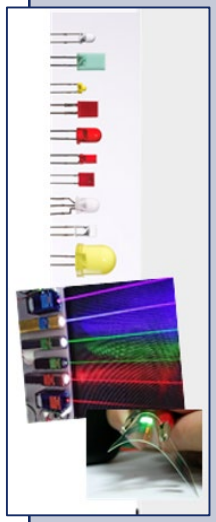
FRIEDRICH-SCHILLER-
UNIVERSITÄT
JENA

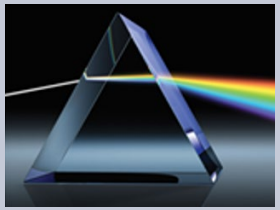
EOS Topical Meeting on Diffractive Optics, September 2019, Jena, Germany

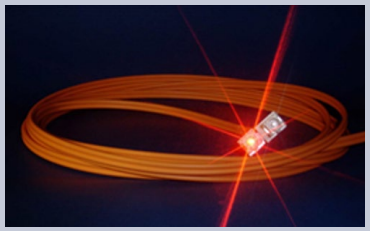
On the importance of homeomorphic operations in physical and geometrical optics

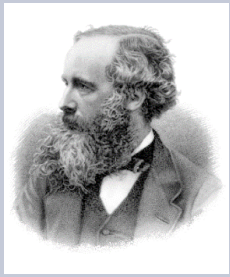
F. Wyrowski*, O. Baladron-Zorita*, Z. Wang*, C. Hellmann**
*University of Jena, **Wyrowski Photonics

Physical-Optics System Modeling









Name	Integral equations	Differential equations
Gauss's law	$\oiint_{\partial\Omega} \mathbf{E}\cdot\mathrm{d}\mathbf{S} = \frac{1}{\varepsilon_0}\iiint_{\Omega}\rho\,\mathrm{d}V$	$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B}\cdot\mathrm{d}\mathbf{S} = 0$	$\nabla\cdot\mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E}\cdot\mathrm{d}\boldsymbol{\ell} = -\frac{d}{dt}\iint_{\Sigma}\mathbf{B}\cdot\mathrm{d}\mathbf{S}$	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$
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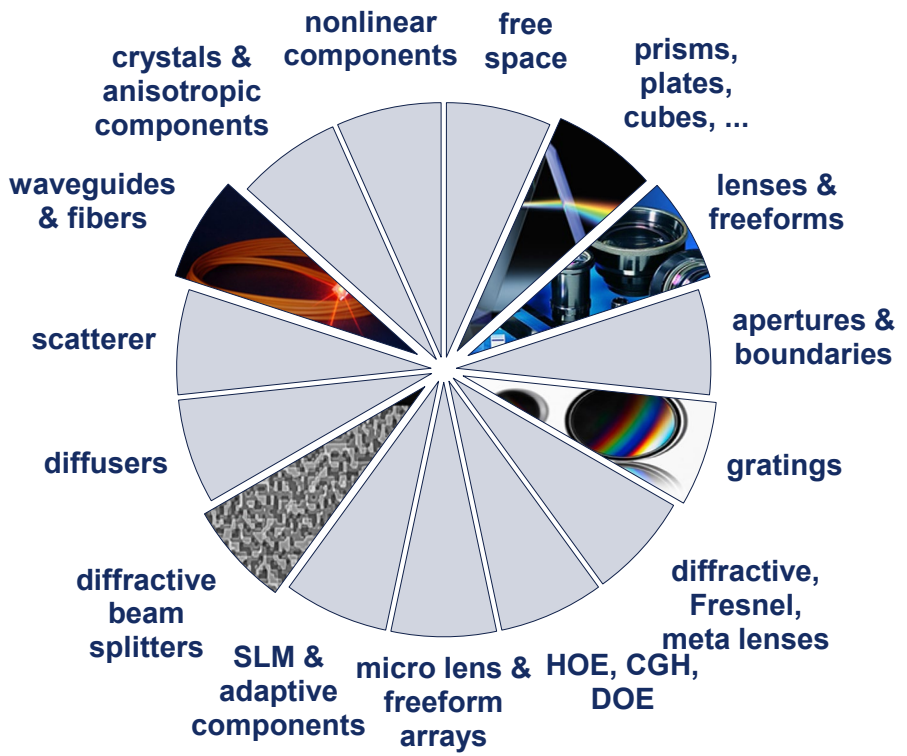
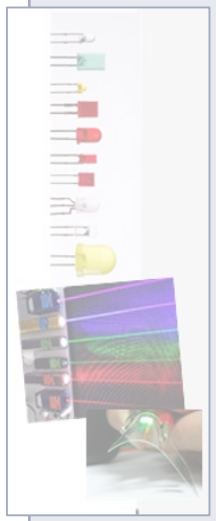
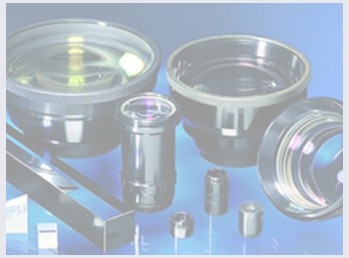
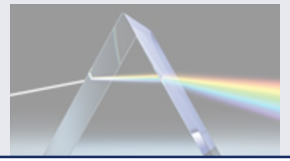
➡ Field Solver

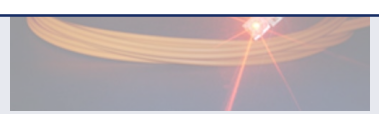


Physical-Optics System Modeling

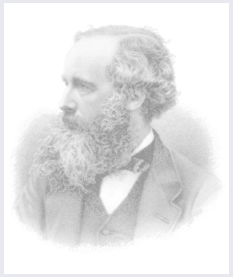








Application of a single field solver, e.g. FDTD, is not a realistic approach in system modeling.



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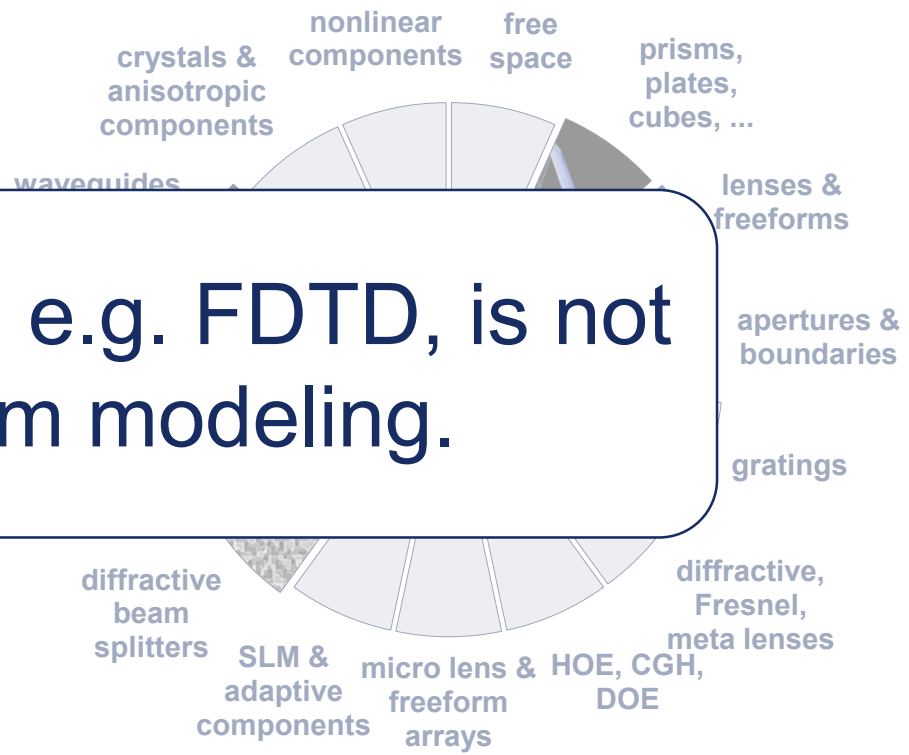

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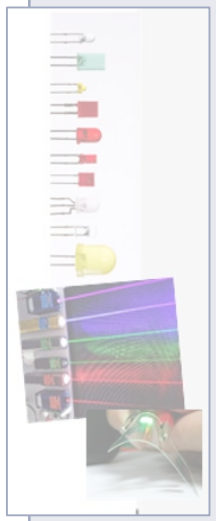

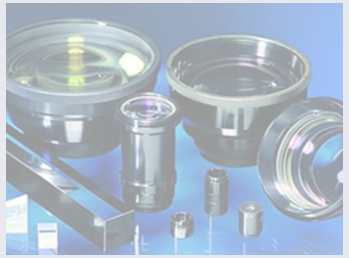

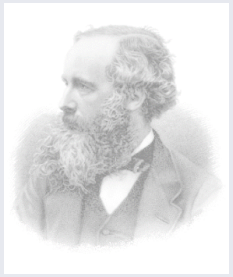

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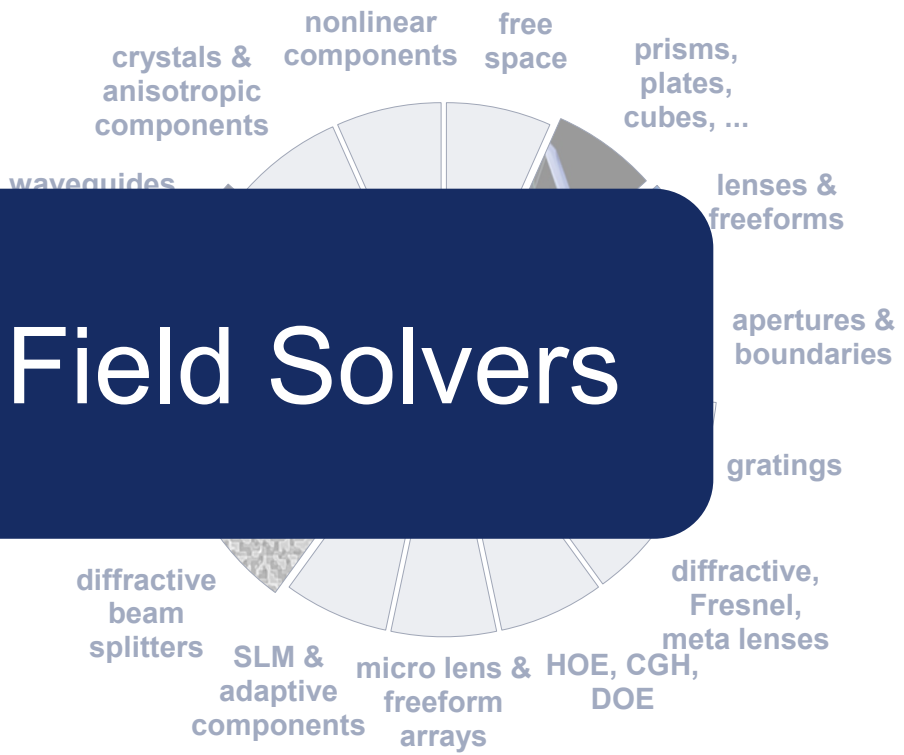
Field Solver

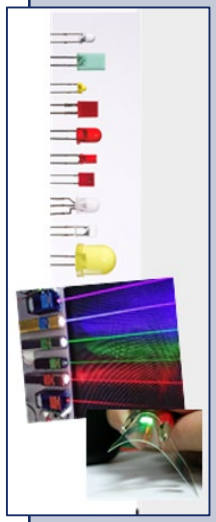


Field Tracing: Connecting Field Solvers

Field Tracing: Connecting Field Solvers

1. Usage of numerically most efficient field solver per component/region

Field Tracing: Connecting Field Solvers

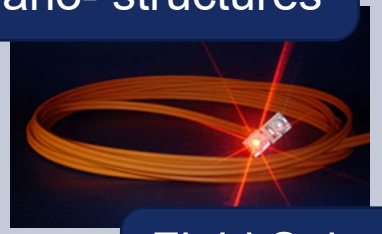


Field Solver Lenses

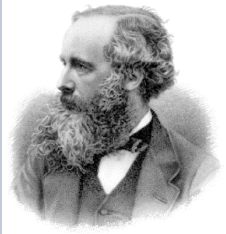
Field Solver Prisms

Field Solver Micro- and Nano- structures

Field Solver Gratings



Field Solver Fibers



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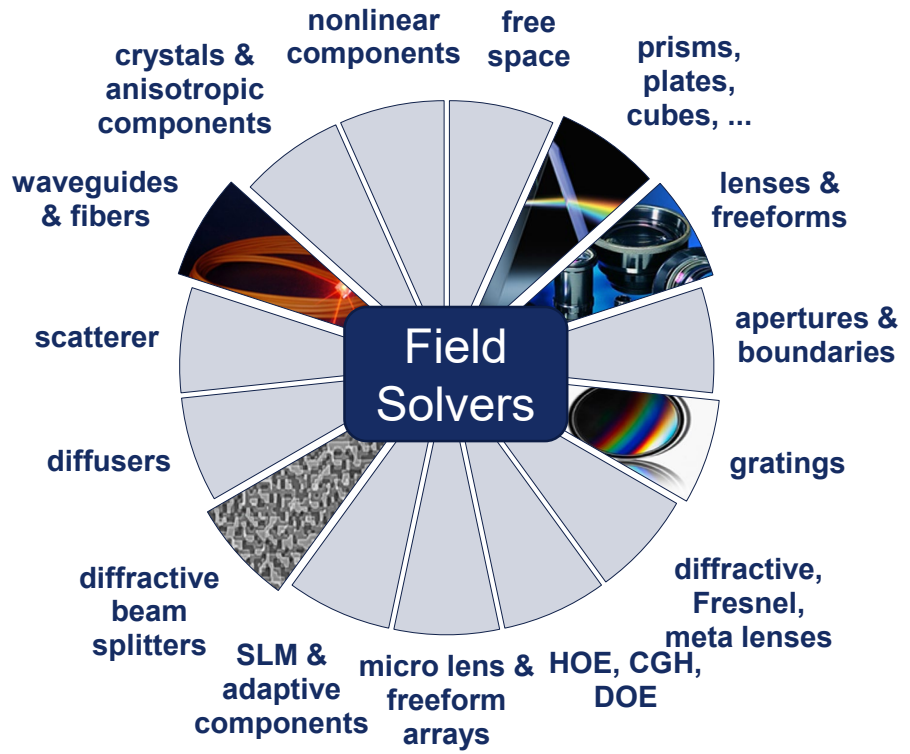
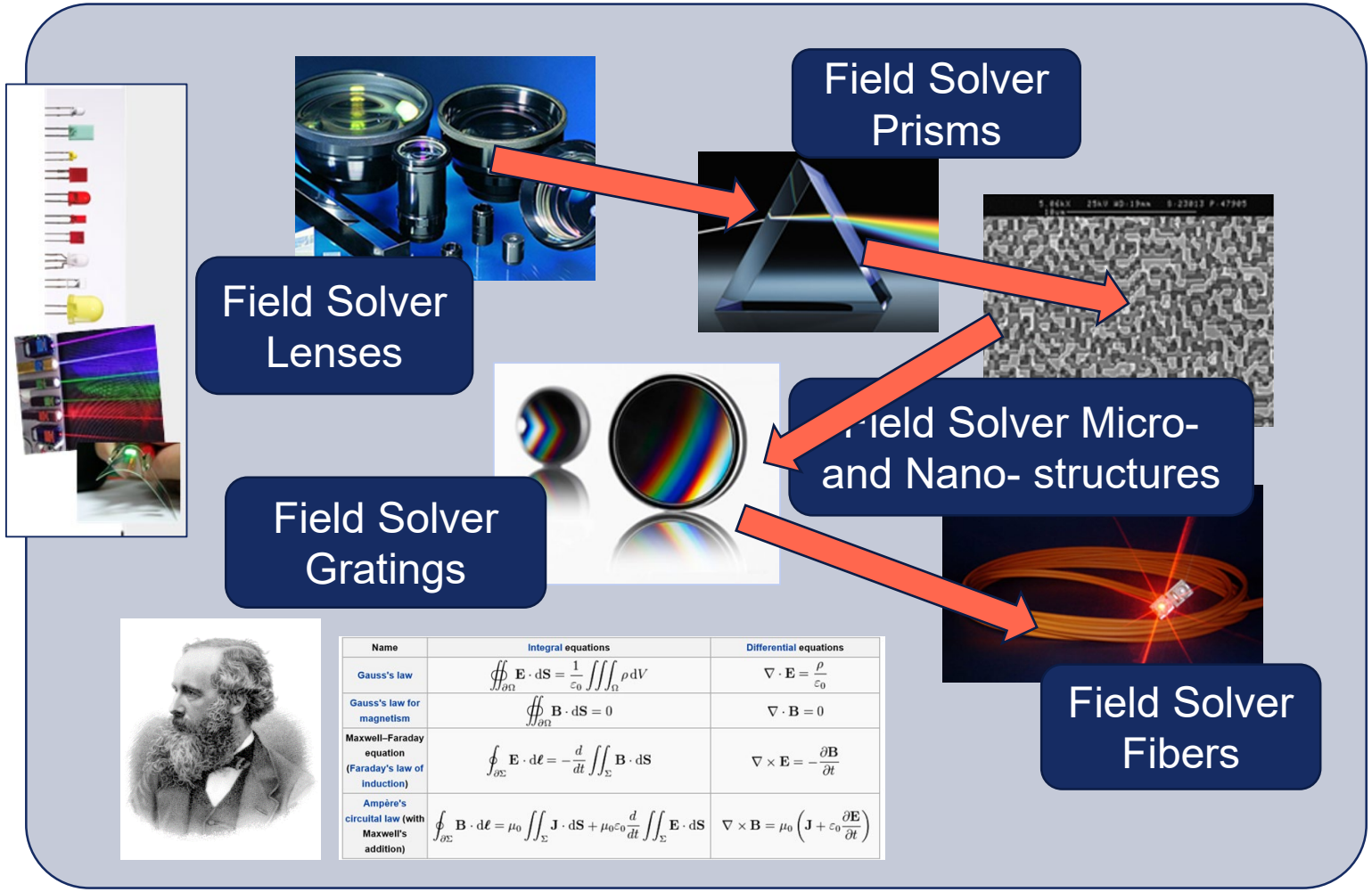


Field Tracing: Connecting Field Solvers

1. Usage of numerically most efficient field solver per component/region
2. Non-sequential connection of regional solvers

Field Tracing: Sequential



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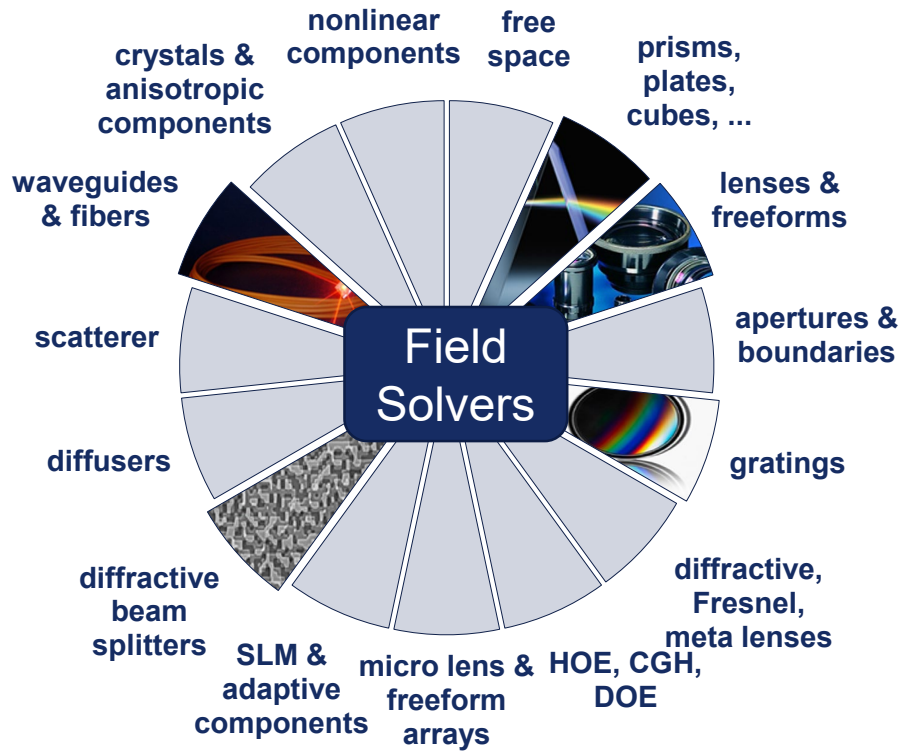
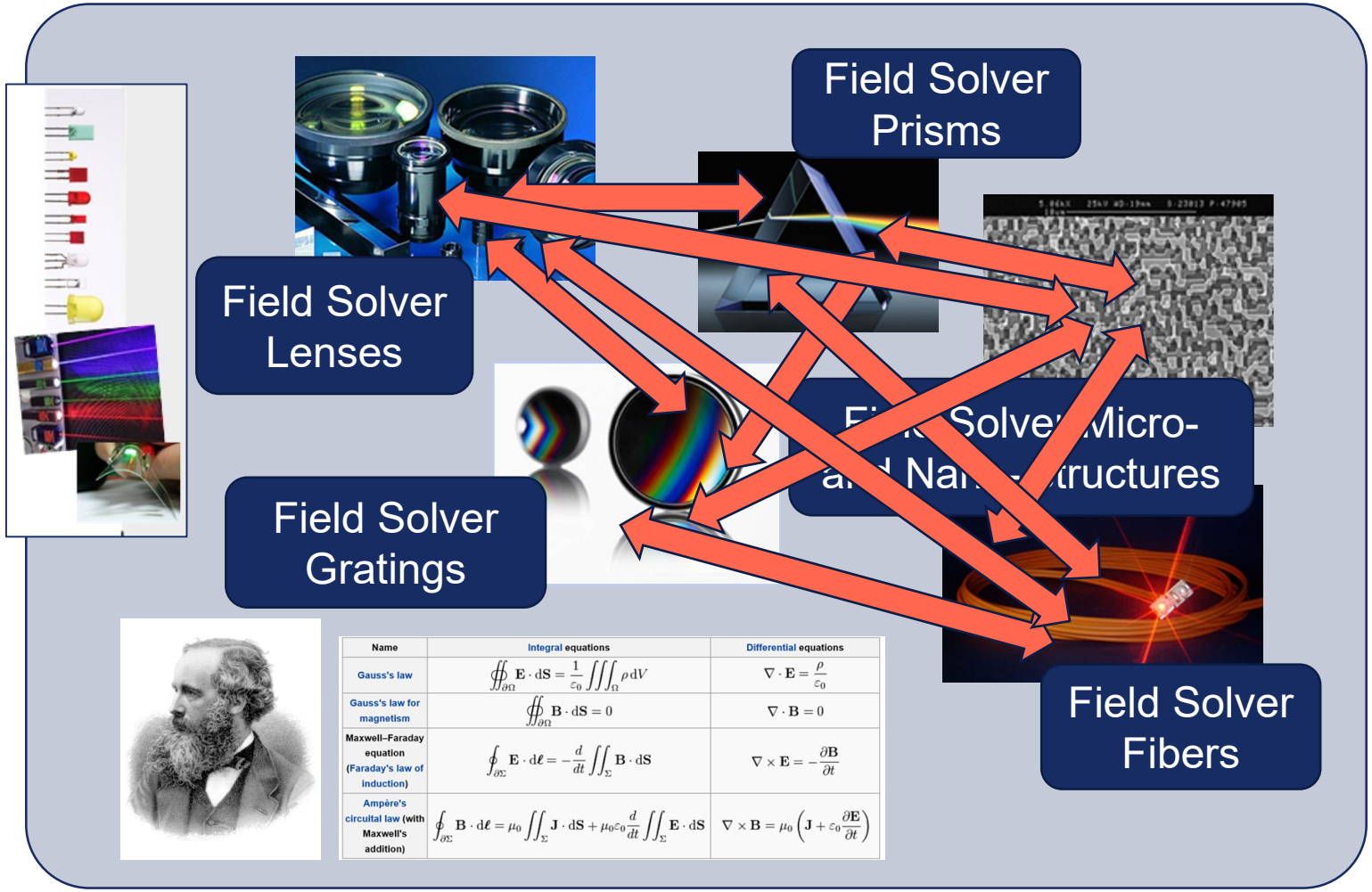


Field Tracing: Non-Sequential



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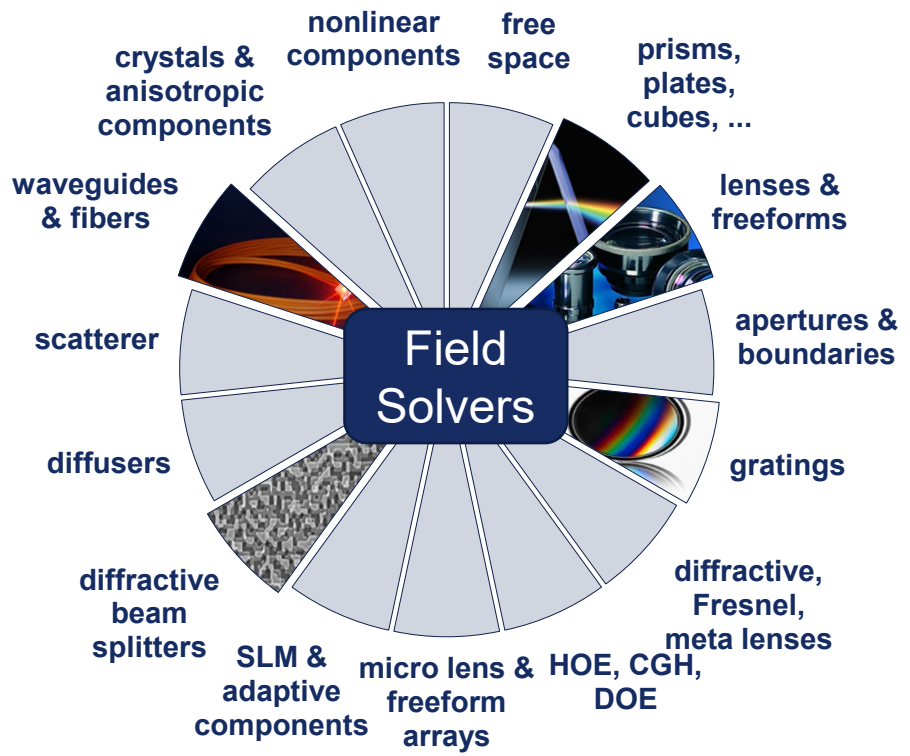


Field Tracing: Connecting Field Solvers

1. Usage of numerically most efficient field solver per component/region

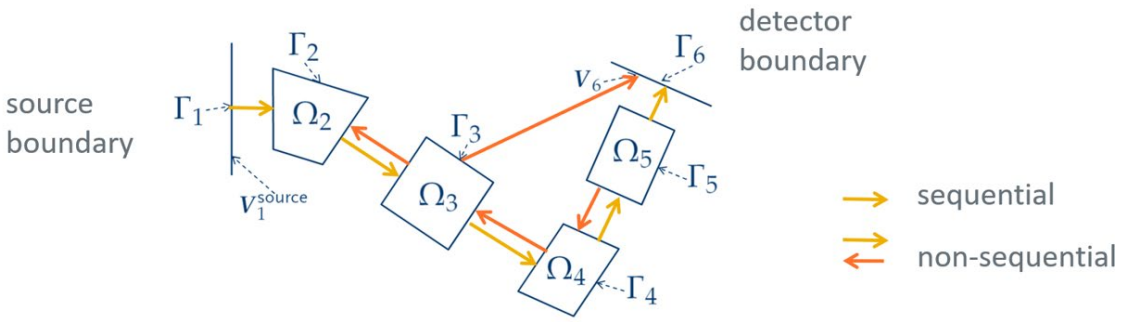
2. Non-sequential connection of regional solvers: **system solver**

2013

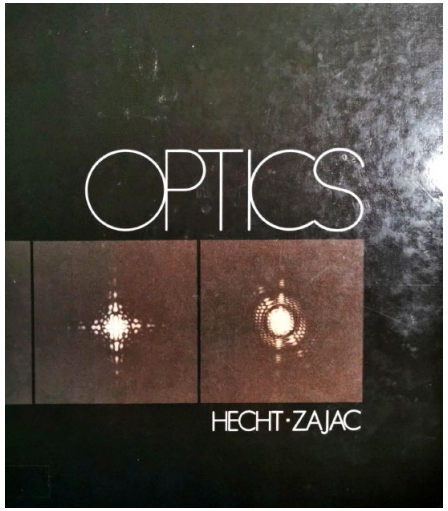
Non-Sequential Optical Field Tracing

Michael Kuhn, Frank Wyrowski, and Christian Hellmann

Kuhn, M.; Wyrowski, F. & Hellmann, C. (2013), Non-sequential optical field tracing, *in* T. Apel & O. Steinbach, ed., 'Finite Element Methods and Applications', Springer-Verlag, Berlin, , pp. 257-274.



