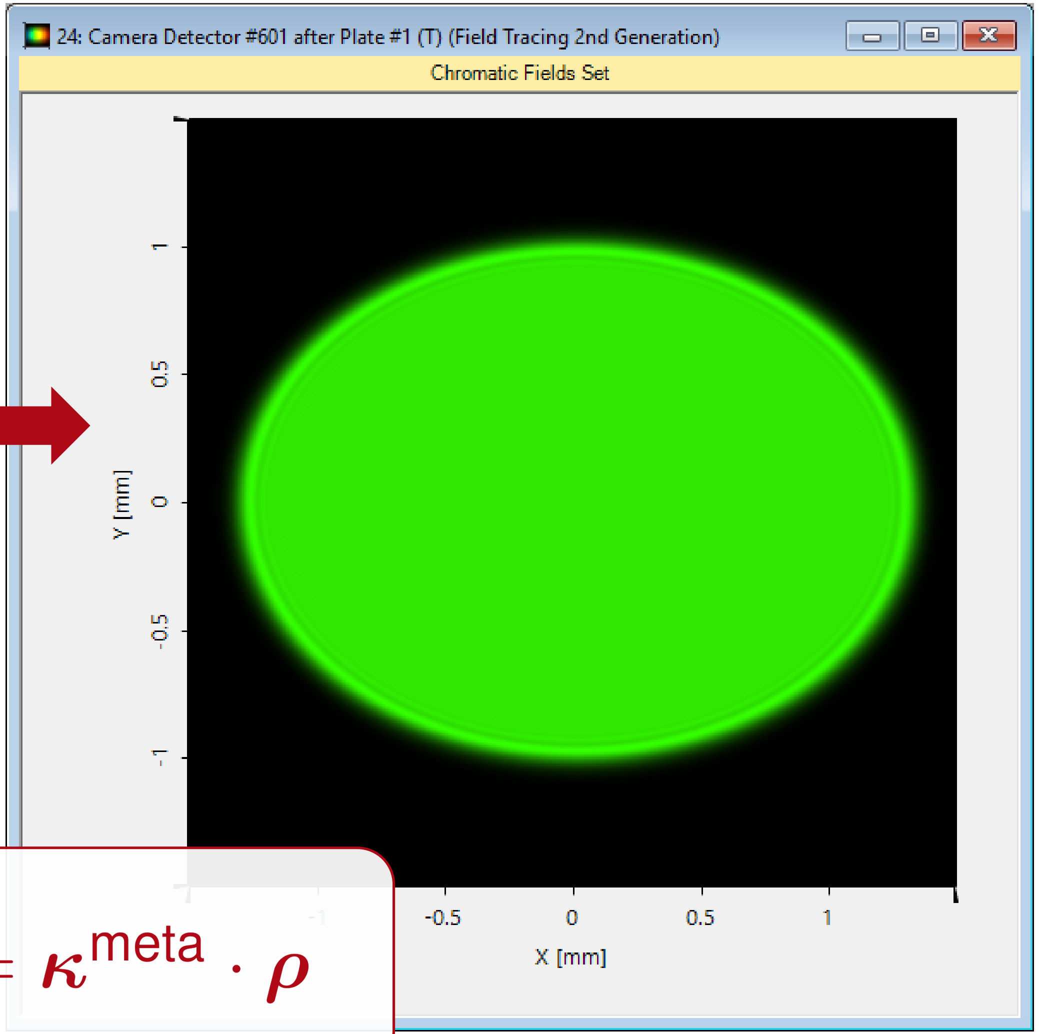



Law of refraction



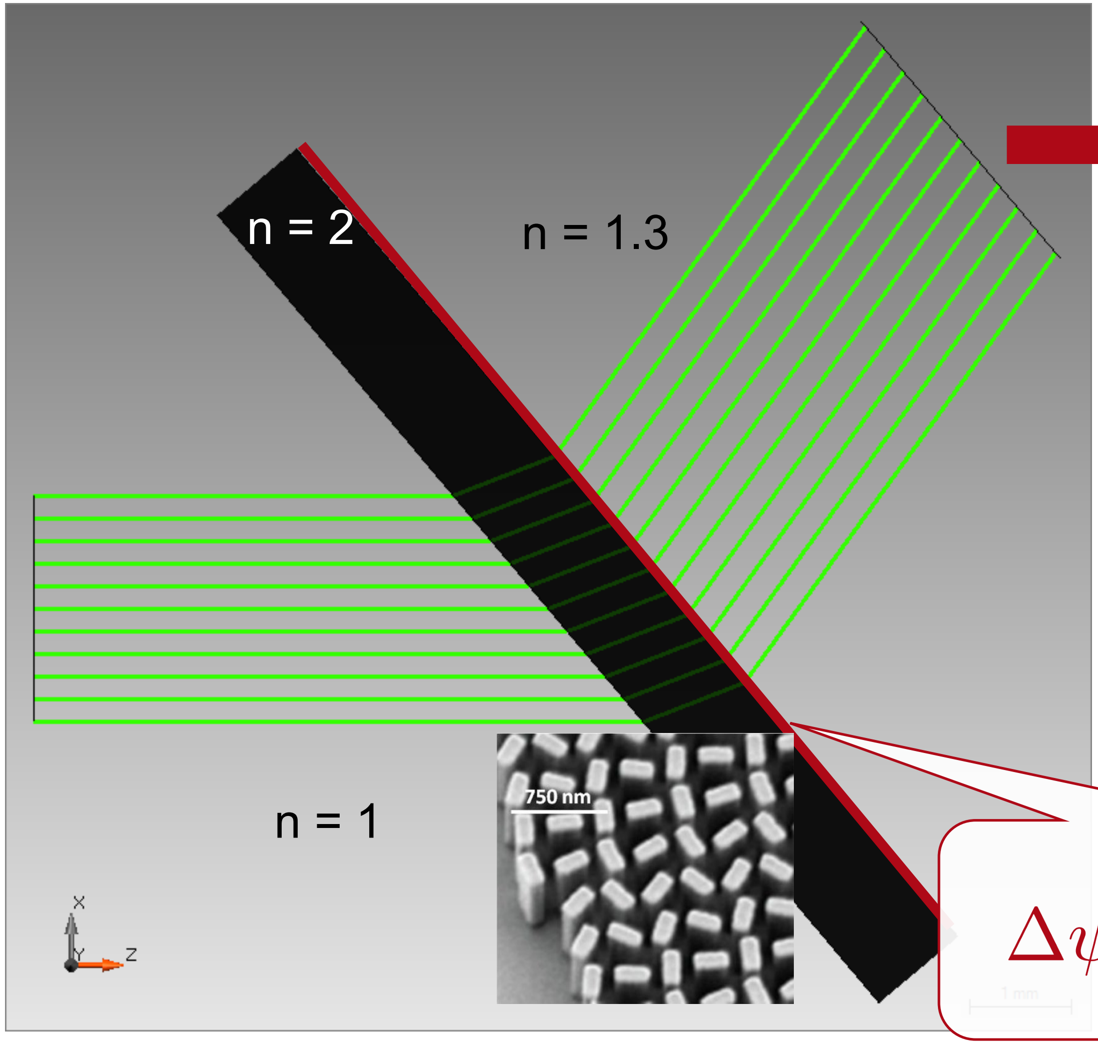
# Plane Wave through Plate: Introducing Metasurface

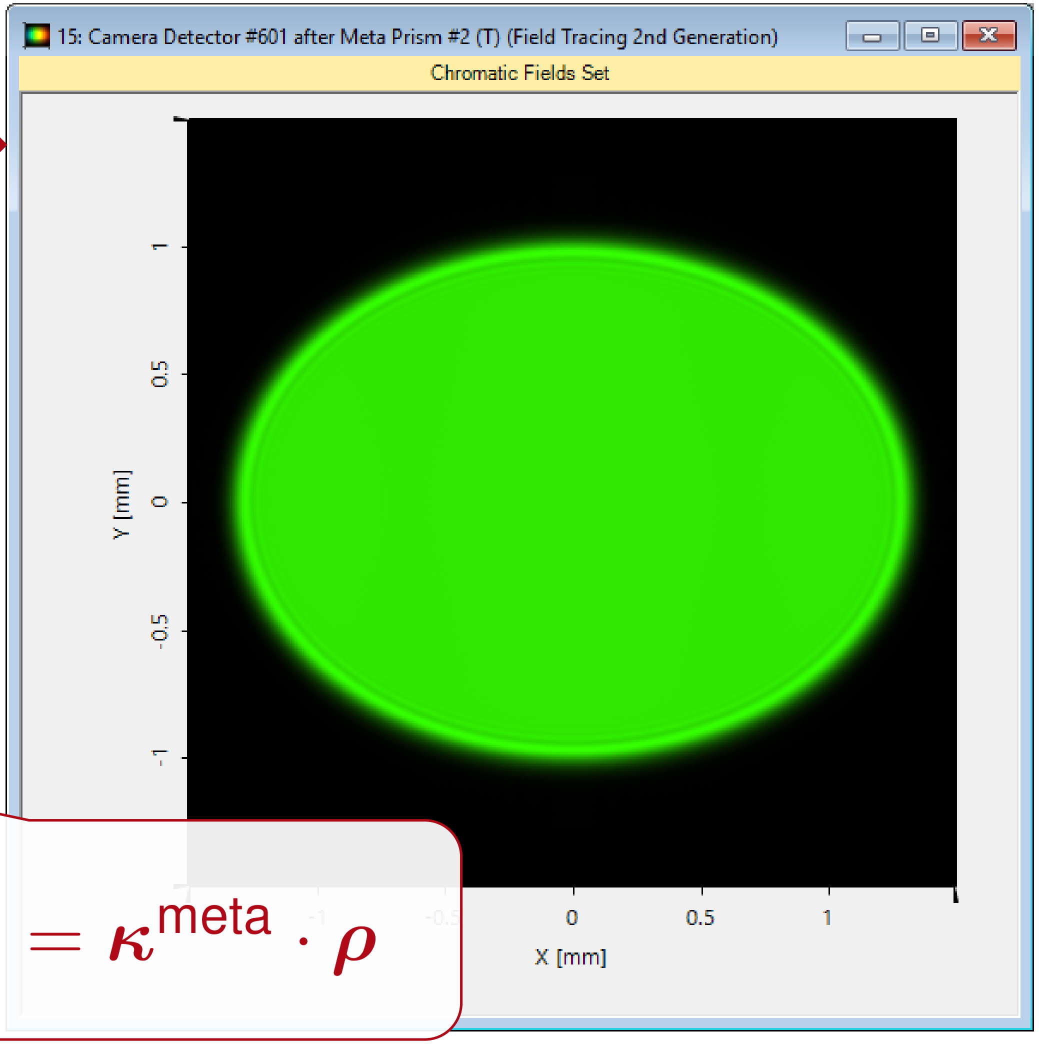



$$\Delta\psi(\boldsymbol{\rho}) = \boldsymbol{\kappa}^{\text{meta}} \cdot \boldsymbol{\rho}$$

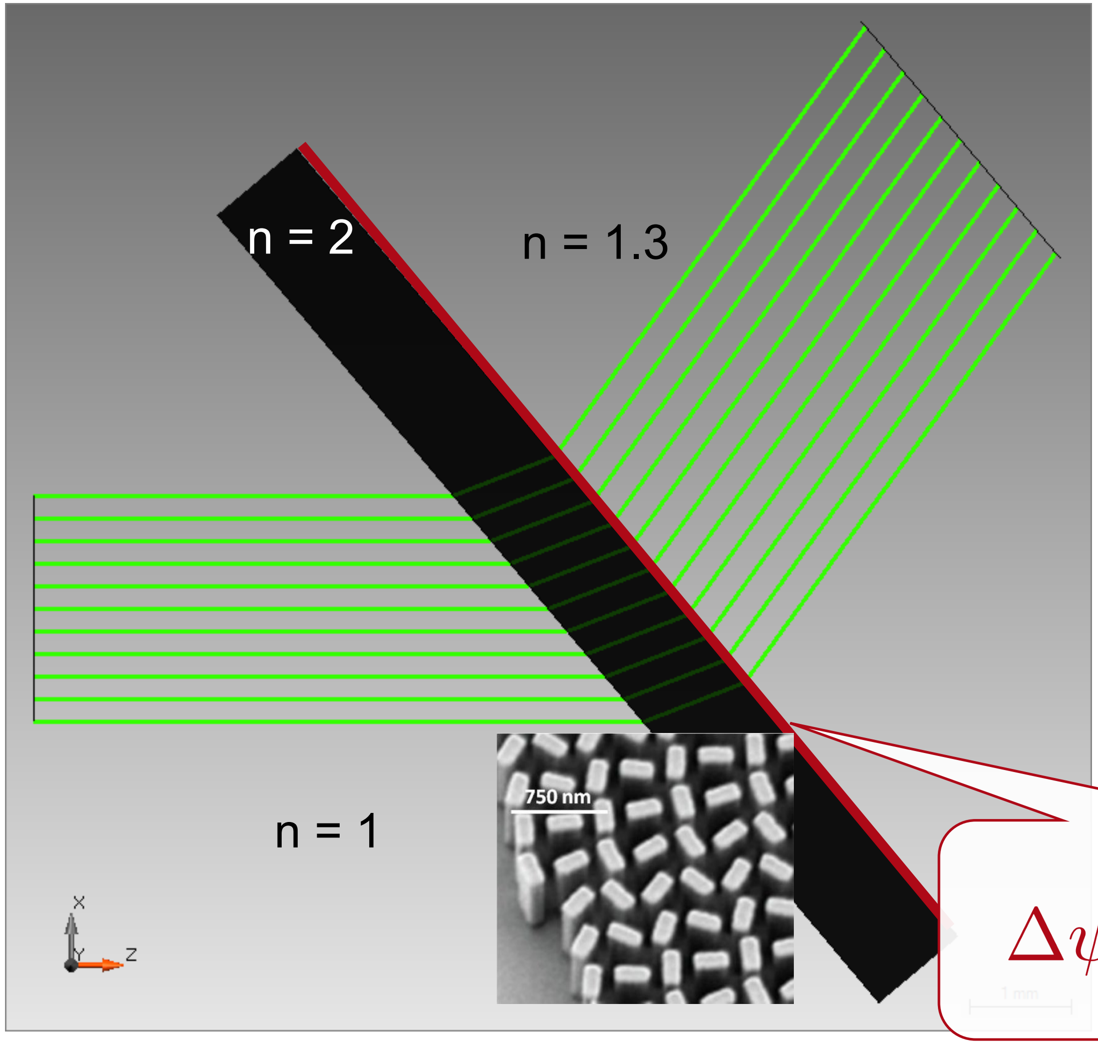
# Propagation Plate with Metasurface: Ray and Field Tracing



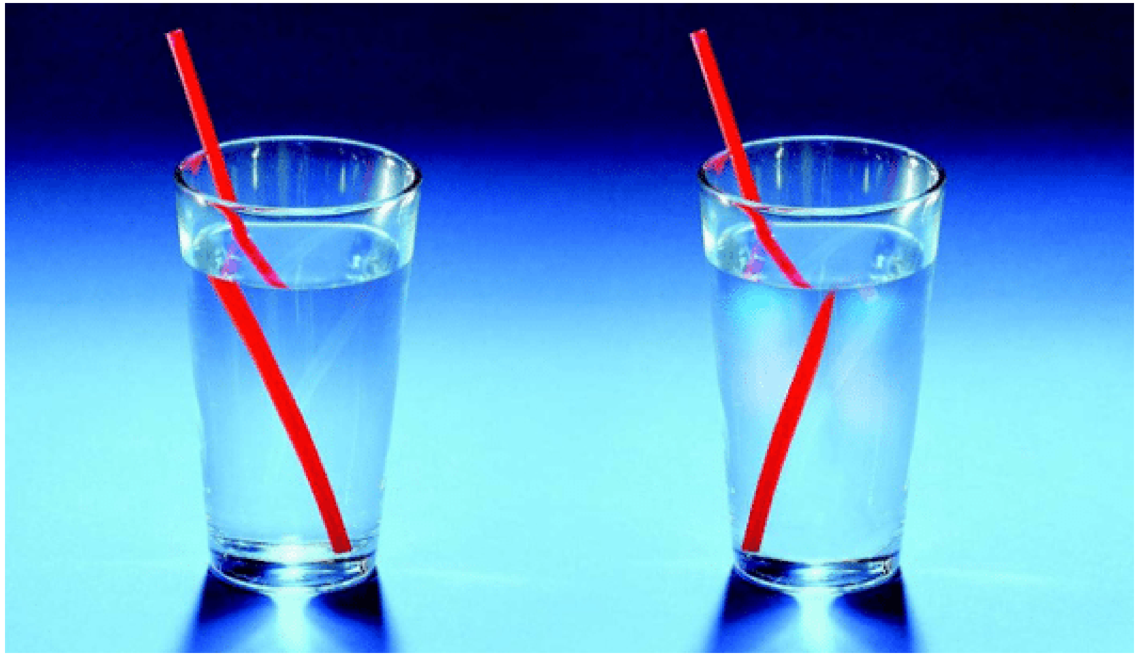




$$\Delta\psi(\boldsymbol{\rho}) = \boldsymbol{\kappa}^{\text{meta}} \cdot \boldsymbol{\rho}$$

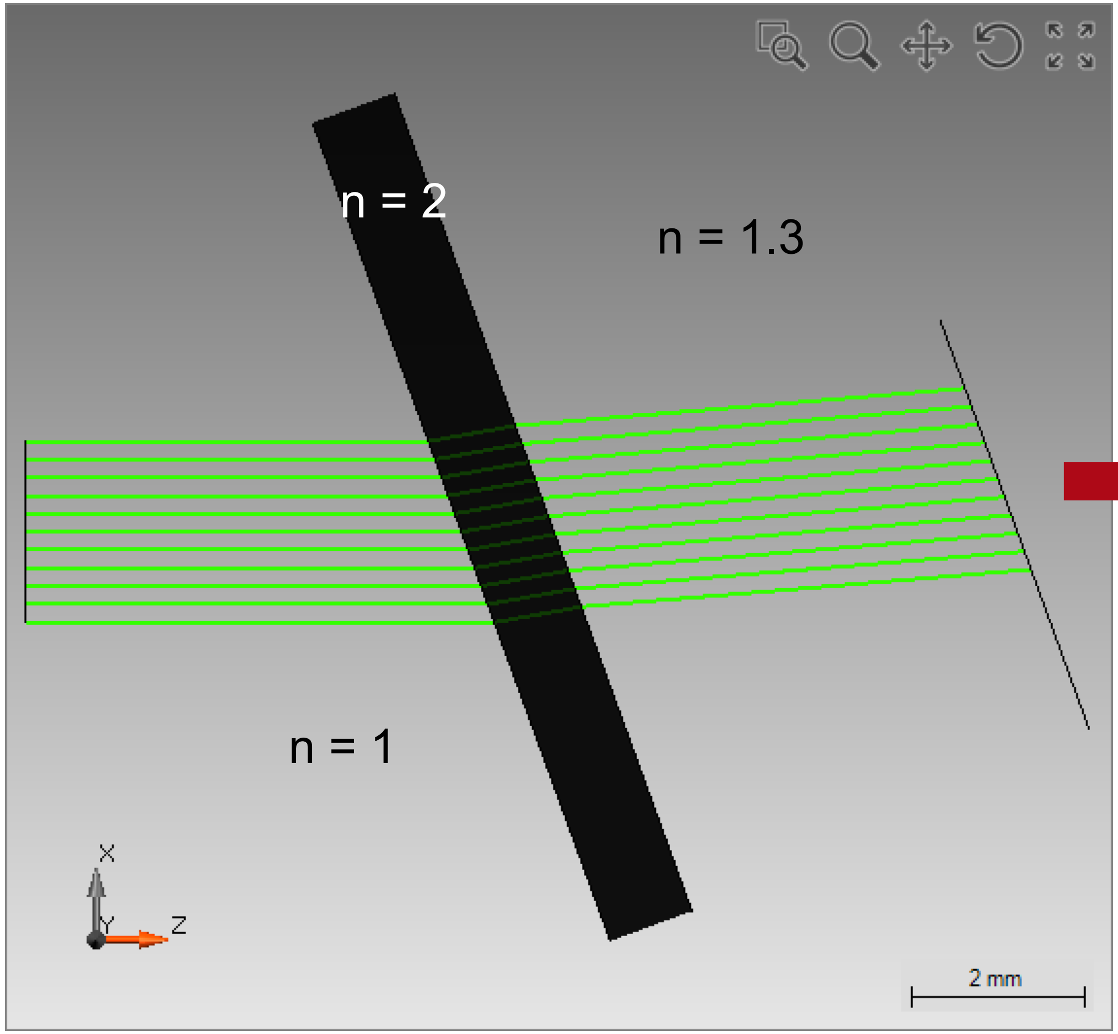
# Propagation Plate with Metasurface: Ray and Field Tracing

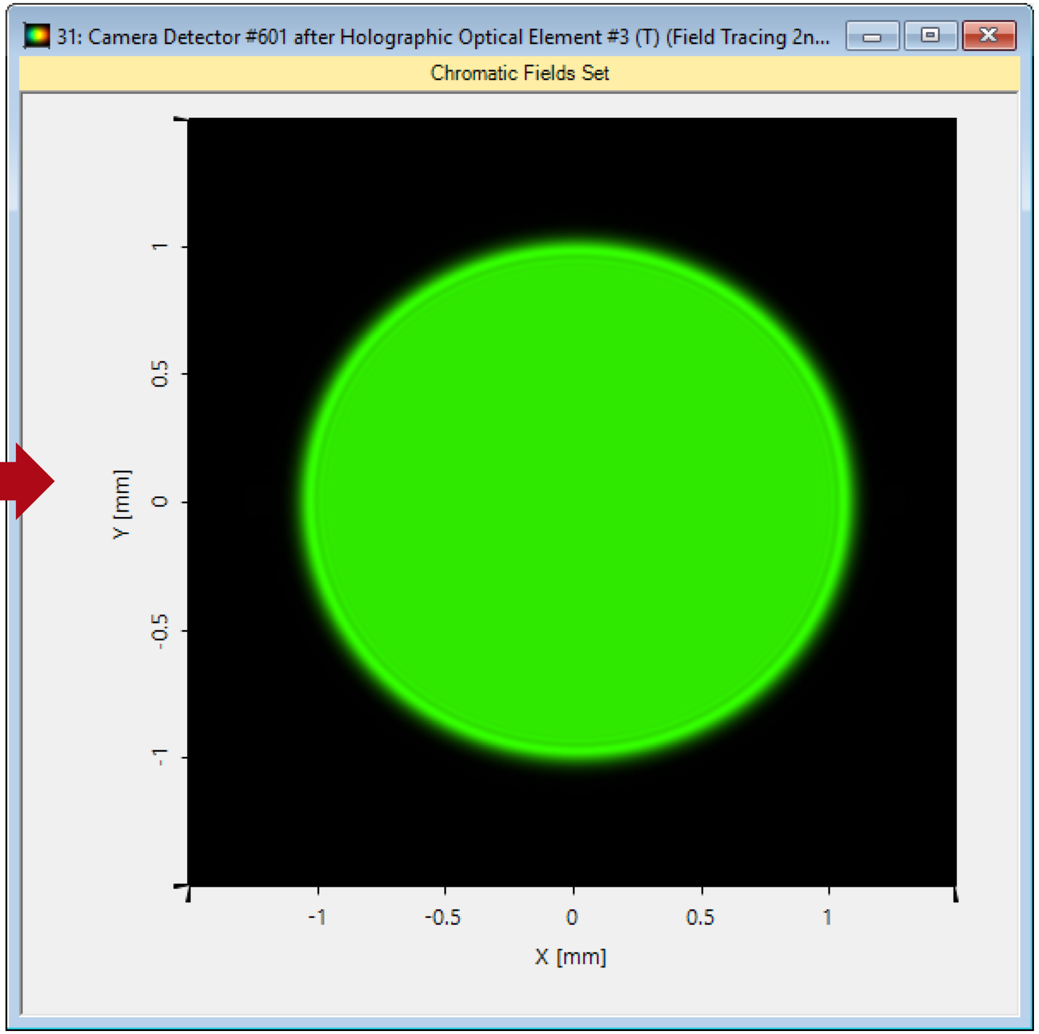



$$\Delta\psi(\boldsymbol{\rho}) = \boldsymbol{\kappa}^{\text{meta}} \cdot \boldsymbol{\rho}$$

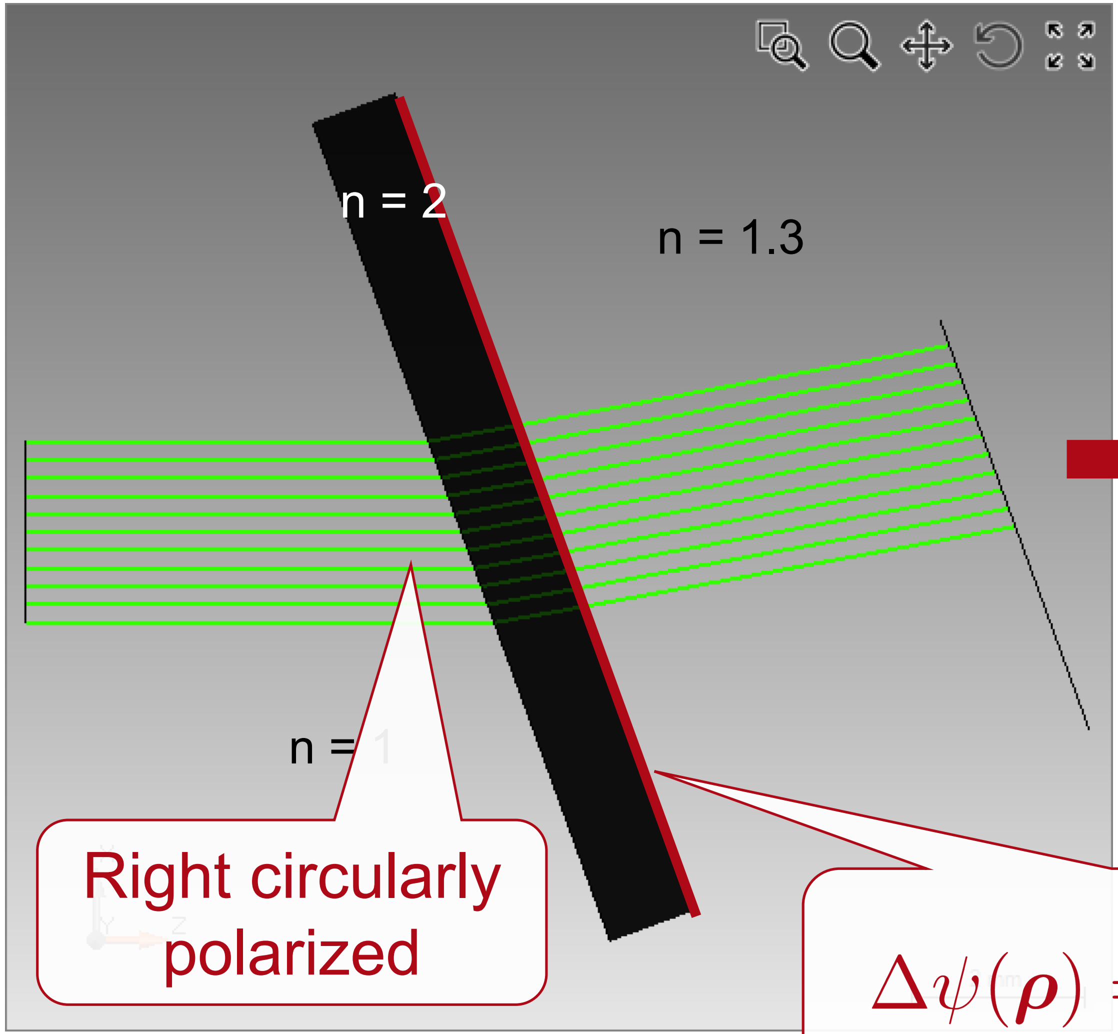


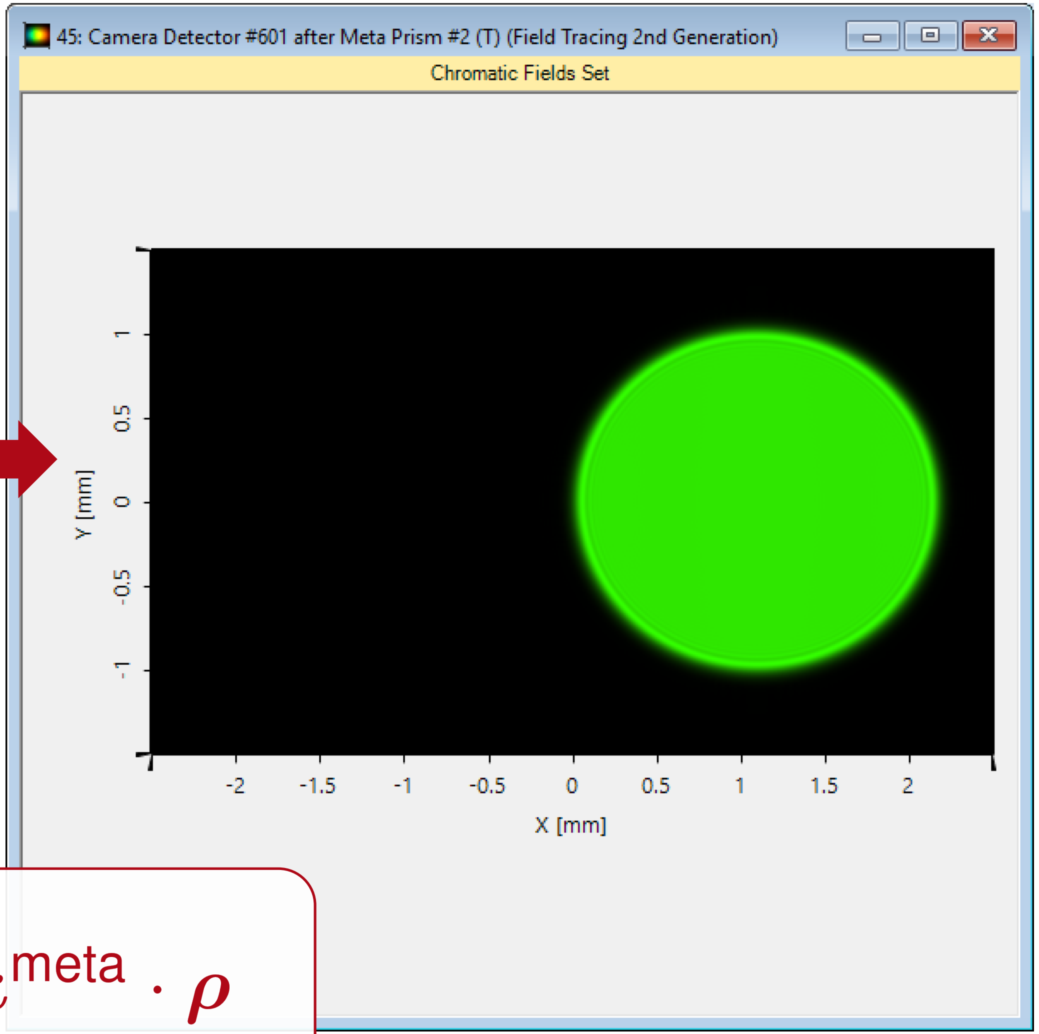
# Plane Wave through Plate: Ray and Field Tracing



# Plate with Metasurface: Ray and Field Tracing

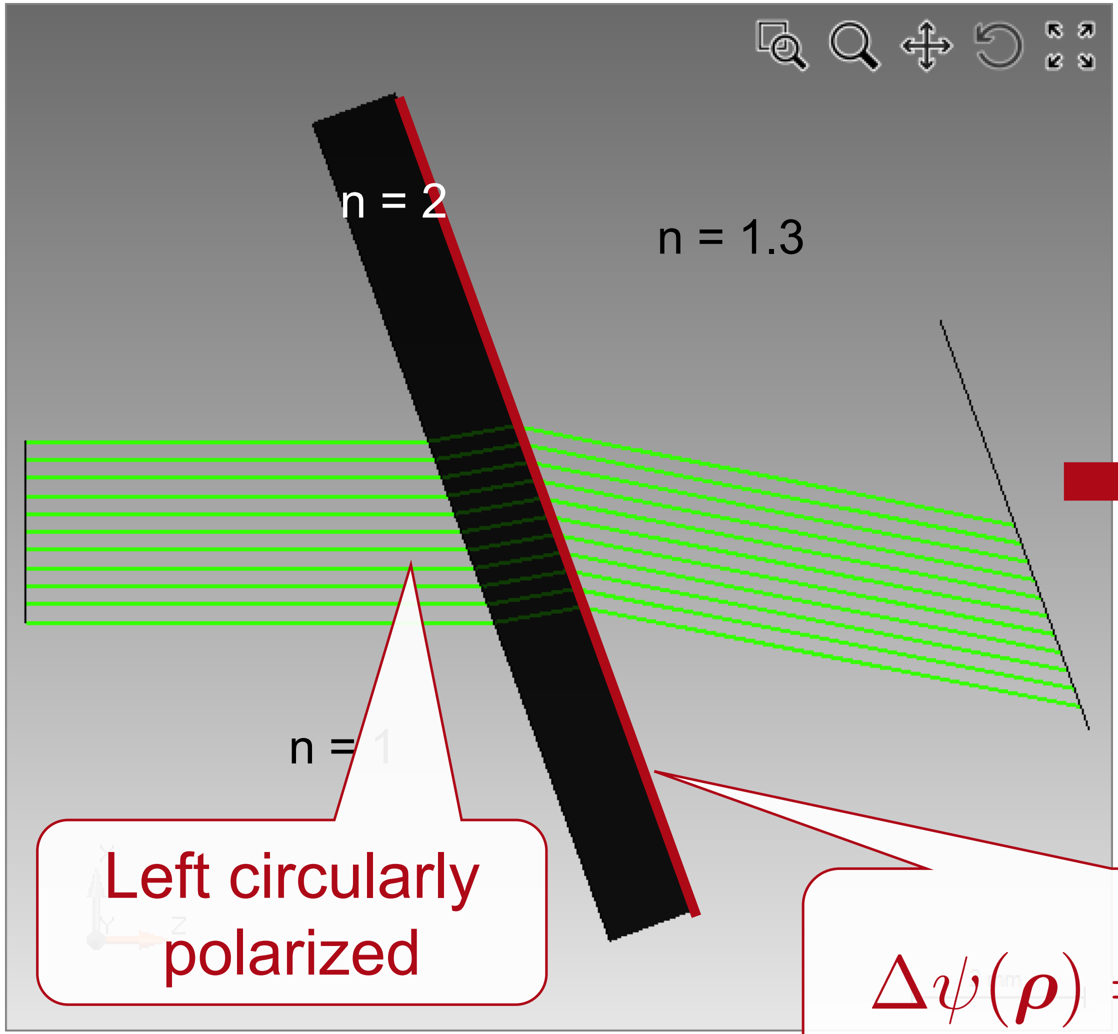


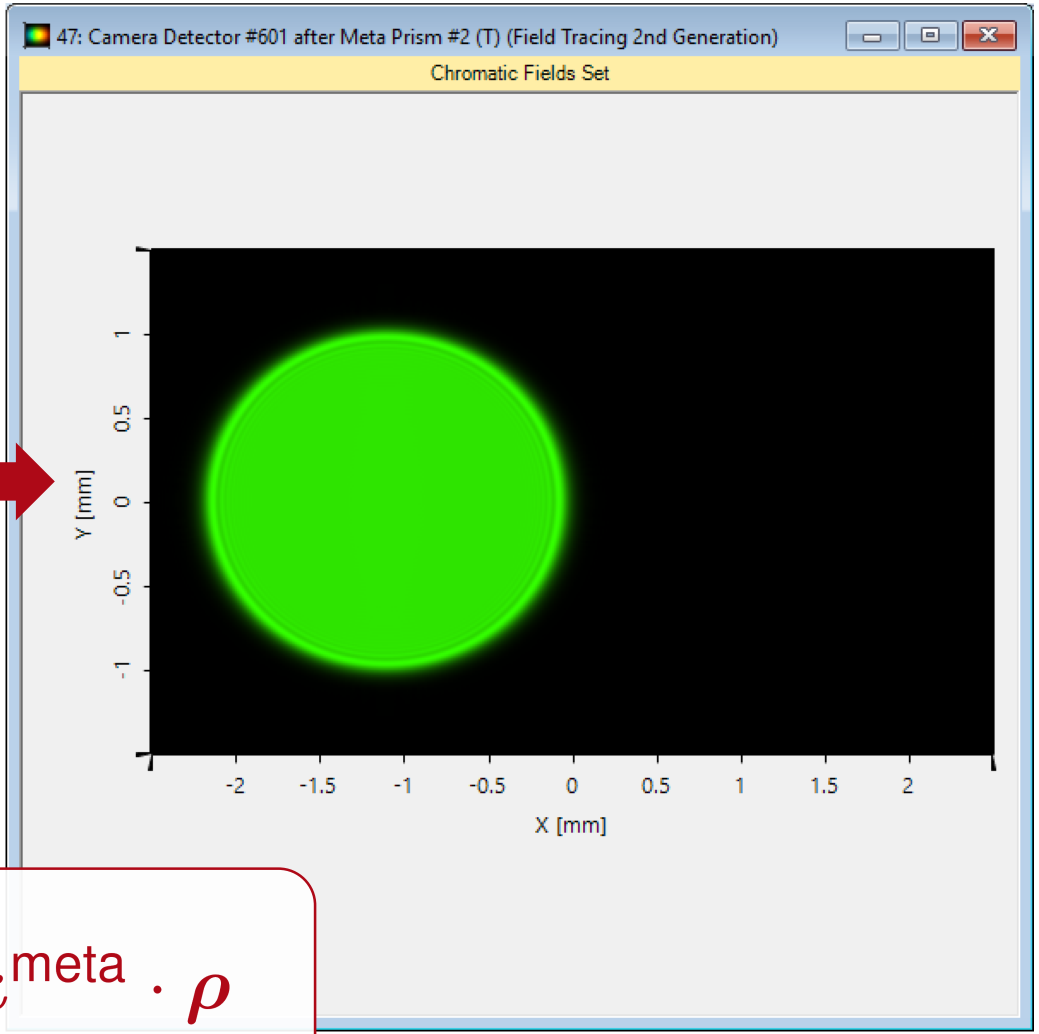




Right circularly polarized

$$\Delta\psi(\boldsymbol{\rho}) = \boldsymbol{\kappa}^{\text{meta}} \cdot \boldsymbol{\rho}$$