Example of Plate/Etalon



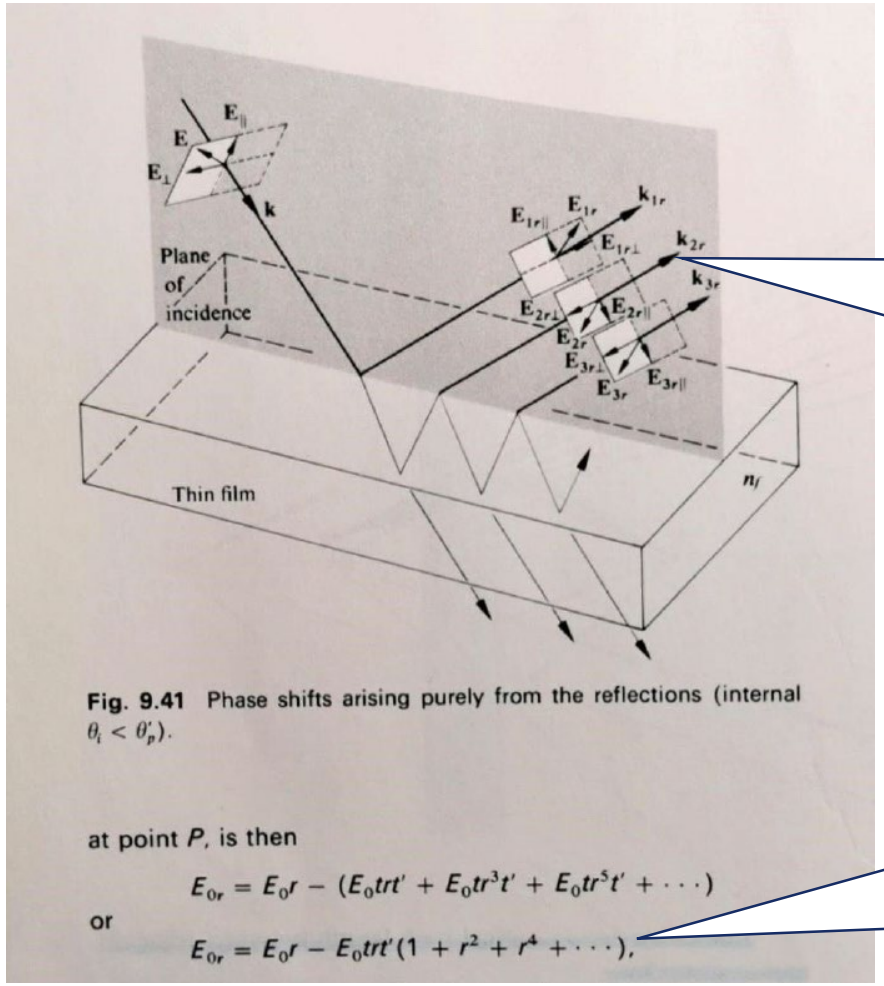


Fig. 9.41 Phase shifts arising purely from the reflections (internal $\theta_i < \theta_p'$).

at point P, is then

$$E_{0r} = E_0 r - (E_0 trt' + E_0 tr^3 t' + E_0 tr^5 t' + \cdots)$$

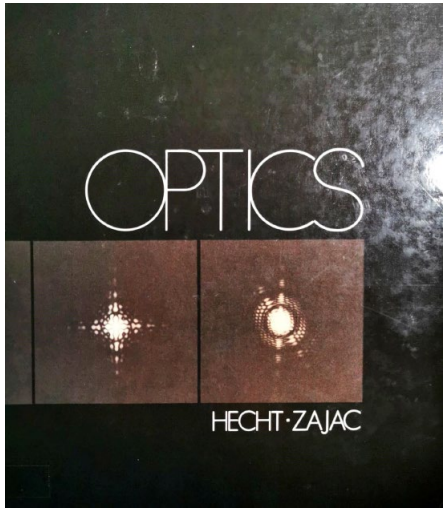
or

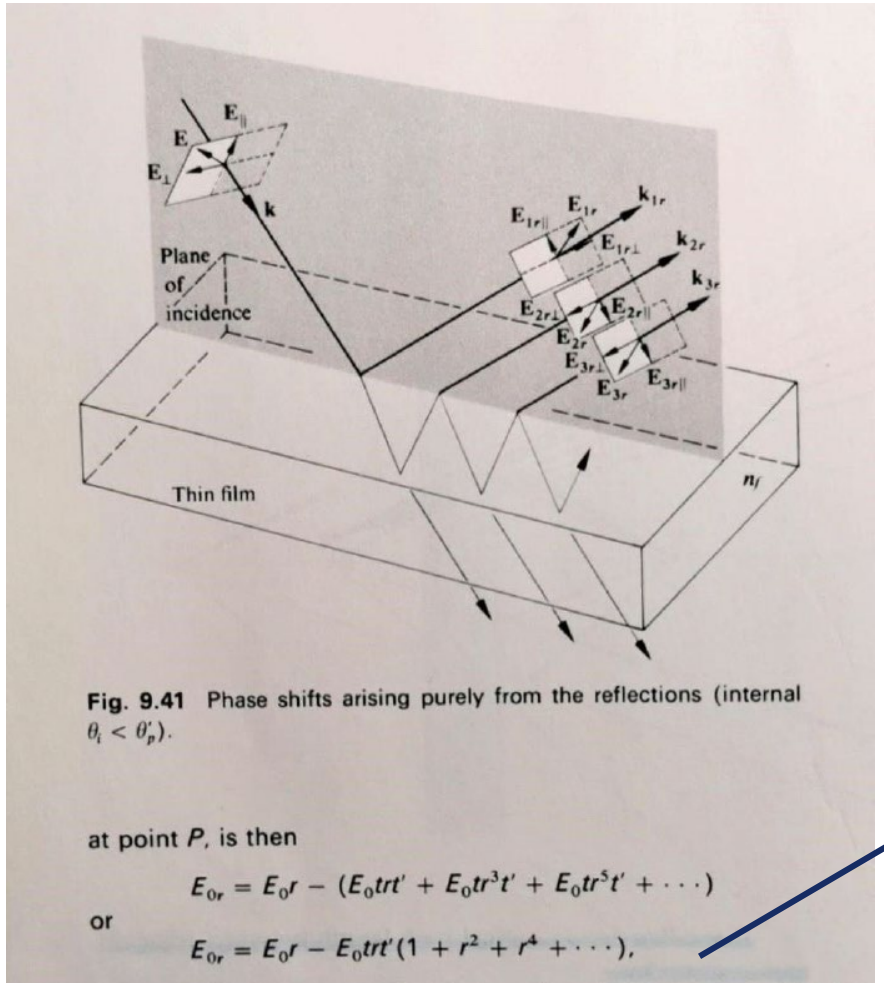
$$E_{0r} = E_0 r - E_0 trt'(1 + r^2 + r^4 + \cdots),$$

Different lightpaths because of non-sequential model

Sum of mutually coherent fields per lightpath

Example of Plate/Etalon





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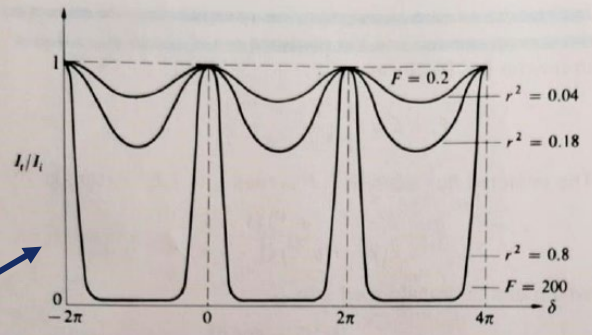


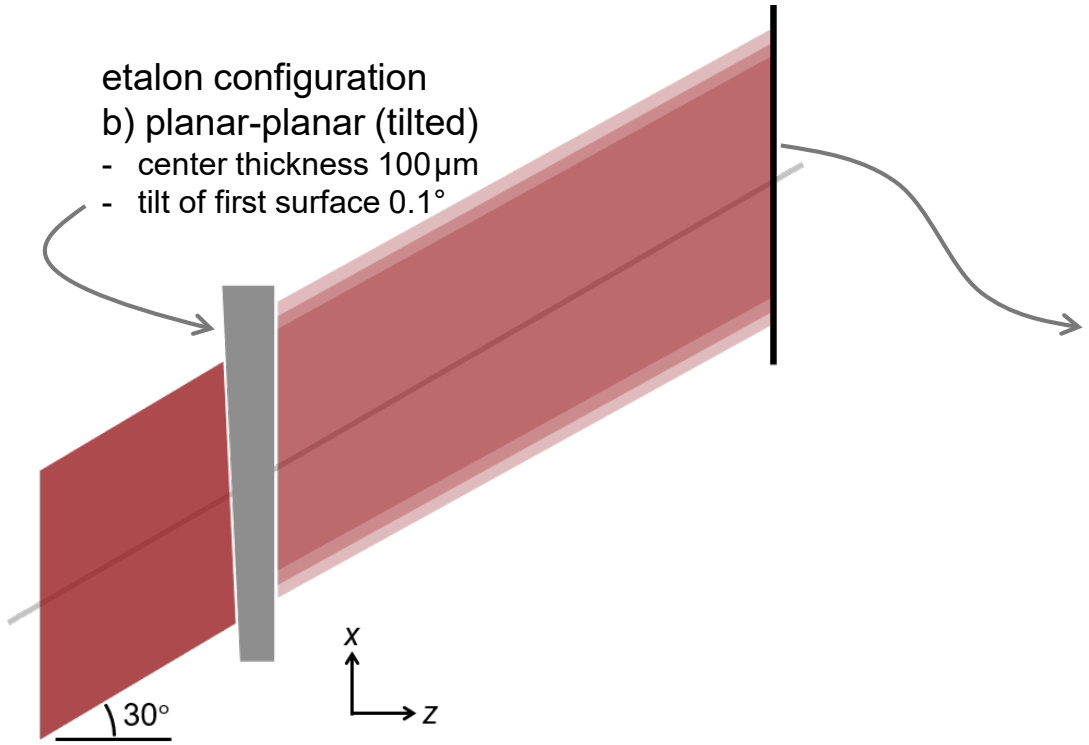
Fig. 9.45 Airy function.

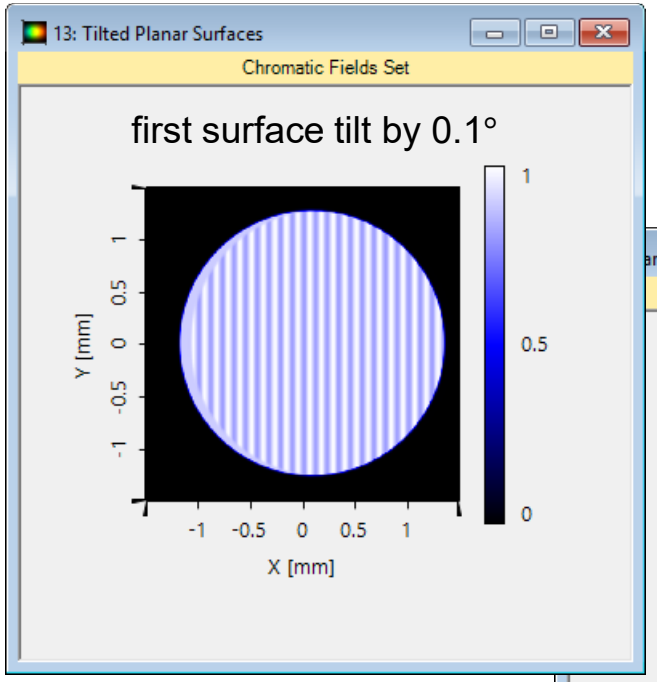
density emerging from the film. It follows from Eqs. (9.33) and (9.34), that this is indeed the case, that is

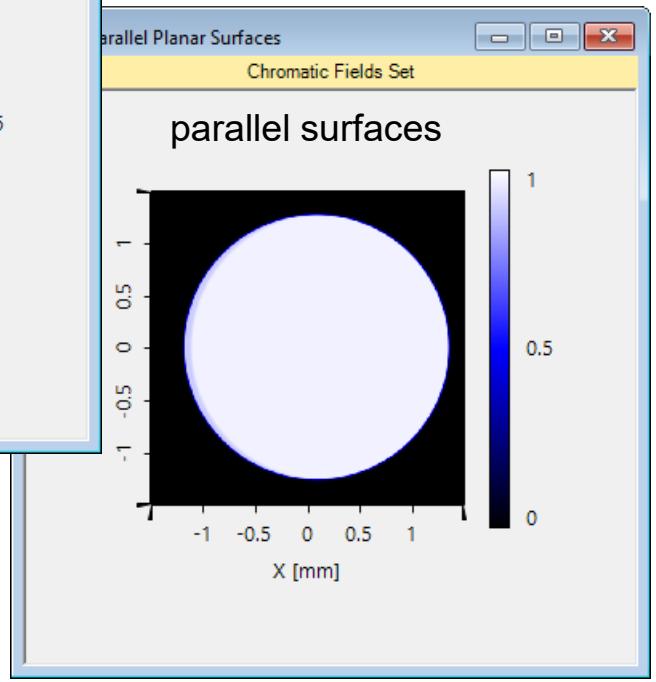
$$I_i = I_r + I_t. \qquad (9.35)$$

Tilted Planar-Planar Surfaces

etalon configuration
b) planar-planar (tilted)
- center thickness 100µm
- tilt of first surface 0.1°



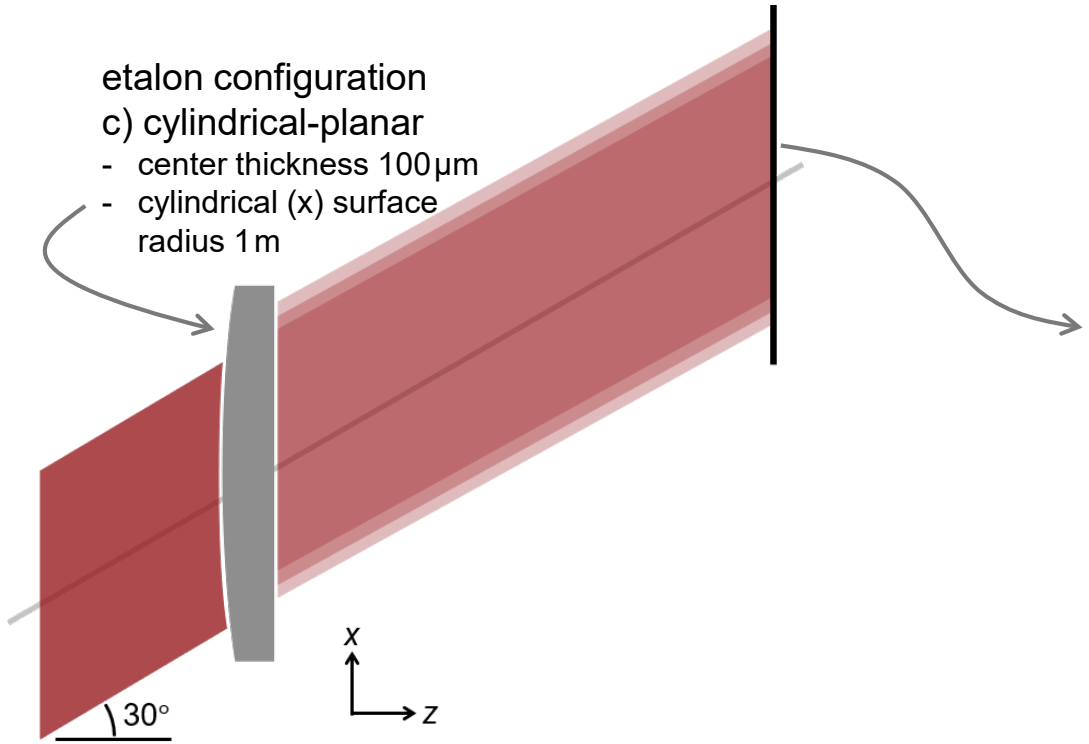


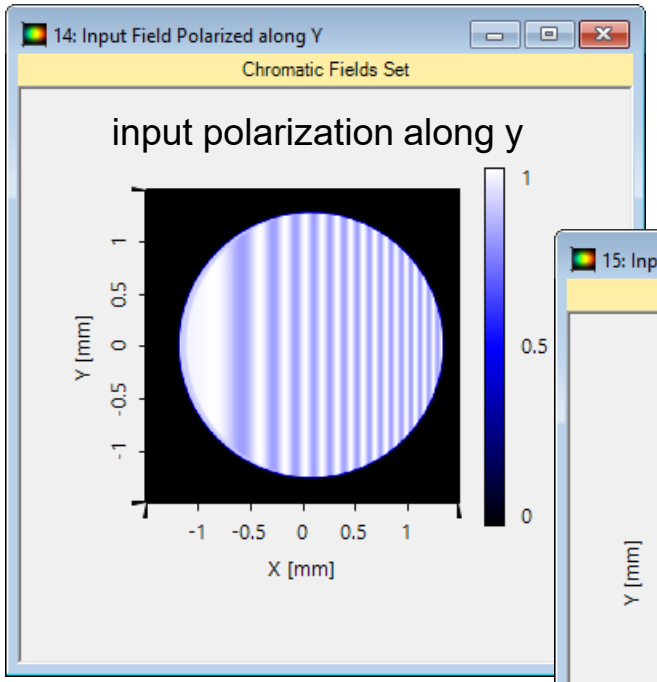


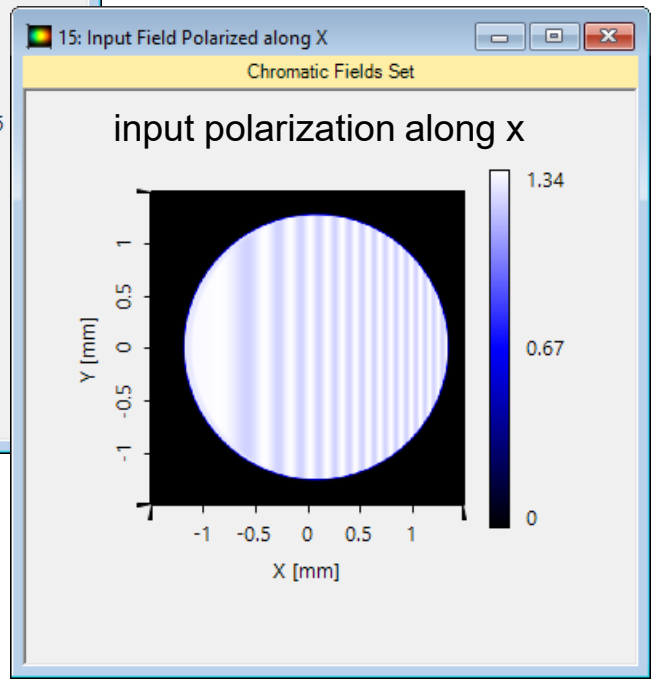
Linear interference fringes appear due to linear change of etalon thickness.

Cylindrical-Planar Surfaces

etalon configuration
c) cylindrical-planar
- center thickness 100 µm
- cylindrical (x) surface radius 1 m

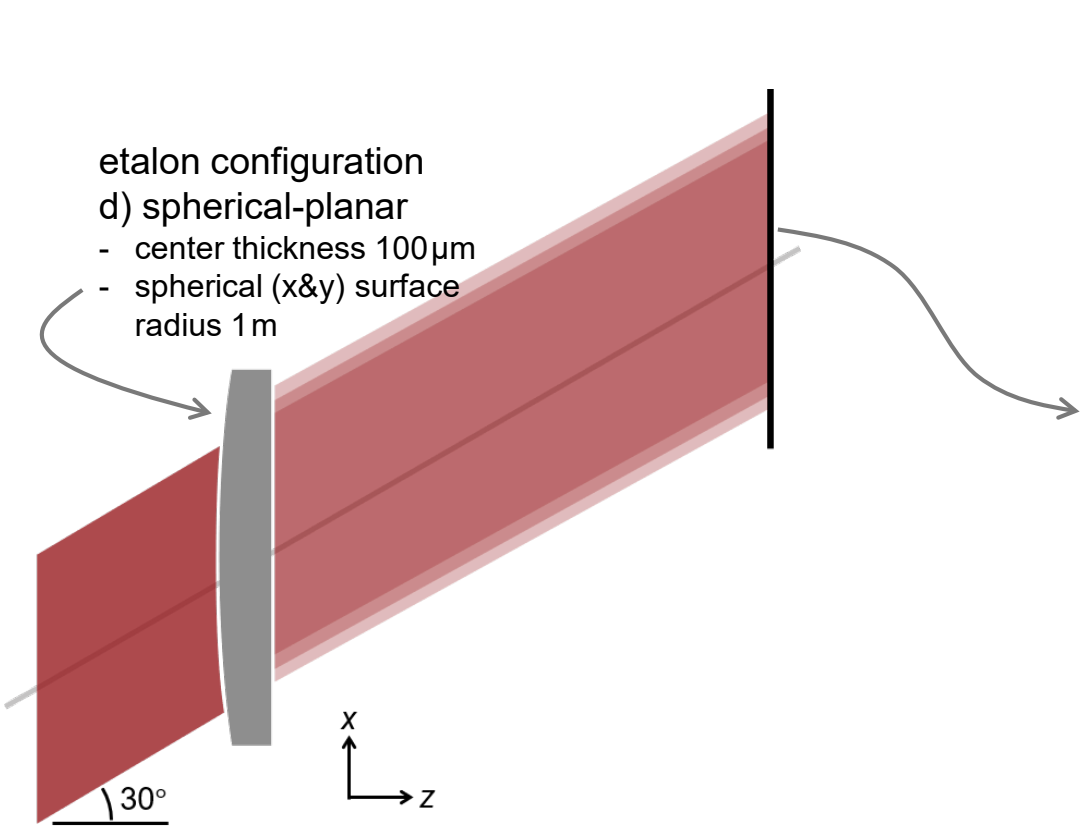


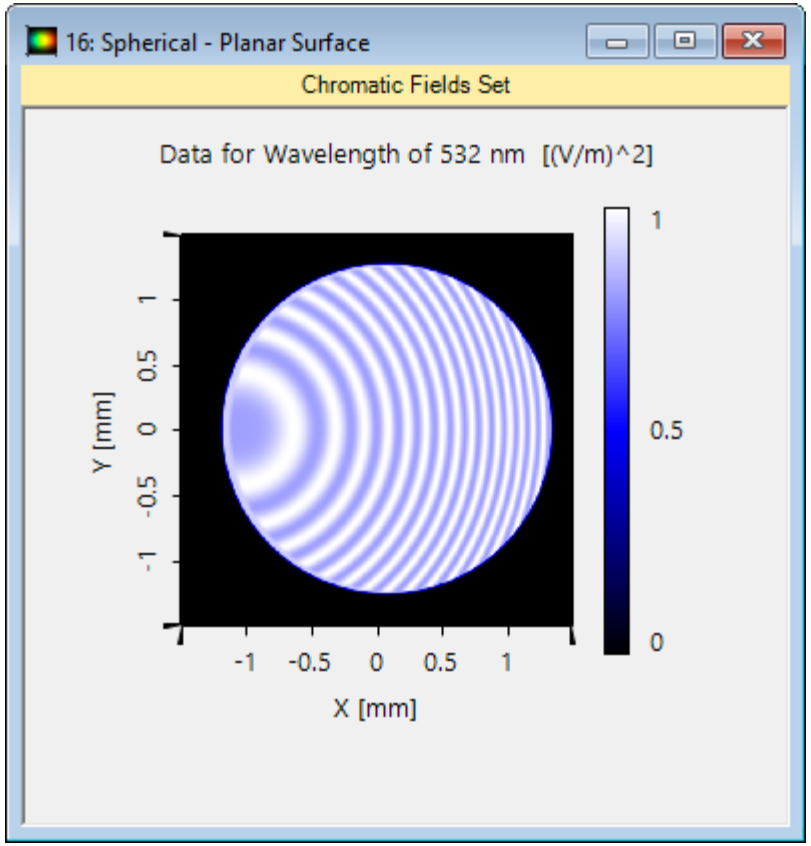




Polarization-dependent effect on the interference is considered in the simulation.

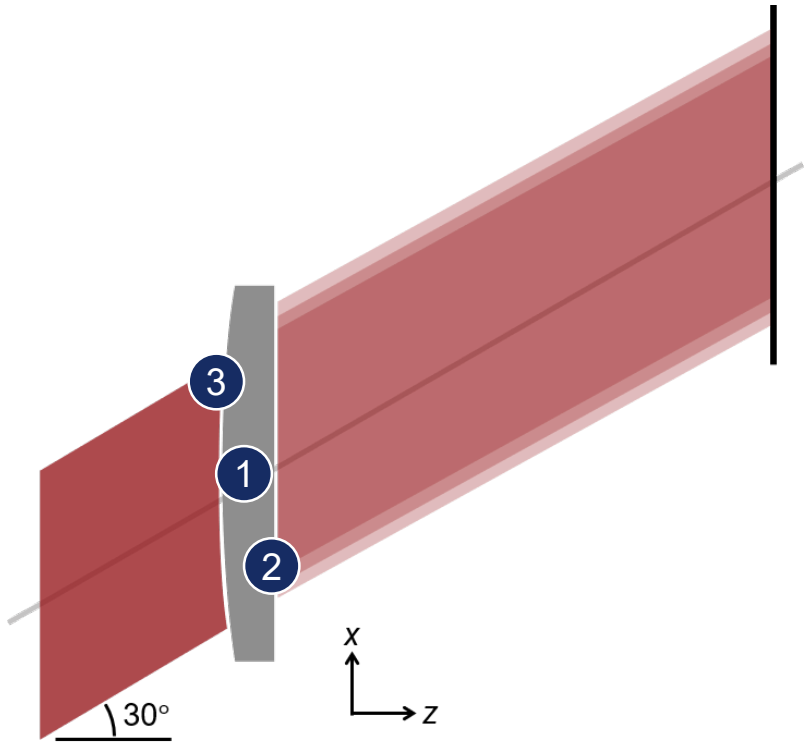
Spherical-Planar Surfaces

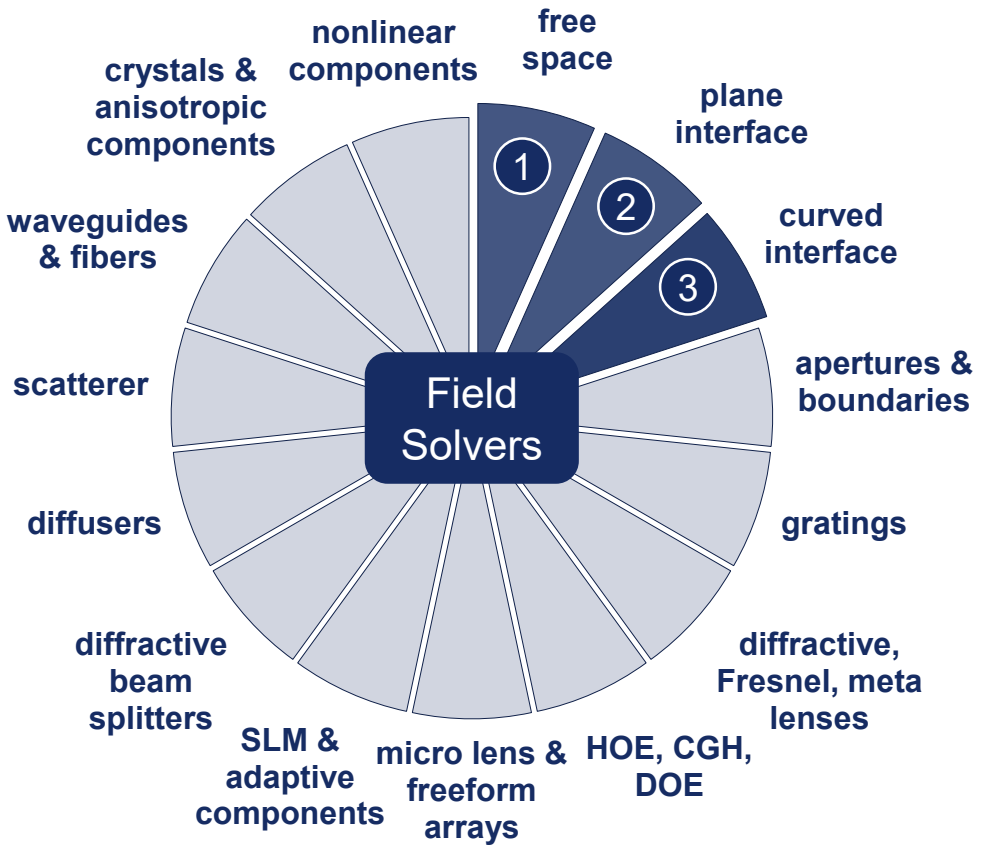




Non-sequential field tracing simulation of etalons allows the consideration of arbitrary surface types.

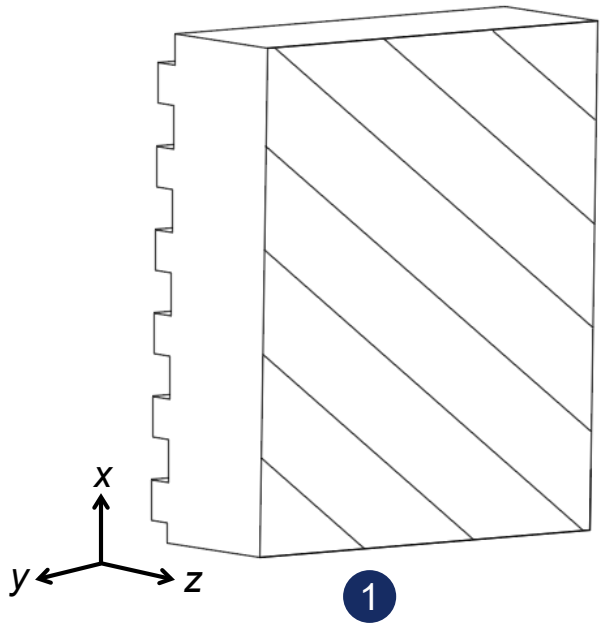
Connecting Field Solvers

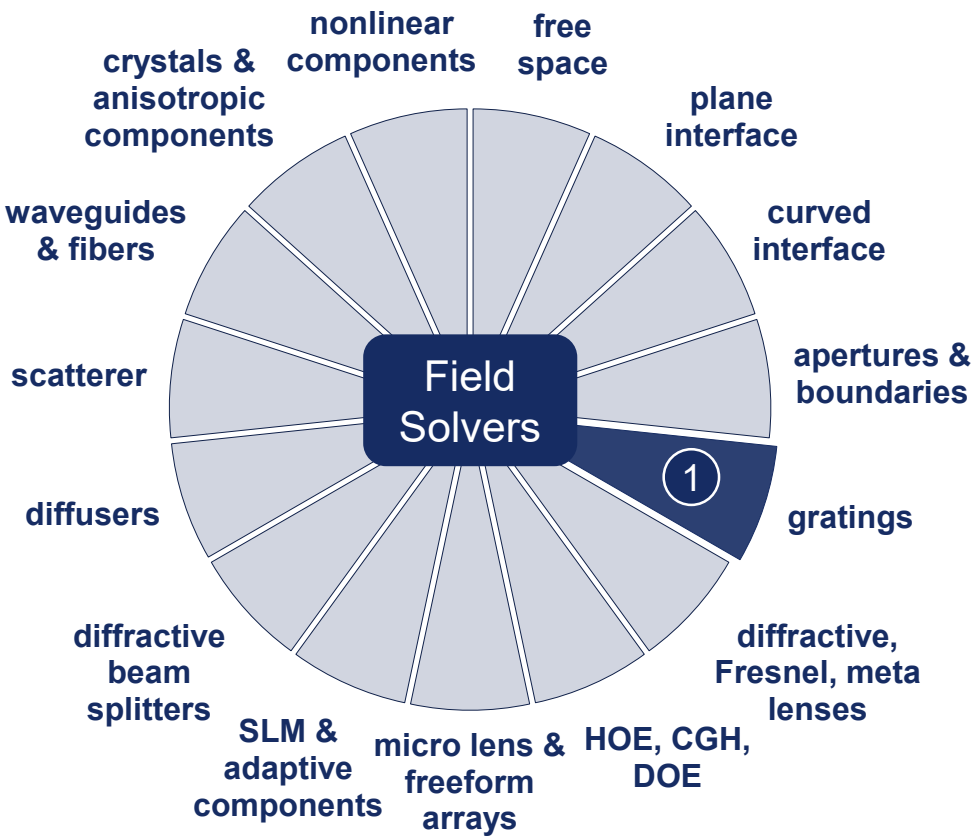




Rectangular + Sawtooth Grating (45° rotated)

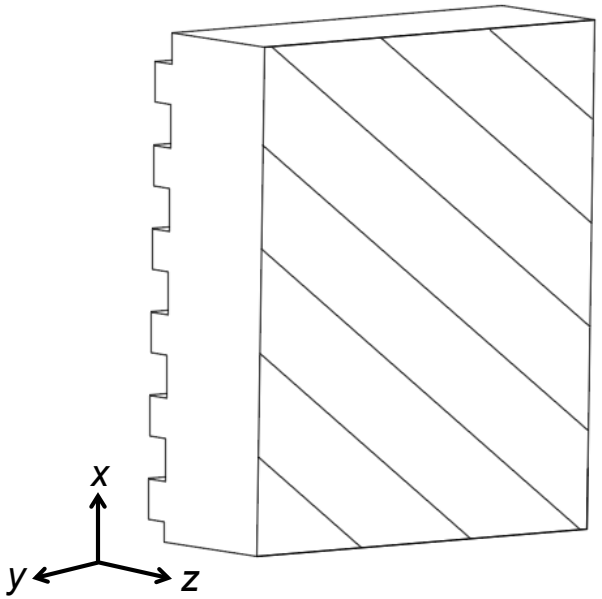
- Structure
  - Front: rectangular grating
    (along *x* direction)
  - Back: sawtooth grating
    (along *x*-*y* diagonal direction)

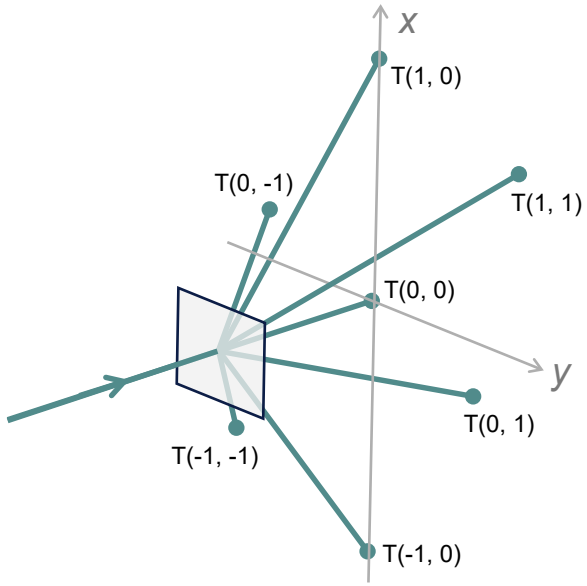




Rectangular + Sawtooth Grating (45° rotated)

- Structure
  - Front: rectangular grating
    (along *x* direction)
  - Back: sawtooth grating
    (along *x*-*y* diagonal direction)

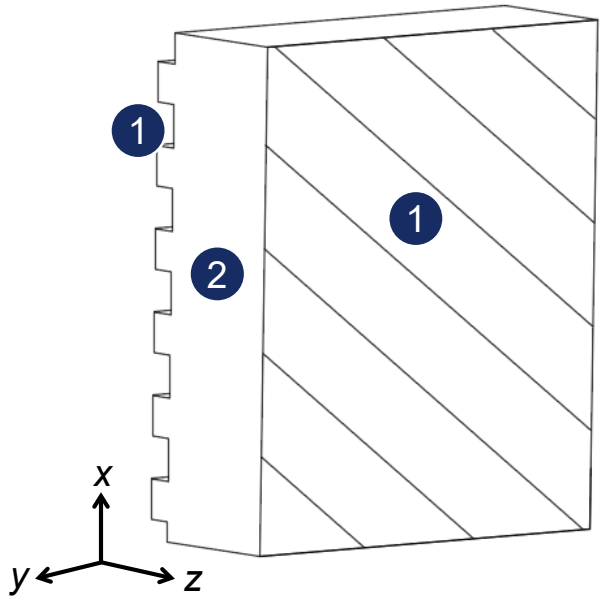


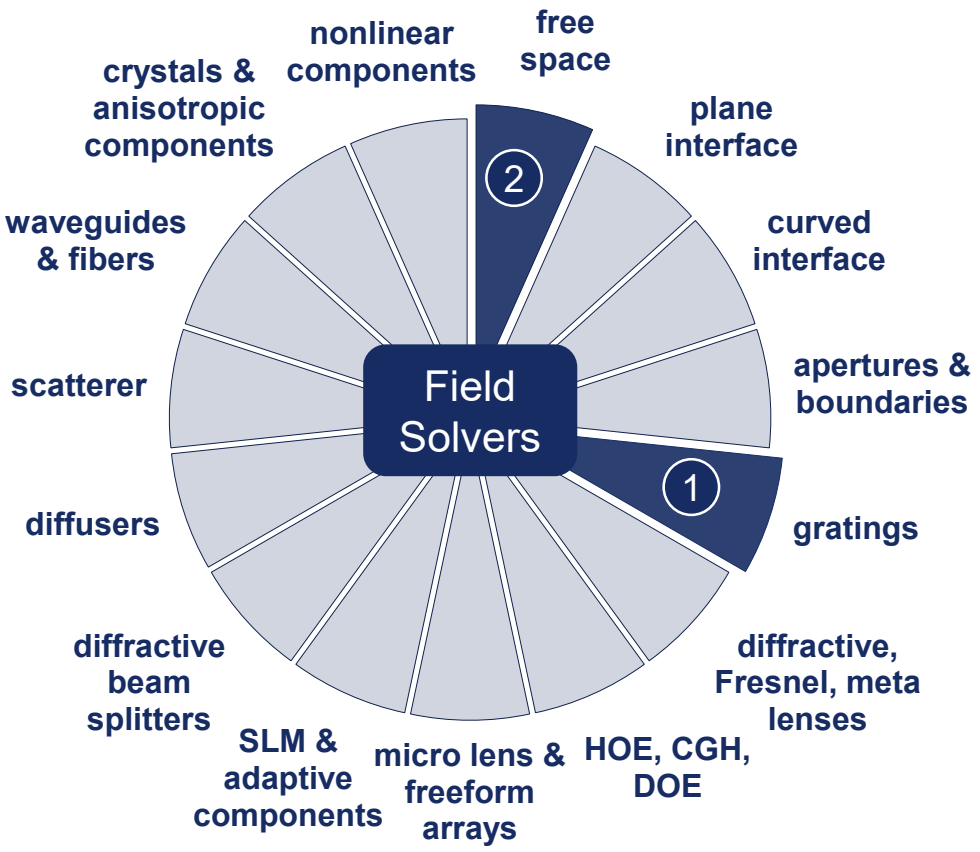


Global S matrix (TM)
➔ **No common period!**
➔ Huge computational effort even with approximated common period

Connecting Field Solvers

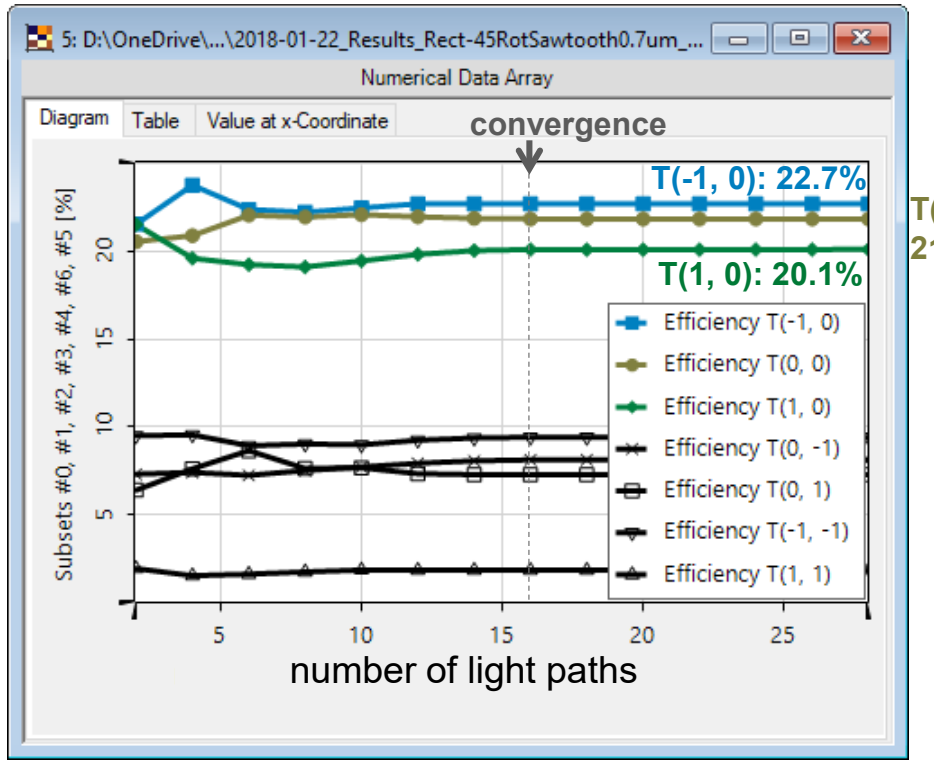
- Structure
  - Front: rectangular grating
    (along *x* direction)
  - Back: sawtooth grating
    (along *x*-*y* diagonal direction)



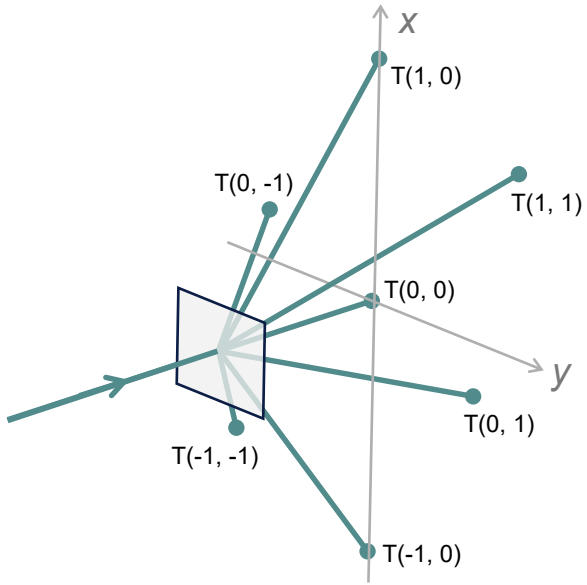


Connecting Field Solvers

- Non-sequential field tracing (TM)



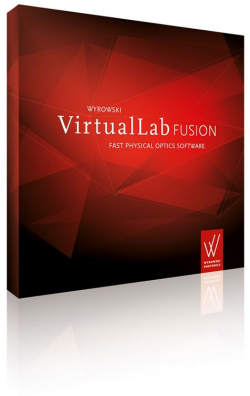
T(0, 0): 21.8%



Global S matrix ***NOT*** possible!
➔ No common period
➔ Huge computational effort even with approximated common period

Field Tracing: Connecting Field Solvers

1. Usage of numerically most efficient field solver per component/region
2. Non-sequential connection of regional solvers: **system solver**
3. Evaluation of all relevant (energy) lightpaths to detectors

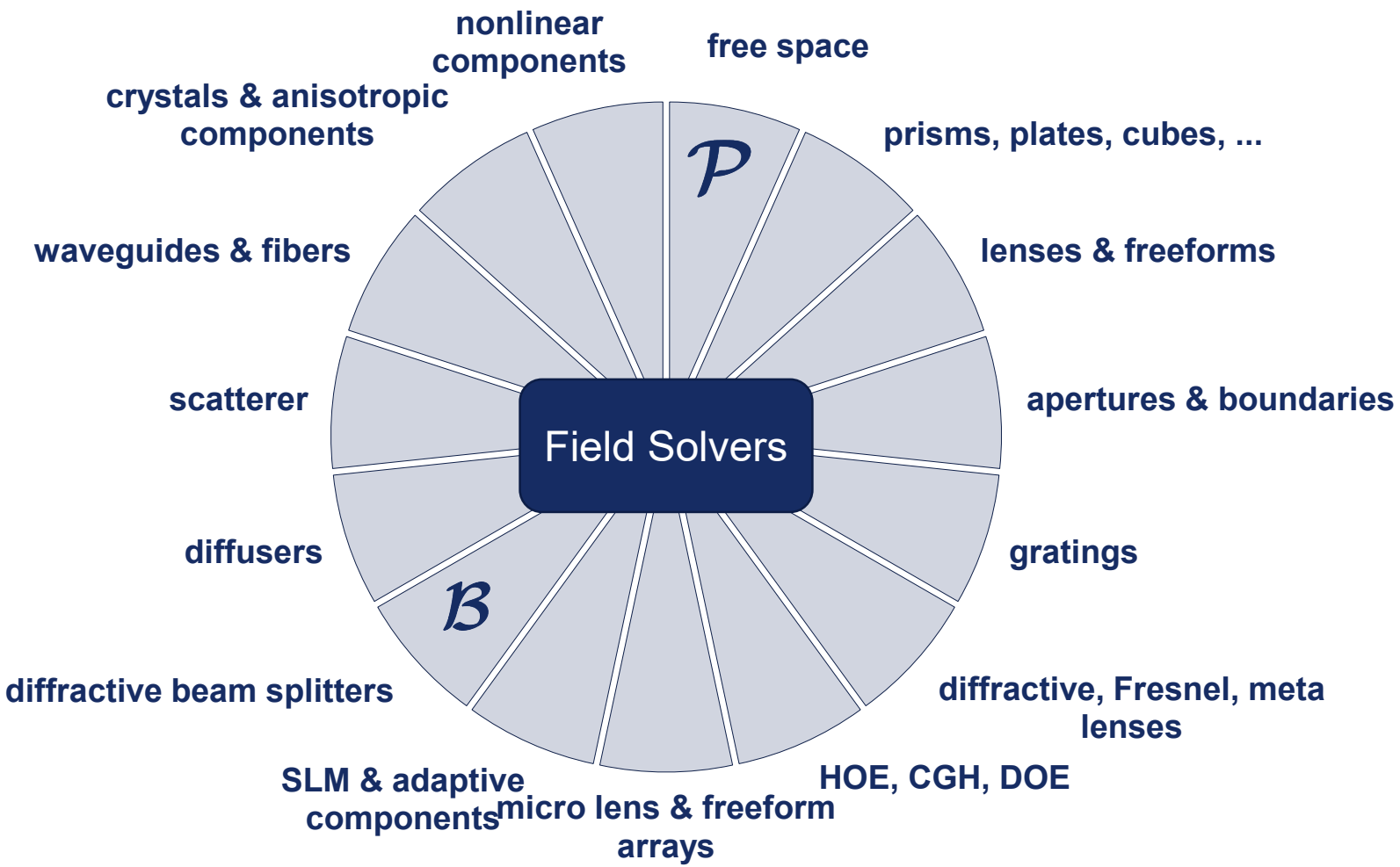


VirtualLab Fusion’s Lightpath Finder provides lightpath “tree” of all relevant lightpaths for further processing.

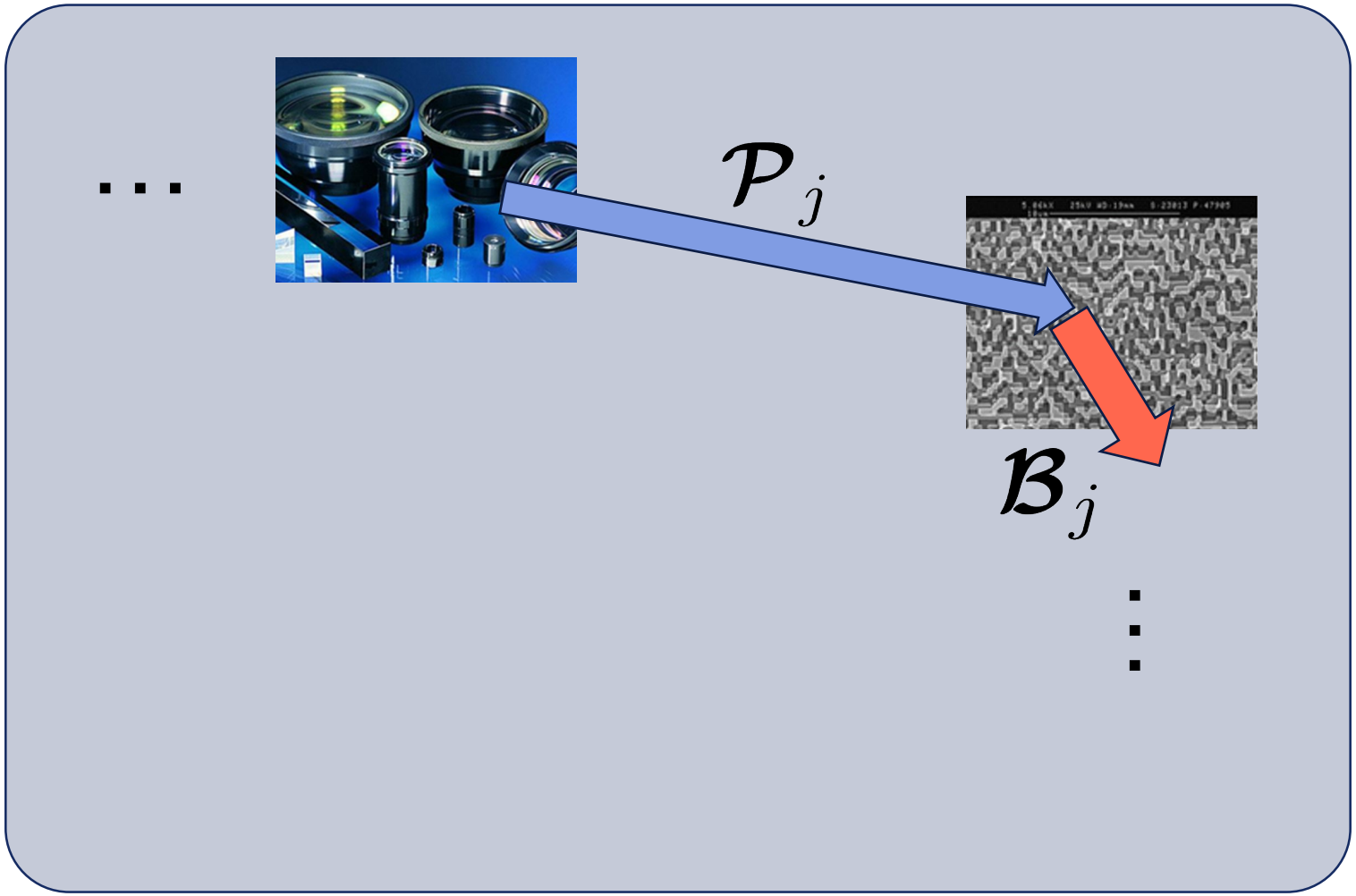
Field Tracing: Connecting Field Solvers

1. Usage of numerically most efficient field solver per component/region
2. Non-sequential connection of regional solvers: **system solver**
3. Evaluation of all relevant (energy) lightpaths to detectors
4. Physical-optics analysis of all lightpaths by operator sequences

Physical-Optics System Modeling: Regional Field Solver



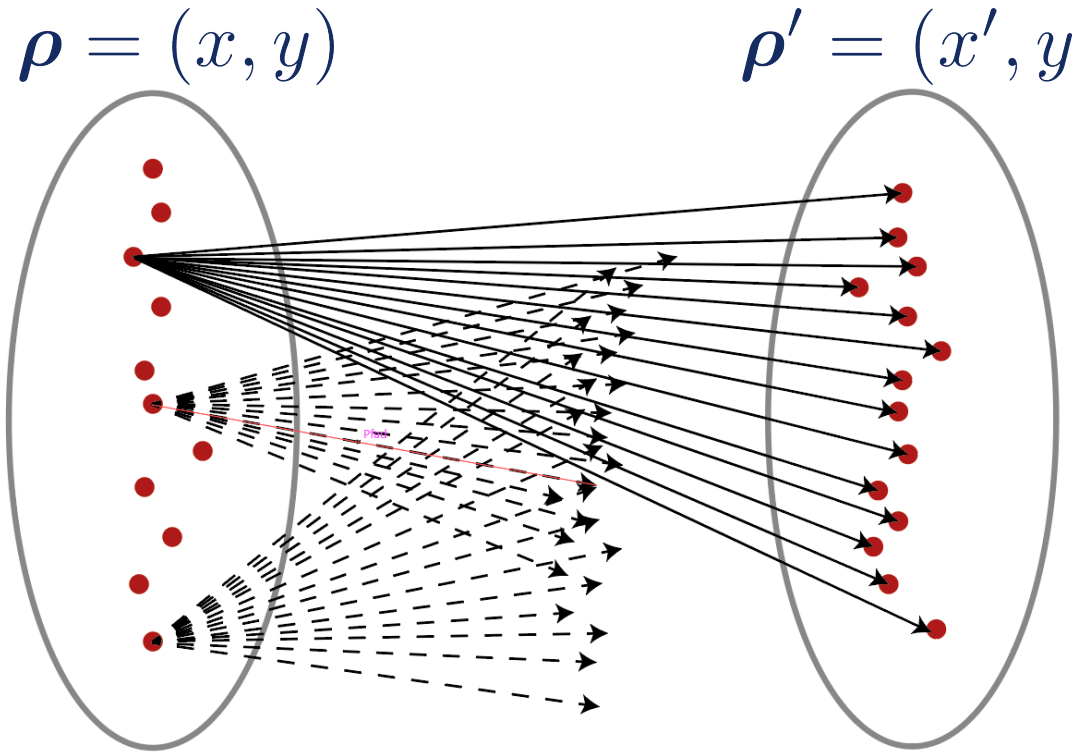
Operator Sequences per Lightpath



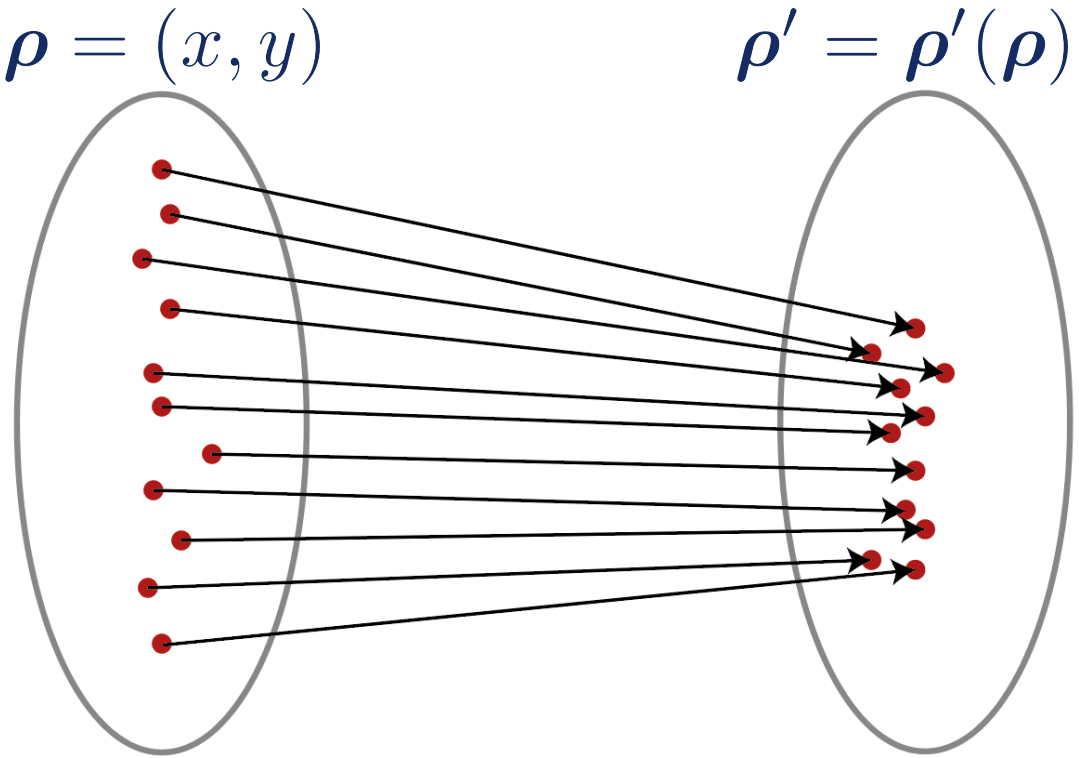
Evaluation of sequence

$$\ldots \left(\mathcal{B}_{j+1}\mathcal{P}_{j+1}\right)\left(\mathcal{B}_j\mathcal{P}_j\right)\ldots$$

Numerical Complexity of Operators



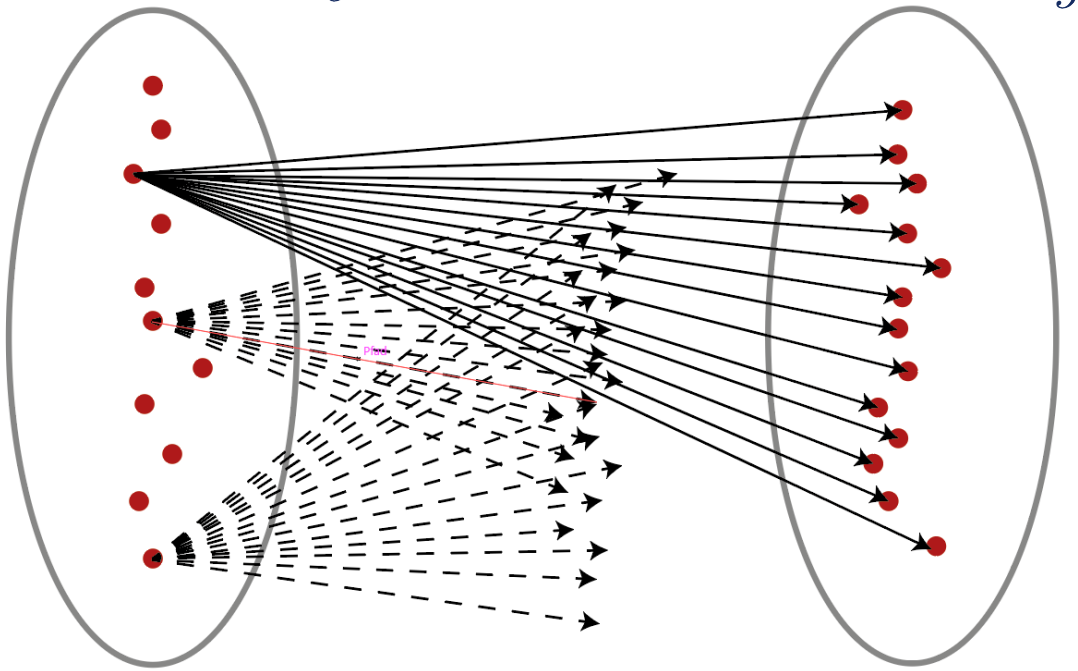
Integral Operator:
Complexity $O(N^2)$



1:1 Mapping-Type Operator:
Complexity O(N)

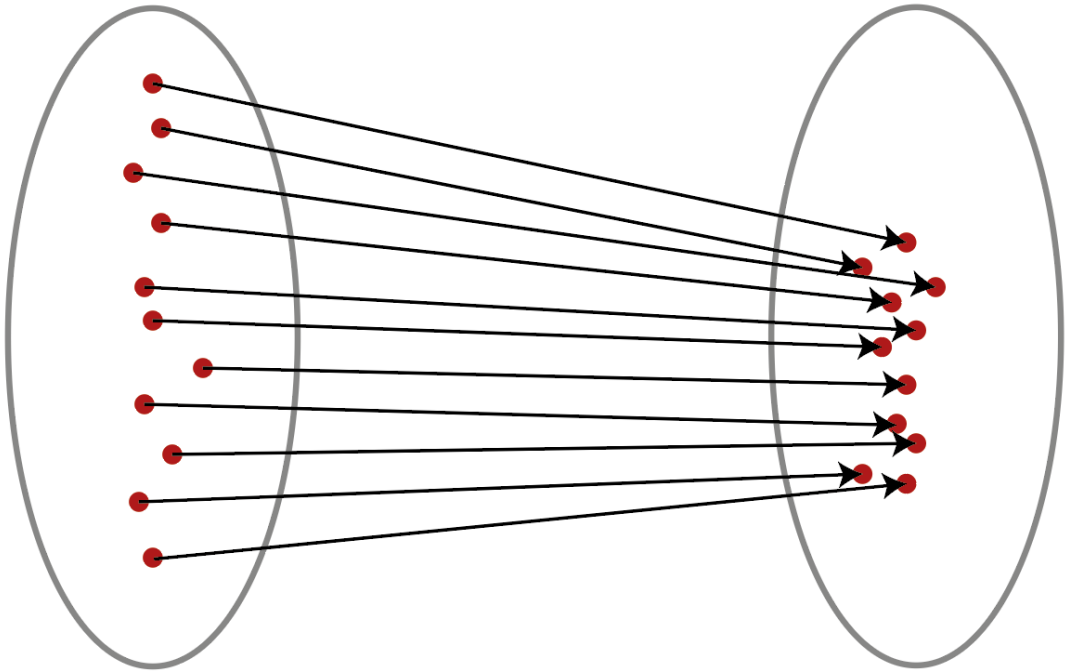
Numerical Complexity of Operators

$\boldsymbol{\kappa} = (k_x, k_y)$ $\boldsymbol{\kappa}' = (k'_x, k'_y)$



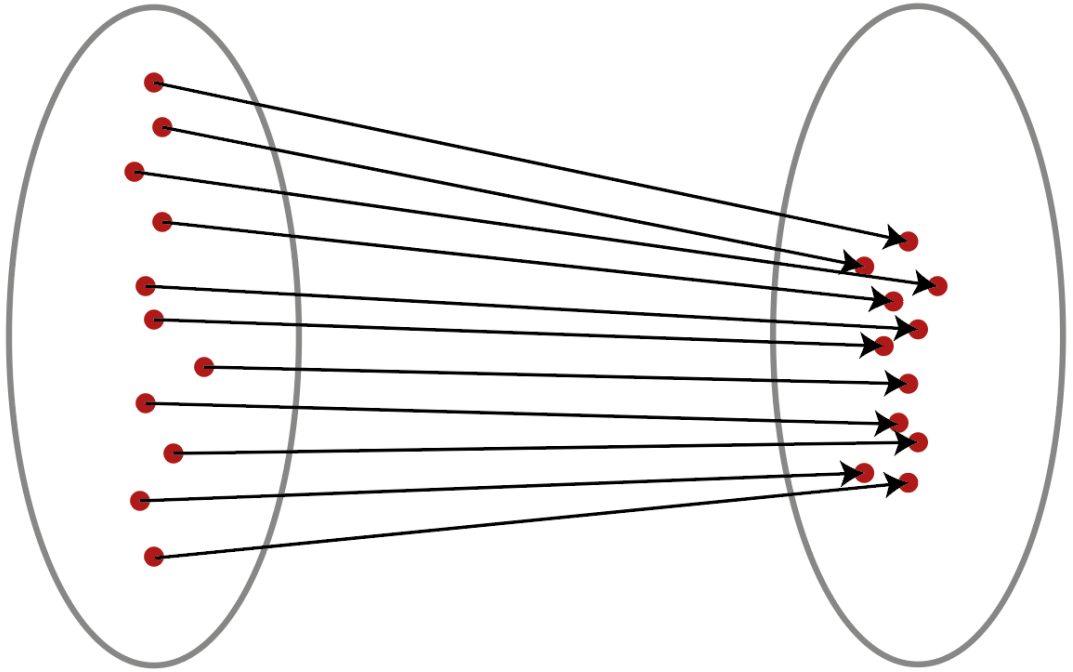
Integral Operator:
Complexity O(N²)

$\boldsymbol{\kappa} = (k_x, k_y)$ $\boldsymbol{\kappa}' = \boldsymbol{\kappa}'(\boldsymbol{\kappa})$



1:1 Mapping-Type Operator:
Complexity O(N)

Homeomorphism

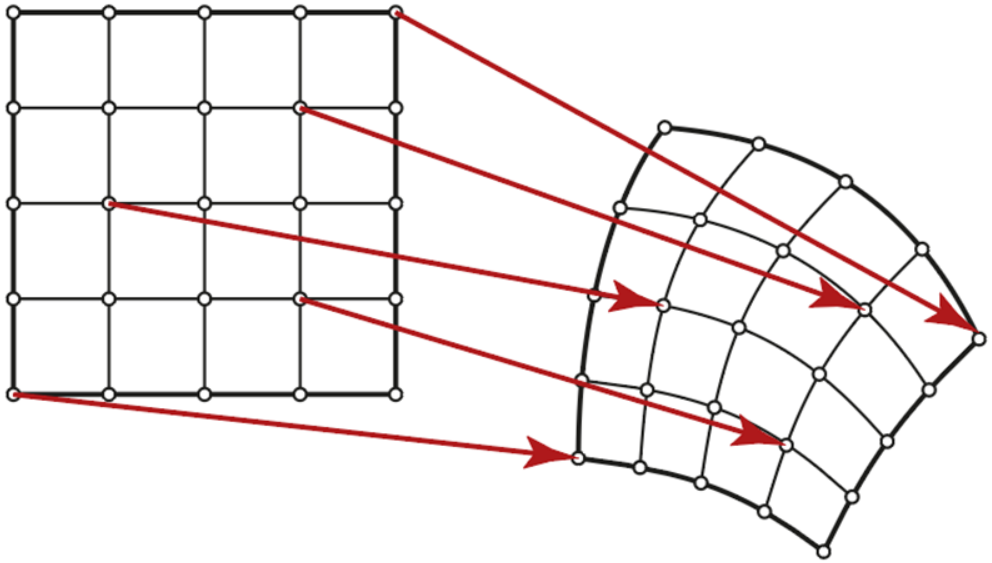


$$\rho' = \rho'(\rho)$$

$$\kappa' = \kappa'(\kappa)$$

$$\kappa = \kappa(\rho)$$

Homeomorphism



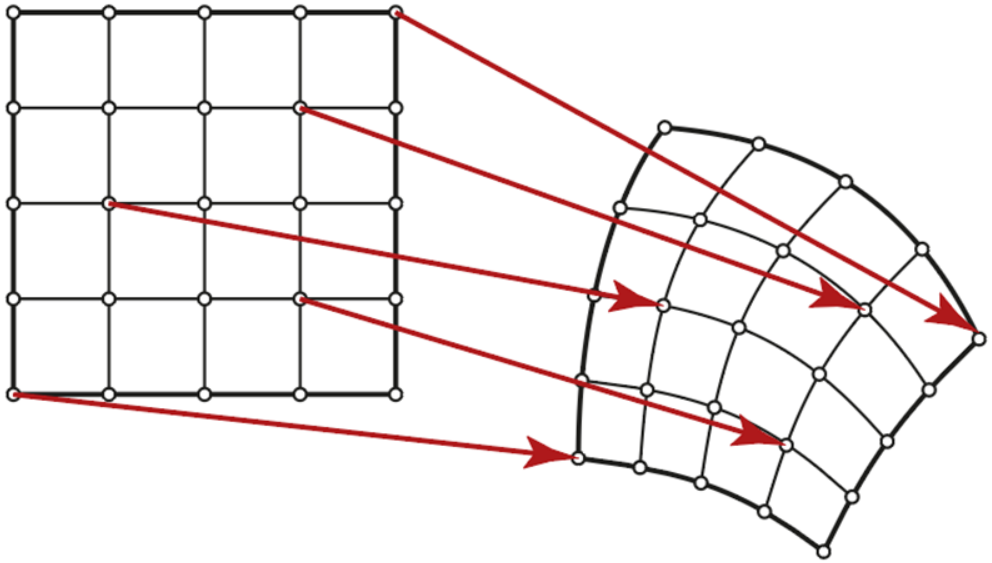
$$\boldsymbol{\rho}' = \boldsymbol{\rho}'(\boldsymbol{\rho})$$

$$\boldsymbol{\kappa}' = \boldsymbol{\kappa}'(\boldsymbol{\kappa})$$

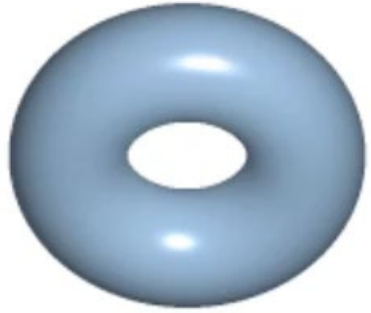
$$\boldsymbol{\kappa} = \boldsymbol{\kappa}(\boldsymbol{\rho})$$

- Homeomorphism: $\boldsymbol{\rho}' = \boldsymbol{\rho}'(\boldsymbol{\rho})$ is bijective
- Homeomorphic mapping: Coordinates change but neighbors remain the same.