# Plate with Metasurface: Ray and Field Tracing

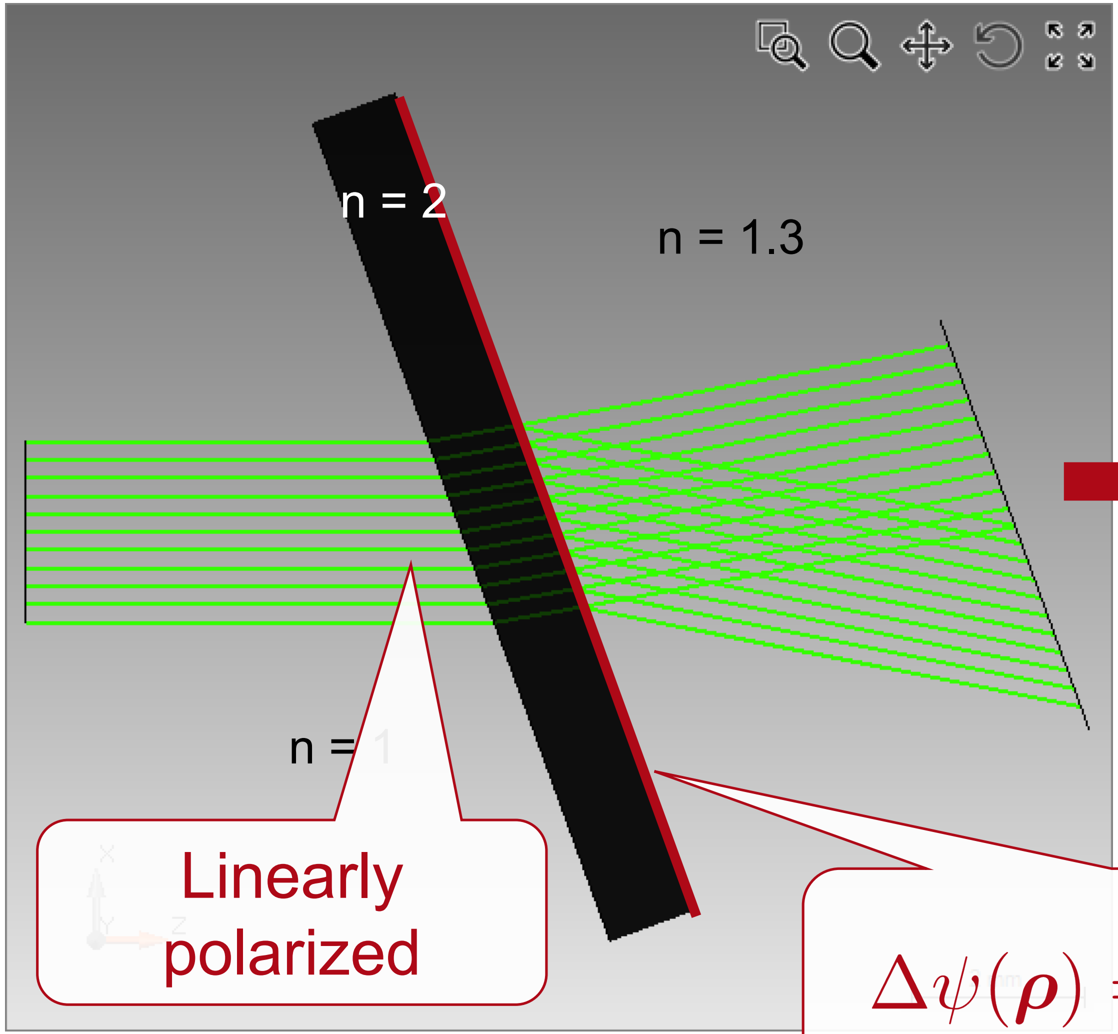


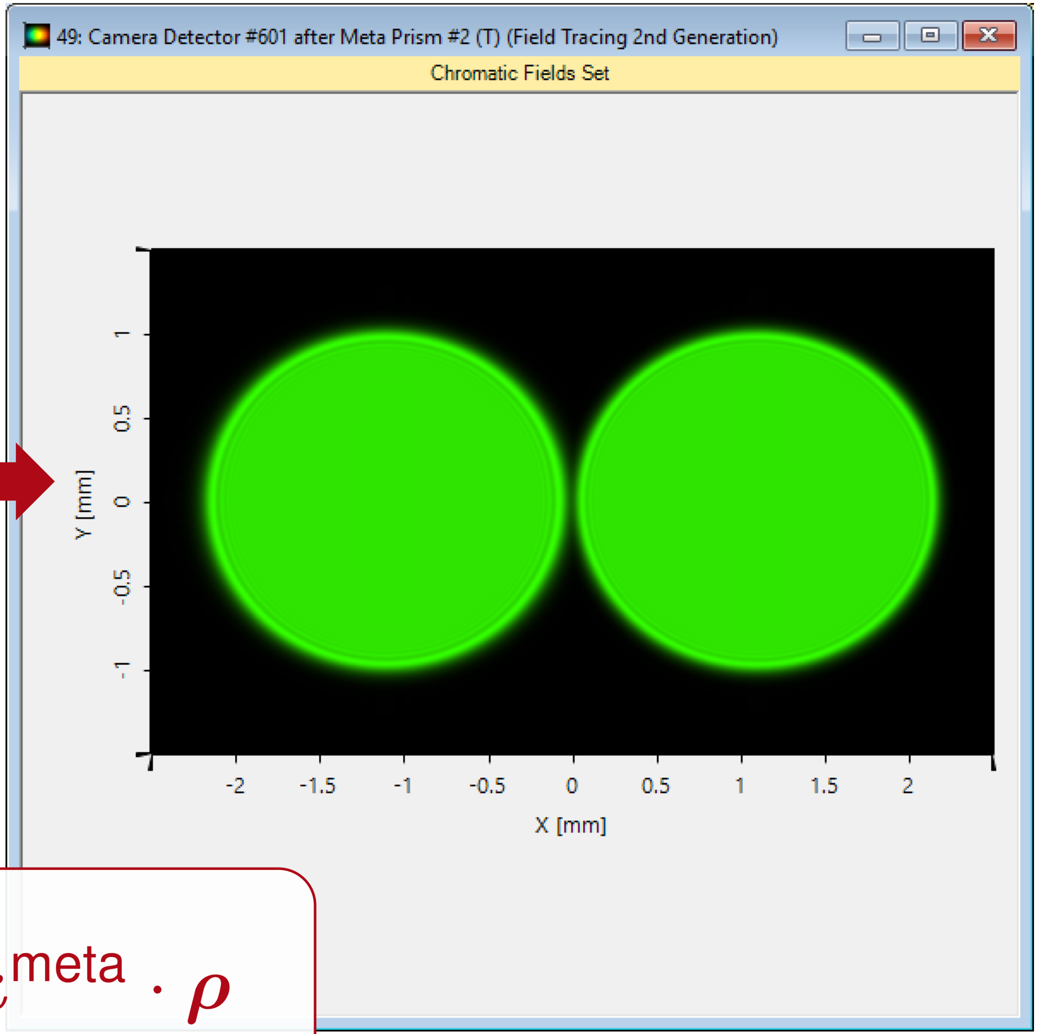




Left circularly polarized

$$\Delta\psi(\boldsymbol{\rho}) = \boldsymbol{\kappa}^{\mathrm{meta}} \cdot \boldsymbol{\rho}$$

# Plate with Metasurface: Ray and Field Tracing



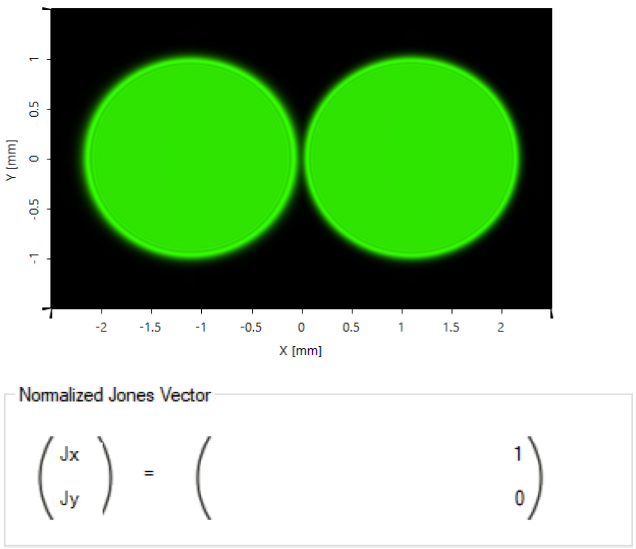




Linearly polarized

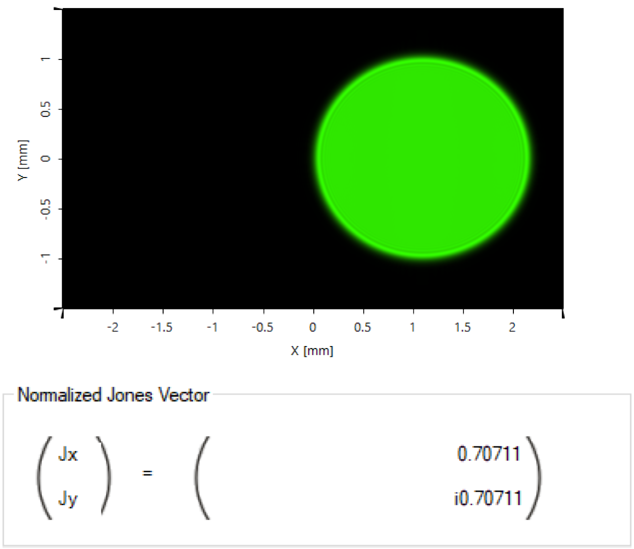
$$\Delta\psi(\boldsymbol{\rho}) = \boldsymbol{\kappa}^{\text{meta}} \cdot \boldsymbol{\rho}$$

# Polarization Dependent Fuction of Nanofin Metalayer

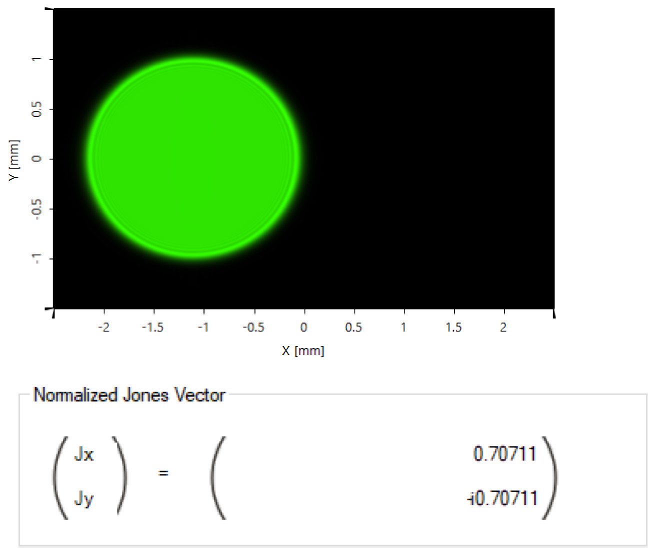
$$\boldsymbol{V}_{\perp}^{\mathrm{out}}(\boldsymbol{\rho}) = \left\{\mathbf{B}^{+}(\boldsymbol{\rho};\psi^{\mathsf{in}})\boldsymbol{U}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho})\right\}\exp\left(\mathrm{i}\psi^{\mathsf{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})\right)$$
$$+\left\{\mathbf{B}^{-}(\boldsymbol{\rho};\psi^{\mathsf{in}})\boldsymbol{U}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho})\right\}\exp\left(\mathrm{i}\psi^{\mathsf{in}}(\boldsymbol{\rho}) - \Delta\psi(\boldsymbol{\rho})\right)$$




Linear Polarized (Ex)



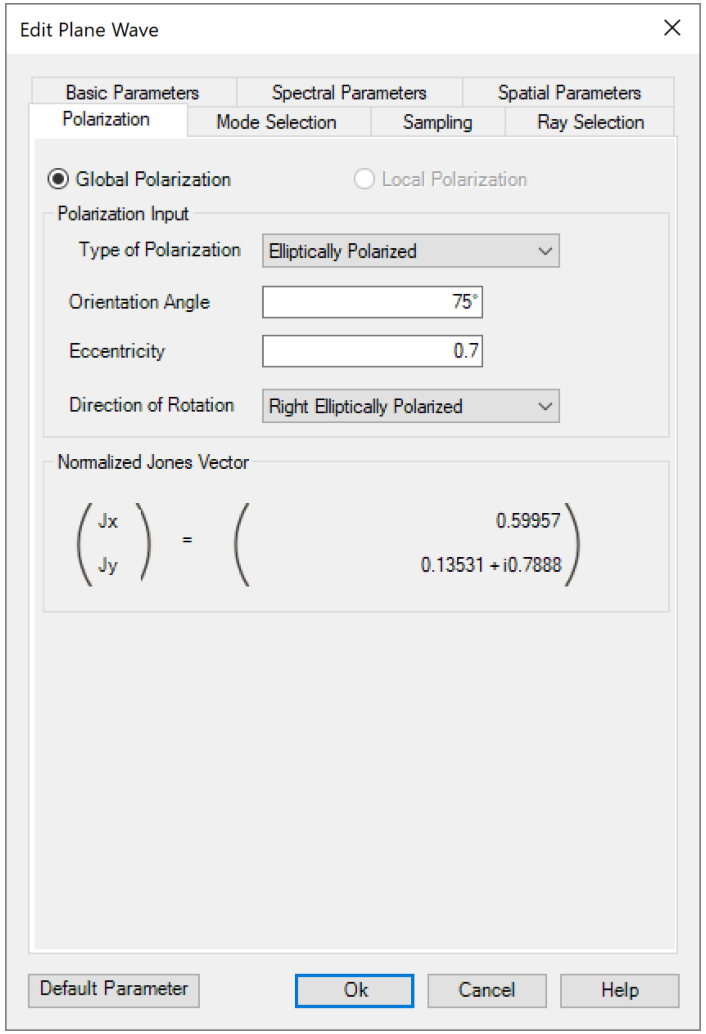

Right Circularly Polarized

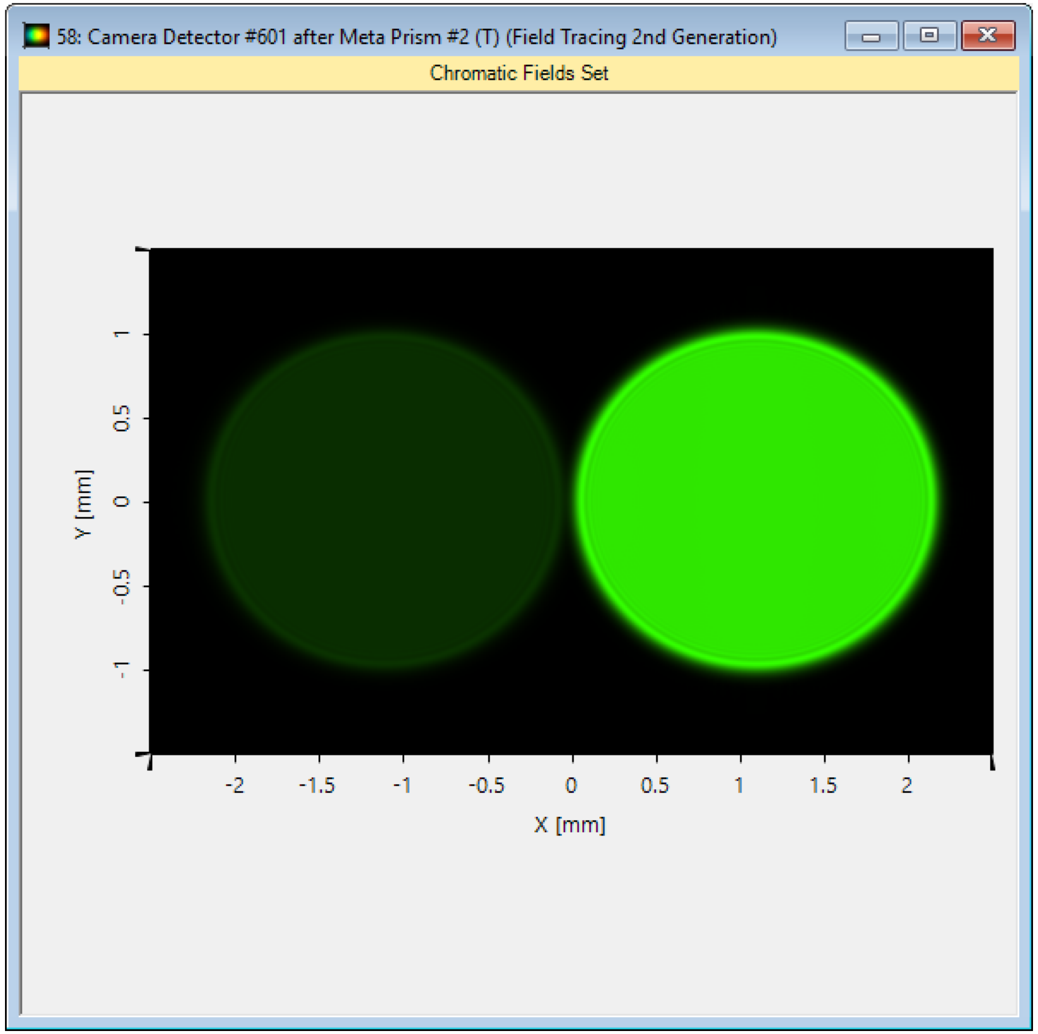



Left Circularly Polarized

# Polarization Dependent Fuction of Nanofin Metalayer

Elliptical Polarization:



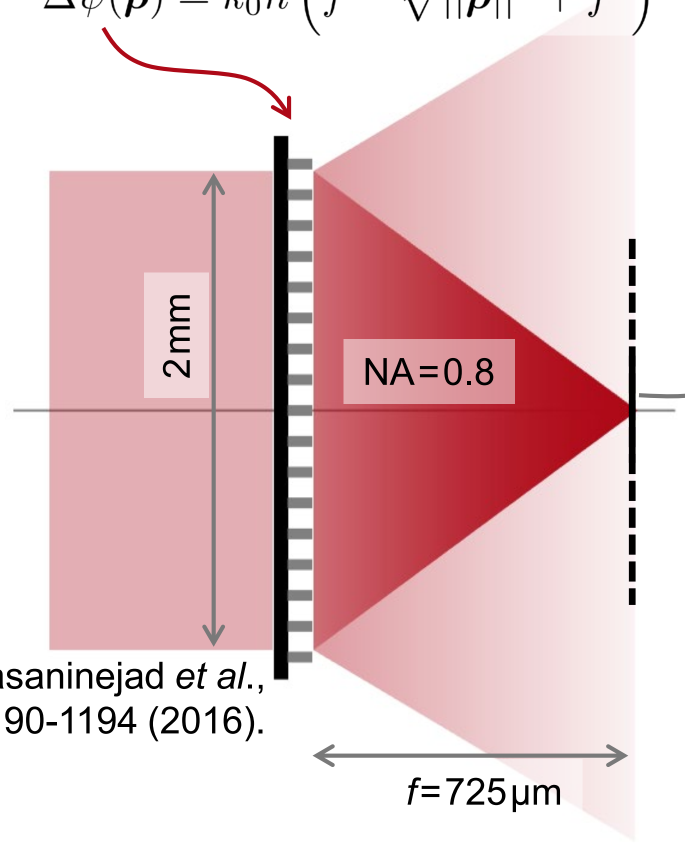
# High-NA Metalens Simulation

$$\Delta\psi(\boldsymbol{\rho}) = k_0 n \left( f - \sqrt{||\boldsymbol{\rho}||^2 + f^2} \right)$$

Desired to function as a focusing lens

input plane wave

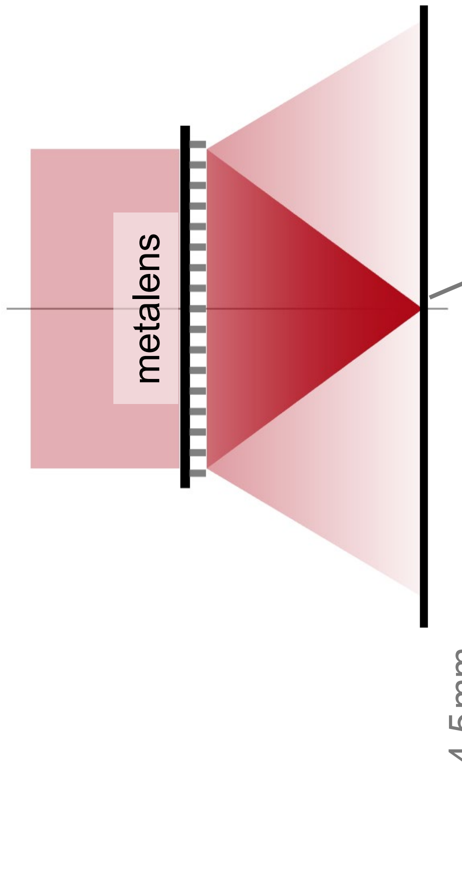
- normal incidence
- wavelength @532 nm
- beam diameter 2 mm
- polarization state
  a) R-circular
  b) Linear
  c) L-circular



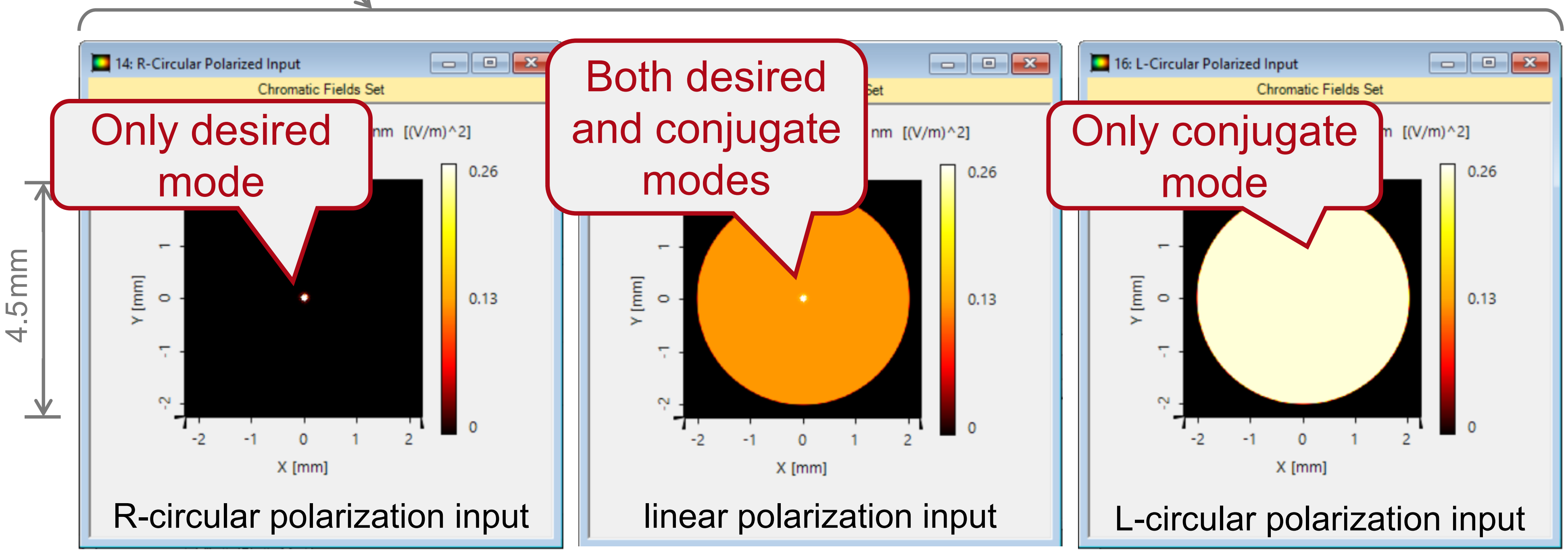

M. Khorasaninejad *et al.*, Science **352**, 1190-1194 (2016).


How to calculate the point spread function (PSF) at the focal plane of a metalens, with polarization effects considered?