Homeomorphism

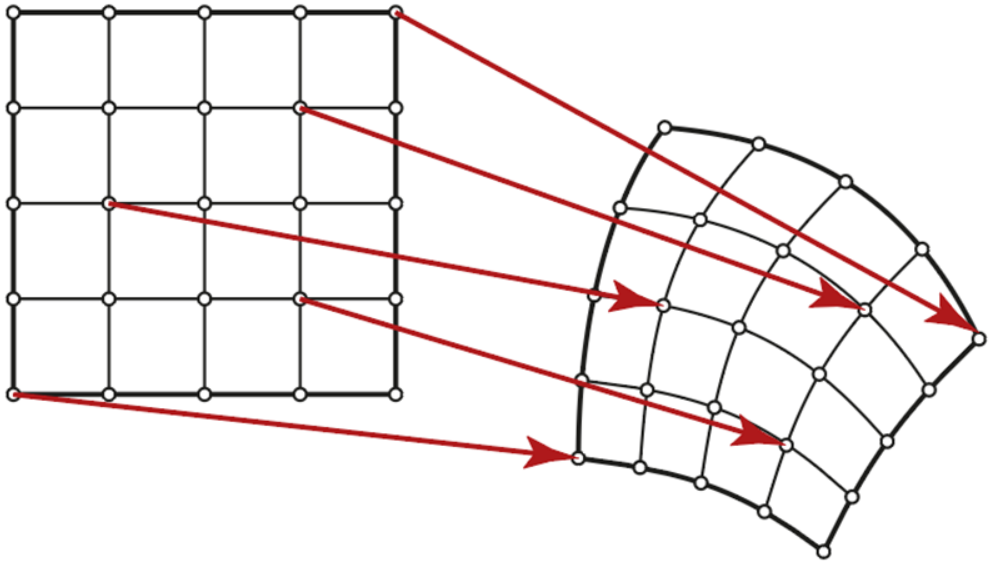


https://en.wikipedia.org/wiki/Homeomorphism



- Homeomorphism: $\boldsymbol{\rho}' = \boldsymbol{\rho}'(\boldsymbol{\rho})$ is bijective
- Homeomorphic mapping: Coordinates change but neighbors remain the same.

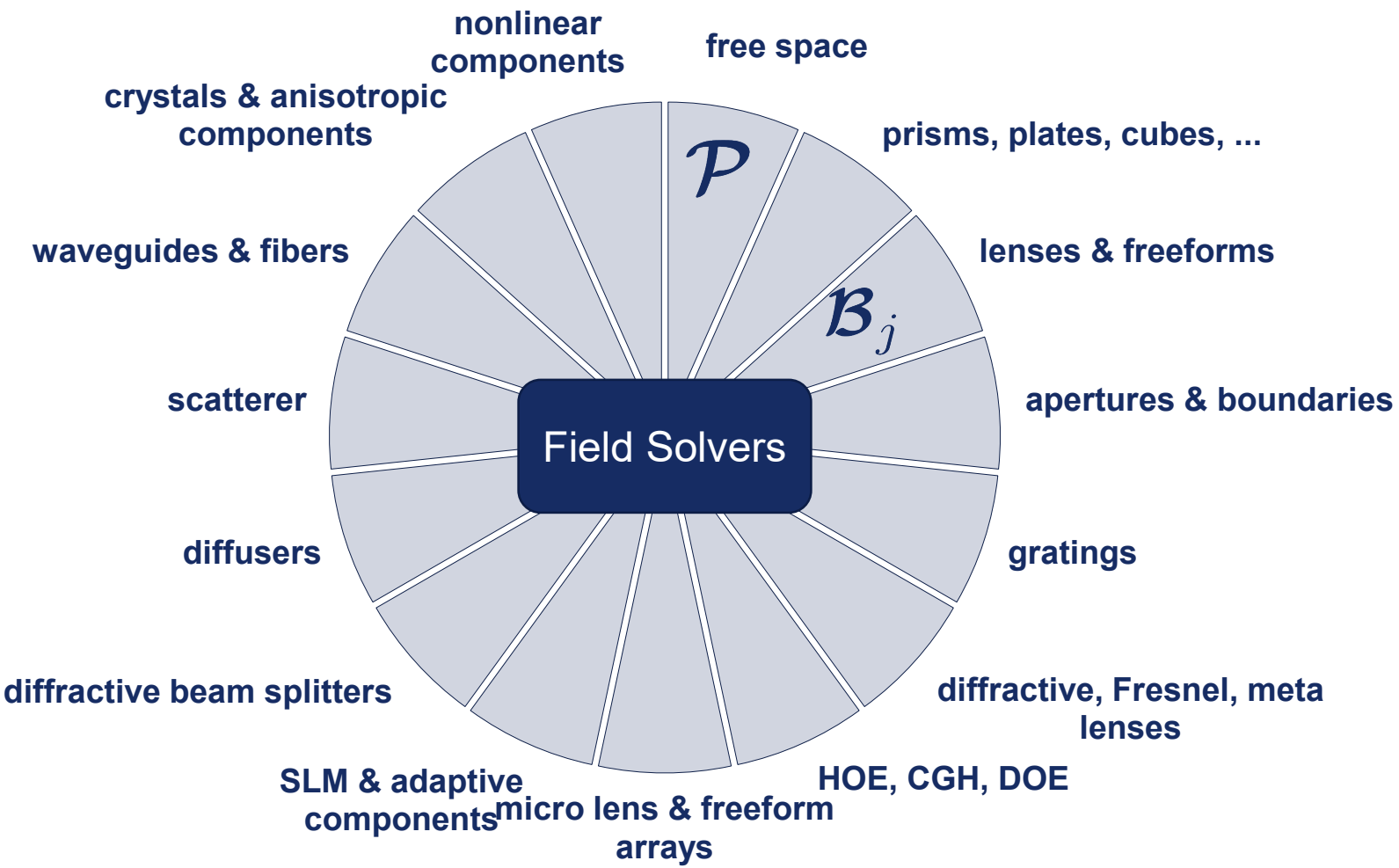
Homeomorphism



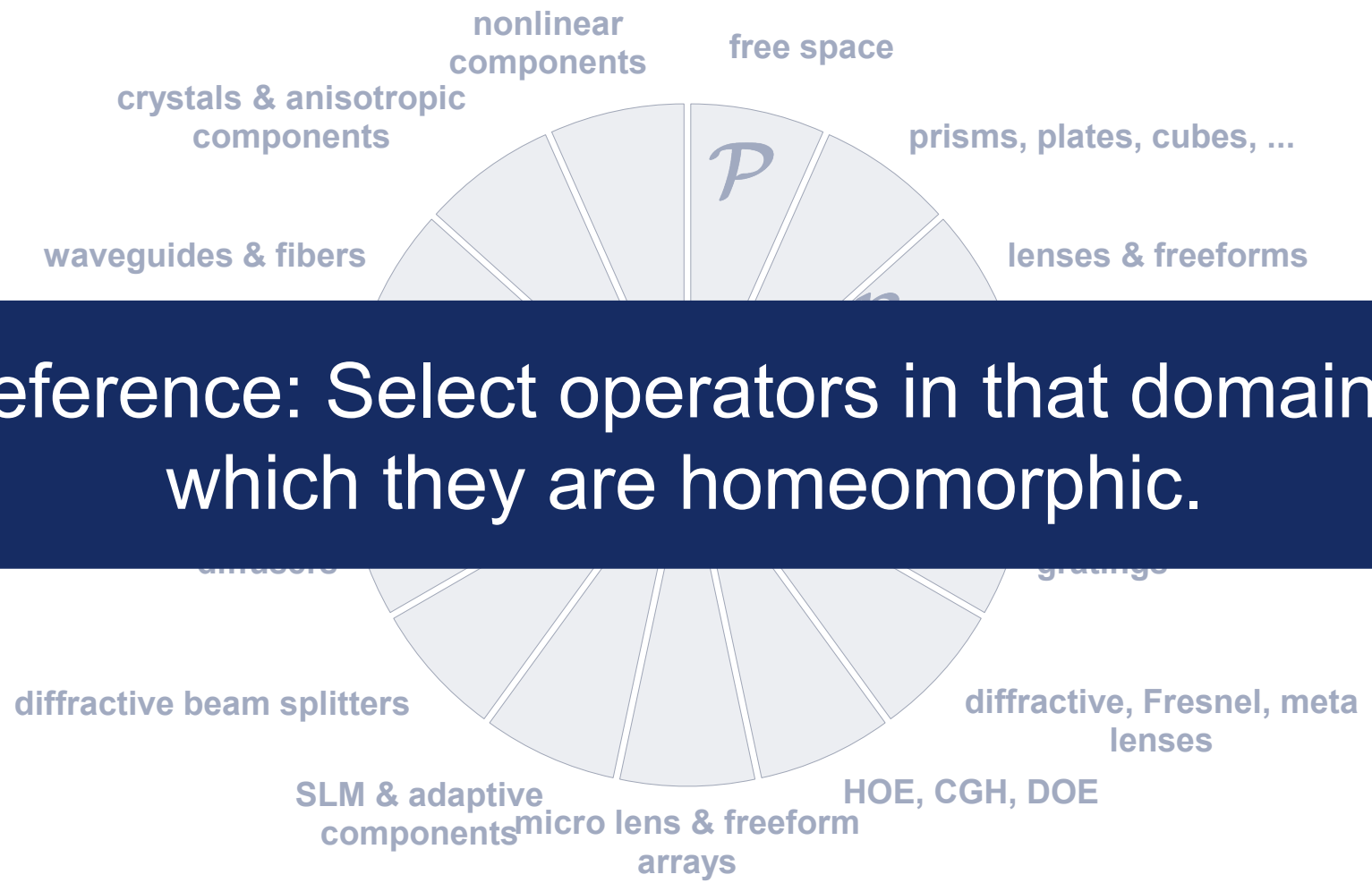
- Homeomorphism: $\boldsymbol{\rho}' = \boldsymbol{\rho}'(\boldsymbol{\rho})$ is bijective
- Homeomorphic mapping: Coordinates change but neighbors remain the same.
- Advantage: Mesh-based technique to interpolate gridless data by fitting and spline interpolation.

Irfan Badar

Physical-Optics System Modeling: Regional Field Solver

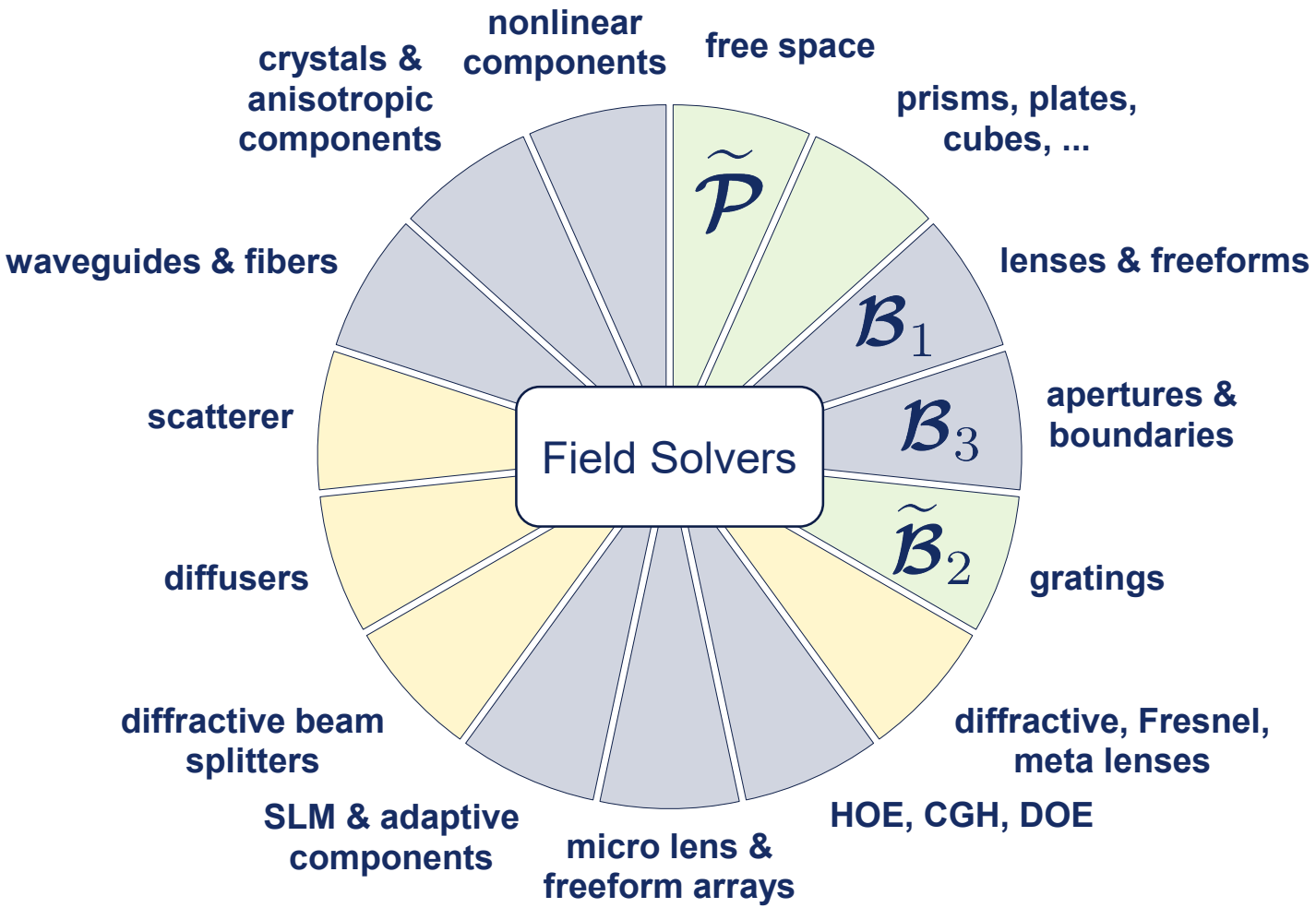


Physical-Optics System Modeling: Regional Field Solver



Preference: Select operators in that domain in which they are homeomorphic.

Regional Field Solvers Applied in Different Domains

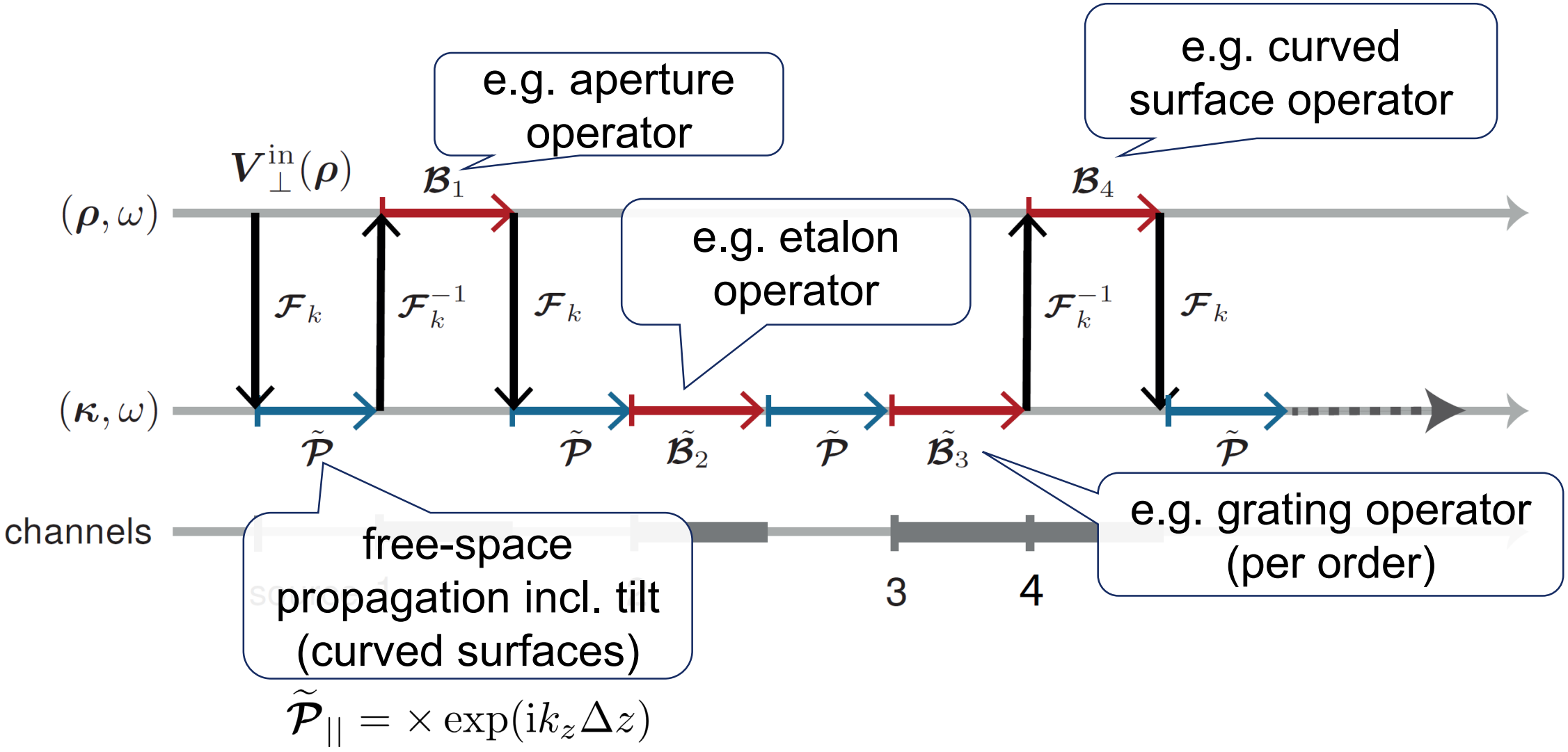


space domain

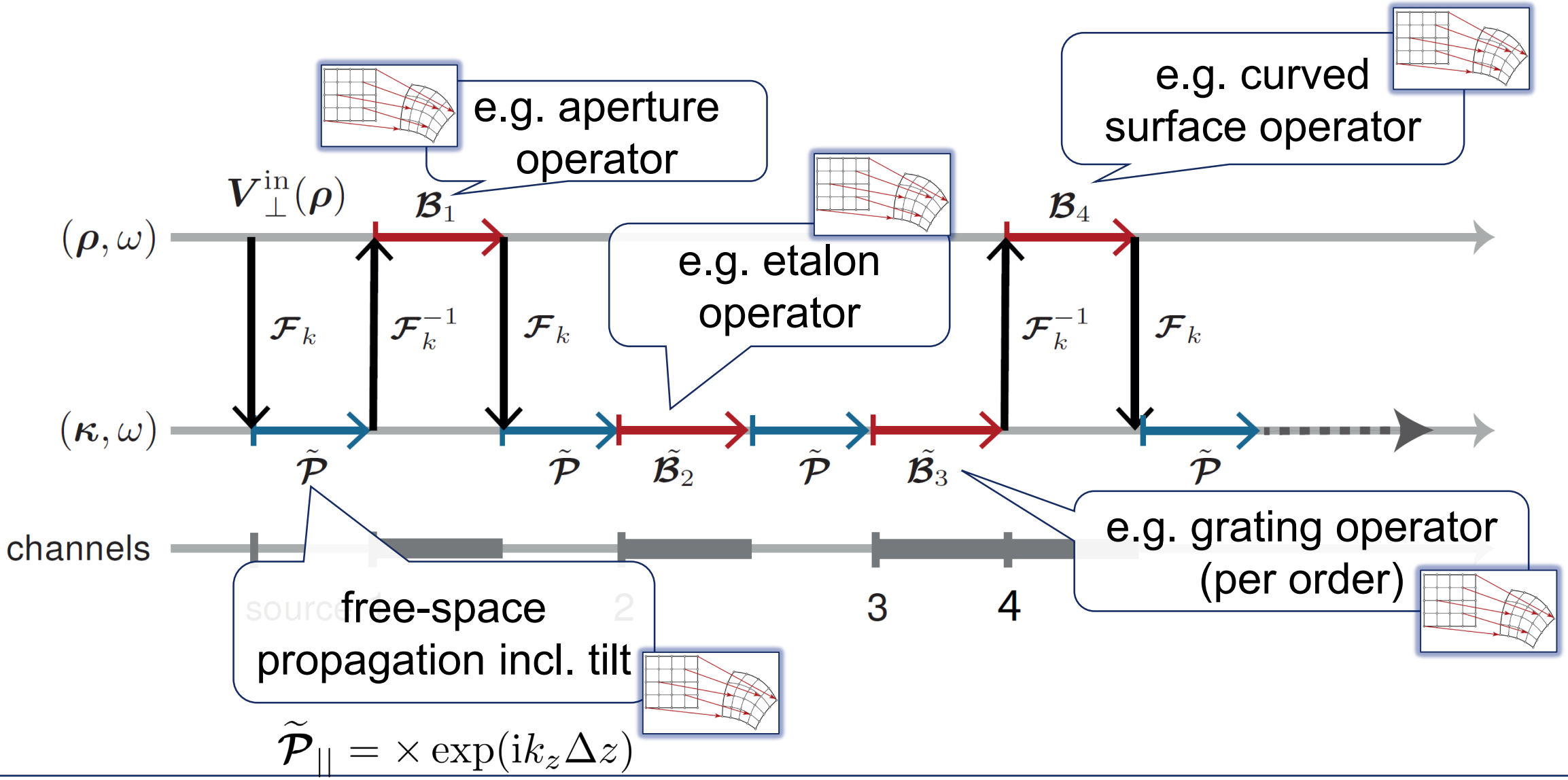
k-domain

space or/and k-domain

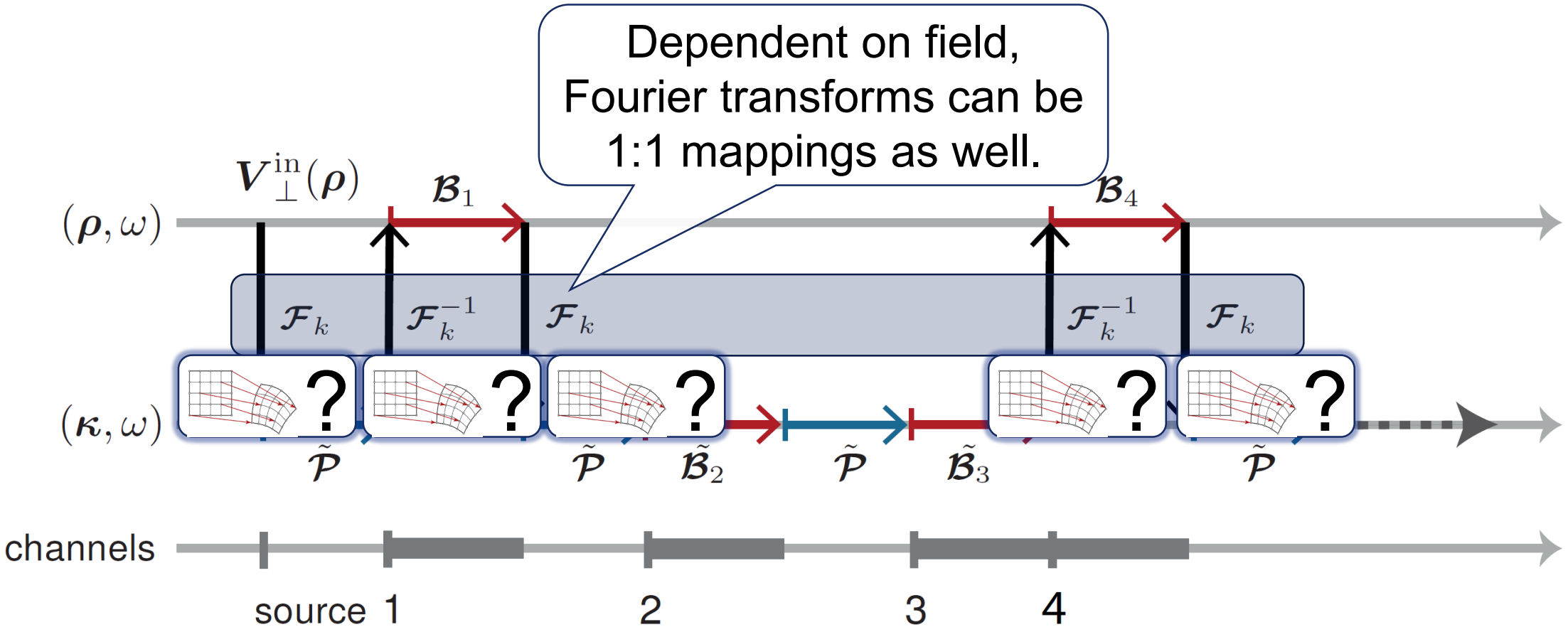
Switching Domains: Preferable Operators Linear in N



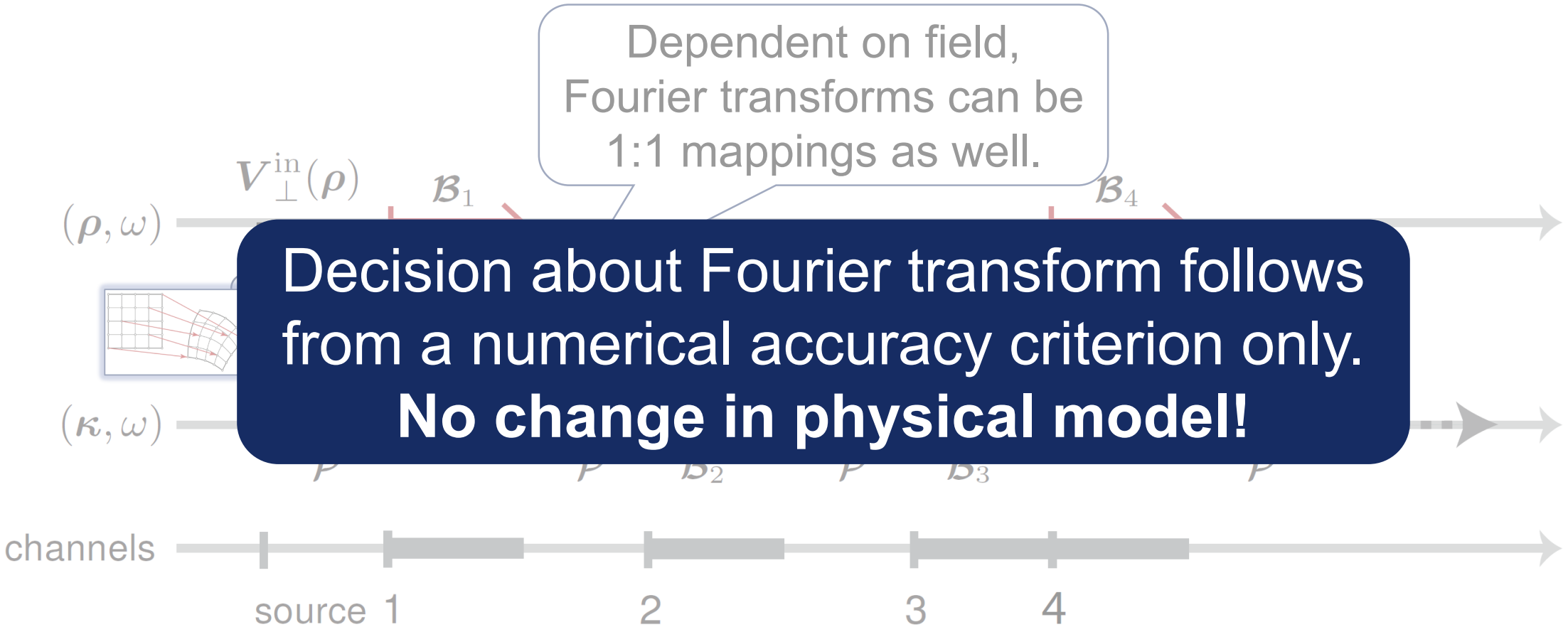
Homeomorphic Operations in Operator Sequence



Fourier Transform Integral Operator

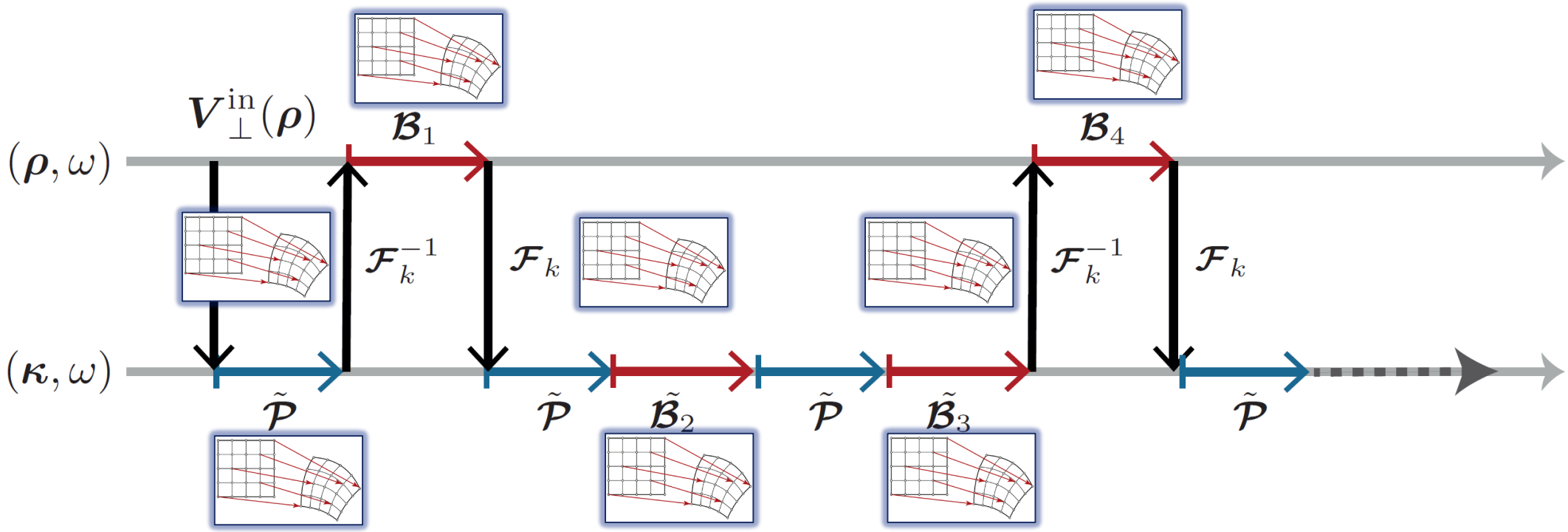


Fourier Transform Integral Operator

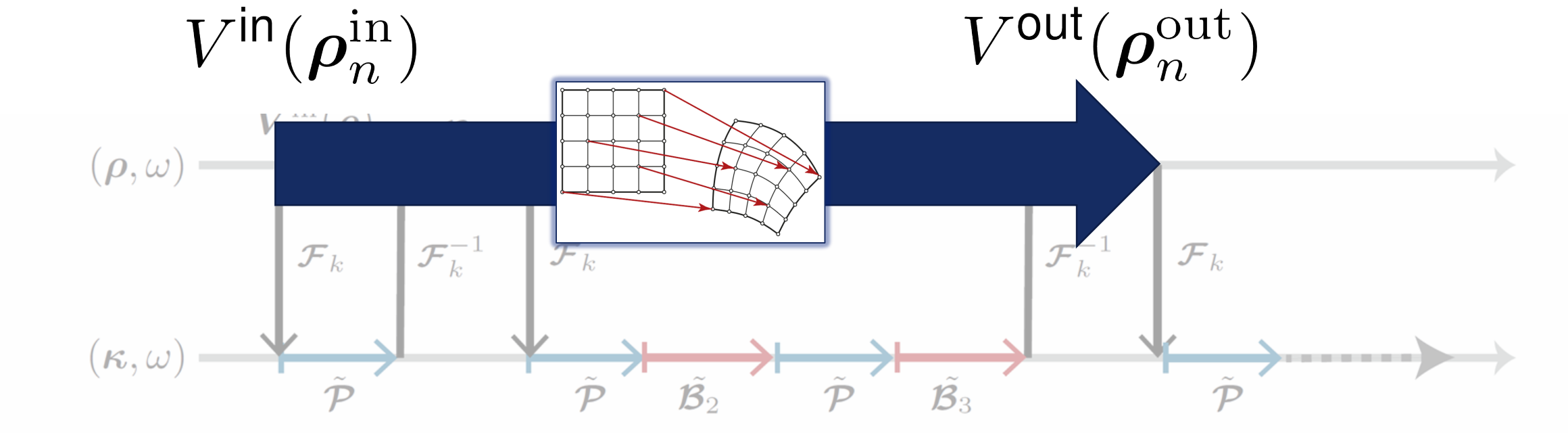


Decision about Fourier transform follows from a numerical accuracy criterion only.
No change in physical model!