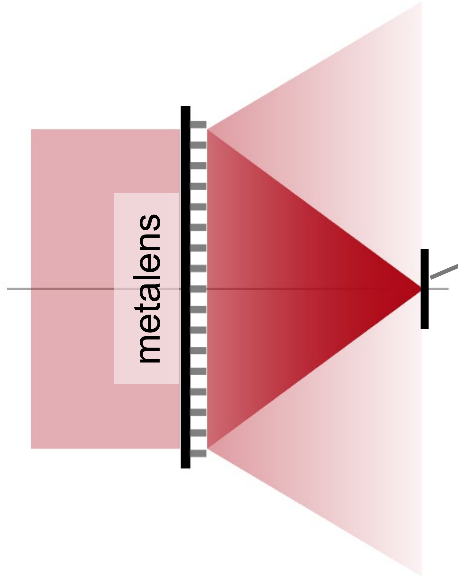
# High-NA Metalens Simulation



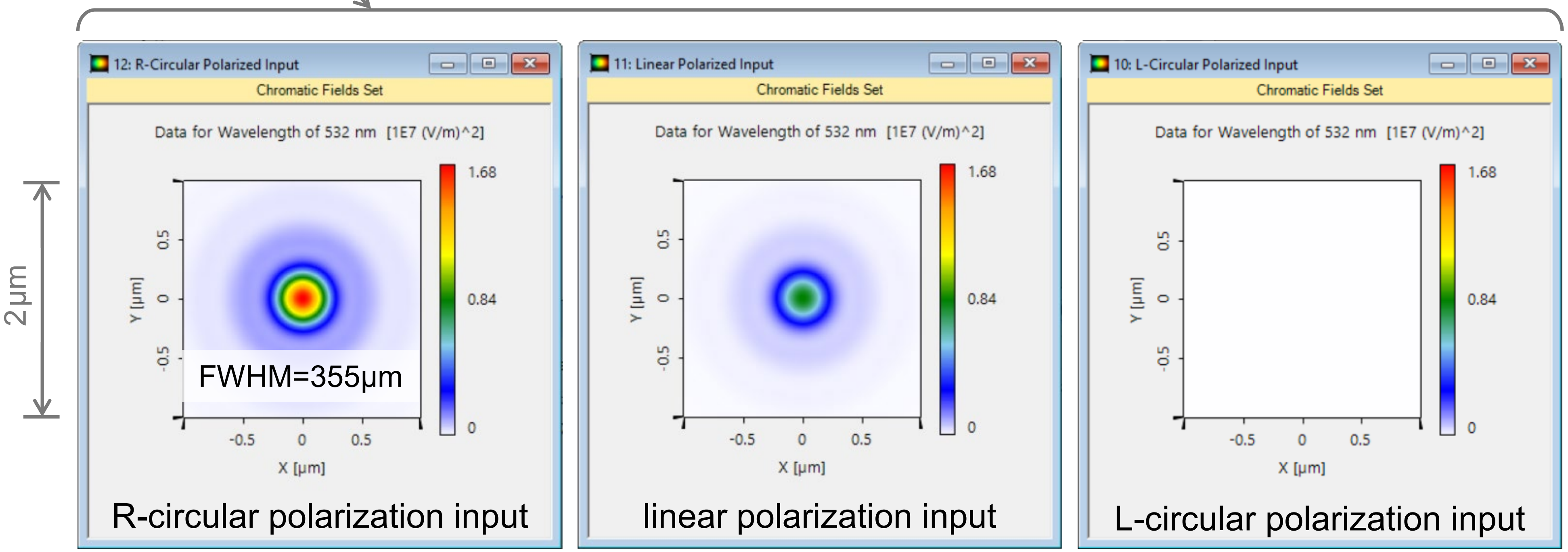

$$
\begin{aligned}
\boldsymbol{V}_{\perp}^{\mathrm{out}}(\boldsymbol{\rho}) = &\left\{\mathbf{B}^{+}(\boldsymbol{\rho};\psi^{\mathrm{in}})\boldsymbol{U}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho})\right\}\exp\left(\mathrm{i}\psi^{\mathrm{in}}(\boldsymbol{\rho})+\Delta\psi(\boldsymbol{\rho})\right)\\
&+\left\{\mathbf{B}^{-}(\boldsymbol{\rho};\psi^{\mathrm{in}})\boldsymbol{U}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho})\right\}\exp\left(\mathrm{i}\psi^{\mathrm{in}}(\boldsymbol{\rho})-\Delta\psi(\boldsymbol{\rho})\right)
\end{aligned}
$$



# High-NA Metalens Simulation




$$\boldsymbol{V}_{\perp}^{\mathrm{out}}(\boldsymbol{\rho}) = \left\{\mathbf{B}^{+}(\boldsymbol{\rho};\psi^{\mathsf{in}})\boldsymbol{U}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho})\right\}\exp\!\left(\mathrm{i}\psi^{\mathsf{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})\right)$$
$$+\left\{\mathbf{B}^{-}(\boldsymbol{\rho};\psi^{\mathsf{in}})\boldsymbol{U}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho})\right\}\exp\!\left(\mathrm{i}\psi^{\mathsf{in}}(\boldsymbol{\rho}) - \Delta\psi(\boldsymbol{\rho})\right)$$



# Diffractive layer

… with application to lenses

# Physical Optics Modeling: Diffractive Layer

- In general a wavefront surface response $\Delta\psi(\boldsymbol{\rho})$ leads to the equation $\nabla_\perp \psi^{\text{out}}(\boldsymbol{\rho}) = \nabla_\perp \psi^{\text{in}}(\boldsymbol{\rho}) + \nabla_\perp \left(\Delta\psi(\boldsymbol{\rho})\right)$ and because of the local plane wave assumption (homeomorphic zone) into

$$\boldsymbol{\kappa}^{\text{out}}(\boldsymbol{\rho}) = \boldsymbol{\kappa}^{\text{in}}(\boldsymbol{\rho}) + \boldsymbol{K}(\boldsymbol{\rho})$$

  with $\boldsymbol{K}(\boldsymbol{\rho}) \stackrel{\text{def}}{=} \nabla_\perp \left(\Delta\psi(\boldsymbol{\rho})\right)$.

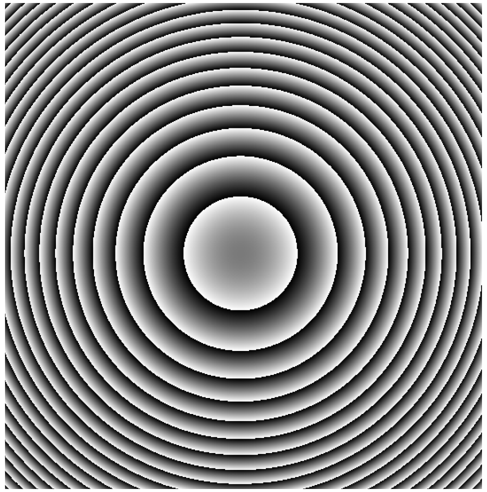
- This equation is directly related to a locally formulated grating equation

$$\boldsymbol{\kappa}^{\text{out}}(\boldsymbol{\rho}) = \boldsymbol{\kappa}^{\text{in}}(\boldsymbol{\rho}) + m\left(2\pi/d_x(\boldsymbol{\rho}), 2\pi/d_y(\boldsymbol{\rho})\right)$$

  with the local grating period $\boldsymbol{d}(\boldsymbol{\rho}) = (d_x(\boldsymbol{\rho}), d_y(\boldsymbol{\rho}))$.

- That leads to the basic principle of a diffractive layer via:

$$\boldsymbol{d}(\boldsymbol{\rho}) = 2\pi\left(\left(\partial\psi(\boldsymbol{\rho})/\partial x\right)^{-1}, \left(\partial\psi(\boldsymbol{\rho})/\partial y\right)^{-1}\right)$$



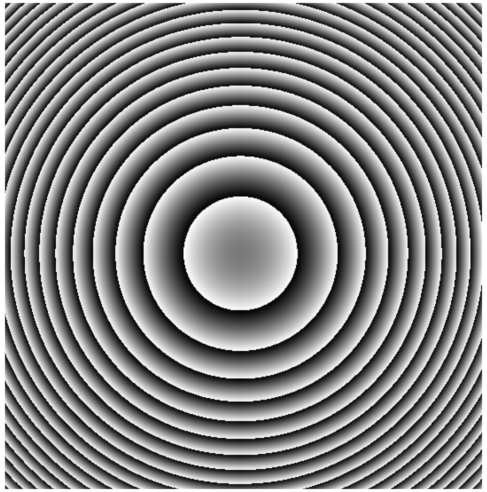
# Physical Optics Modeling: Diffractive Layer

- This design and modeling understanding results in the decomposition of the output field into a series of local grating orders:

$$\boldsymbol{V}_{\perp}^{\mathrm{out}}(\boldsymbol{\rho}) = \left\{\mathbf{B}^{(1)}(\boldsymbol{\rho};\psi^{\mathsf{in}})\boldsymbol{U}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho})\right\}\exp\!\left(\mathrm{i}\psi^{\mathsf{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})\right)$$
$$+ \sum_{m=-\infty, m\neq 1}^{\infty} \left\{\mathbf{B}^{(m)}(\boldsymbol{\rho};\psi^{\mathsf{in}})\boldsymbol{U}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho})\right\}\exp\!\left(\mathrm{i}\psi^{\mathsf{in}}(\boldsymbol{\rho}) + m\Delta\psi(\boldsymbol{\rho})\right)$$

- The $2 \times 2$ matrix $\mathbf{B}^{(m)}(\boldsymbol{\rho};\psi^{\mathsf{in}})$ expresses the Rayleigh-matrix of grating theory and is rigorously calculated per direction and period by the Fourier Modal Method (FMM). Design should minimize Rayleigh coefficients for all undesired orders.

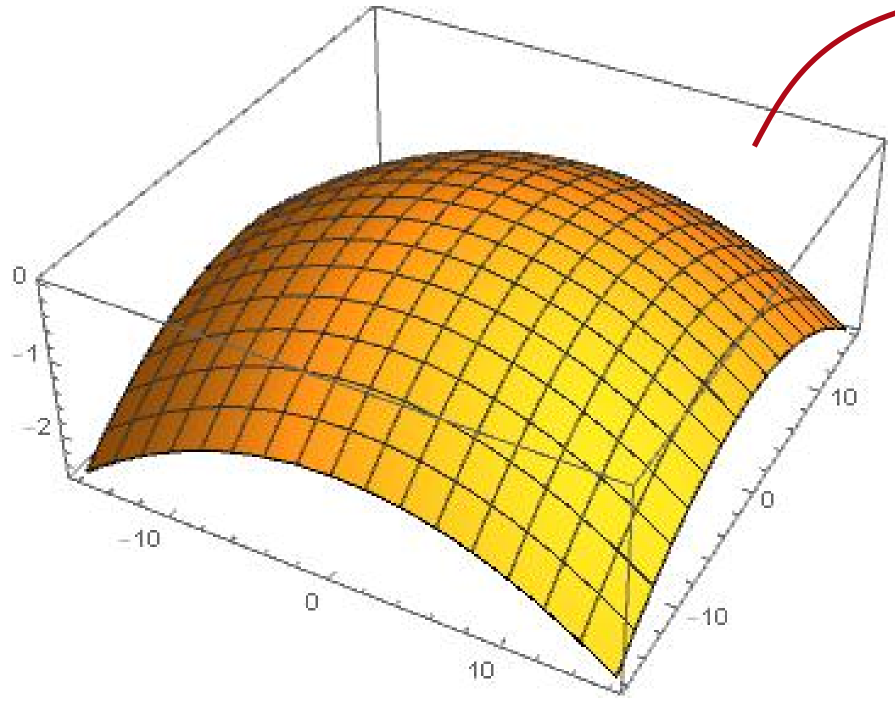
- Complications: Period drastically varies over surface, which results in a laterally varying number of propagating subfields. Full treatment available in **VirtualLab Lens Solutions**.

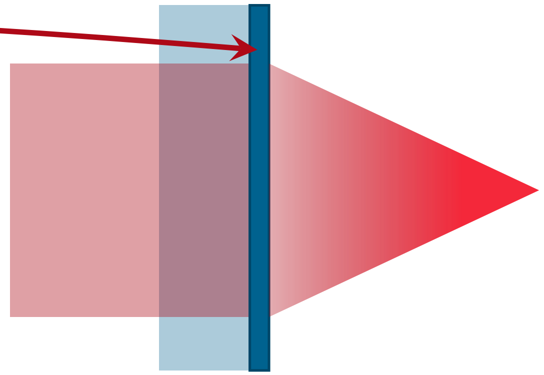


# Wavefront Surface Response of Focusing Lens

- In order to transform a plane incident field into a spherical convernegt one the wavefront surface response should be:

$$\Delta\psi(\boldsymbol{\rho}) = k_0 n \left( f - \sqrt{\|\boldsymbol{\rho}\|^2 + f^2} \right)$$

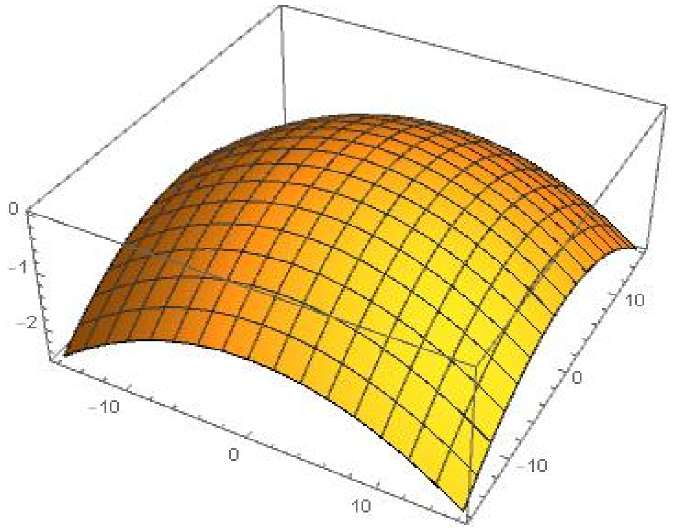


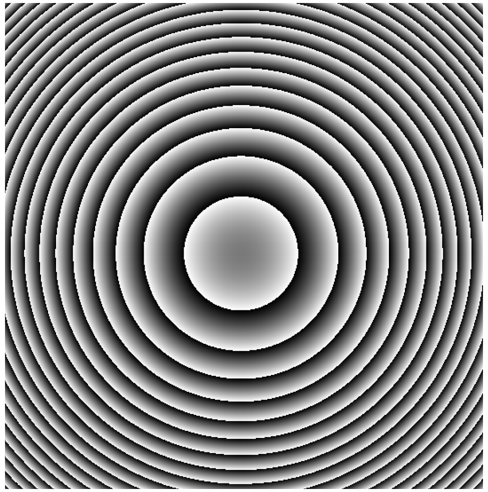



# Structure Design

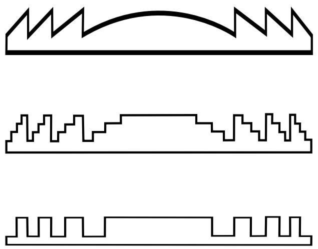
- Wrap the WSR: $(\Delta\psi(\boldsymbol{\rho}))^{\text{DOE}} = \mod_{p2\pi}\left\{k_0 n\left(f - \sqrt{\|\boldsymbol{\rho}\|^2 + f^2}\right)\right\}$ with $p \in \mathbb{N}$.
- For $p = 1$ local radial period follows with $d(\rho) = 2\pi/\Delta\psi'(\rho)$.
- Structure design by inverse Thin Element Approximation (TEA): The height profile $h^{\text{DOE}}$ is given by:

$$h^{\text{DOE}}(\rho) = \frac{\lambda}{2\pi\Delta n}\Delta\psi(\rho)^{\text{DOE}}$$

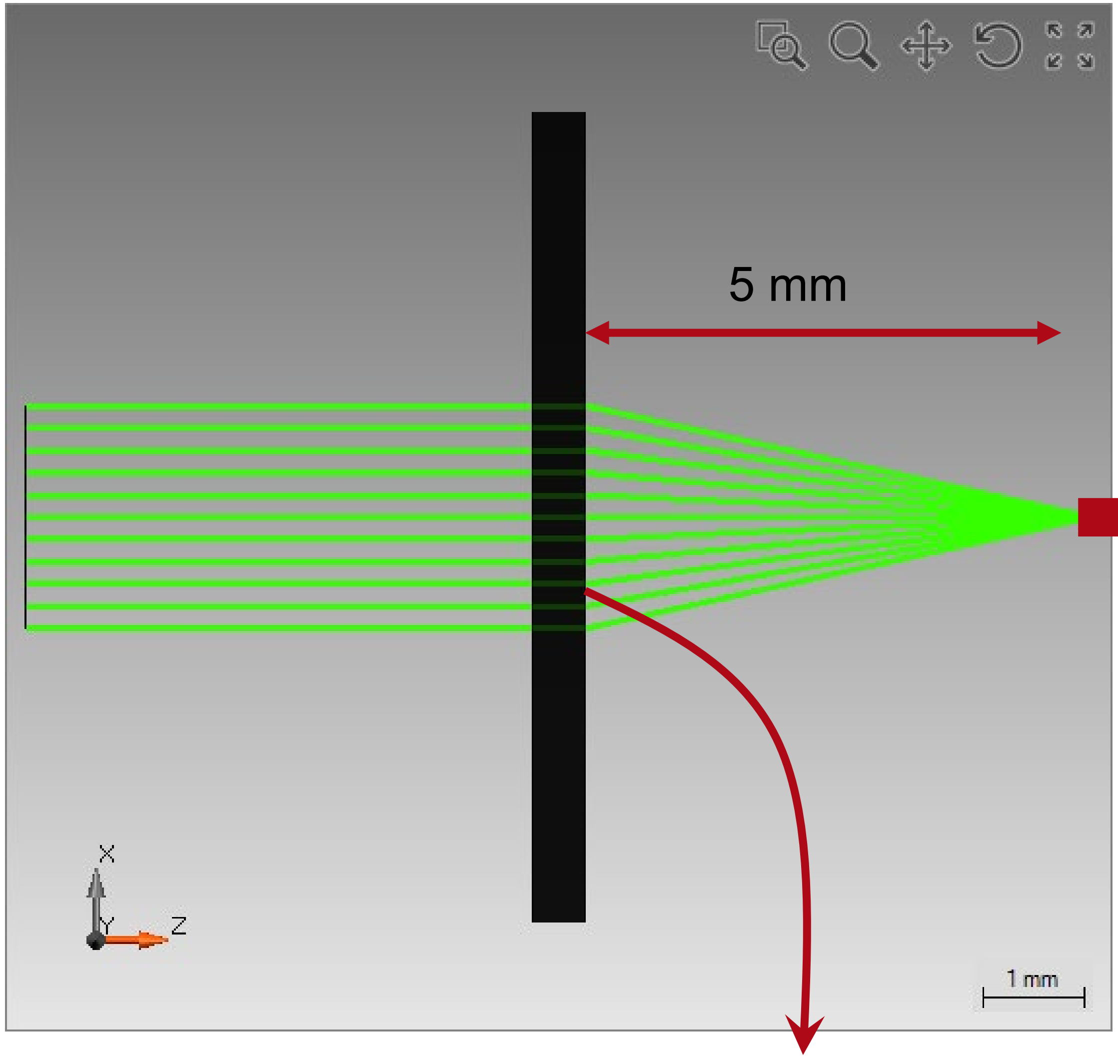


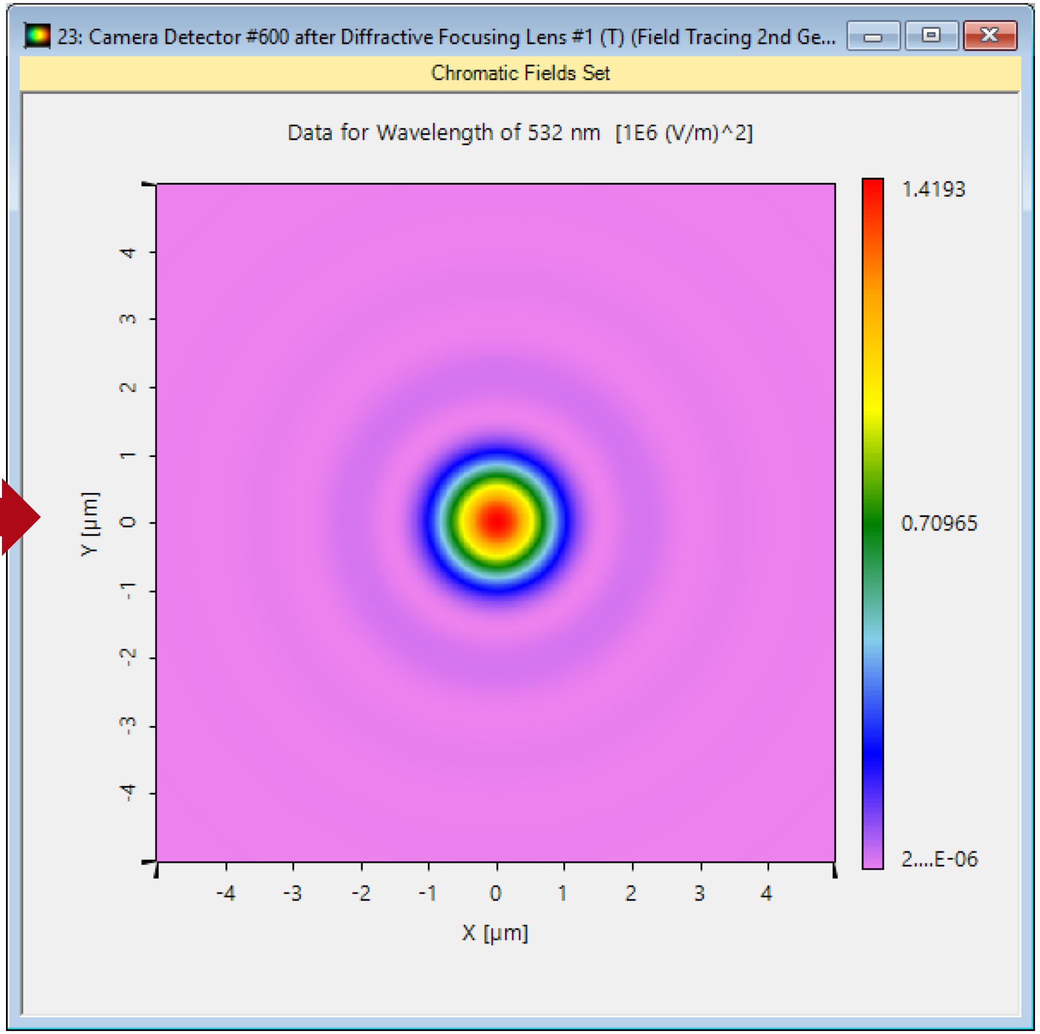



Profile quantization



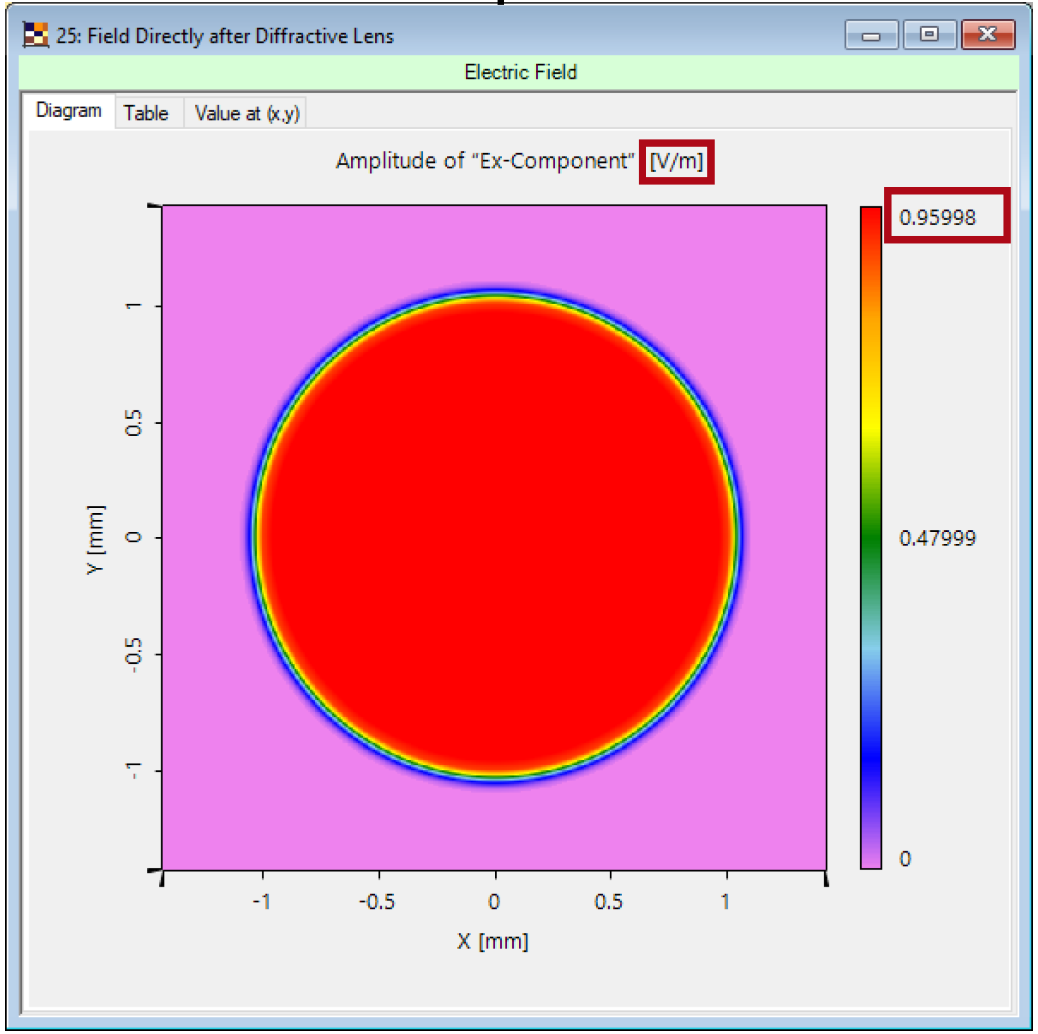
# Focusing Diffractive Lens with NA=0.2: Ray and Field Tracing



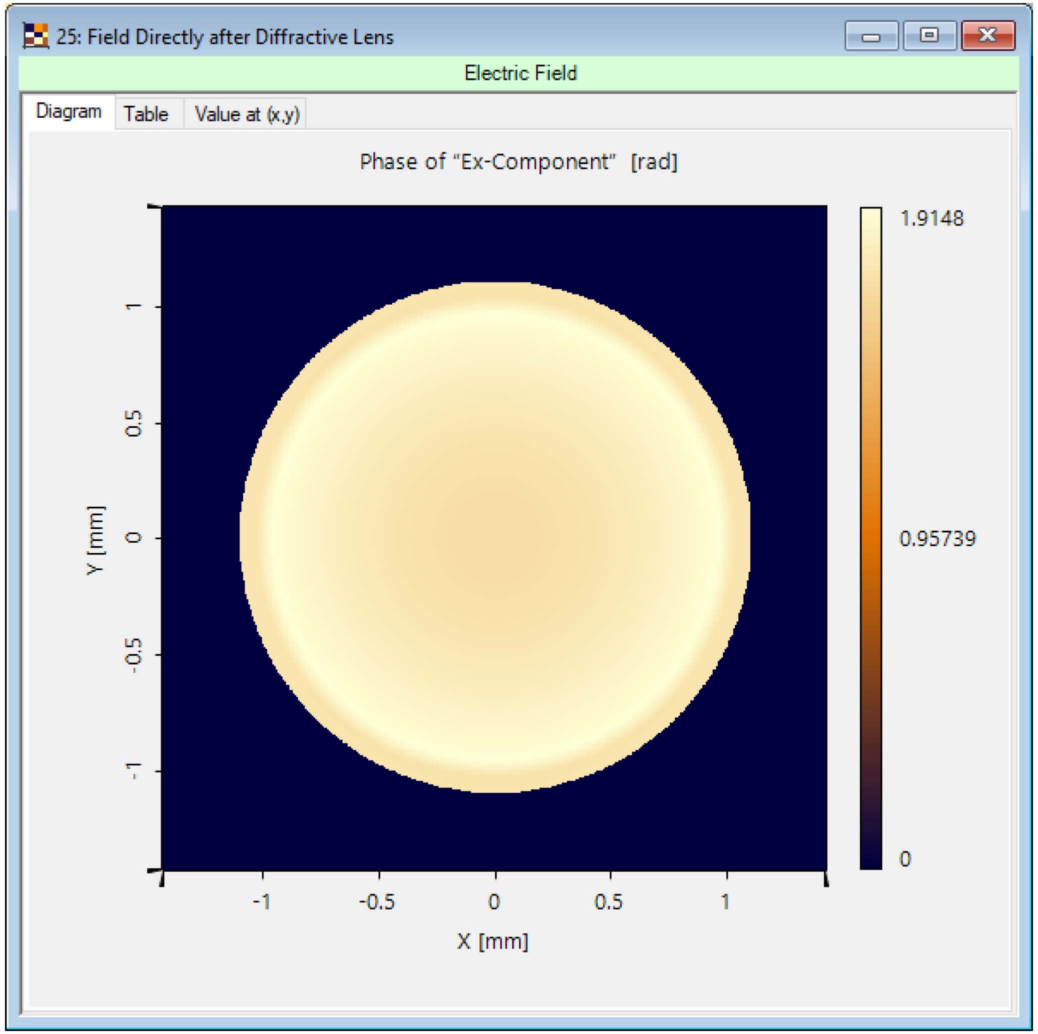
# Focusing Diffractive Lens with NA=0.2: Ray and Field Tracing

Amplitude

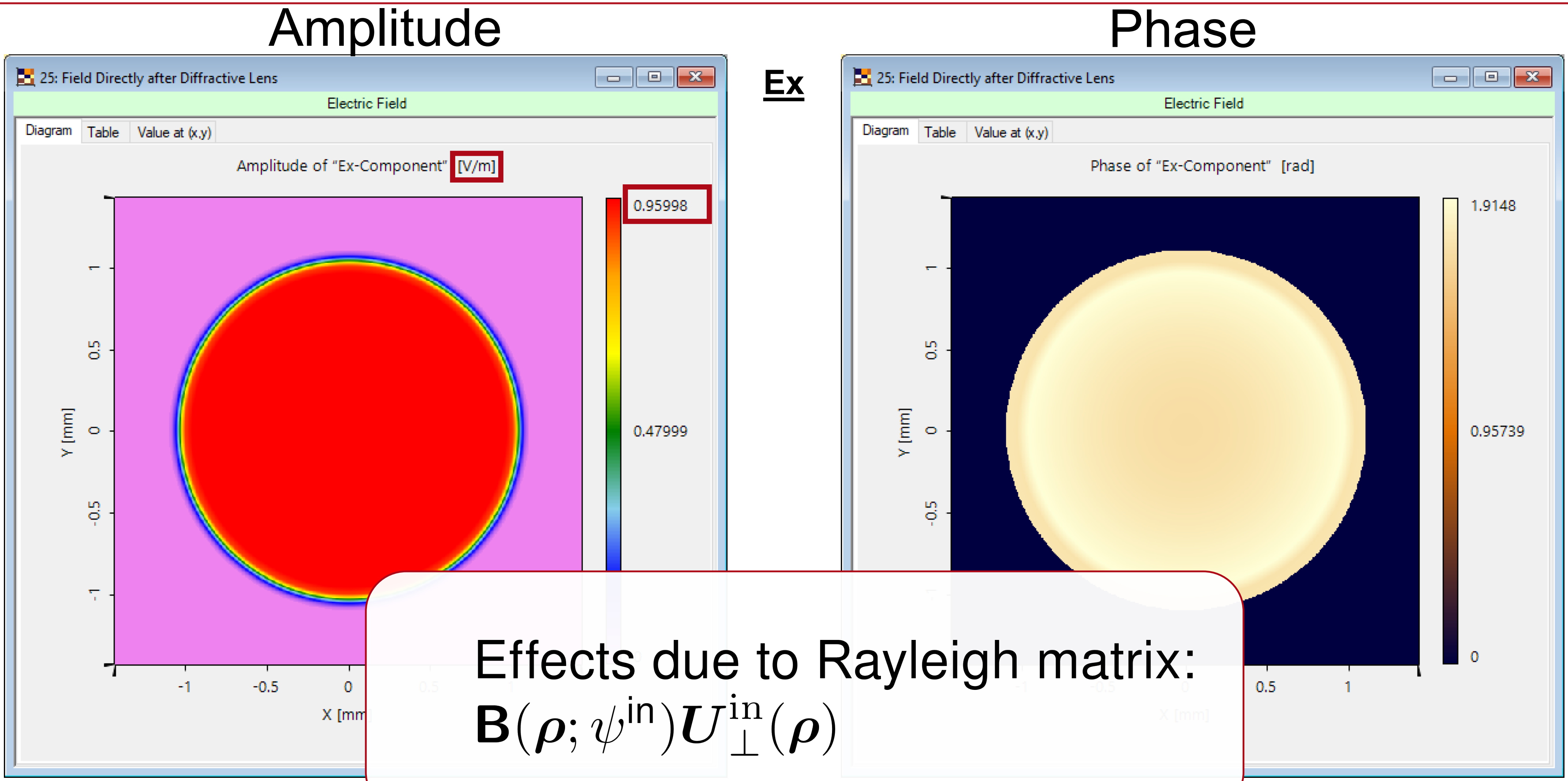



**Ex**

Phase

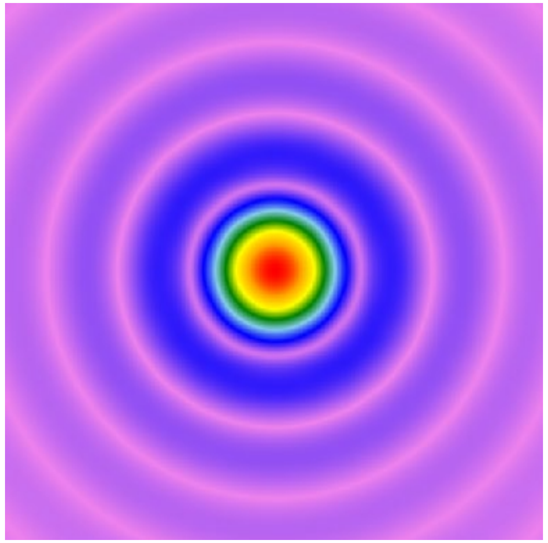


# Focusing Diffractive Lens with NA=0.2: Ray and Field Tracing

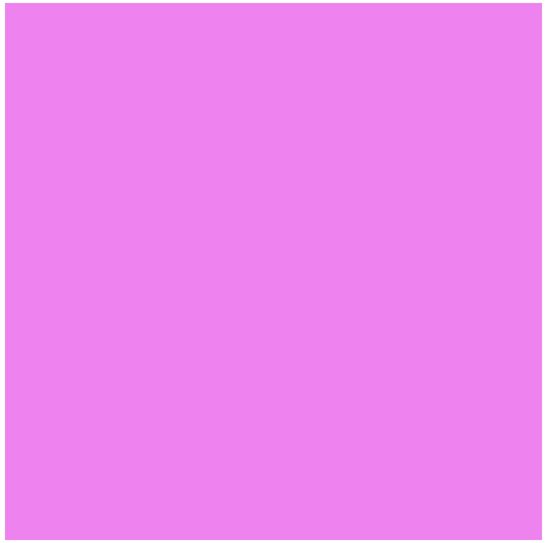



Effects due to Rayleigh matrix:
$\mathbf{B}(\boldsymbol{\rho};\psi^{\text{in}})\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})$

# Focusing Diffractive Lens with NA=0.2: Ray and Field Tracing



Amplitude Ex

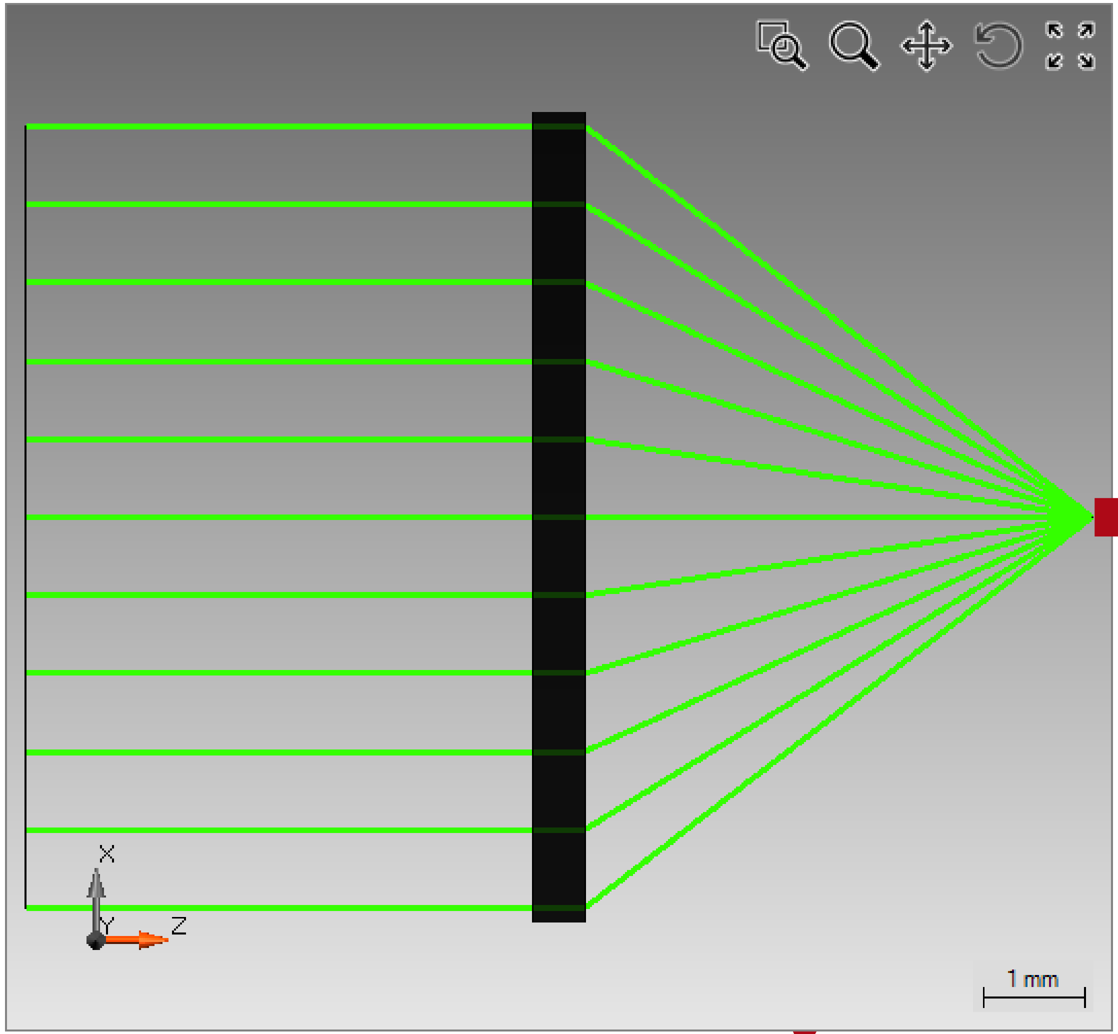


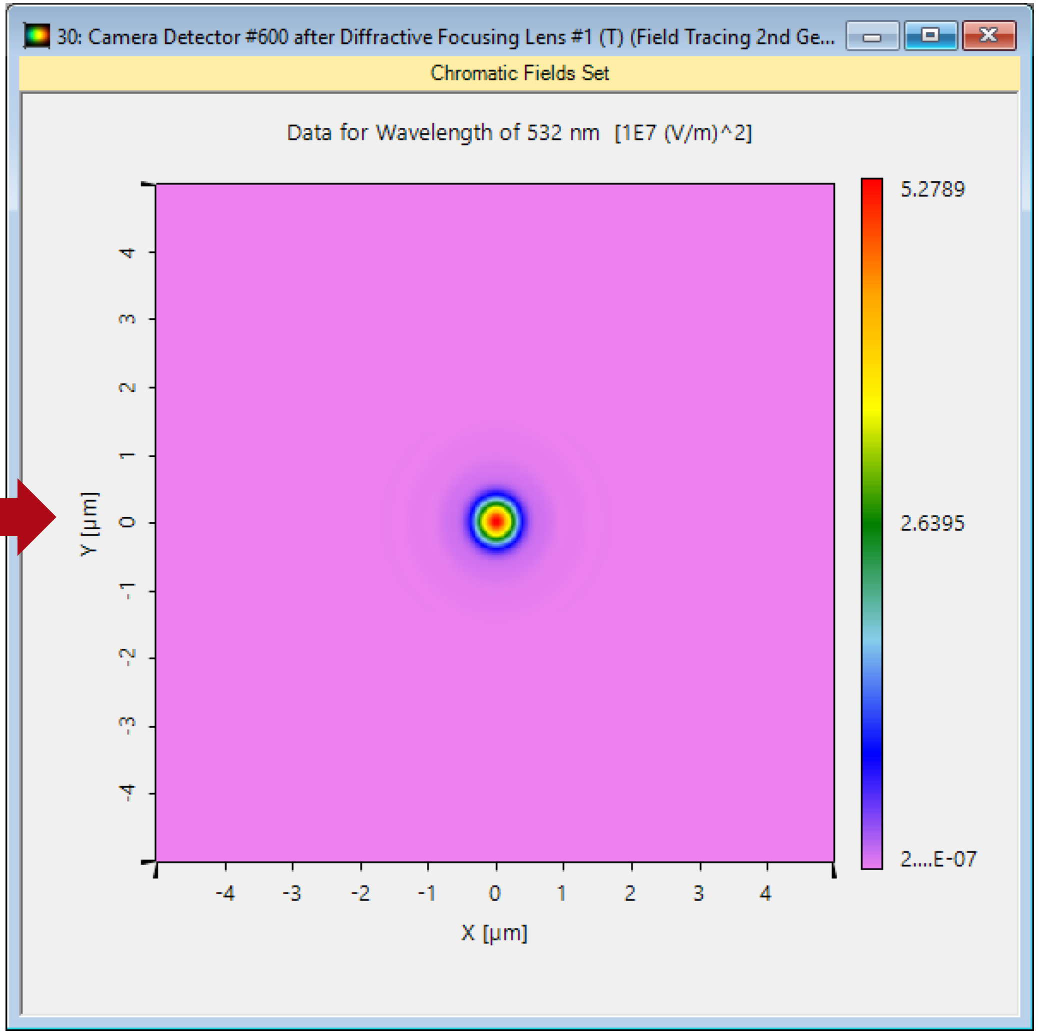
Amplitude Ey

Amplitude Ez

- Amplitudes in Focus (Same scaling!)