1:1 Mapping Sequence in Physical Optics

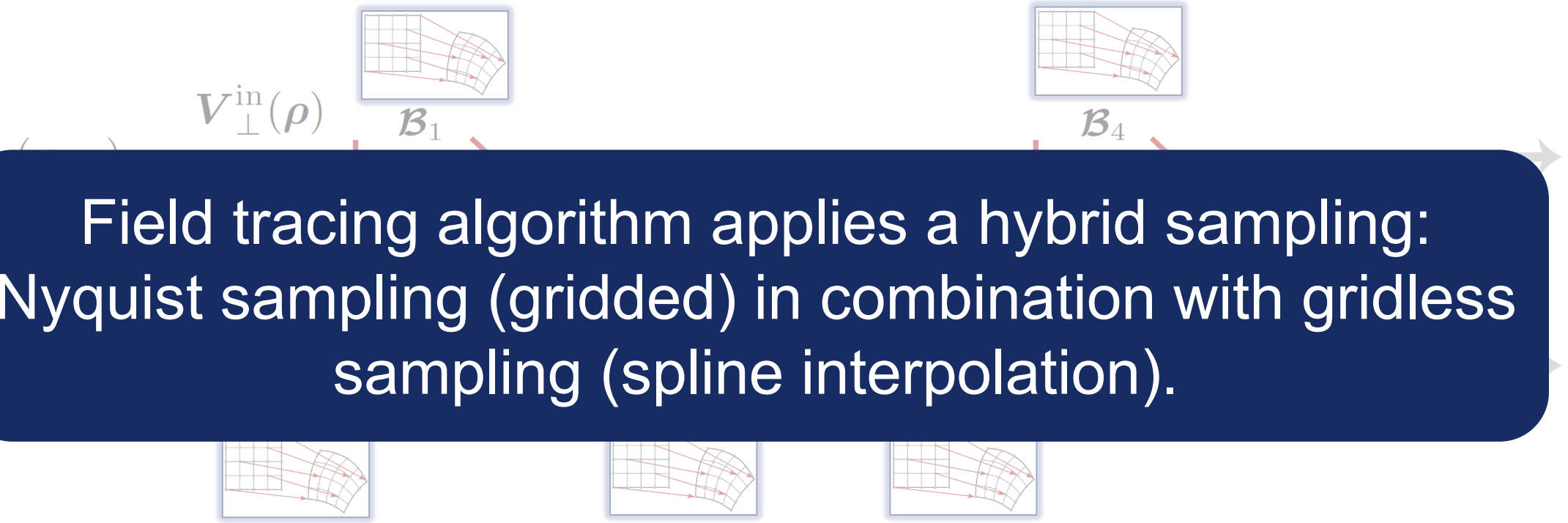


1:1 Mapping Sequence in Physical Optics



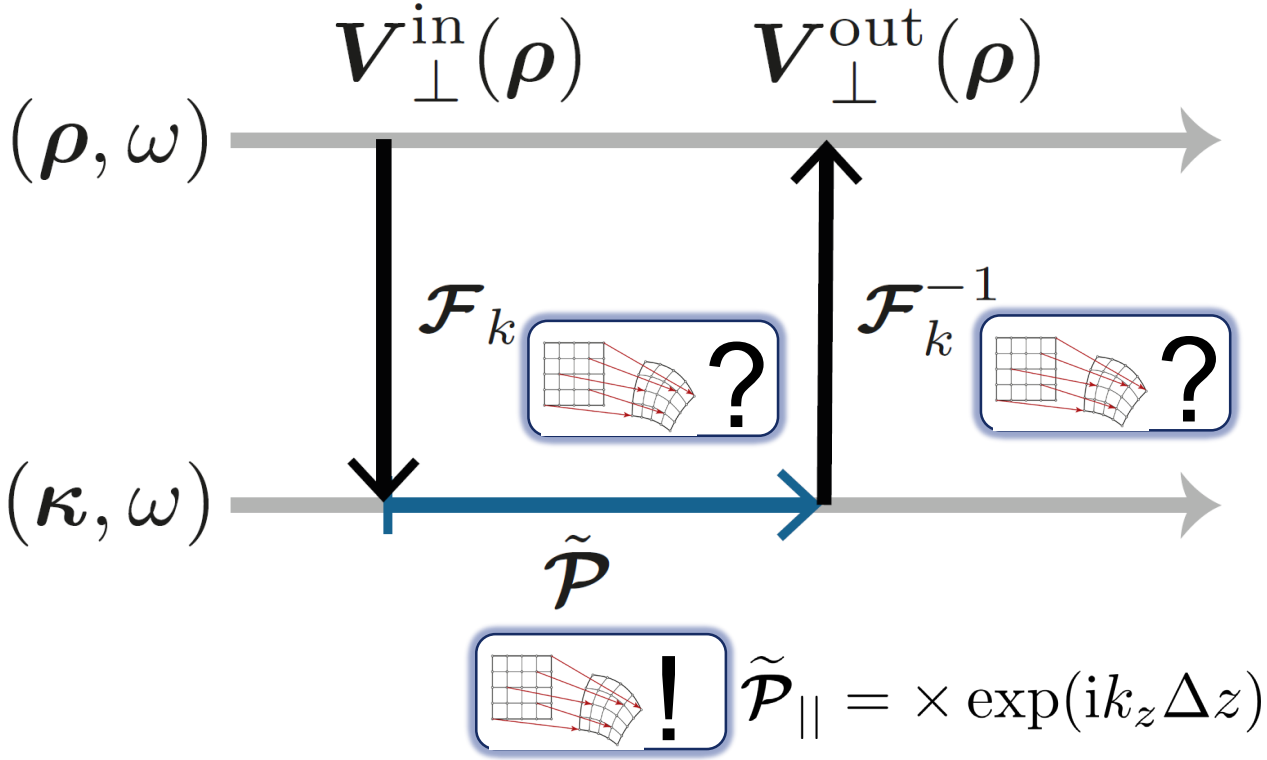
Ray tracing: $\boldsymbol{\rho}_n^{\text{out}}(\boldsymbol{\rho}_n^{\text{in}})$

Field Tracing and Sampling



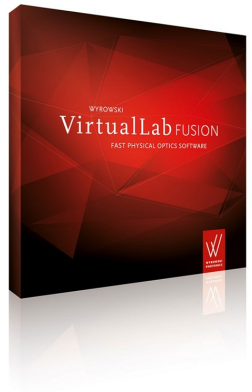
Field tracing algorithm applies a hybrid sampling: Nyquist sampling (gridded) in combination with gridless sampling (spline interpolation).

Selection of Fourier Transforms: Free-Space Propagation



Homeomorphic Sequences in Physical Optics

1. Modeling operators ***B*** are often homeomorphic in one domain.
2. If the homeomorphic Fourier transform connects such 1:1 operators, homeomorphic sequences are obtained in physical optics.
3. Homeomorphic sequences enable fast physical optics modeling.
4. Fast physical optics: field tracing algorithm identifies and performs as many homeomorphic operations as possible.
5. VirtualLab Fusion decides about application of homeomorphic Fourier transform automatically on basis of numerical accuracy!



Field Tracing: Connecting Field Solvers

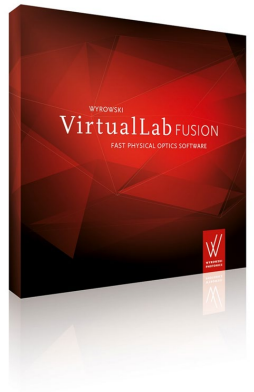
1. Modeling operators ***B*** are often homeomorphic in one domain.

2. If the homeomorphic Fourier transform connects such 1:1 operators

Resulting algorithm stays always in a vectorial electromagnetic modeling with all physical-optics effects included.

4. Ray tracing is embedded in 1:1 mapping sequences in physical optics!

5. VirtualLab Fusion decides about application of homeomorphic Fourier transform automatically on basis of numerical accuracy!



Field Tracing: Connecting Field Solvers

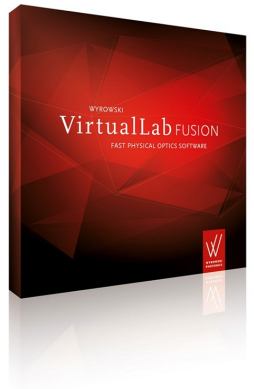
1. Modeling operators ***B*** are often homeomorphic in one domain.

2. If the homeomorphic Fourier transform connects such 1:1 operators,

By OPTIONALLY enforcing homeomorphic Fourier transform in (part of) the system, diffraction effects can be excluded from physical-optics modeling on demand!

4. Ray tracing is embedded in 1:1 mapping sequences in physical optics!

5. VirtualLab Fusion decides about application of homeomorphic Fourier transform automatically on basis of numerical accuracy!



Homeomorphic Sequences in Physical Optics

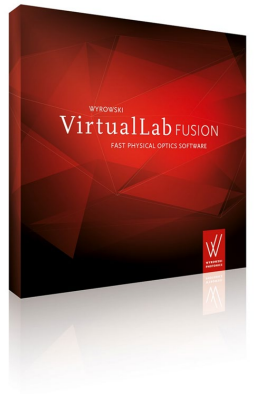
1. Modeling operators ***B*** are often homeomorphic in one domain.

2. If the homeomorphic Fourier transform connects such 1:1 operators,

Ray tracing is properly formulated and accessible through physical optics!

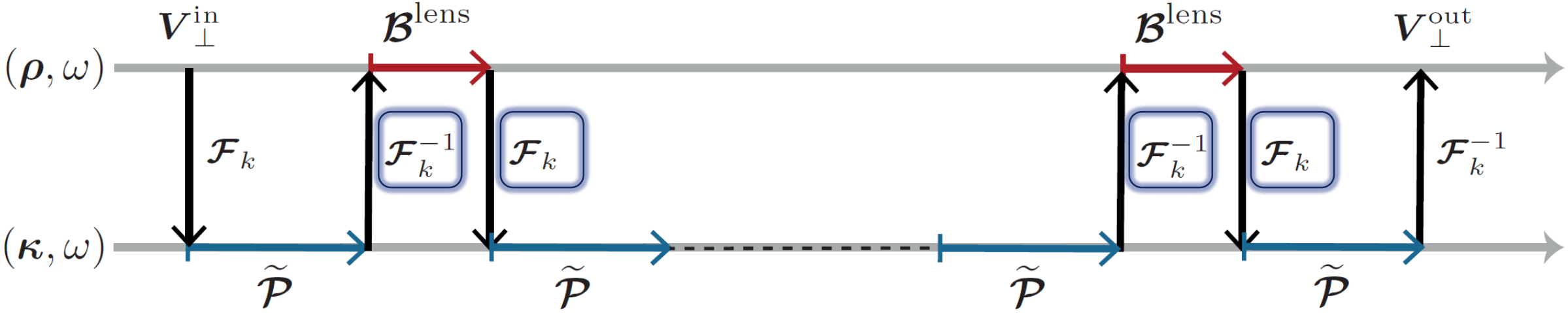
4. Ray tracing is embedded in 1:1 mapping sequences in physical optics!

5. VirtualLab Fusion decides about application of homeomorphic Fourier transform automatically on basis of numerical accuracy!

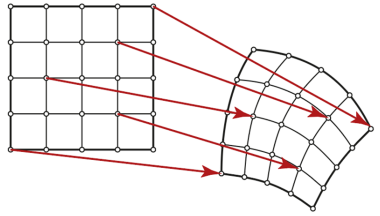


Examples

Conventional Lens System Modeling

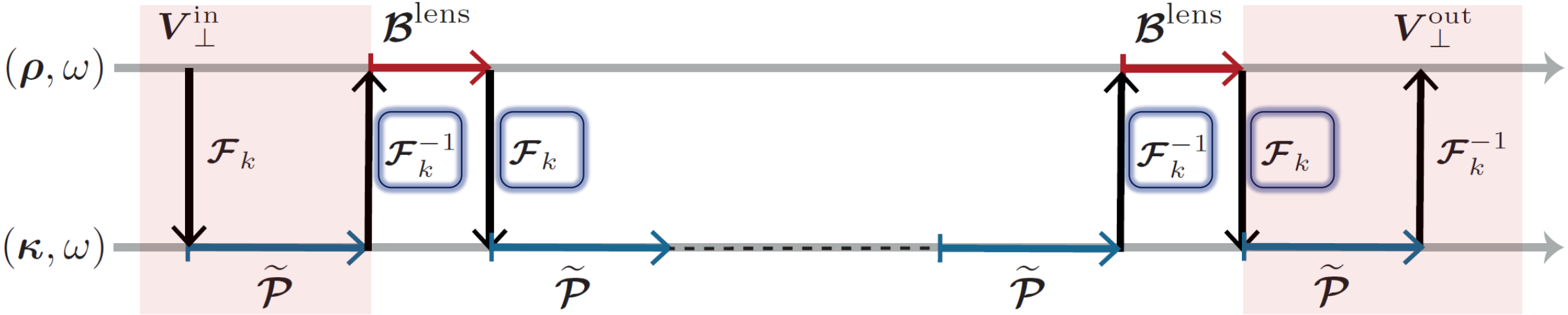


$\mathcal{F}$ Homeomorphic Fourier Transform

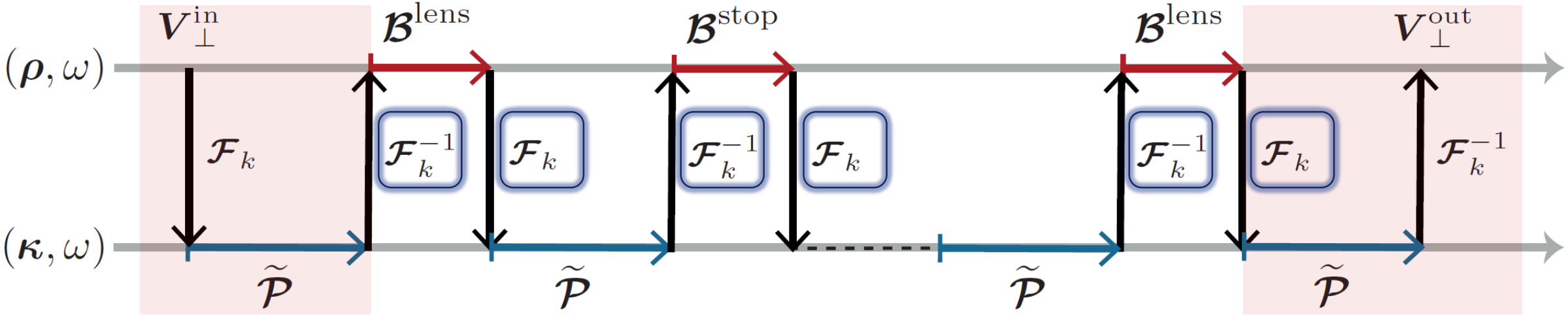


Conventional Lens System Modeling

$\mathcal{F}$ Homeomorphic Fourier Transform

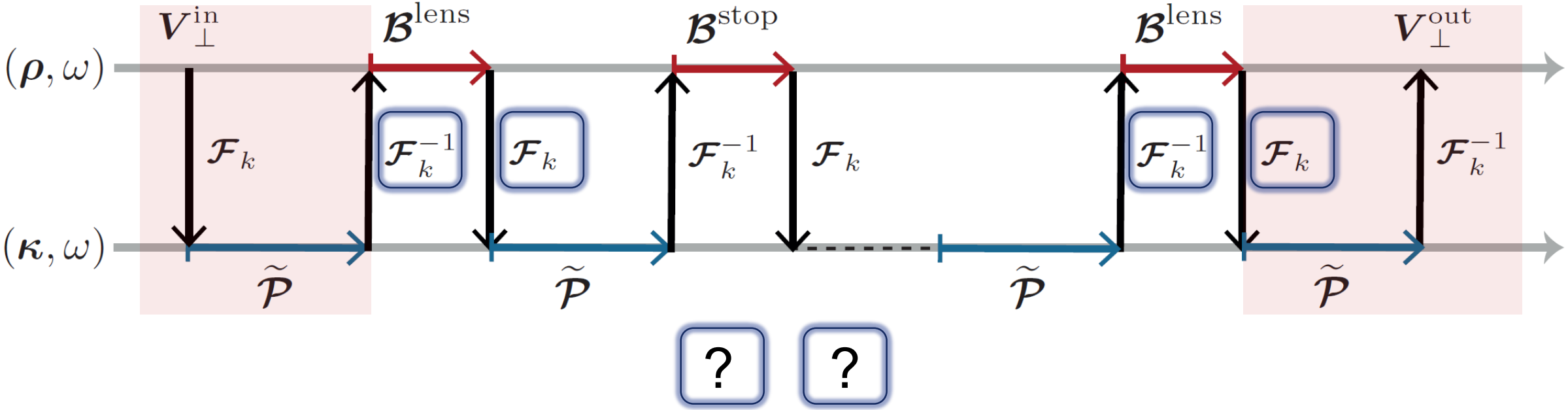


Conventional Lens System Modeling

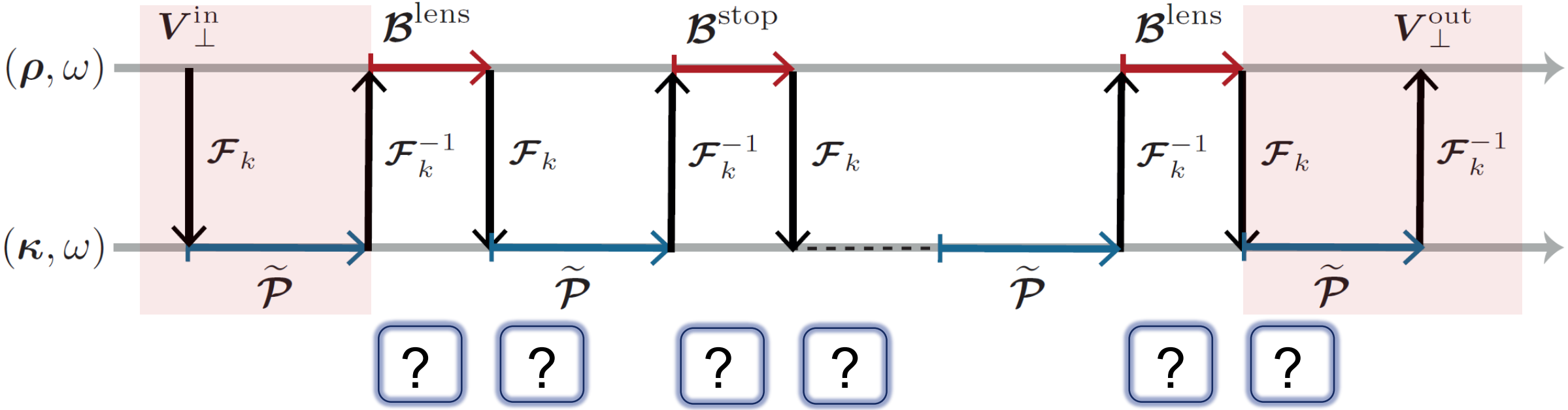


Diffraction Inside Lens System?

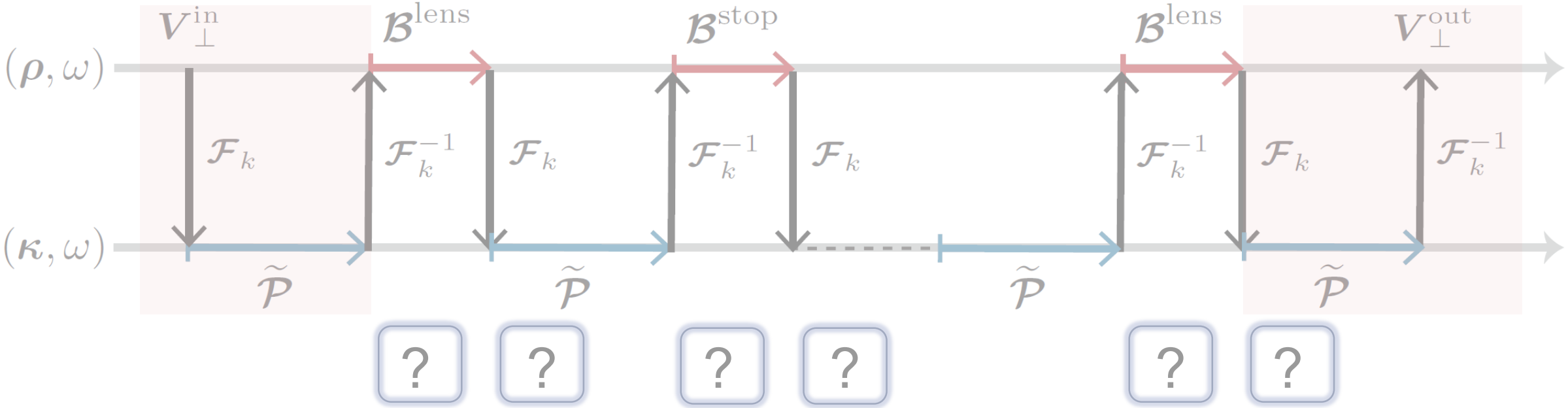
Homeomorphic Fourier Transform



Diffraction Inside Lens System?



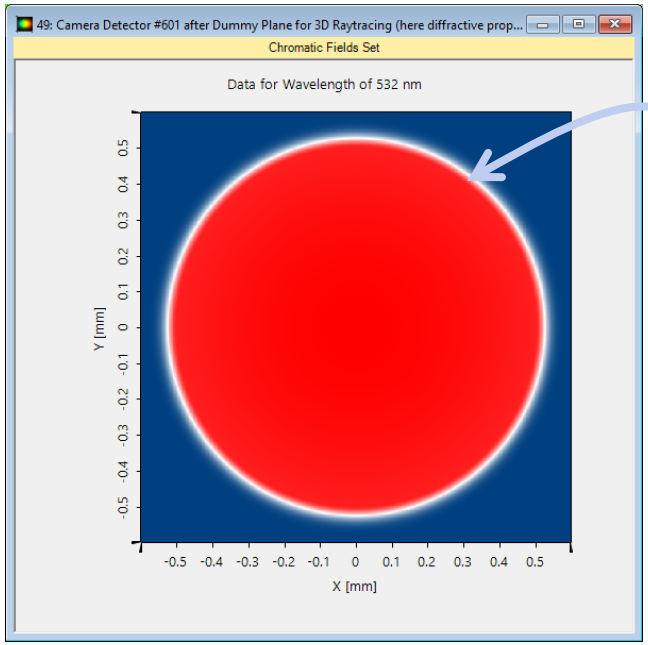
Diffraction Inside Lens System?



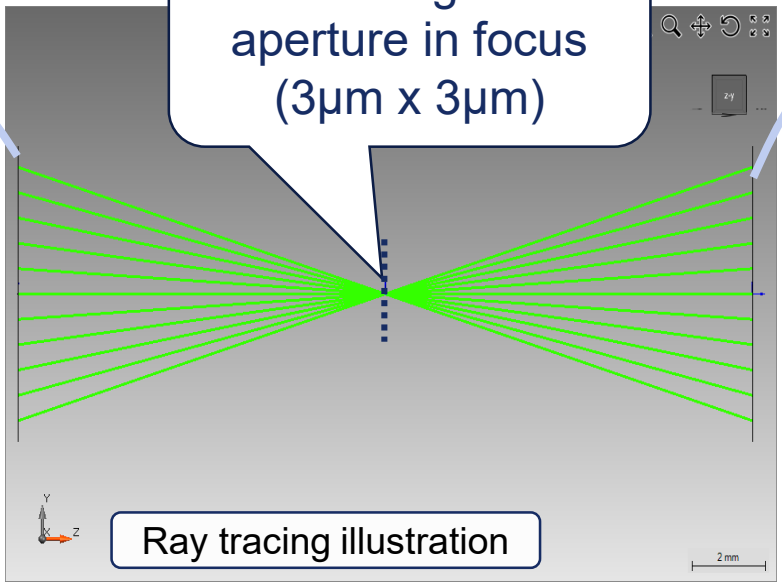
Field tracing considers diffraction of one or more stops/apertures within lens systems.

Propagation of Field Through Aperture: In Focus

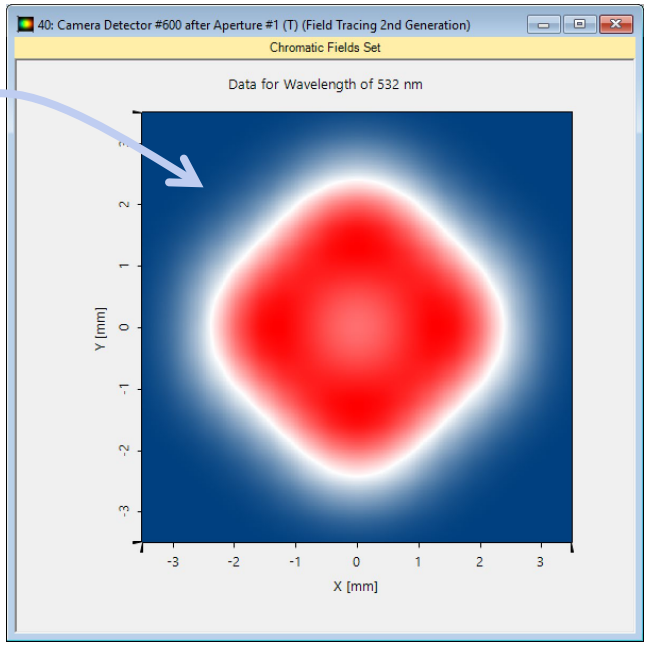
$$I(\boldsymbol{\rho}) = |E_x(\boldsymbol{\rho})|^2 + |E_y(\boldsymbol{\rho})|^2 + |E_z(\boldsymbol{\rho})|^2$$

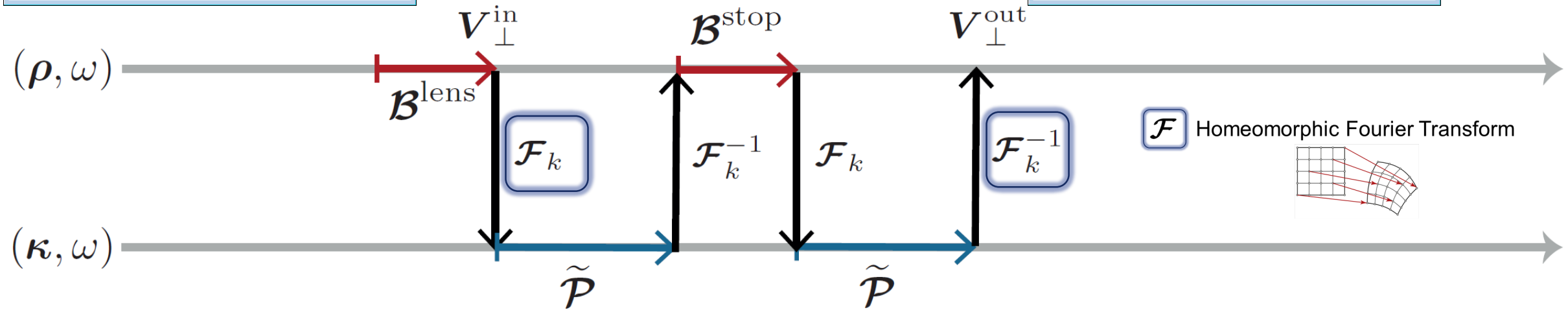


Rectangular aperture in focus (3µm x 3µm)



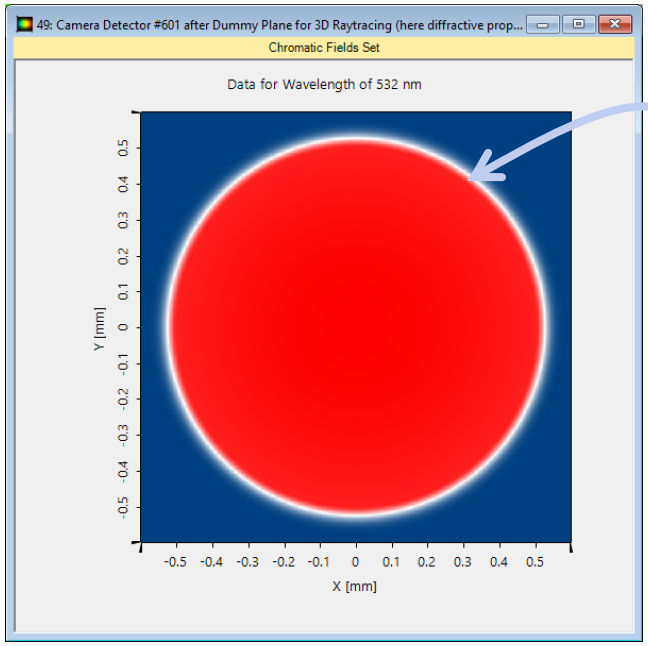
$$I(\boldsymbol{\rho}) = |E_x(\boldsymbol{\rho})|^2 + |E_y(\boldsymbol{\rho})|^2 + |E_z(\boldsymbol{\rho})|^2$$