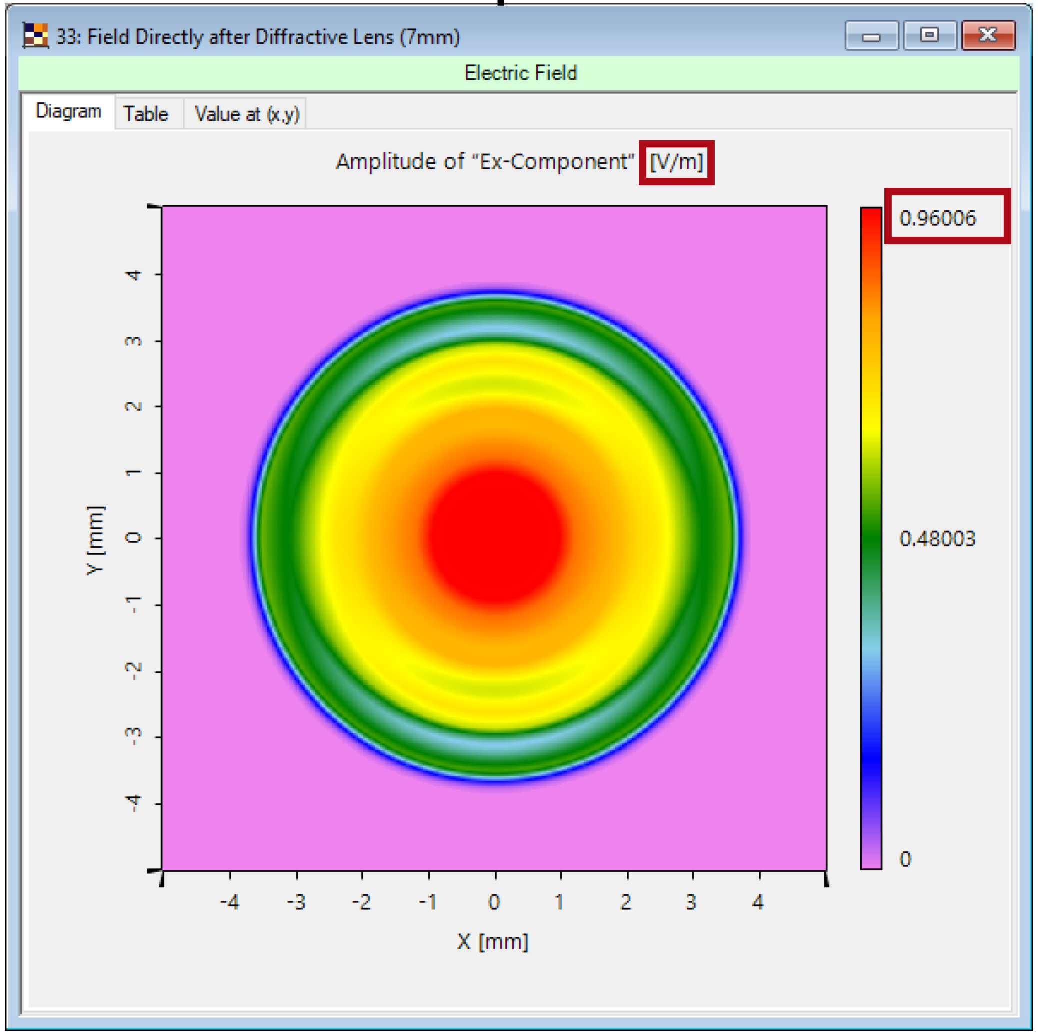
# Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing



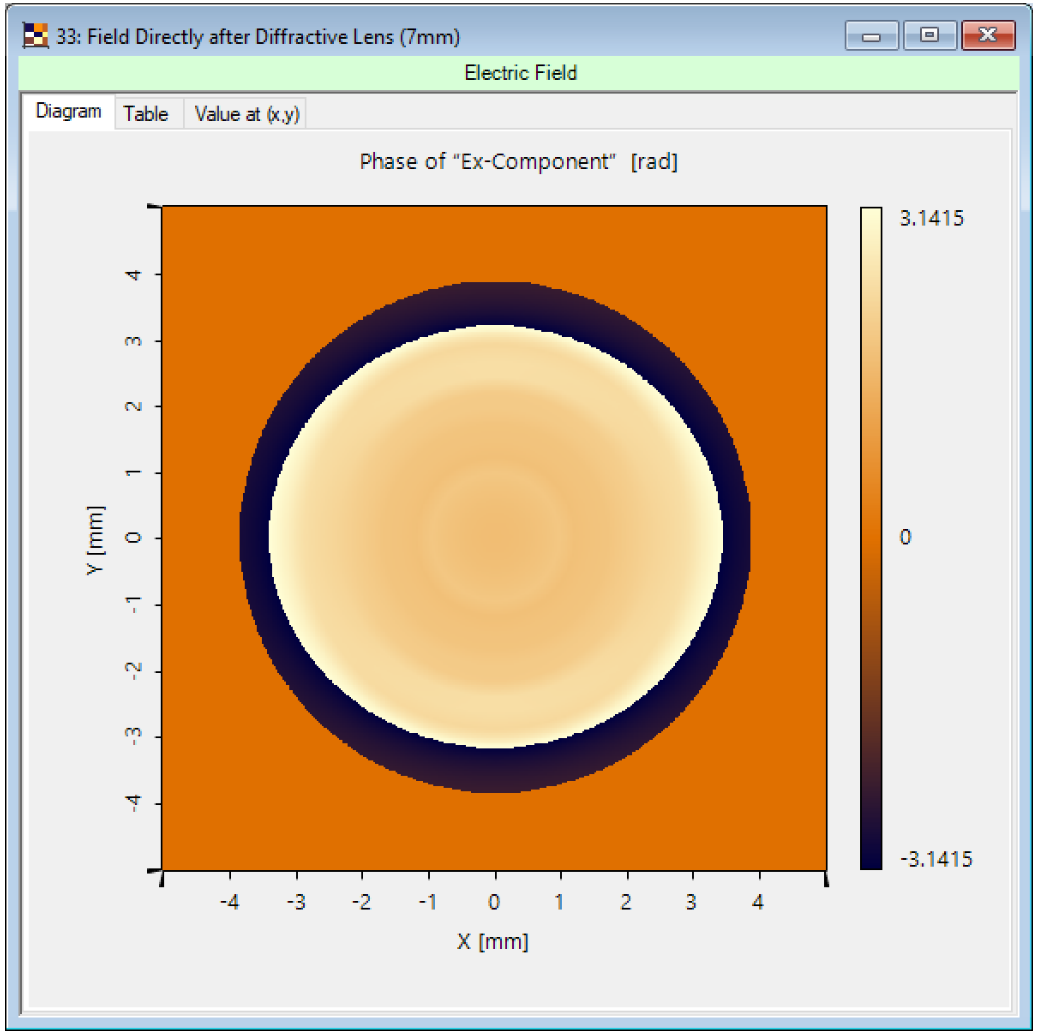
# Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing

Amplitude

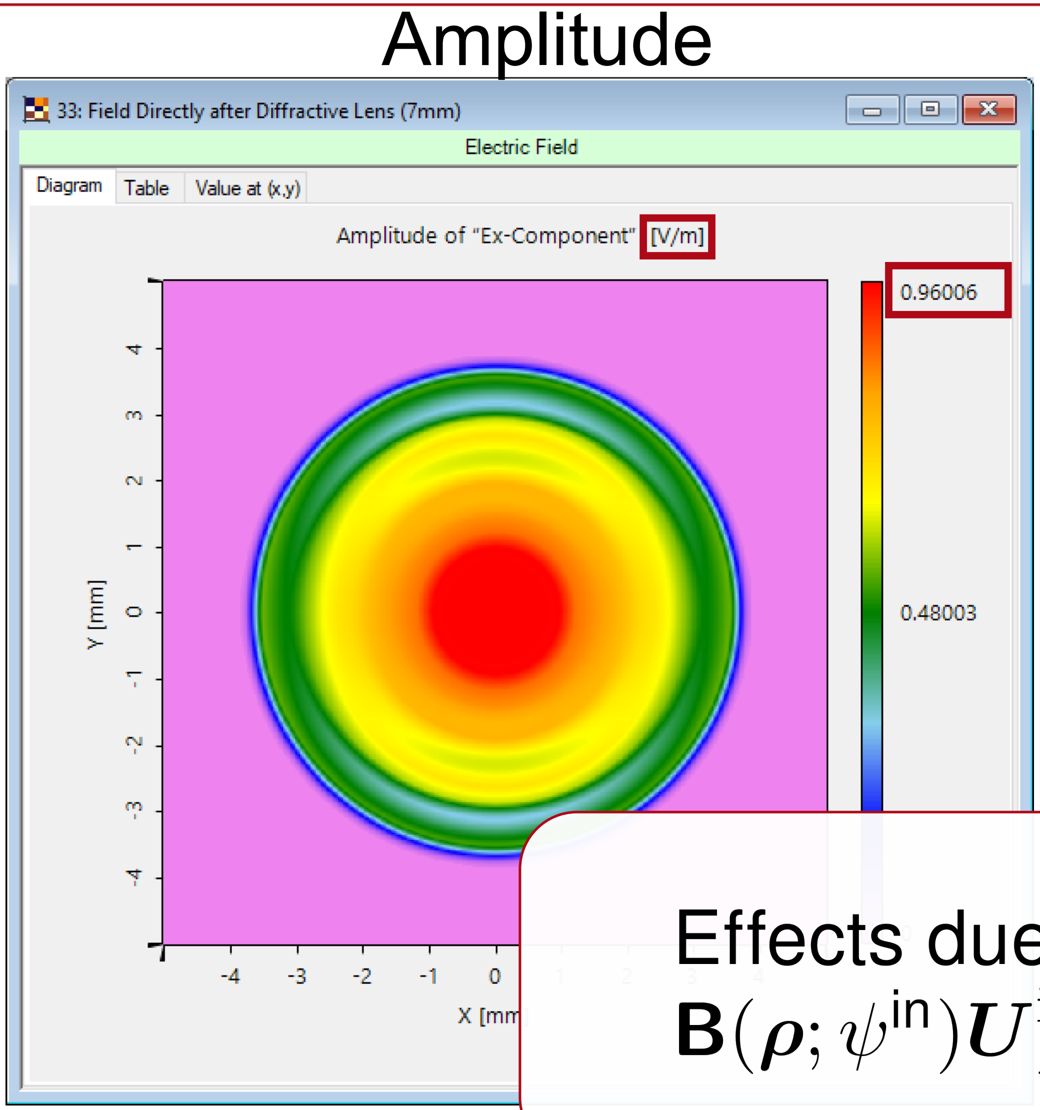



**Ex**

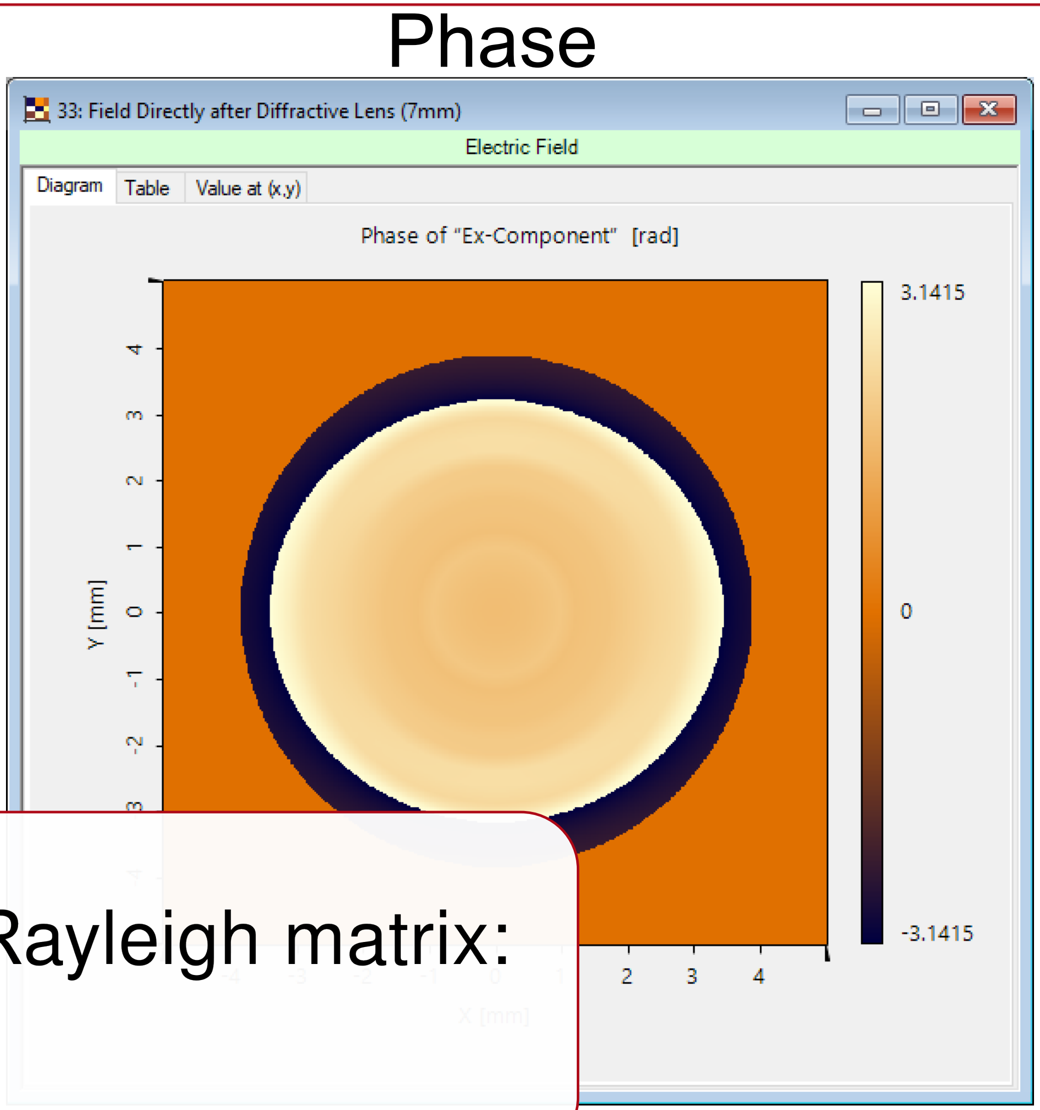
Phase



# Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing

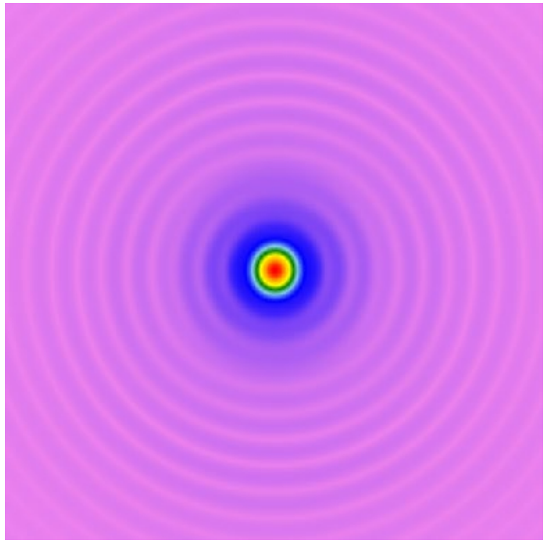



**Ex**



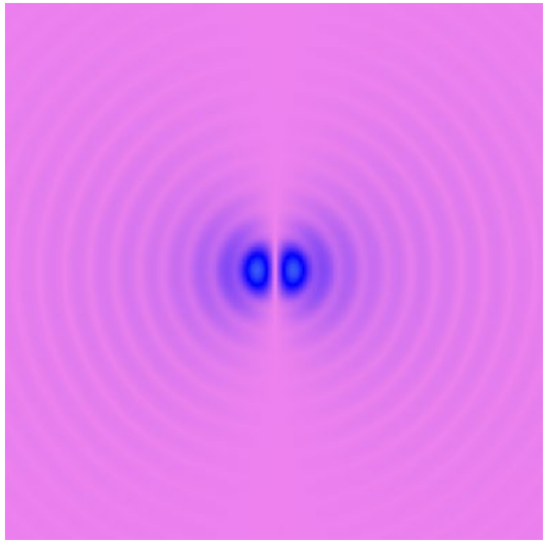

Effects due to Rayleigh matrix:
$\mathbf{B}(\boldsymbol{\rho};\psi^{\text{in}})\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})$

# Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing



Amplitude Ex

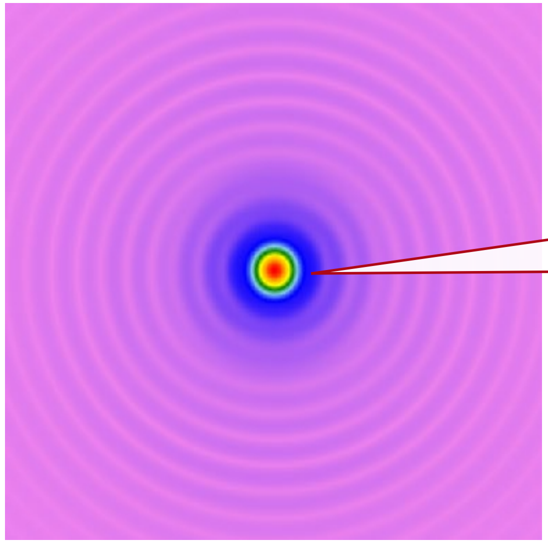
Amplitude Ey



Amplitude Ez

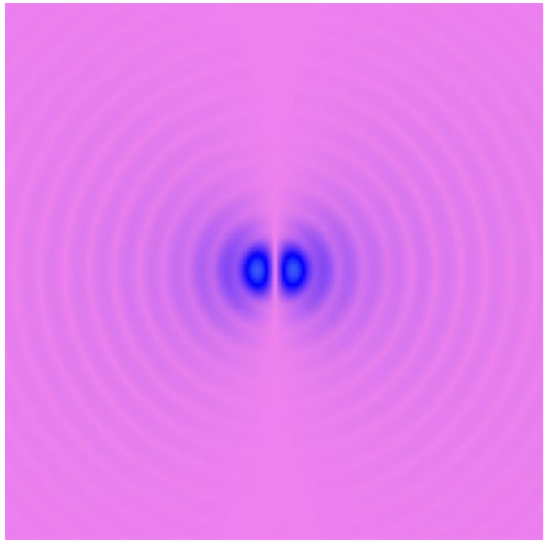
- Amplitudes in Focus (Same scaling!)

# Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing



Amplitude Ex

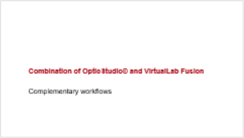
Amplitude Ey



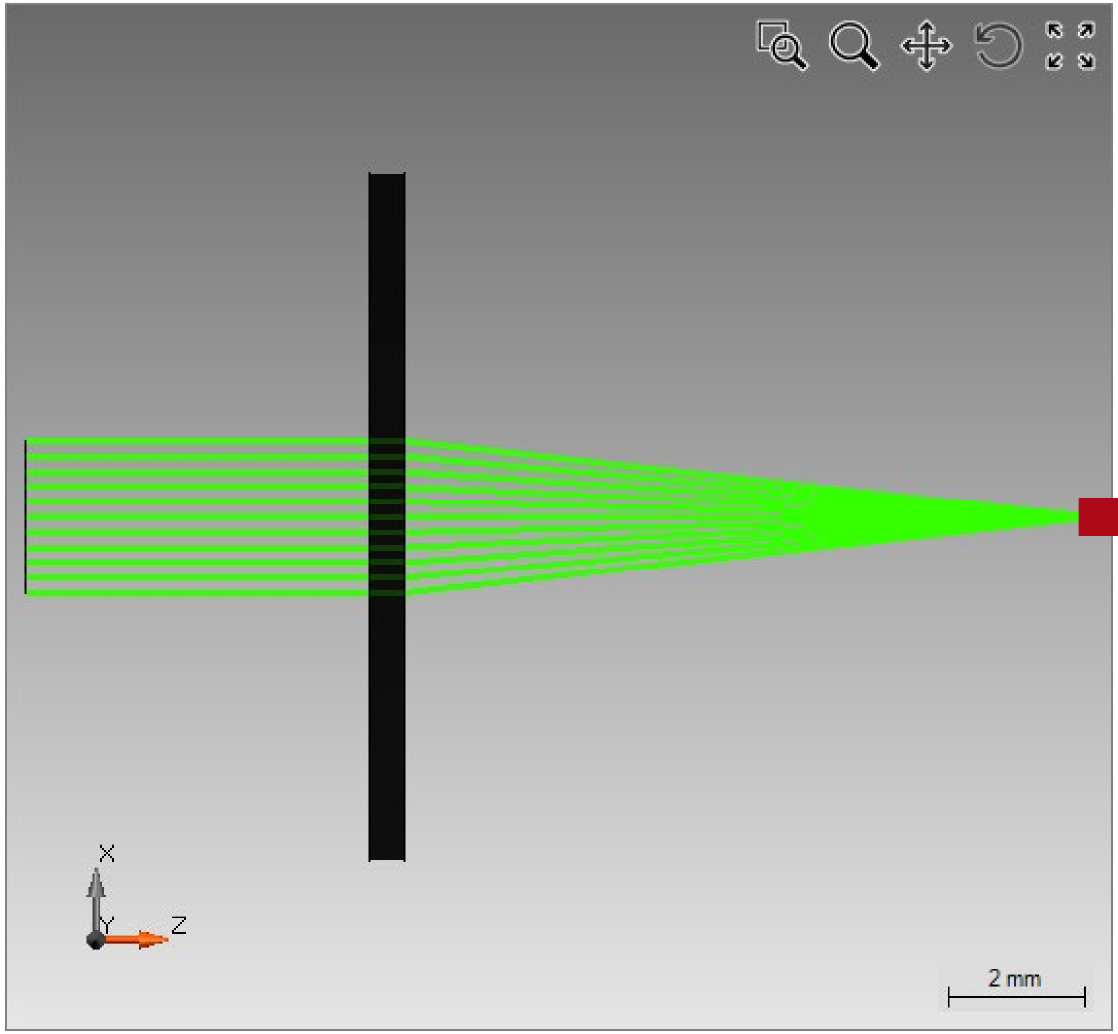
Amplitude Ez

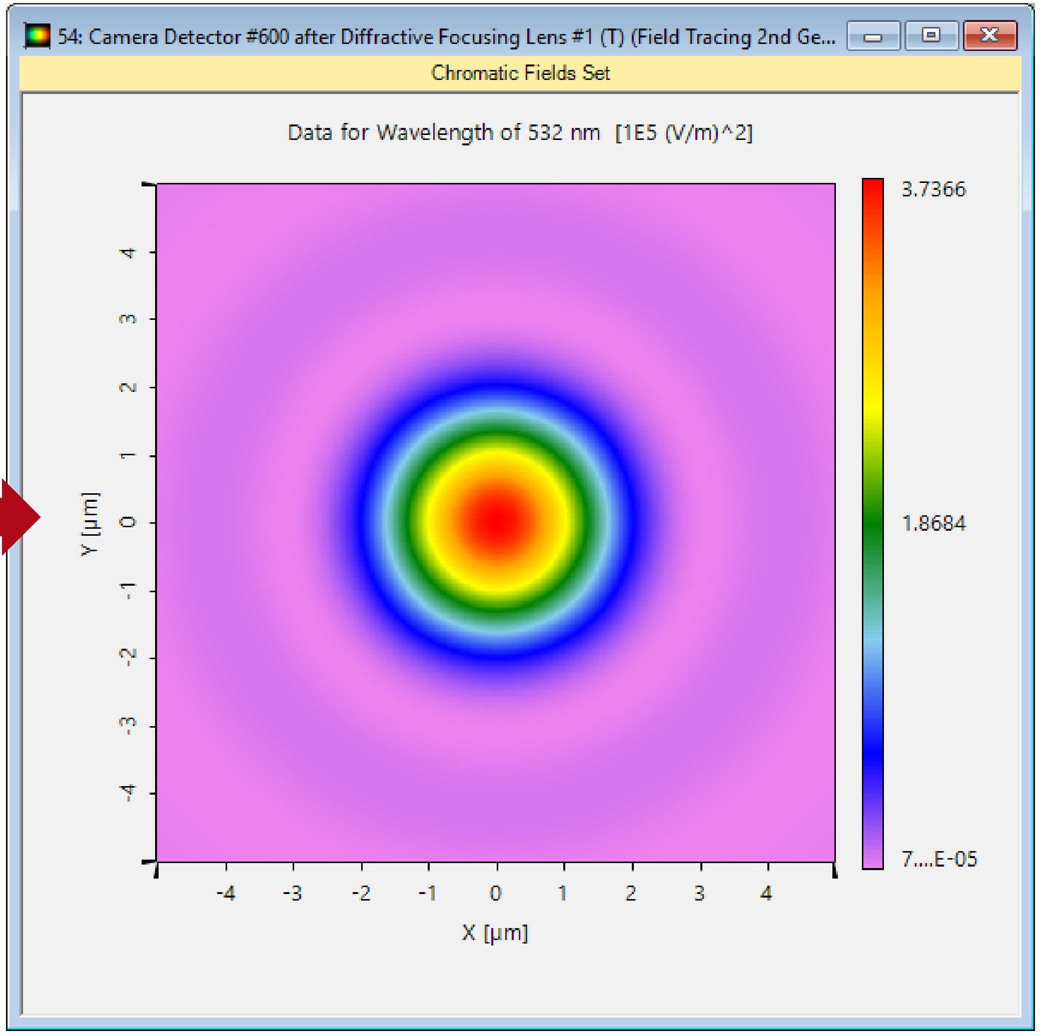
Vectorial grating effects reduce spot quality for high NA

- Amplitudes in Focus (Same scaling!)

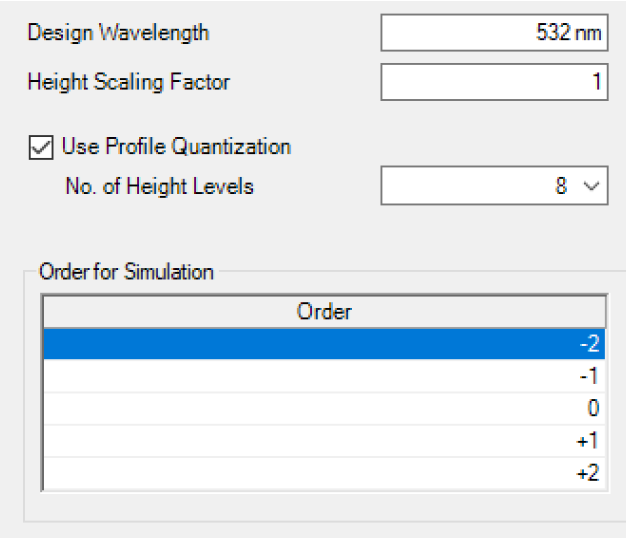


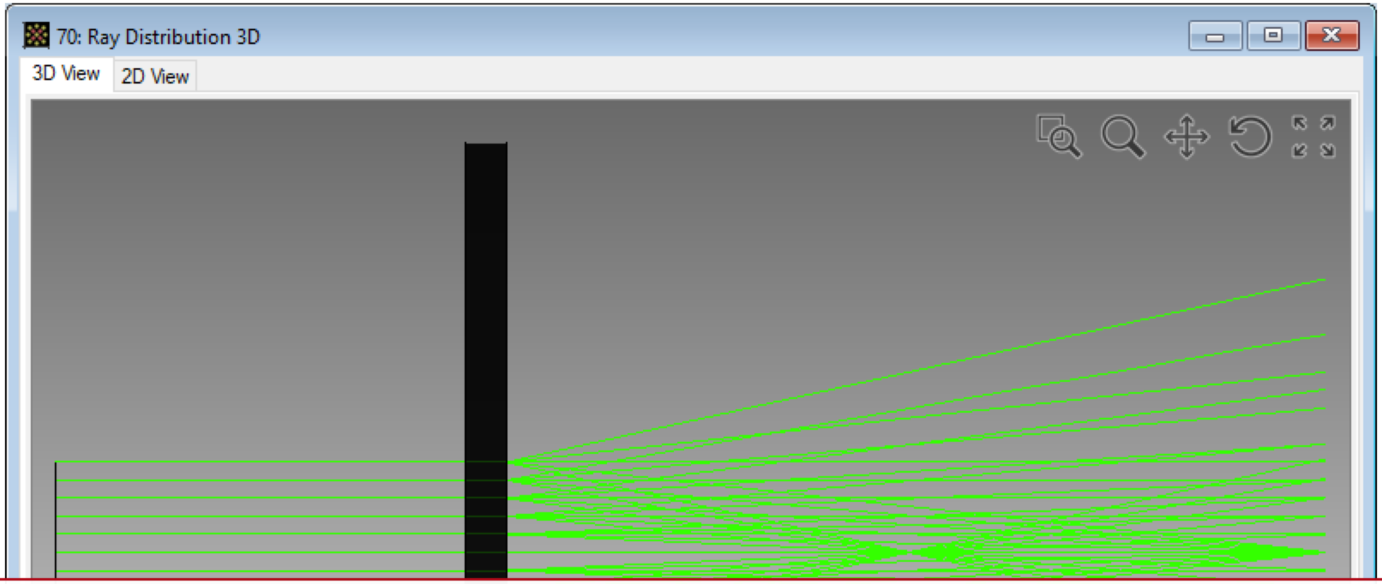
# Lens with NA=0.1 and 8 Height Values: Ray and Field Tracing



# Inclusion of Higher Orders

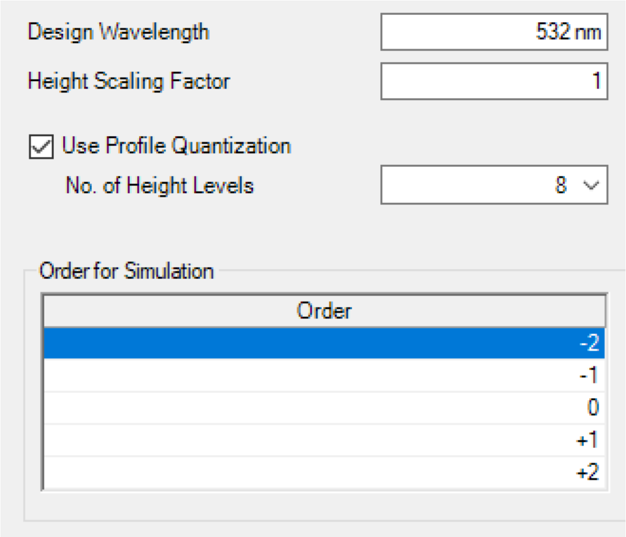


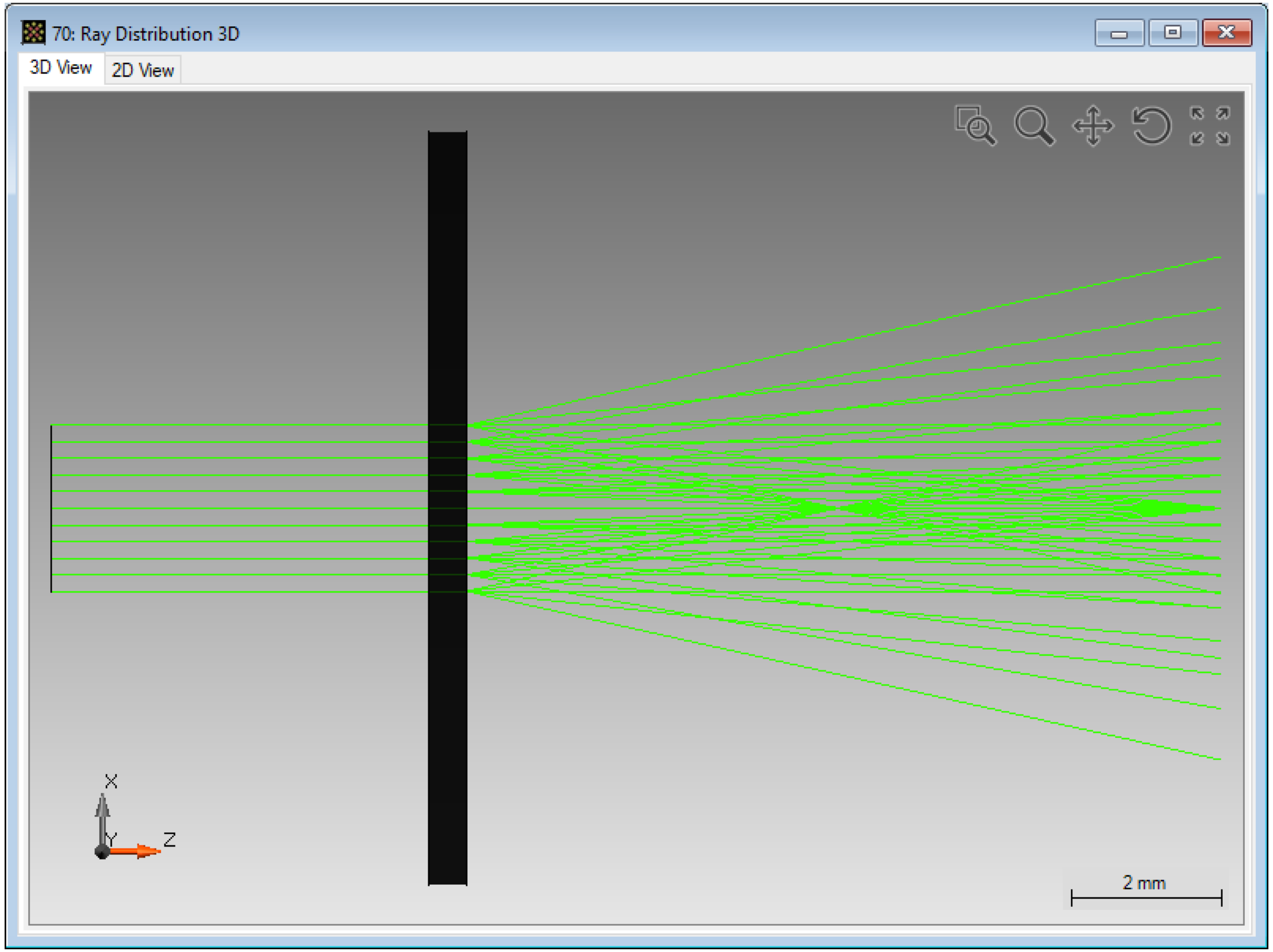




$$\boldsymbol{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \left\{\mathbf{B}^{(1)}(\boldsymbol{\rho};\psi^{\text{in}})\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})\right\}\exp\left(\mathrm{i}\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})\right)$$

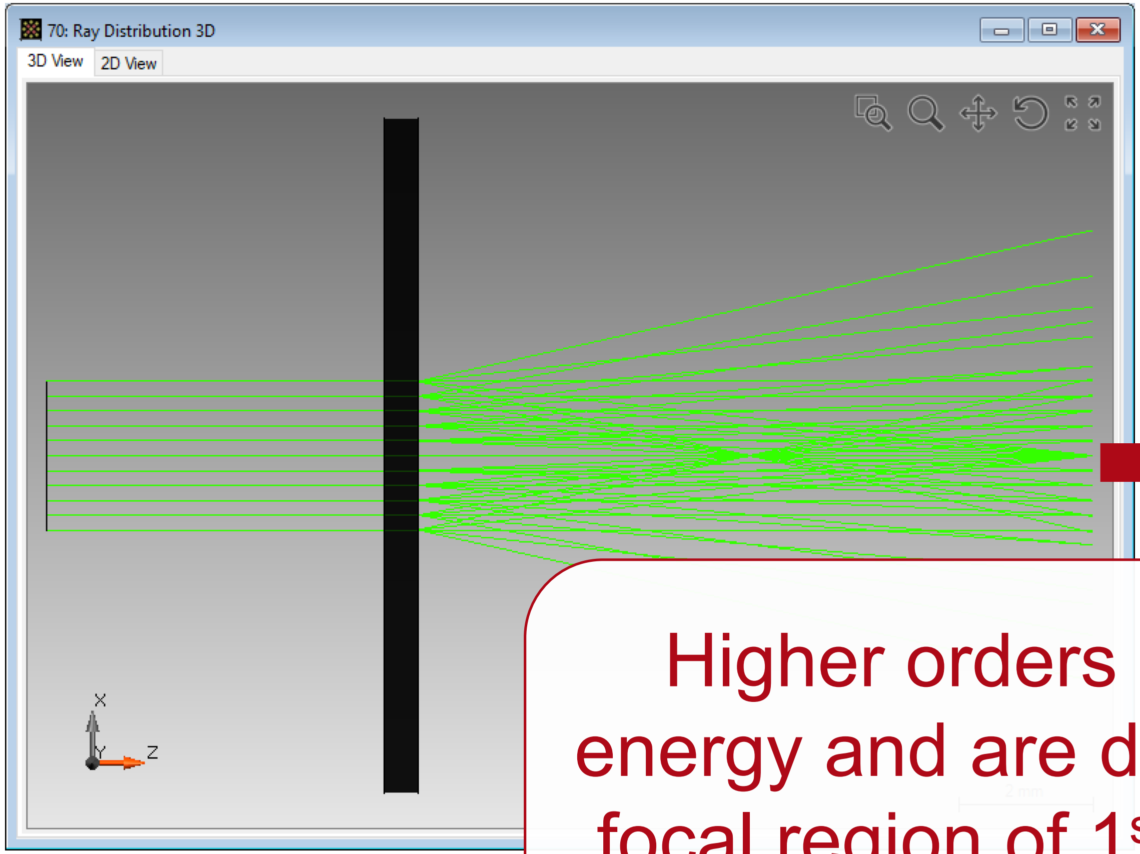
$$+ \sum_{m=-\infty, m\neq 1}^{\infty} \left\{\mathbf{B}^{(m)}(\boldsymbol{\rho};\psi^{\text{in}})\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})\right\}\exp\left(\mathrm{i}\psi^{\text{in}}(\boldsymbol{\rho}) + m\Delta\psi(\boldsymbol{\rho})\right)$$

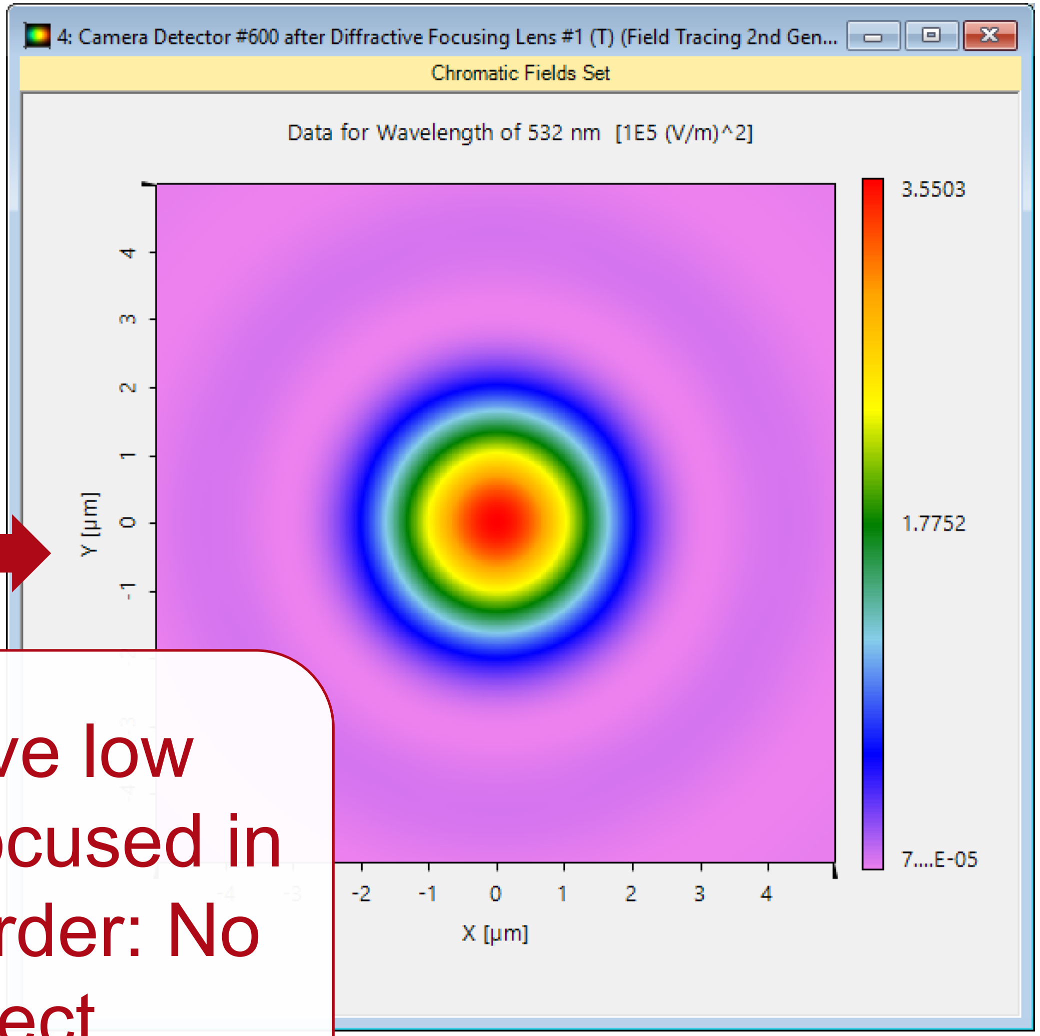
# Inclusion of Higher Orders



# Lens with NA=0.1 and 8 Height Values: Ray and Field Tracing



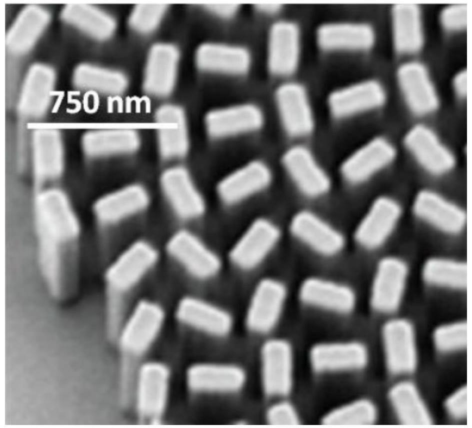

Higher orders have low energy and are defocused in focal region of 1st order: No detrimental effect

# Combination of OpticStudio® and VirtualLab Fusion

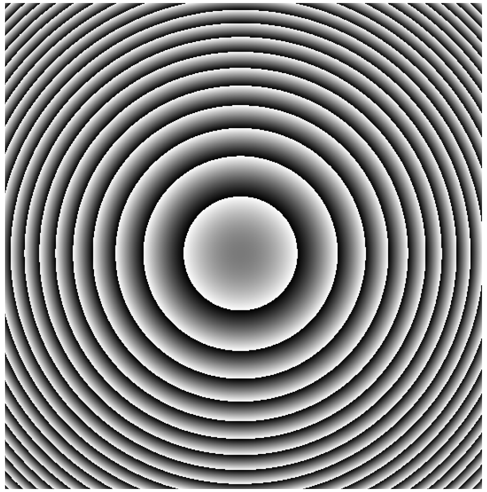
Complementary workflows

# Standard Workflow

- Design of system with optional inclusion of ideal Wavefront Surface Response (WSR) function.
- Decision about most suitable flat optics layer to realize WSR.
- Design of layer structure.
- Analysis of the performance and detrimental additional subfields.
- Tolerancing.
- Export of fabrication data.