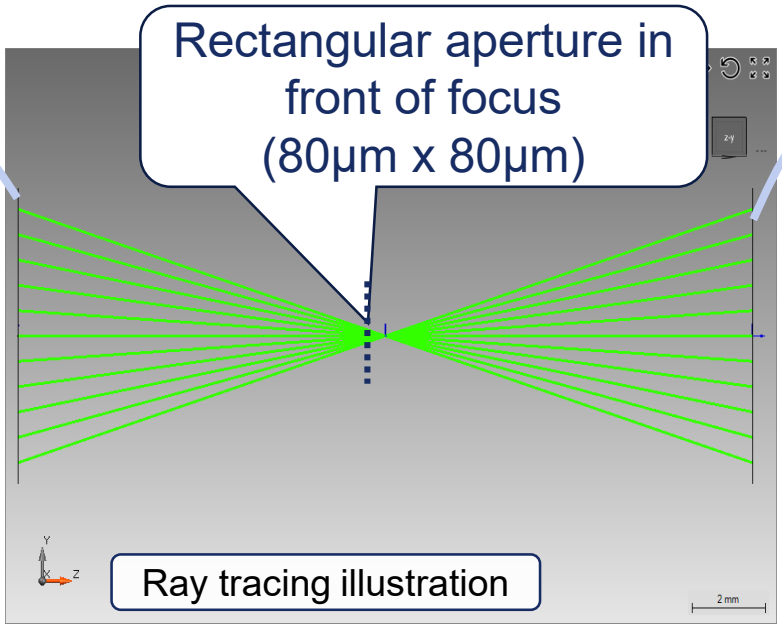




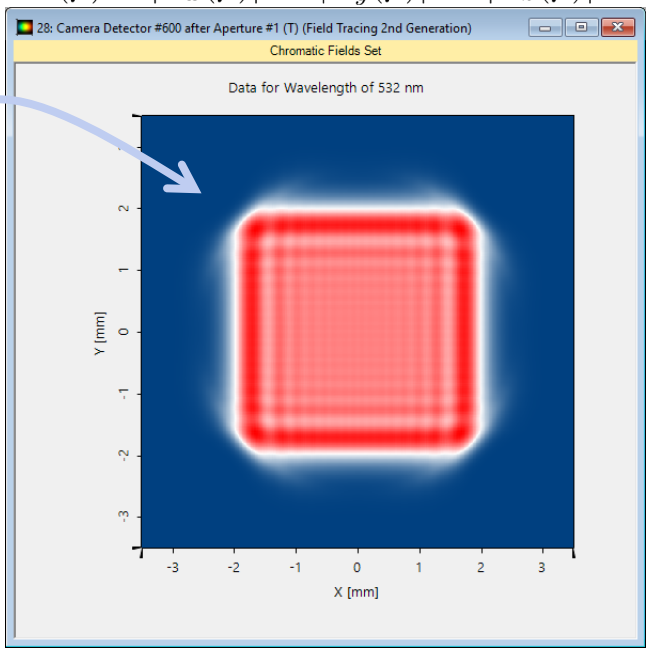
Propagation of Field Through Aperture: Front of Focus

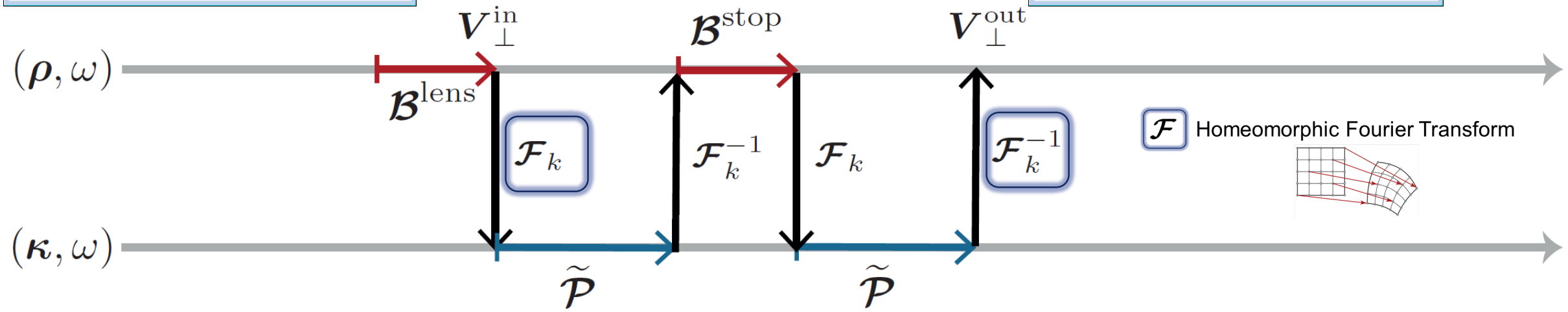
$$I(\boldsymbol{\rho}) = |E_x(\boldsymbol{\rho})|^2 + |E_y(\boldsymbol{\rho})|^2 + |E_z(\boldsymbol{\rho})|^2$$





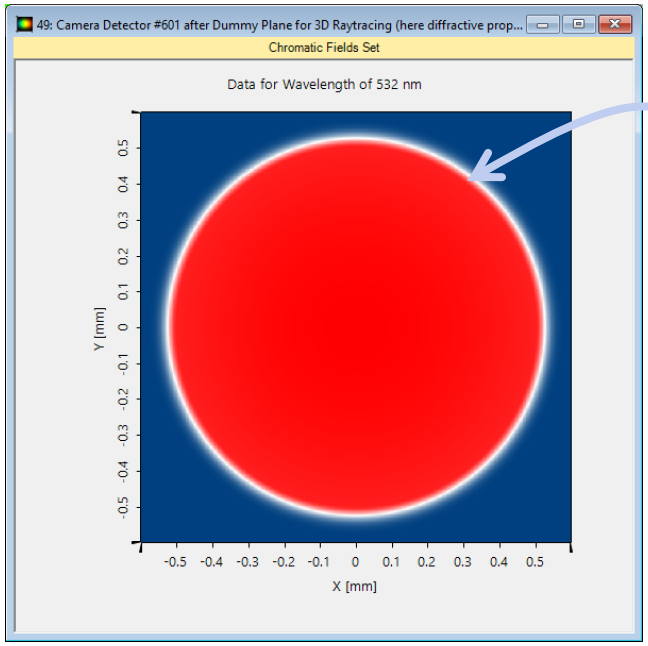
$$I(\boldsymbol{\rho}) = |E_x(\boldsymbol{\rho})|^2 + |E_y(\boldsymbol{\rho})|^2 + |E_z(\boldsymbol{\rho})|^2$$

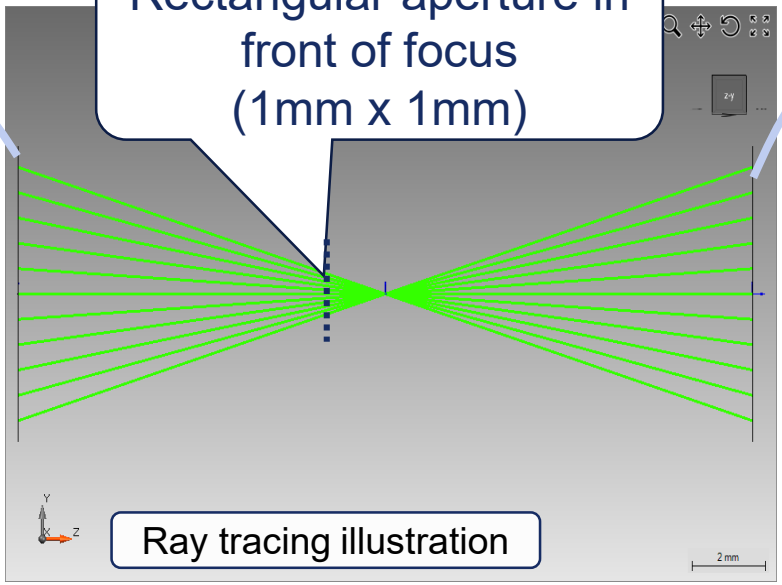




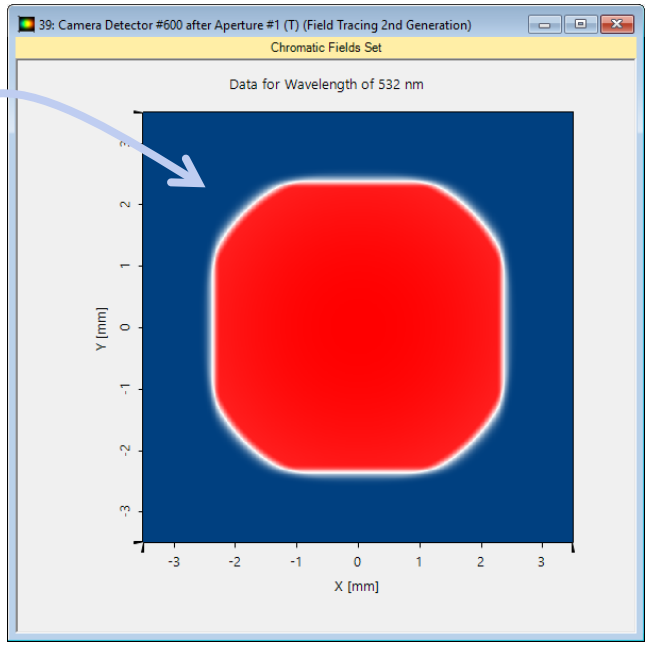
Propagation of Field Through Aperture: Front of Focus

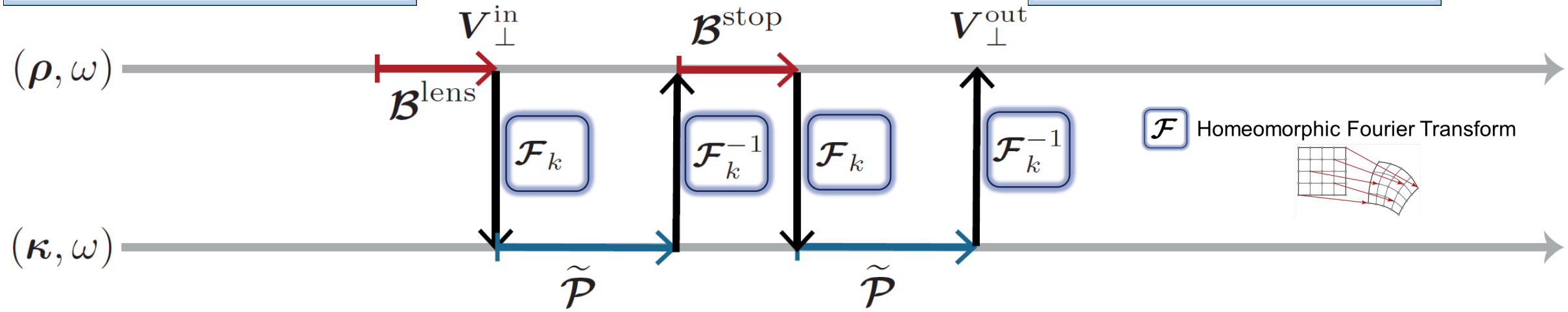
$$I(\boldsymbol{\rho}) = |E_x(\boldsymbol{\rho})|^2 + |E_y(\boldsymbol{\rho})|^2 + |E_z(\boldsymbol{\rho})|^2$$



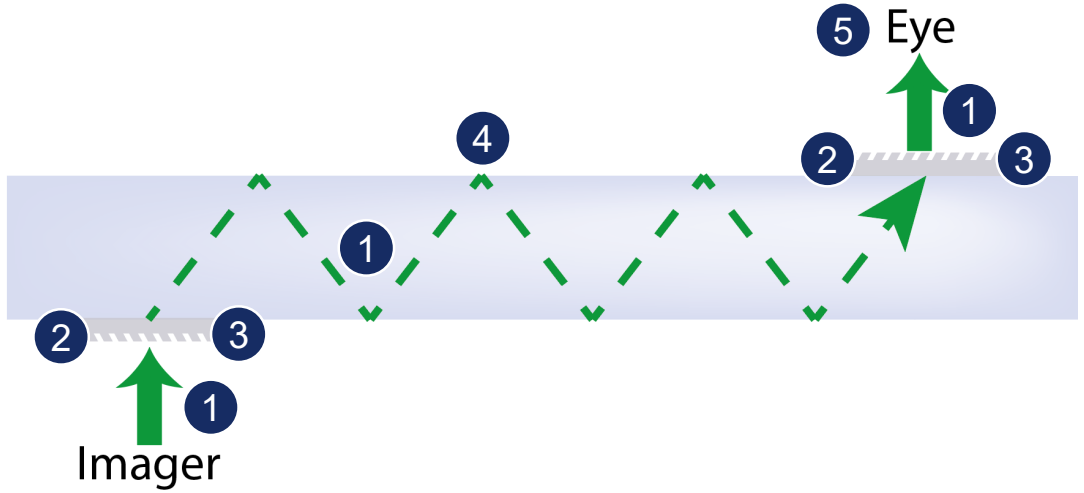


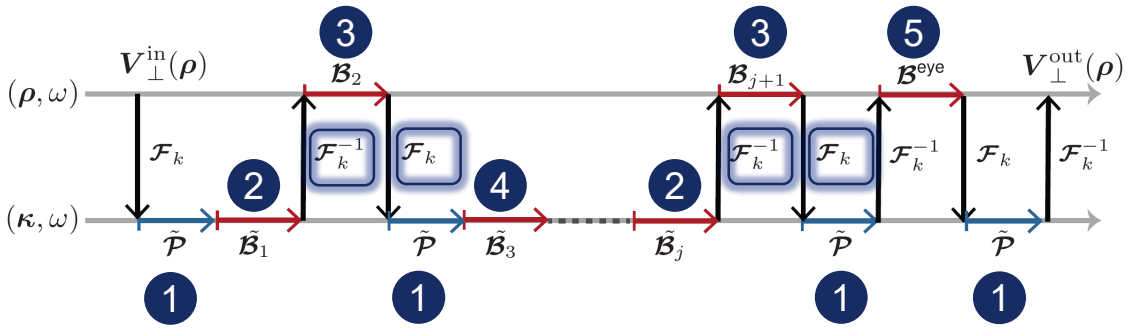
$$I(\boldsymbol{\rho}) = |E_x(\boldsymbol{\rho})|^2 + |E_y(\boldsymbol{\rho})|^2 + |E_z(\boldsymbol{\rho})|^2$$



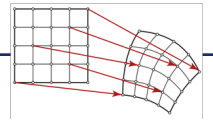


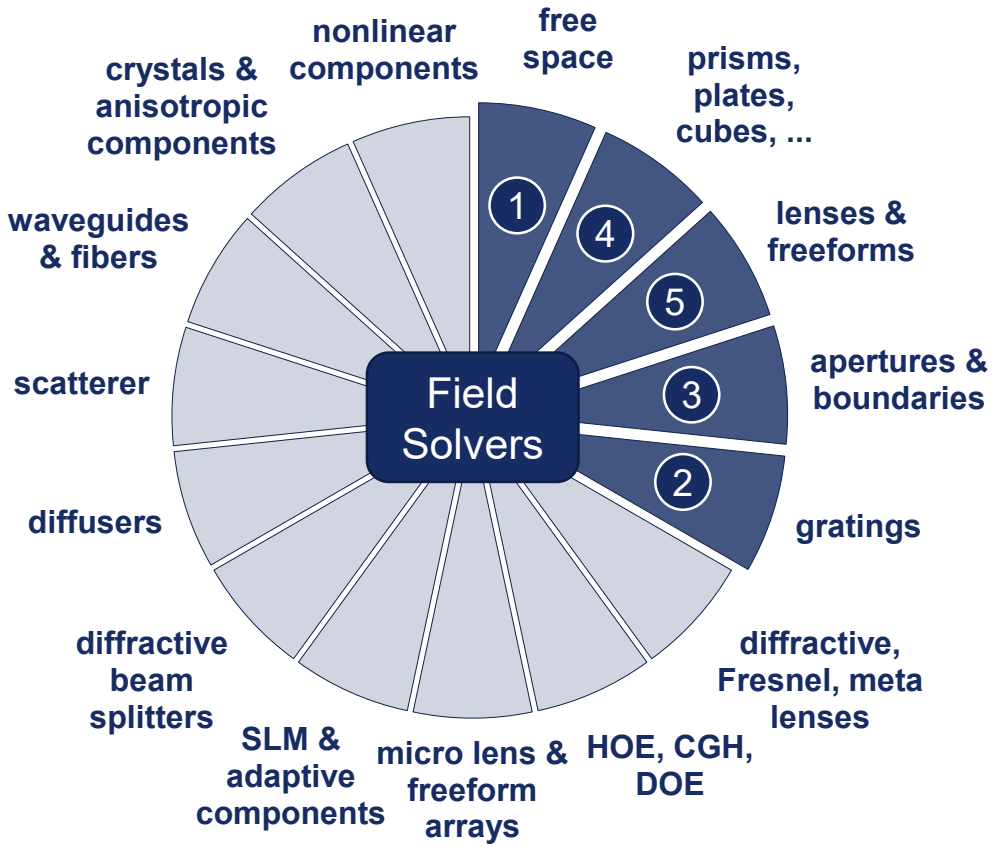
Typical Modeling Situation for AR&MR Lightguide





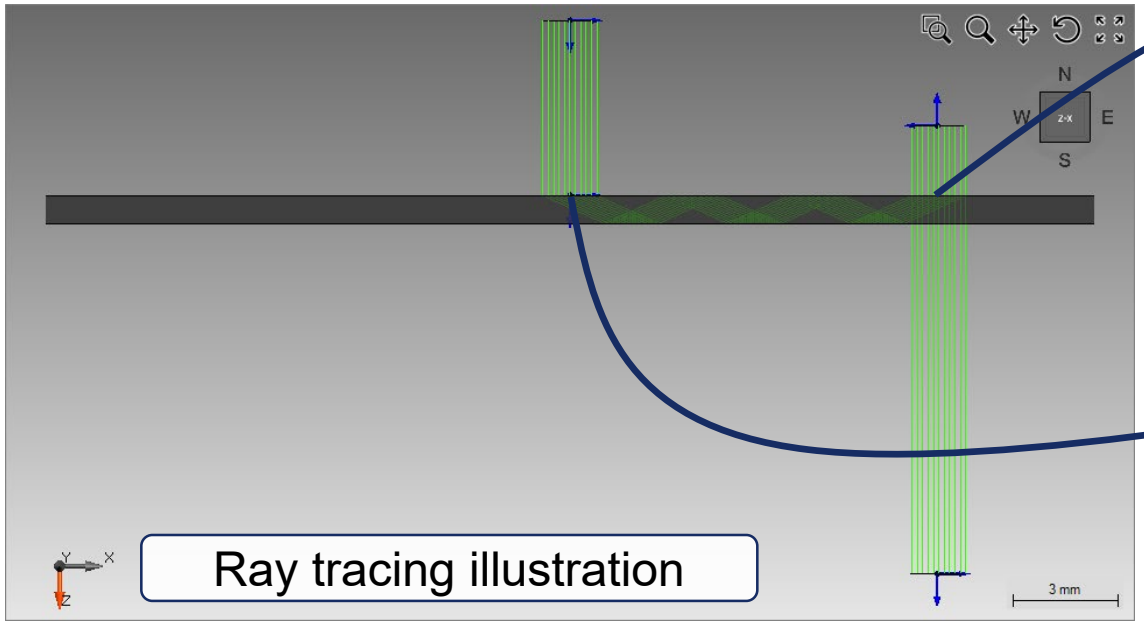
$\mathcal{F}$ Homeomorphic Fourier Transform



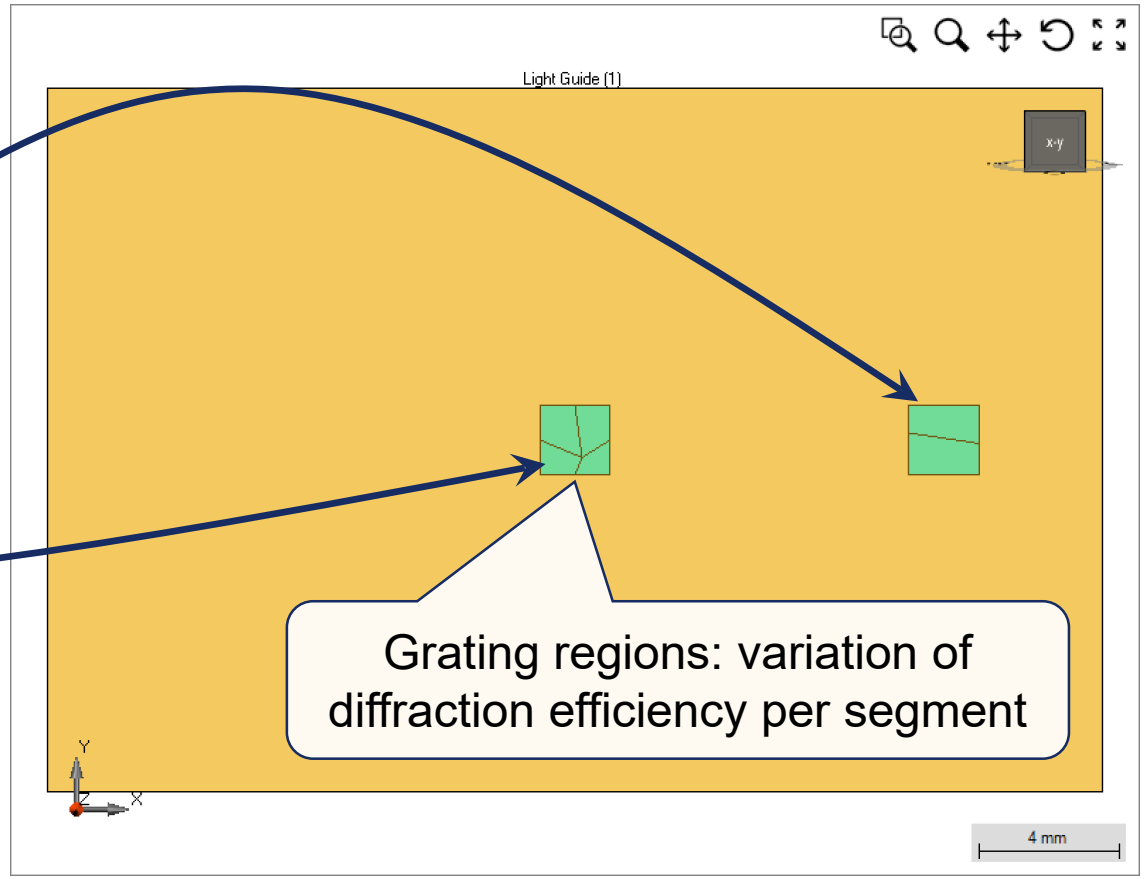


Cascaded Diffraction in Lightguide Modeling: Layout

Lightguide: Front View

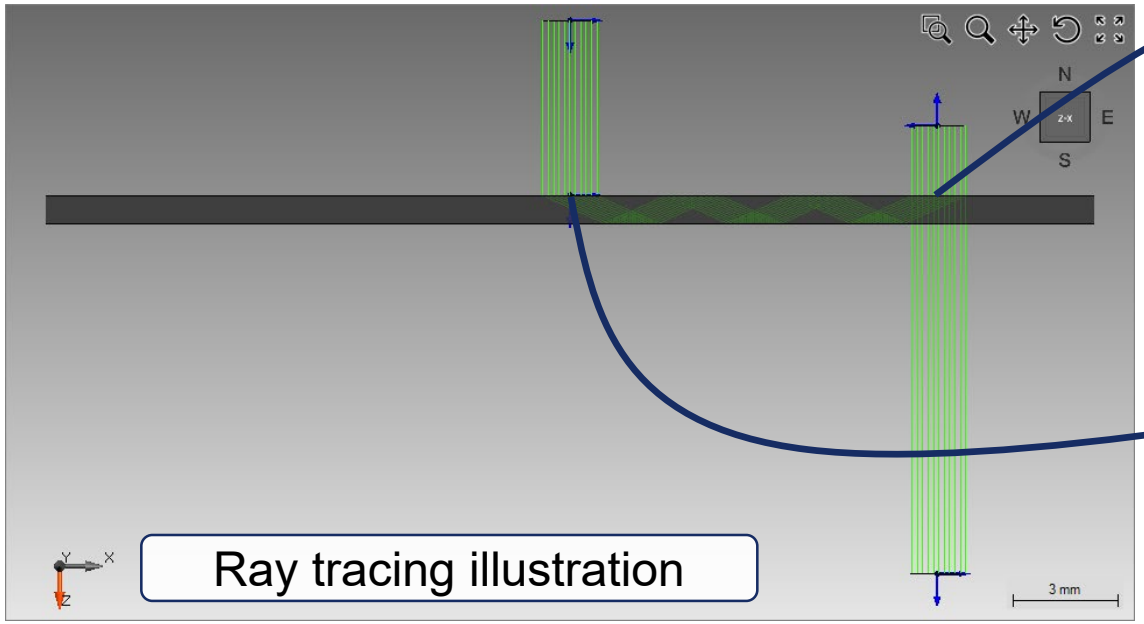


Lightguide: Top View

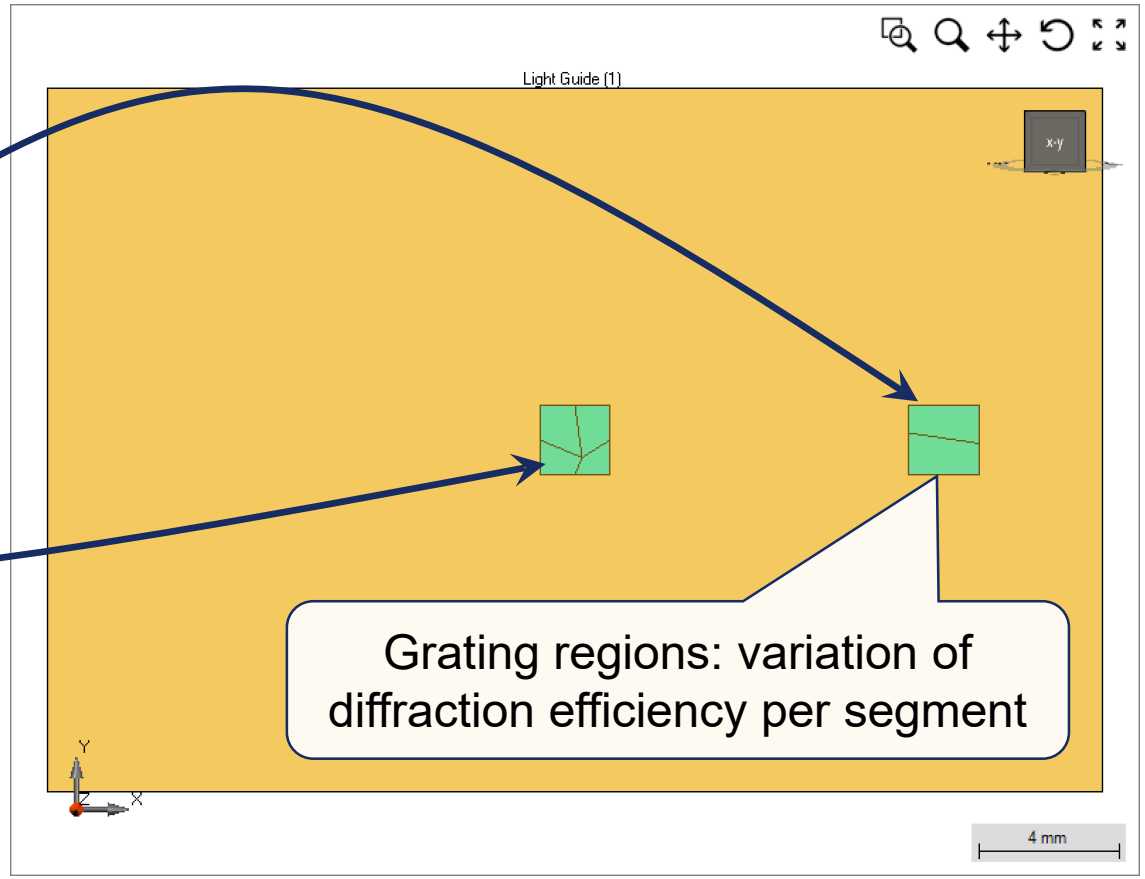


Cascaded Diffraction in Lightguide Modeling: Layout

Lightguide: Front View

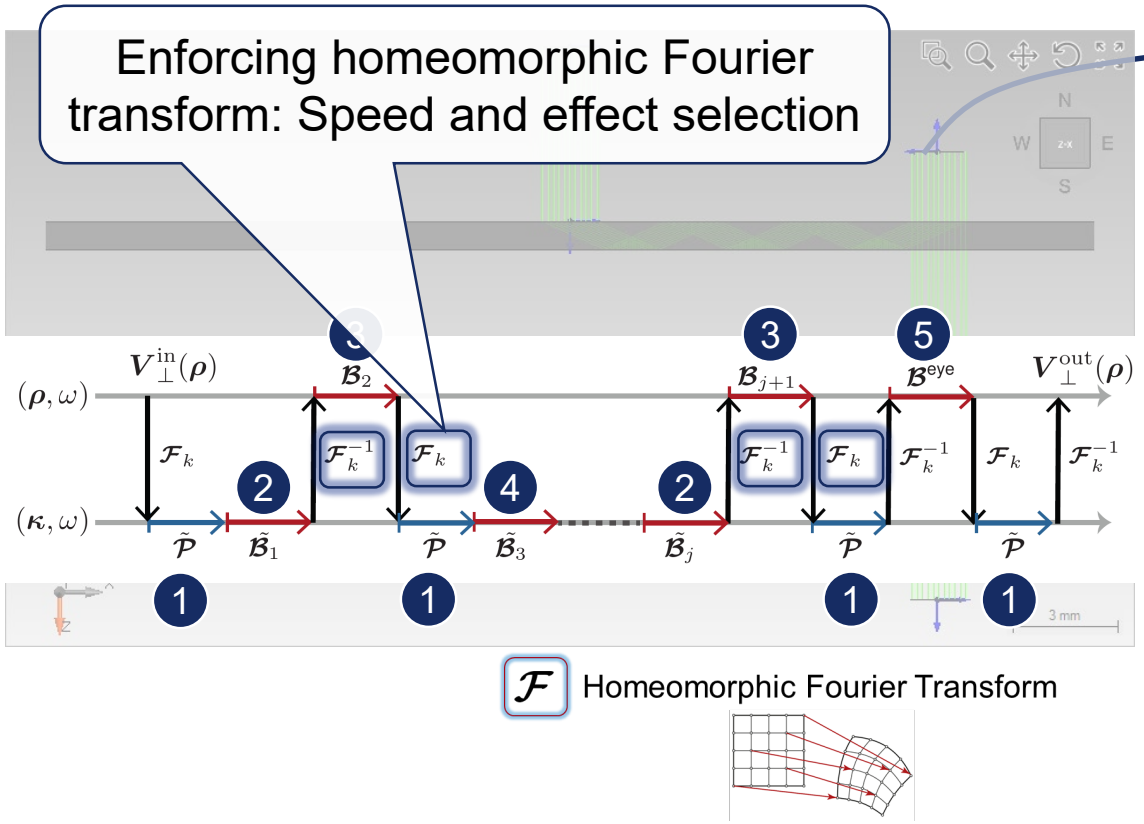


Lightguide: Top View



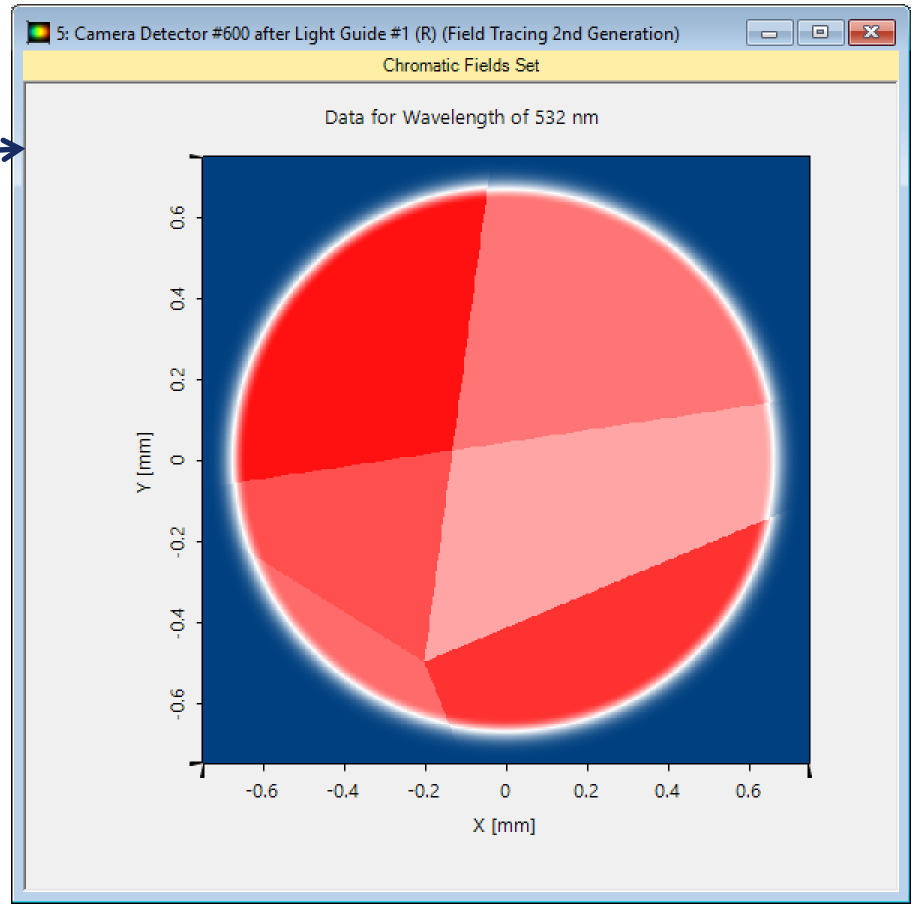
Lightguide Modeling: Suppressed Diffraction Effects (Homeomorphic)

Lightguide: Front View



$\mathcal{F}$ Homeomorphic Fourier Transform

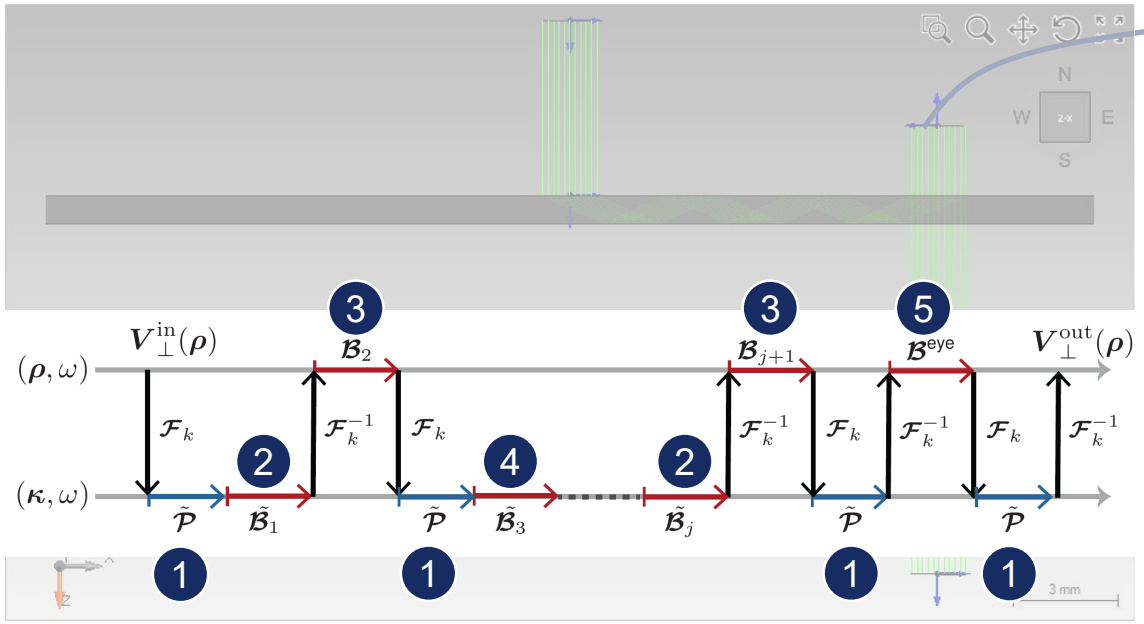
Intensity (homeomorphic model)



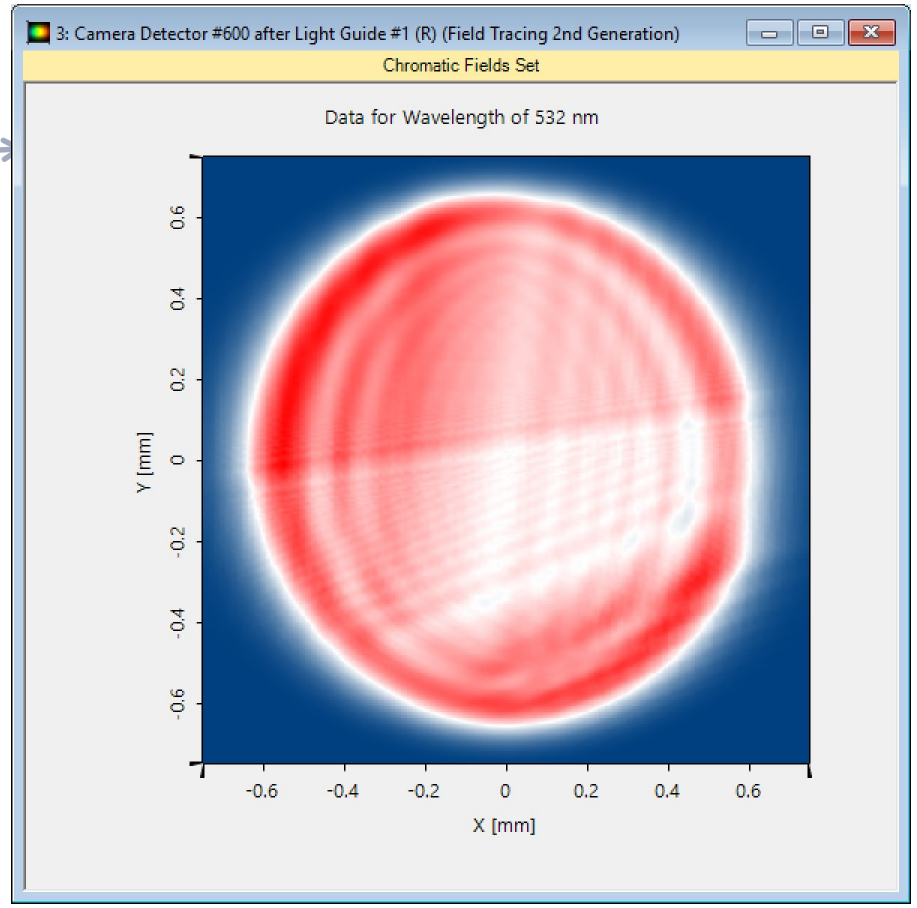
$$I(\boldsymbol{\rho}) = |E_x(\boldsymbol{\rho})|^2 + |E_y(\boldsymbol{\rho})|^2 + |E_z(\boldsymbol{\rho})|^2$$

Lightguide Modeling: Diffraction Effects Included

Lightguide: Front View

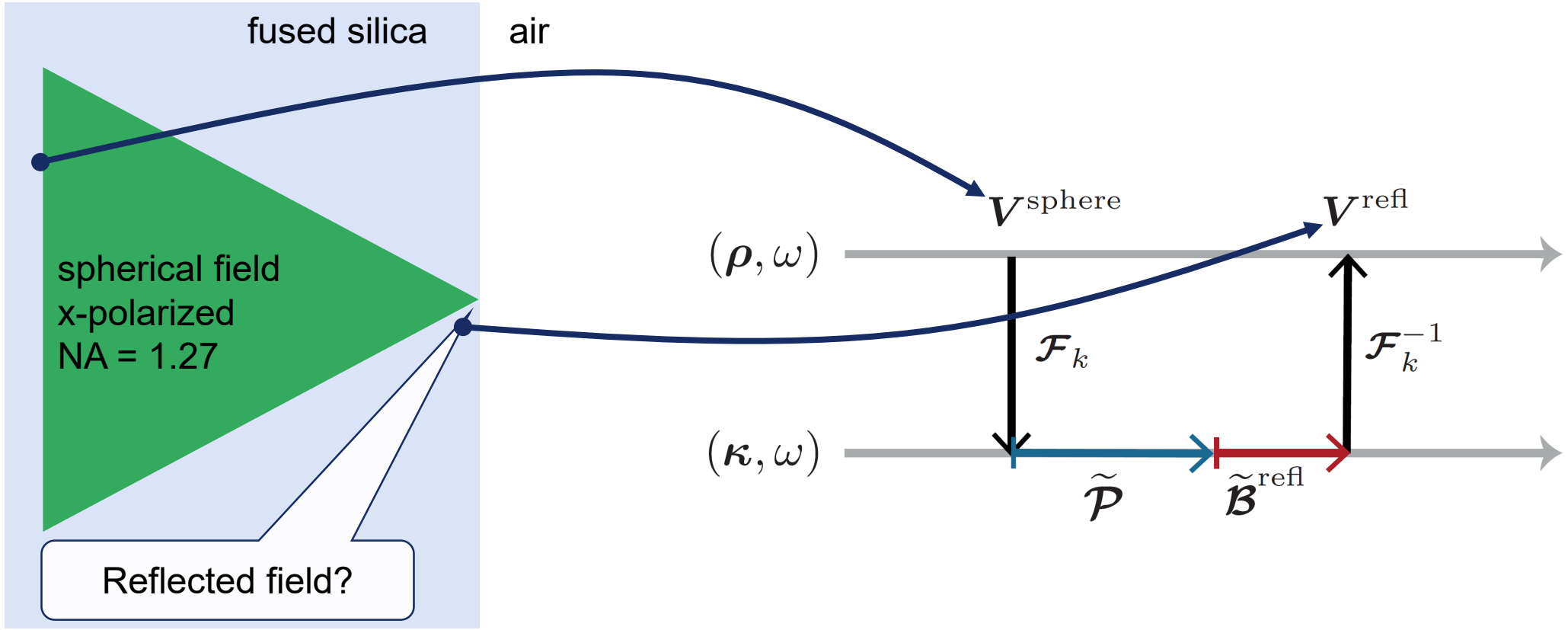


Intensity



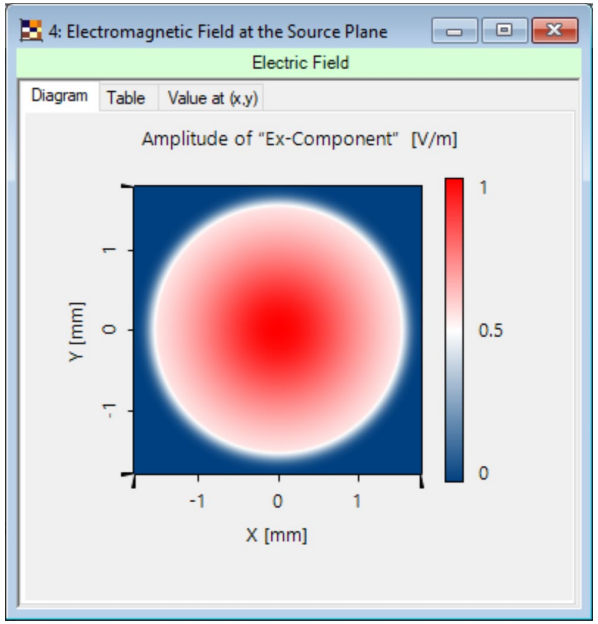
$$I(\boldsymbol{\rho}) = |E_x(\boldsymbol{\rho})|^2 + |E_y(\boldsymbol{\rho})|^2 + |E_z(\boldsymbol{\rho})|^2$$

Reflected Field at Interface Fused Silica to Air

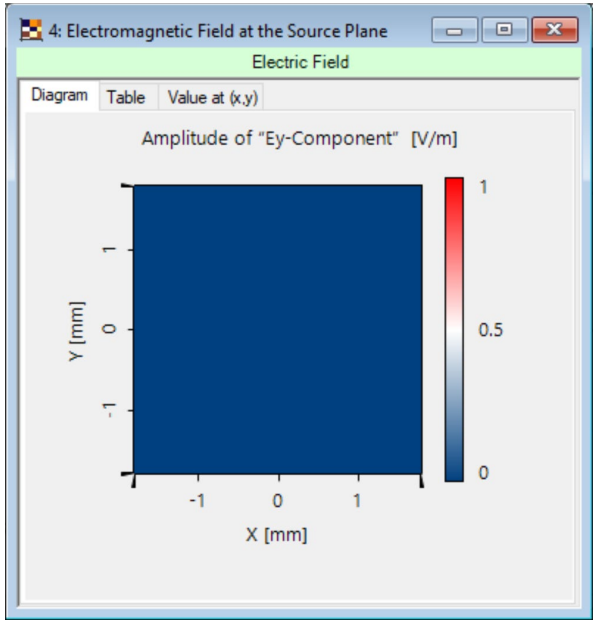


Electromagnetic Input Field

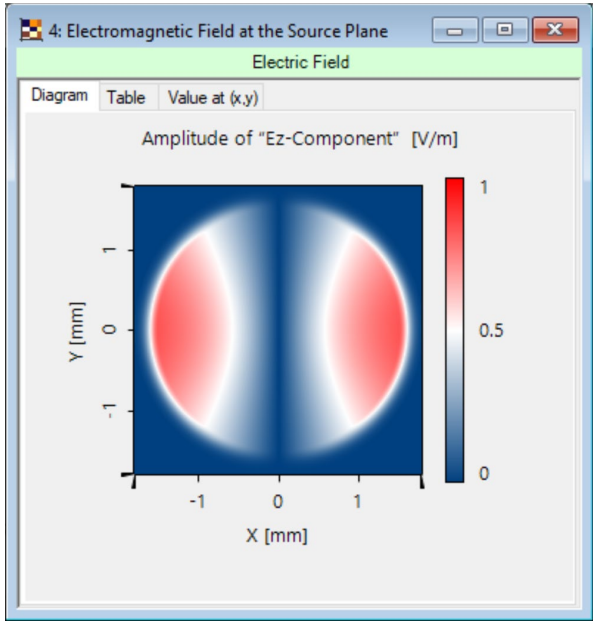
$|E_x(\boldsymbol{\rho})|$



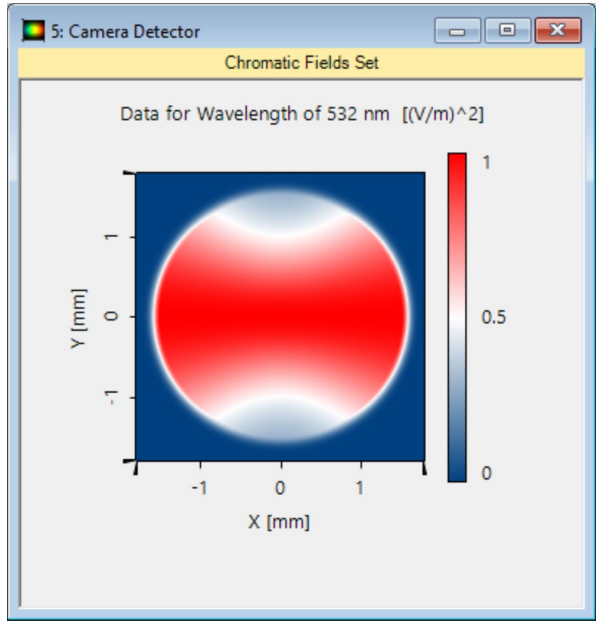
$|E_y(\boldsymbol{\rho})|$

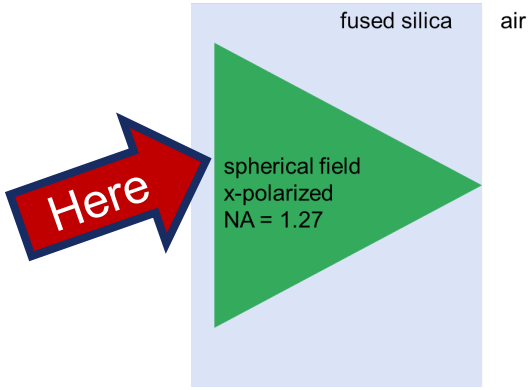


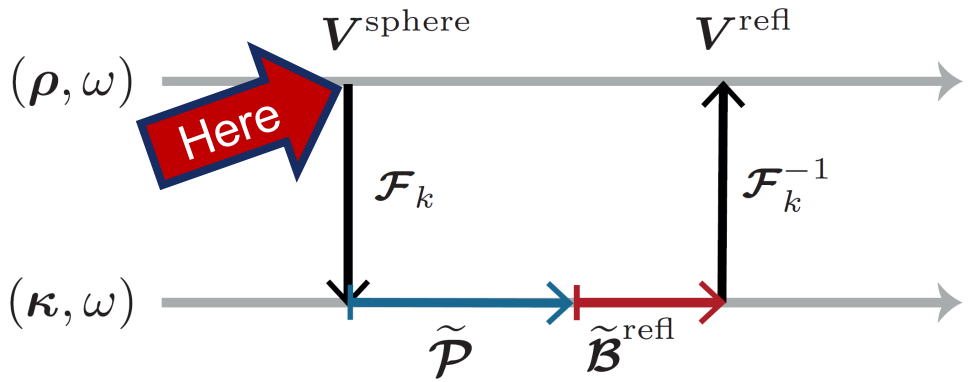
$|E_z(\boldsymbol{\rho})|$



$|E_x(\boldsymbol{\rho})|^2 + |E_y(\boldsymbol{\rho})|^2 + |E_z(\boldsymbol{\rho})|^2$

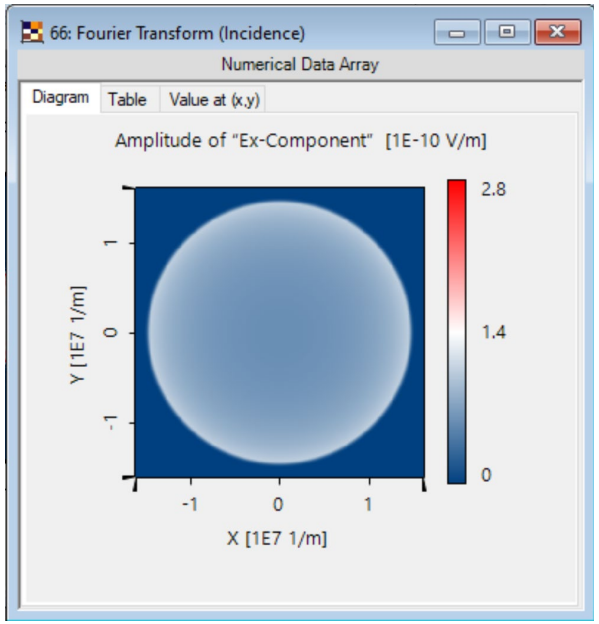




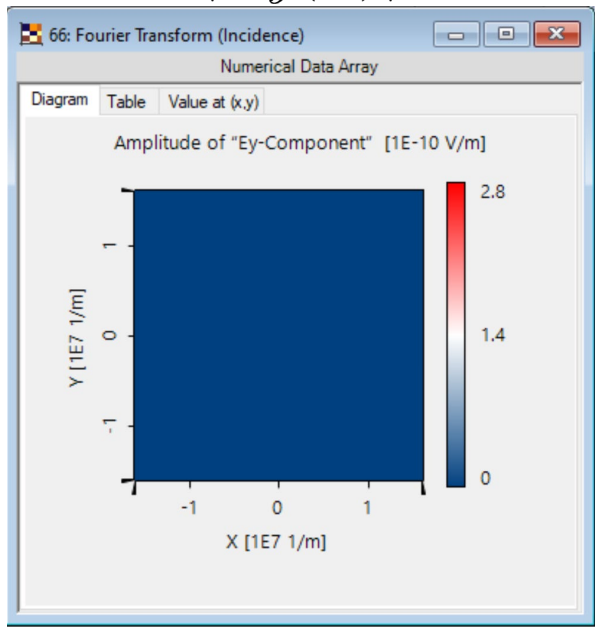


Electromagnetic Spherical Field: *k*-Domain

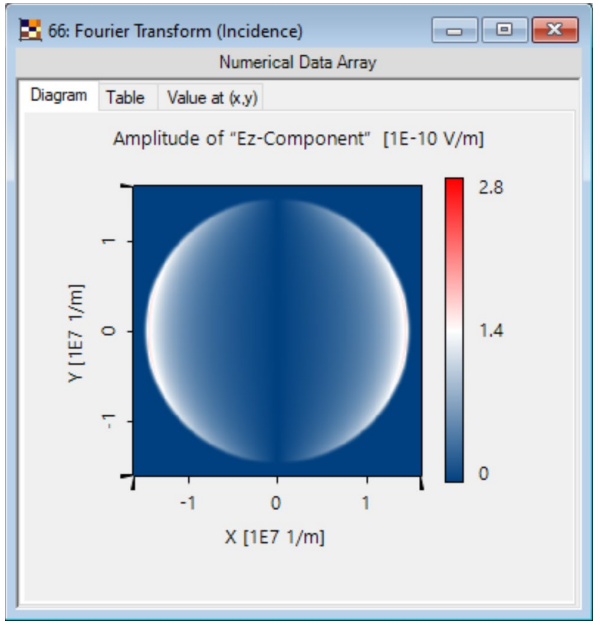
$|\tilde{E}_x(\boldsymbol{\kappa})|$

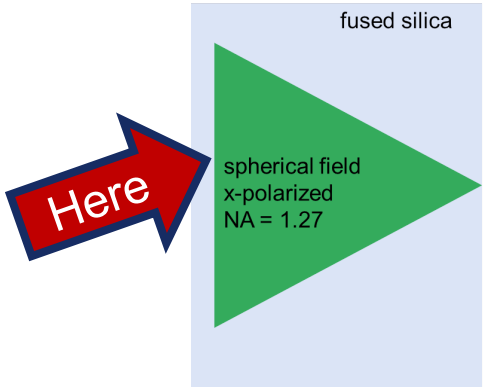


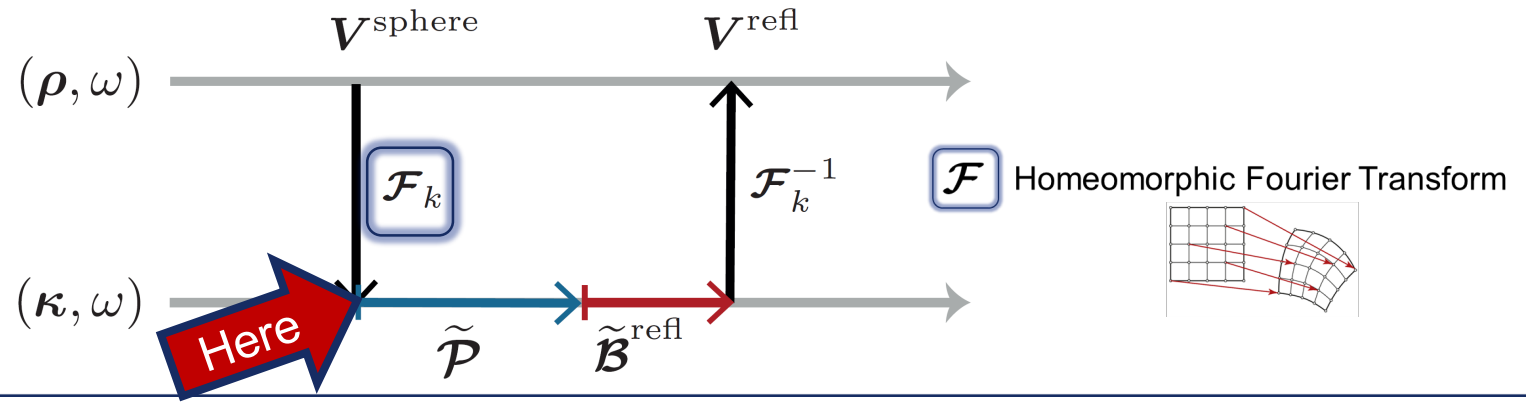
$|\tilde{E}_y(\boldsymbol{\kappa})|$



$|\tilde{E}_z(\boldsymbol{\kappa})|$

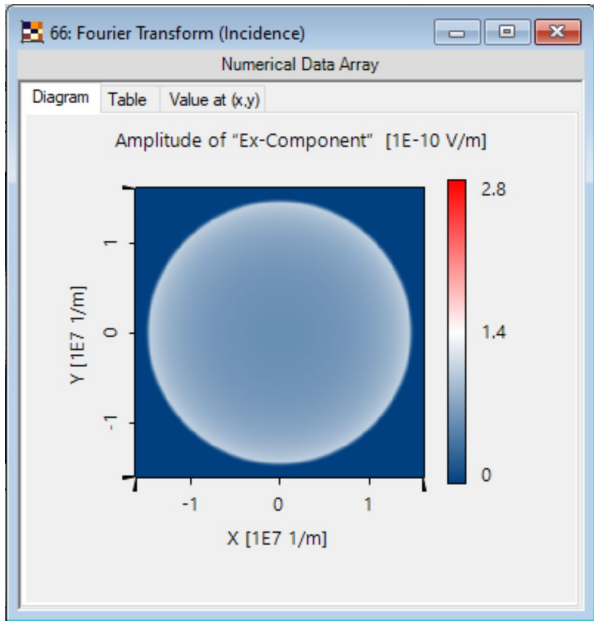




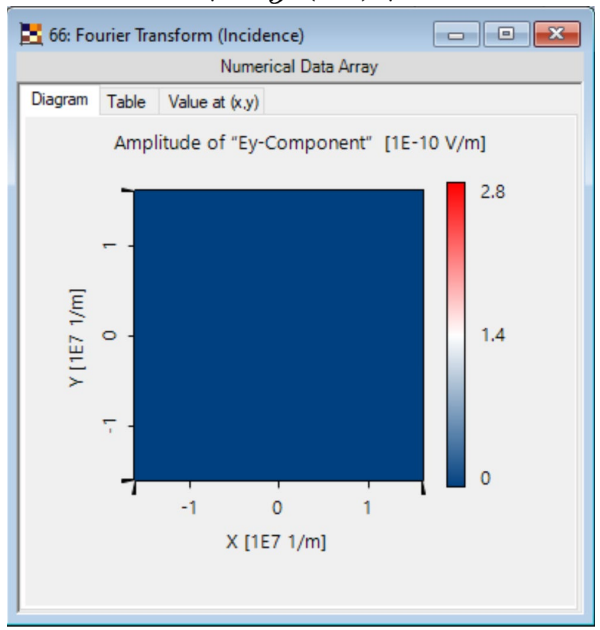


Electromagnetic Spherical Field: *k*-Domain

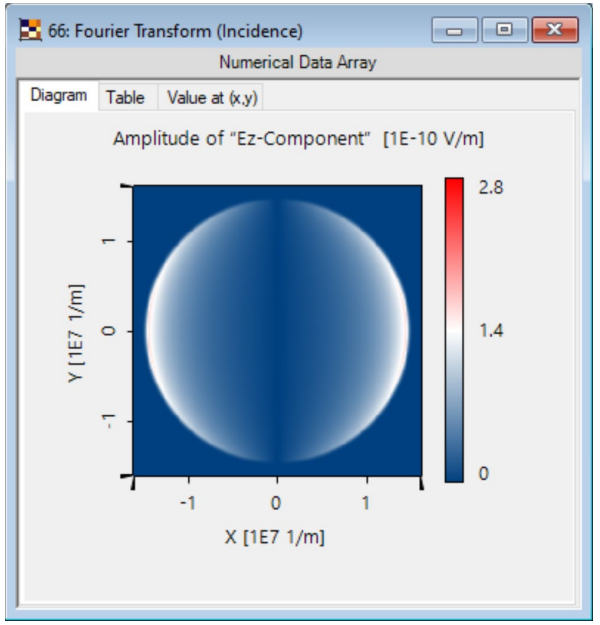
$|\tilde{E}_x(\boldsymbol{\kappa})|$

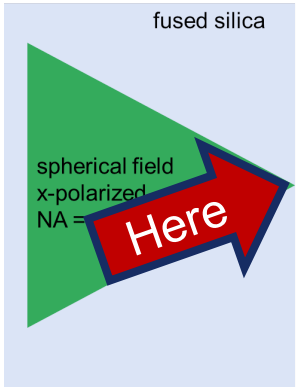


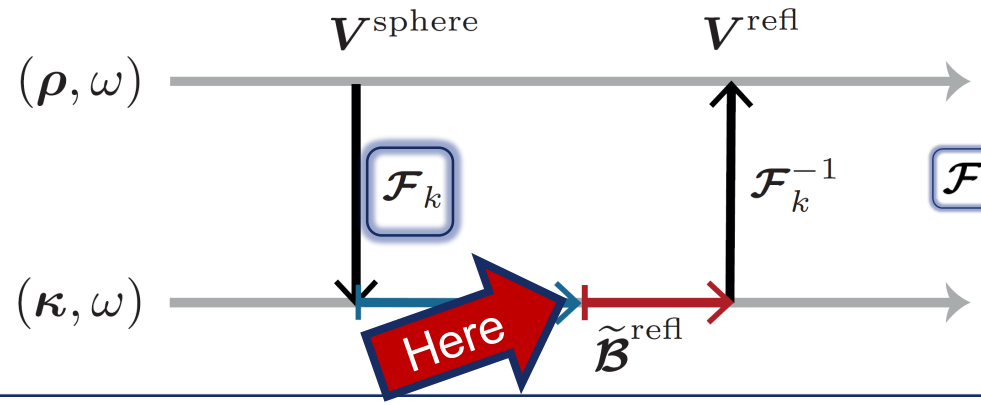
$|\tilde{E}_y(\boldsymbol{\kappa})|$



$|\tilde{E}_z(\boldsymbol{\kappa})|$

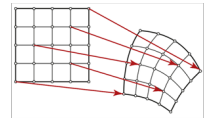






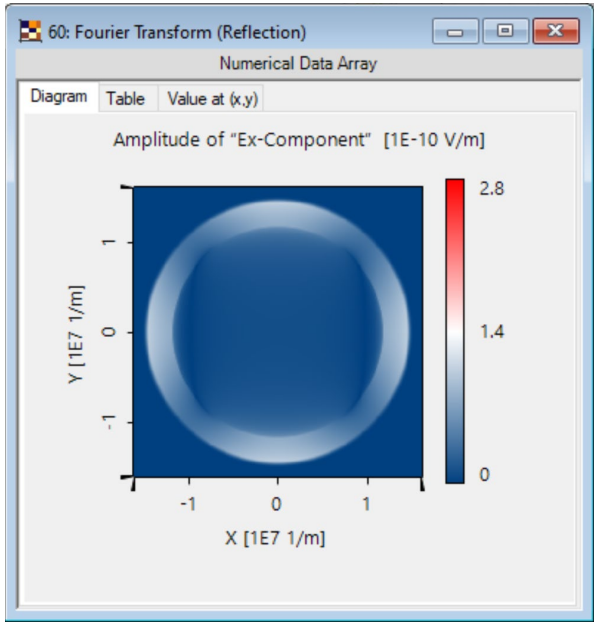
$$\tilde{\boldsymbol{P}}_{||} = \times \exp(\mathrm{i}k_z \Delta z)$$