M. Khorasaninejad *et al.*, Science **352**, 1190-1194 (2016).



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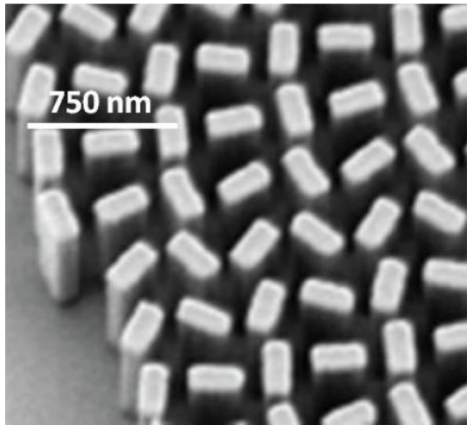
Combination with OpticStudio® via Binary Surfaces, which represent a special form of a WSR.

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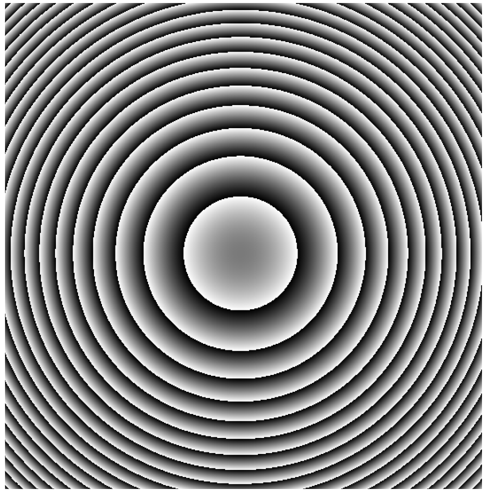
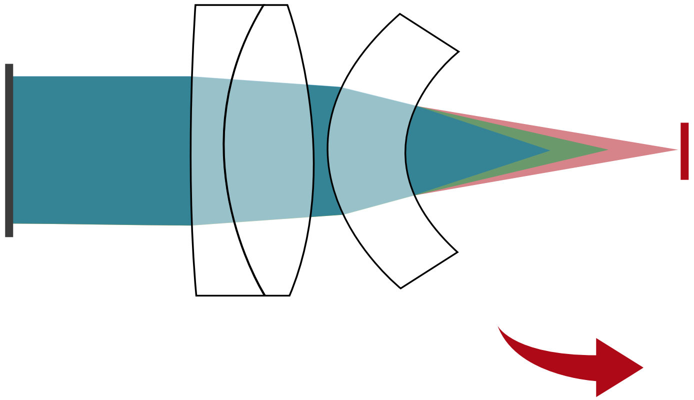
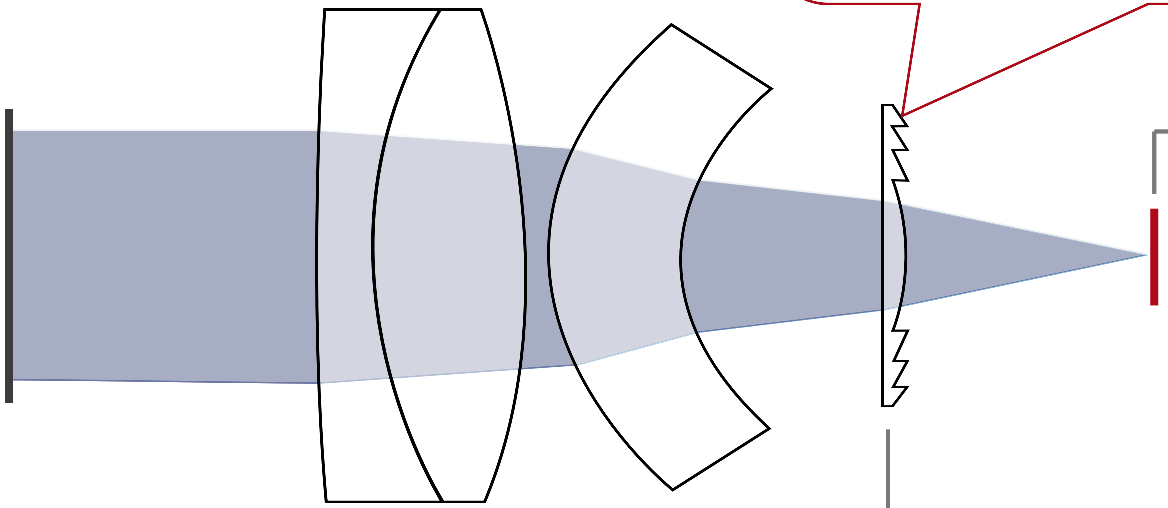


# Color Aberration Correction by DOE

Optimization of Binary Surface = Wavefront Surface Response in OpticStudio®





**detectors**

- point spread function
- modulation transfer function (MTF)

**plane wave**

- wavelength (486, 587, 656)nm
- field of view along y-axis (0; 20)°
- linearly polarized along x-axis
- aperture 5mm×5mm

**lens solution**

- wavefront response function
- diffractive lens
- meta lens

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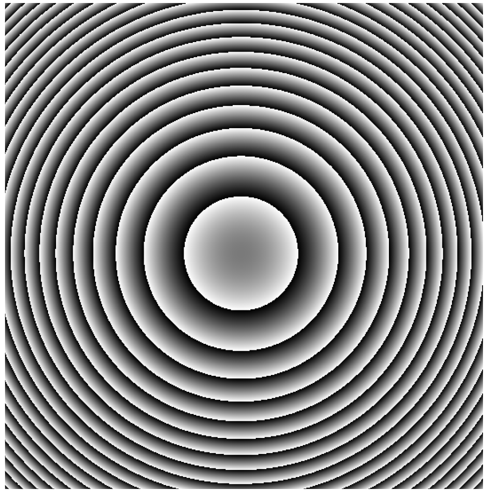

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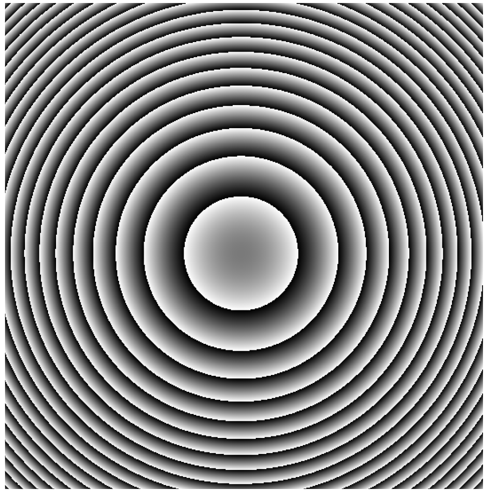

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- Decision about most suitable flat optics layer to realize WSR.
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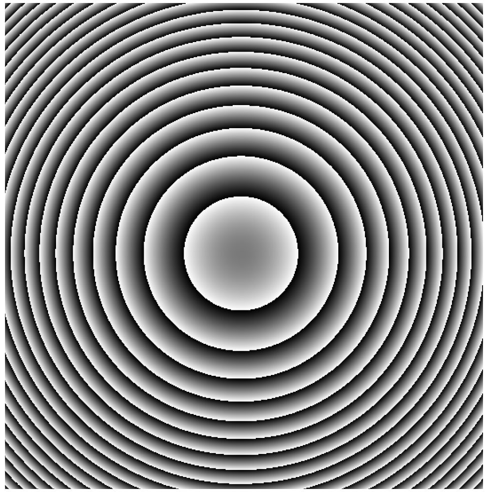
Import into VirtualLab Fusion and perform design.



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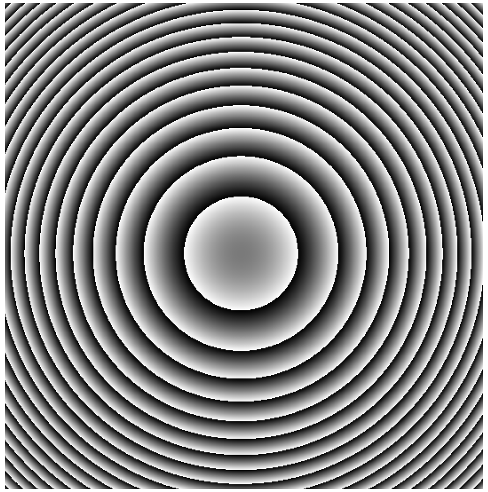
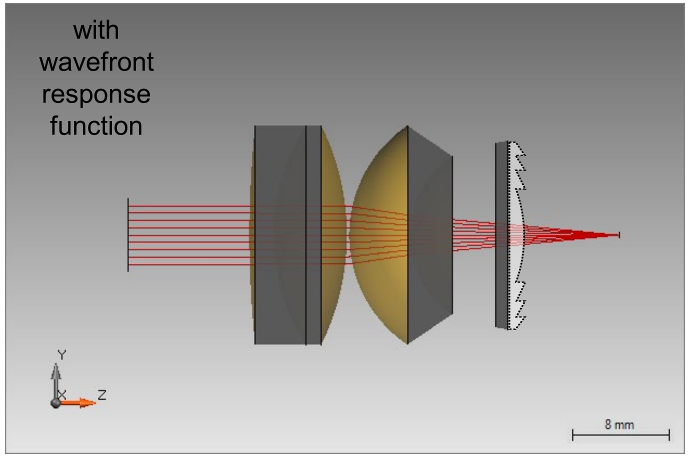
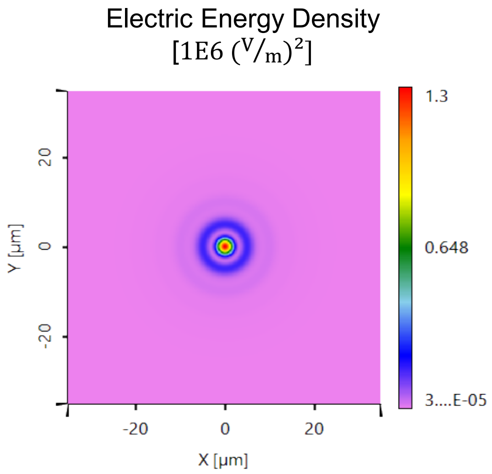
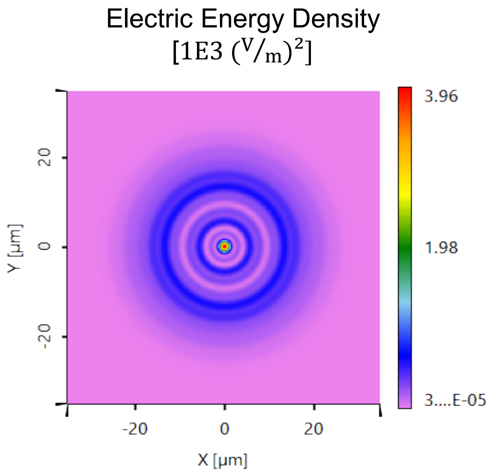
Analysis in VirtualLab Fusion.

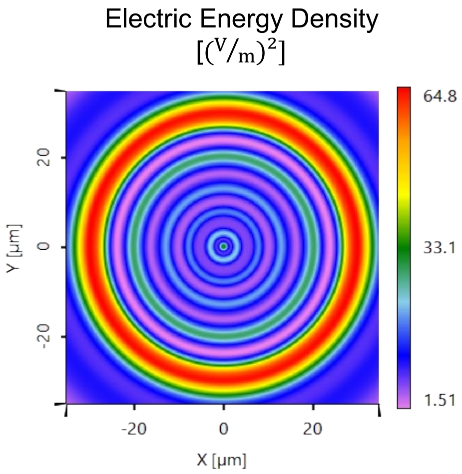


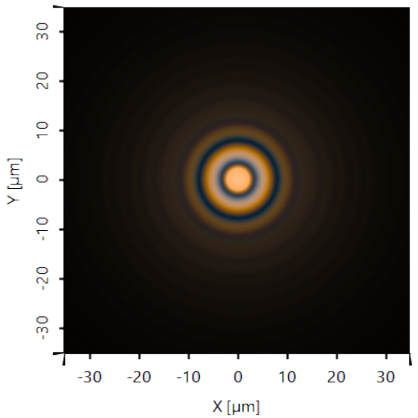
# Inclusion of Higher Orders: On-Axis



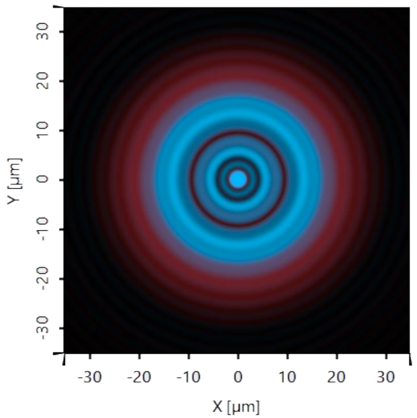



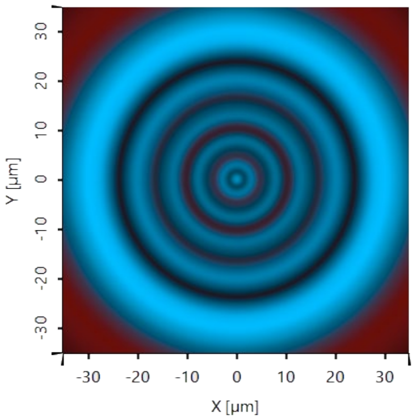




+1$^{st}$ diffraction order



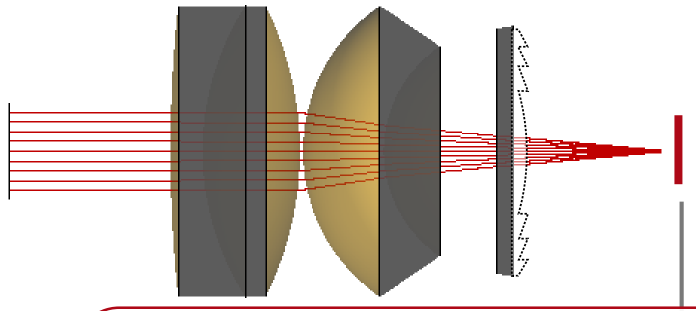

0$^{th}$ diffraction order

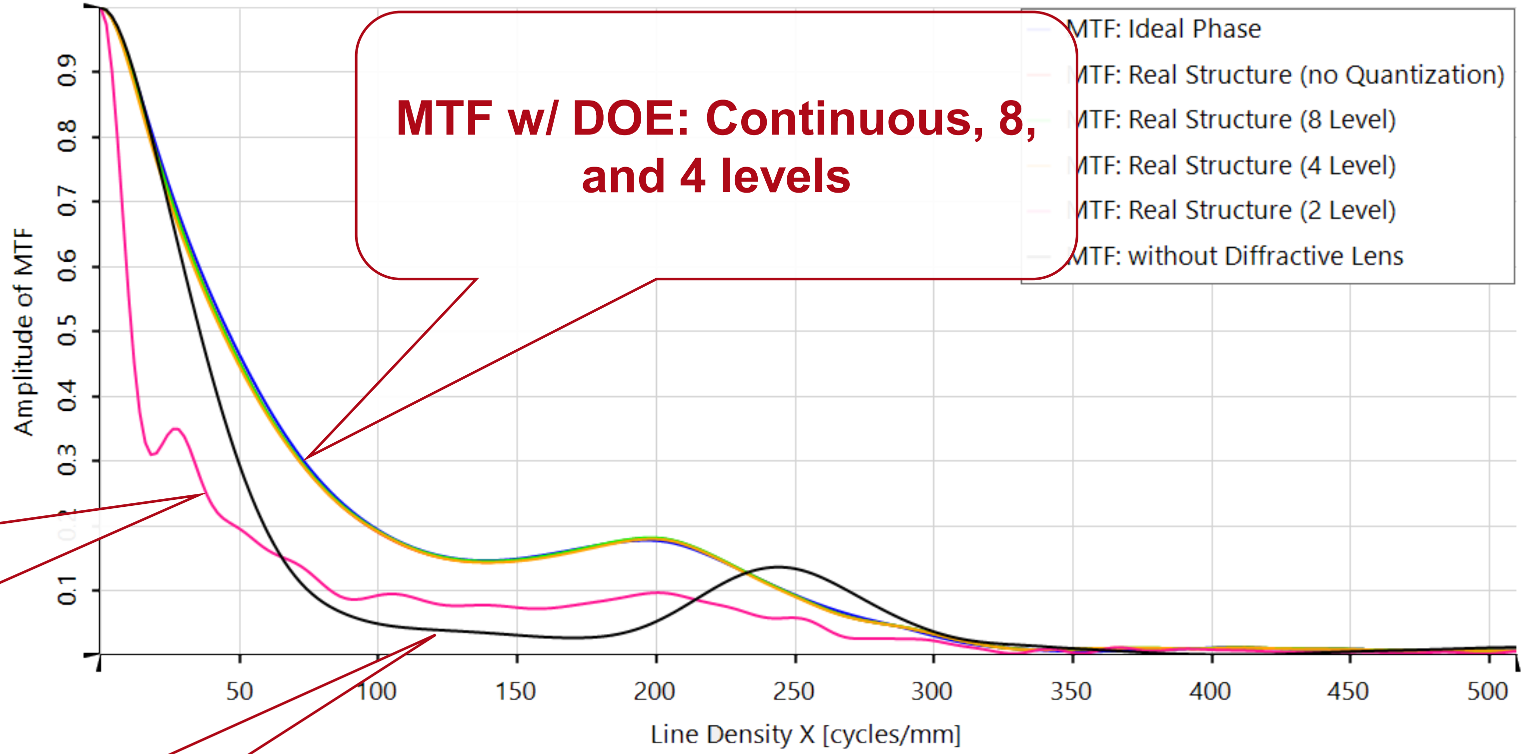



-1$^{st}$ diffraction order

simulation time per order ~seconds

# Results: MTF for Various Diffractive Lens Structures






**MTF w/ DOE: Continuous, 8, and 4 levels**

**MTF w/ DOE: Binary**

**MTF w/o DOE**

# VirtualLab Meta- and Diffractive Surface Solutions

- We prepare a new VirtualLab product for design and modeling meta- and diffractive surfaces to be released in 2019.
- It will be based on the theory presented in this talk. The examples shown in this talk will be included as special Use Cases together with suitable workflows.