$\mathcal{F}$ Homeomorphic Fourier Transform

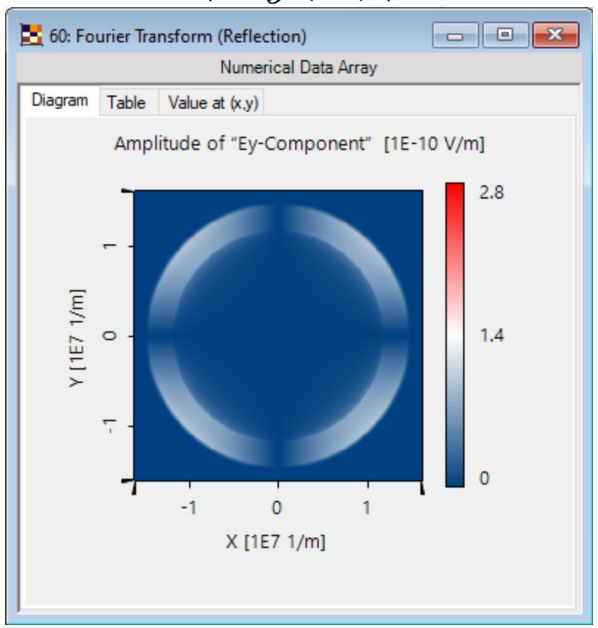


Electromagnetic Spherical Field: *k*-Domain

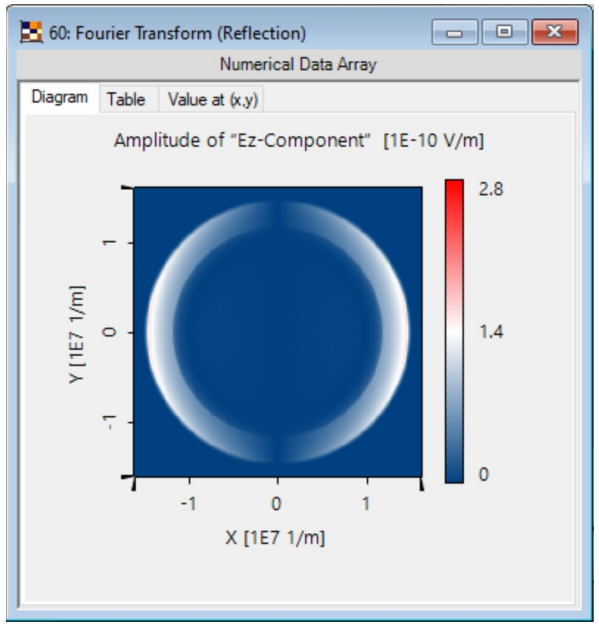
$|\tilde{E}_x(\boldsymbol{\kappa})|$

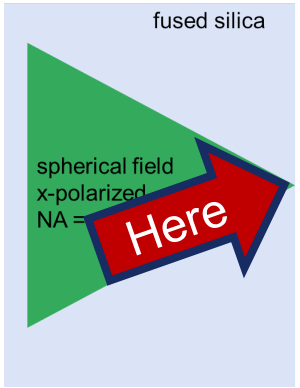


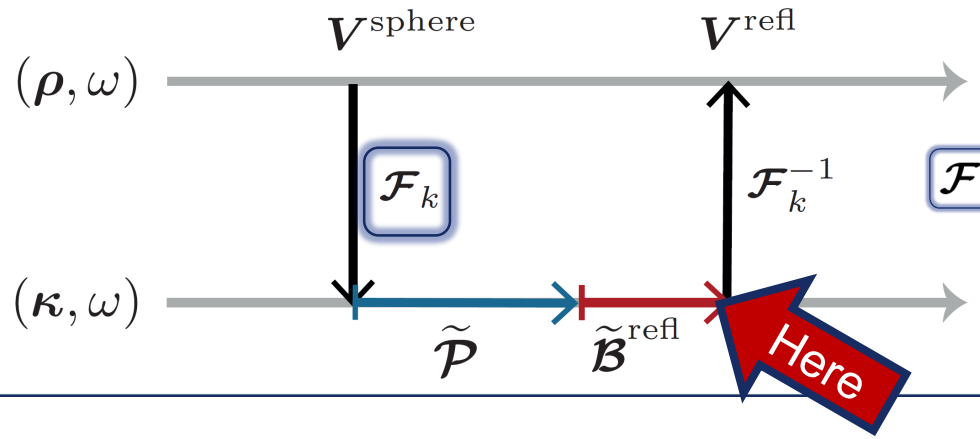
$|\tilde{E}_y(\boldsymbol{\kappa})|$



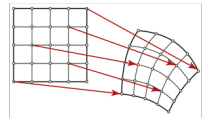
$|\tilde{E}_z(\boldsymbol{\kappa})|$





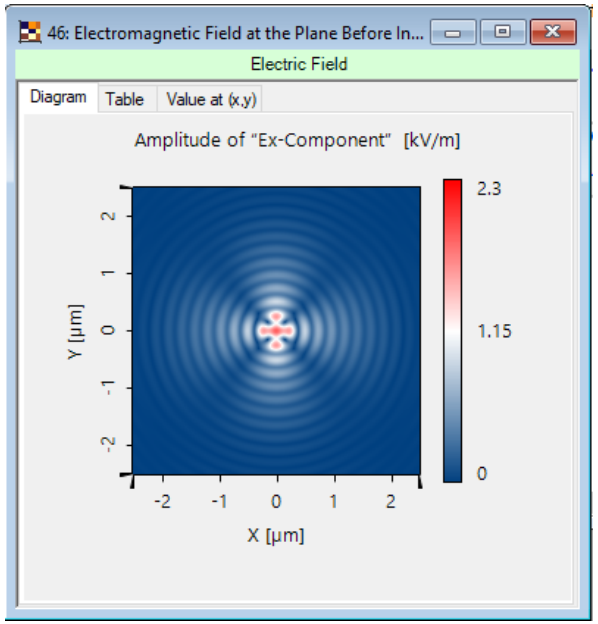


$\mathcal{F}$ Homeomorphic Fourier Transform

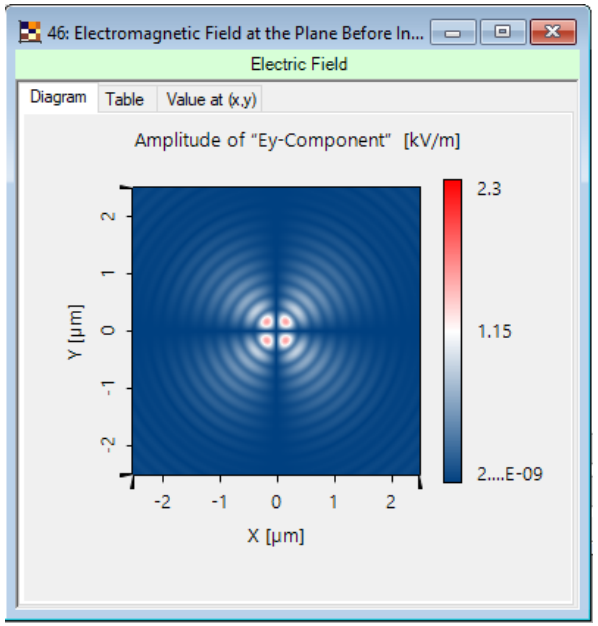


Electromagnetic Spherical Field: *k*-Domain

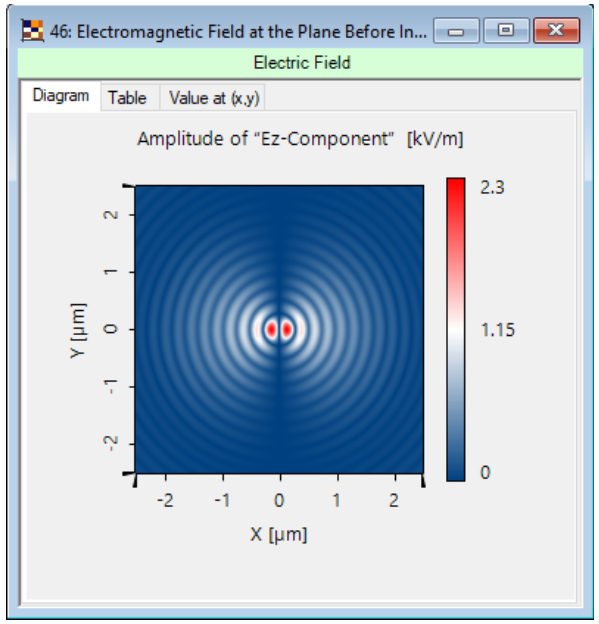
$|E_x(\boldsymbol{\rho})|$



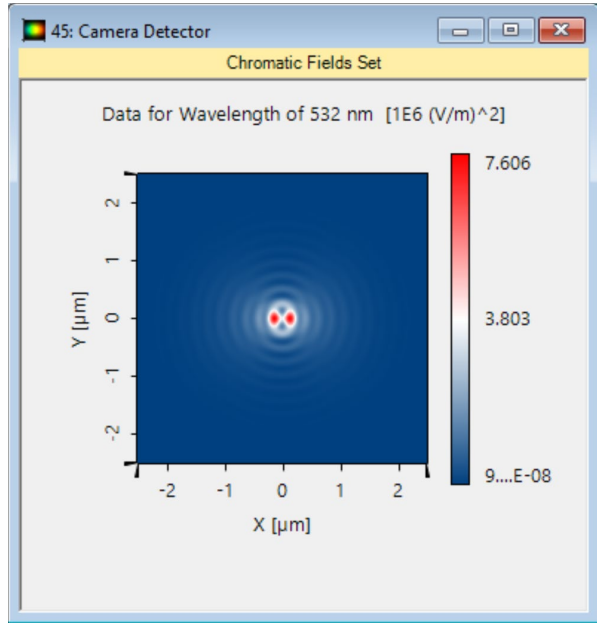
$|E_y(\boldsymbol{\rho})|$

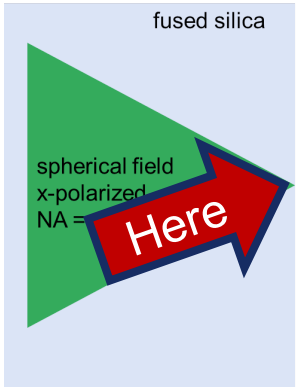


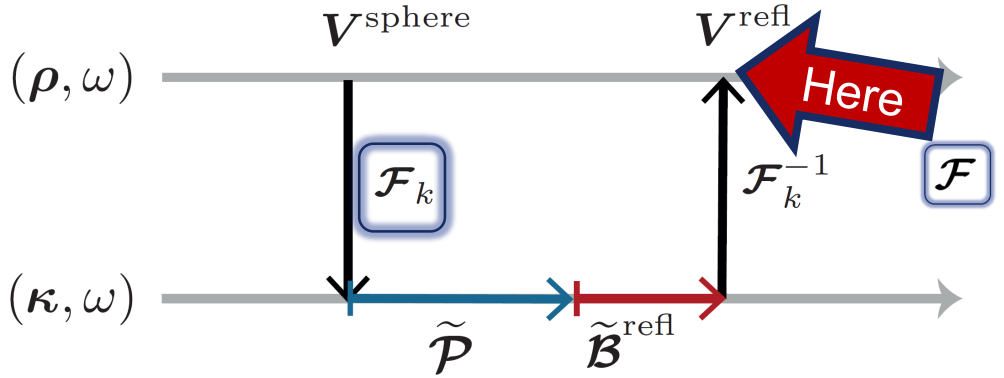
$|E_z(\boldsymbol{\rho})|$



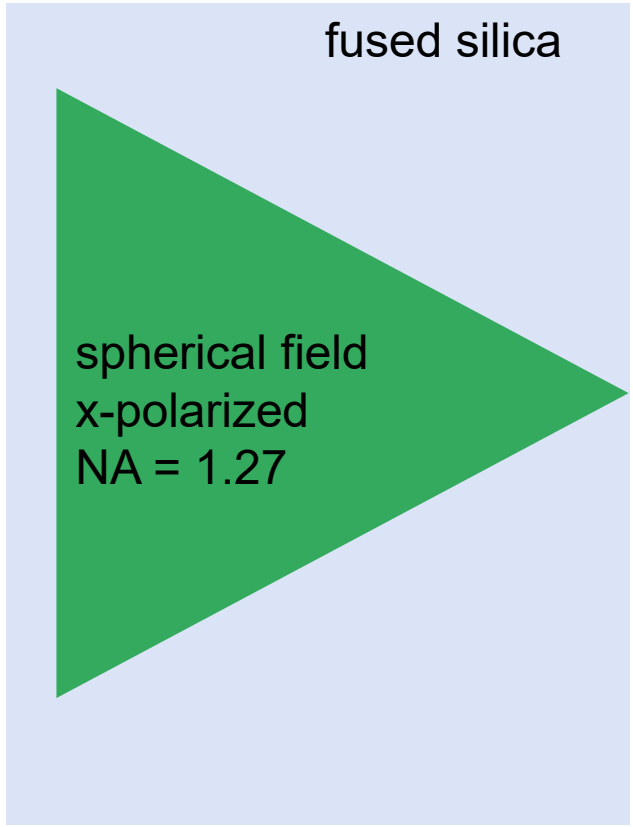
$|E_x(\boldsymbol{\rho})|^2 + |E_y(\boldsymbol{\rho})|^2 + |E_z(\boldsymbol{\rho})|^2$

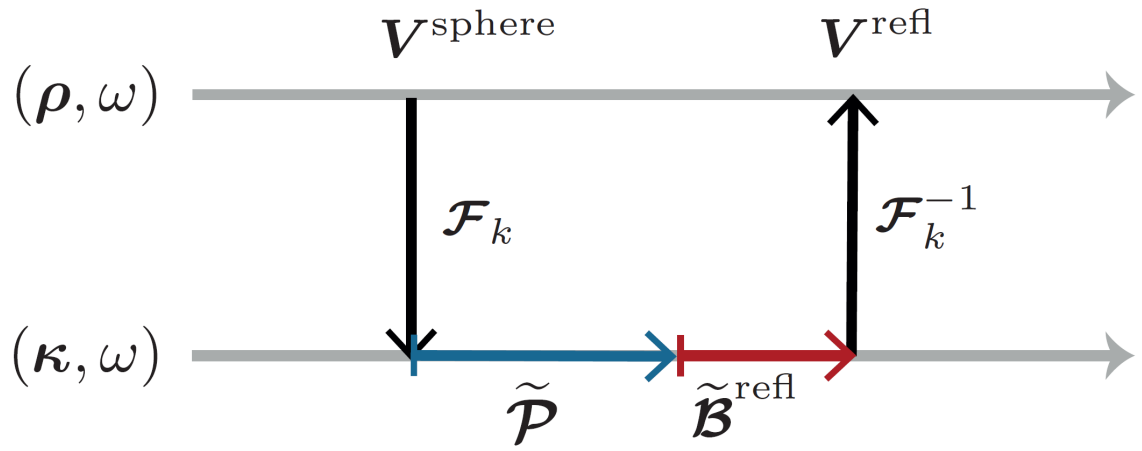




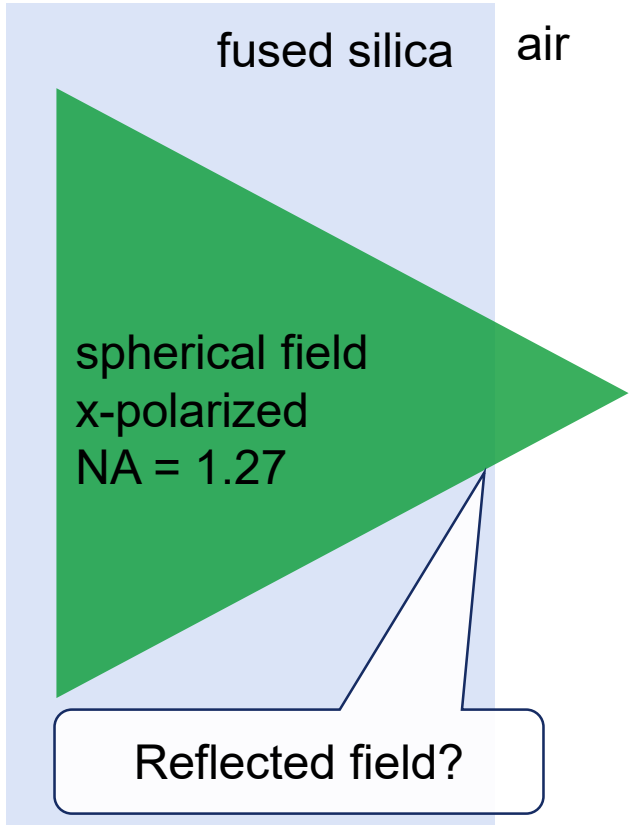


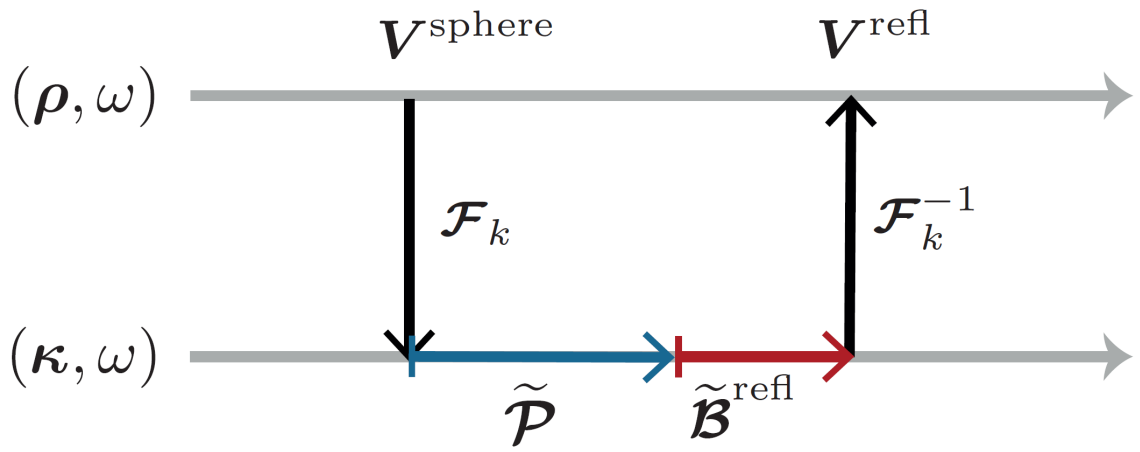
Reflected Field at Interface Fused Silica to Air





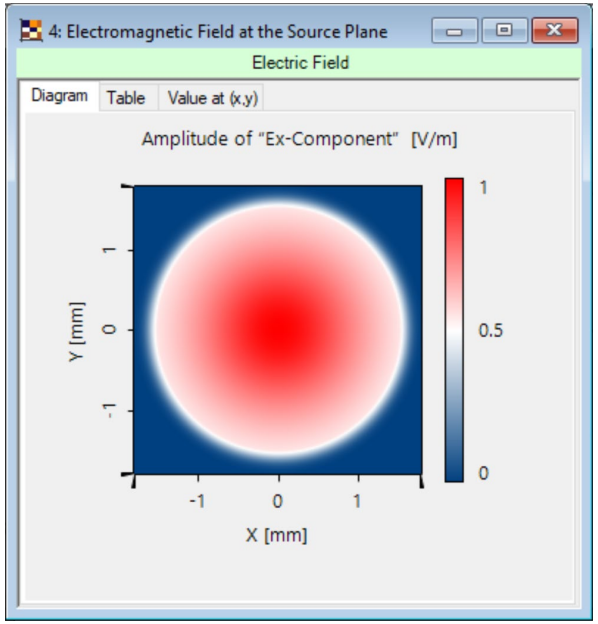
Reflected Field at Interface Fused Silica to Air



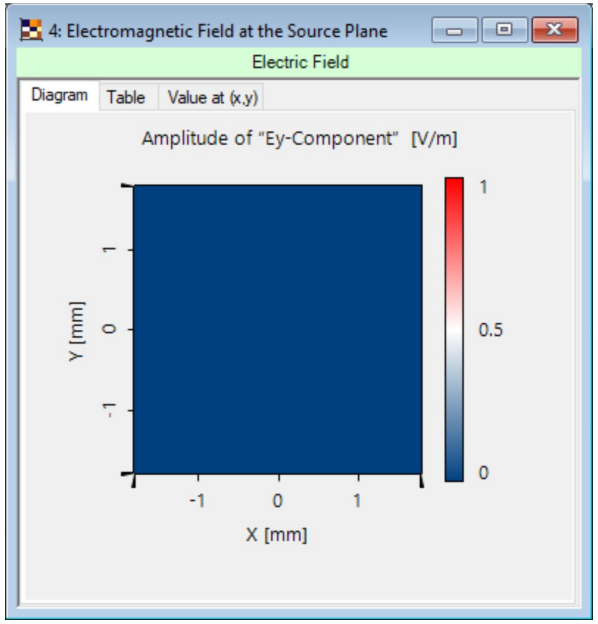


Electromagnetic Input Field

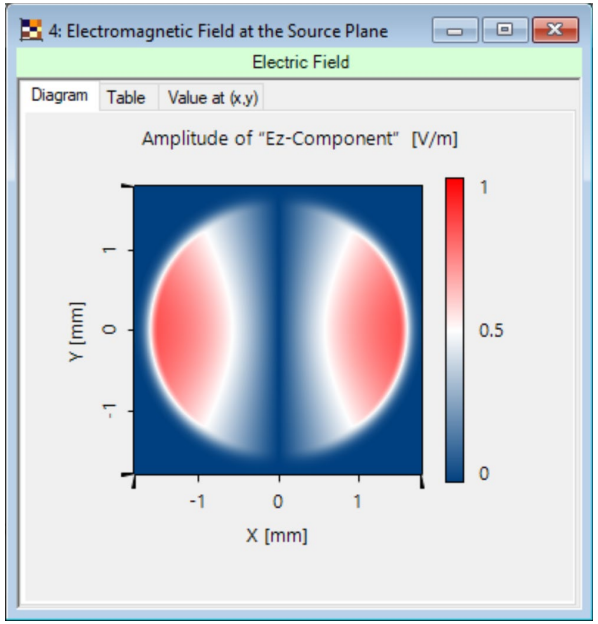
$|E_x(\boldsymbol{\rho})|$



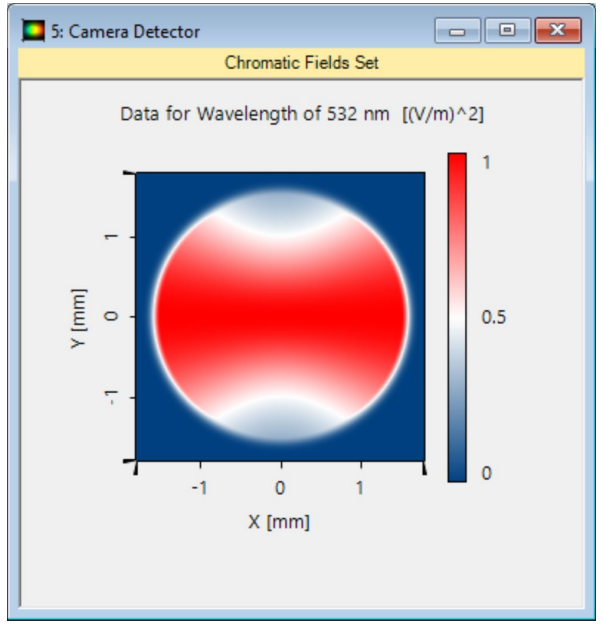
$|E_y(\boldsymbol{\rho})|$

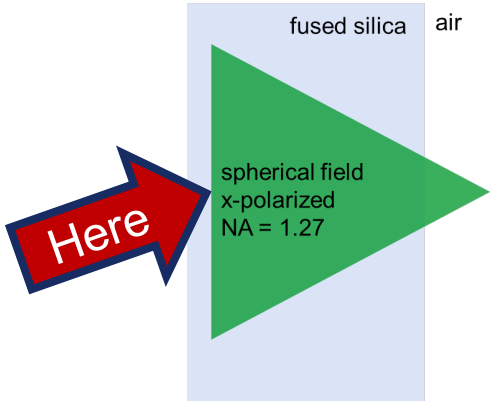


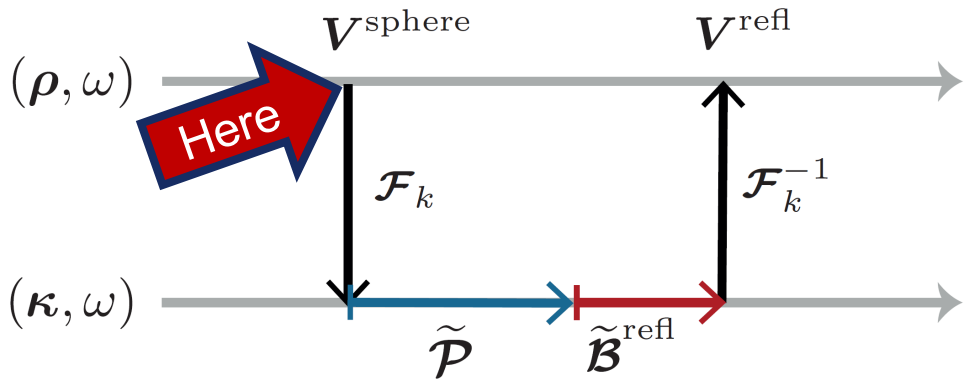
$|E_z(\boldsymbol{\rho})|$



$|E_x(\boldsymbol{\rho})|^2 + |E_y(\boldsymbol{\rho})|^2 + |E_z(\boldsymbol{\rho})|^2$

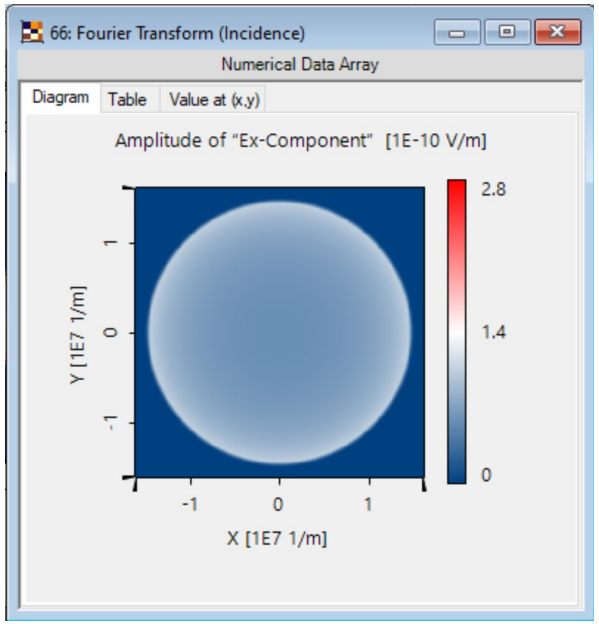




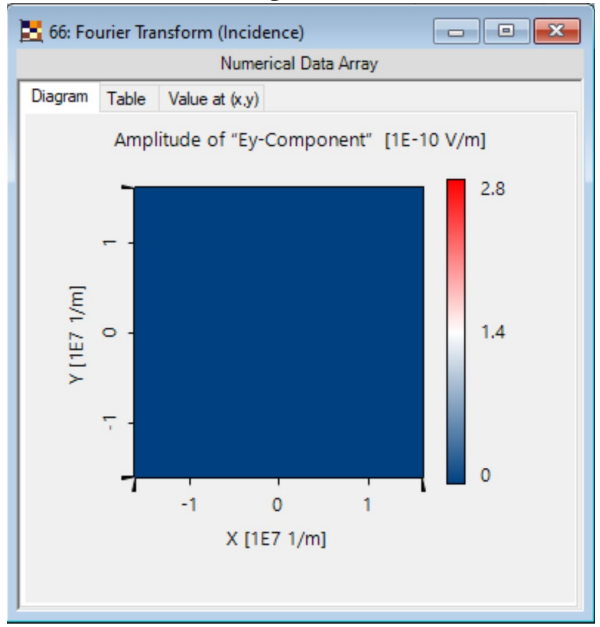


Electromagnetic Spherical Field: *k*-Domain

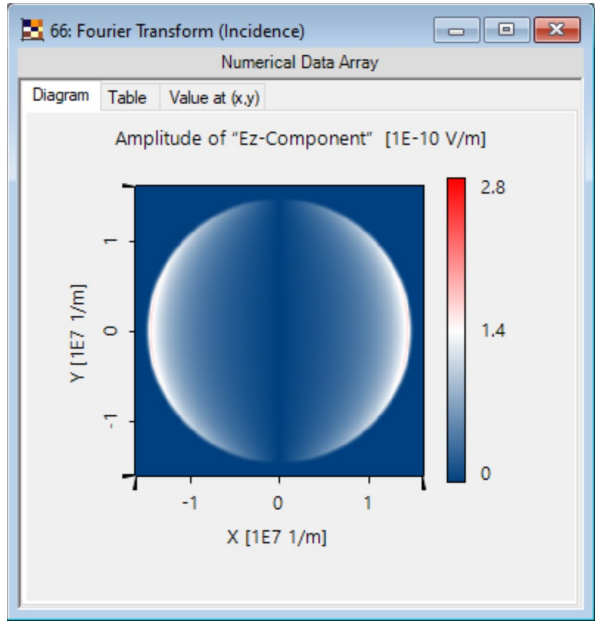
$|\tilde{E}_x(\boldsymbol{\kappa})|$

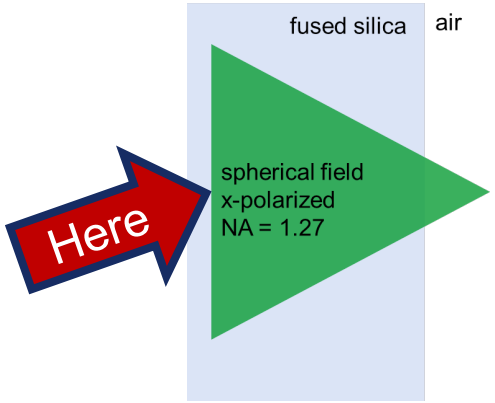


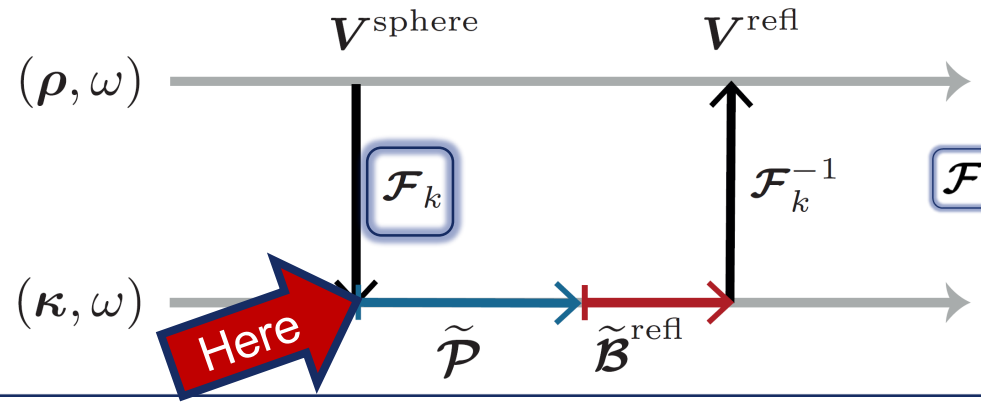
$|\tilde{E}_y(\boldsymbol{\kappa})|$



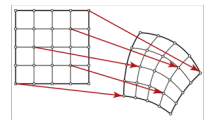
$|\tilde{E}_z(\boldsymbol{\kappa})|$





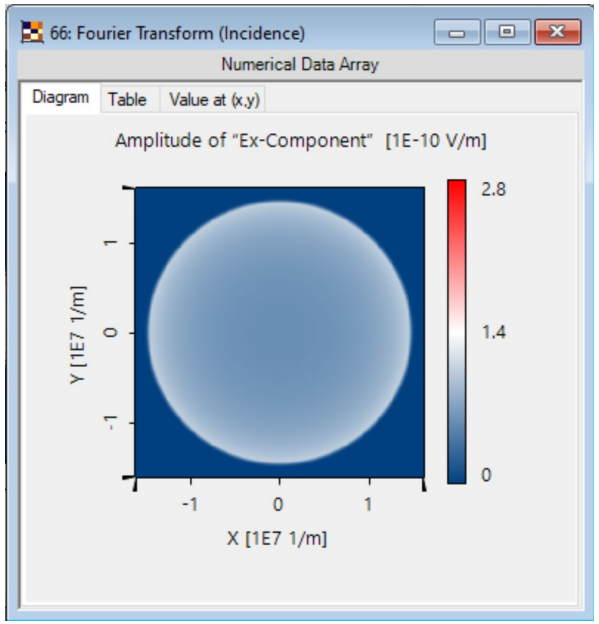


$\mathcal{F}$ Homeomorphic Fourier Transform

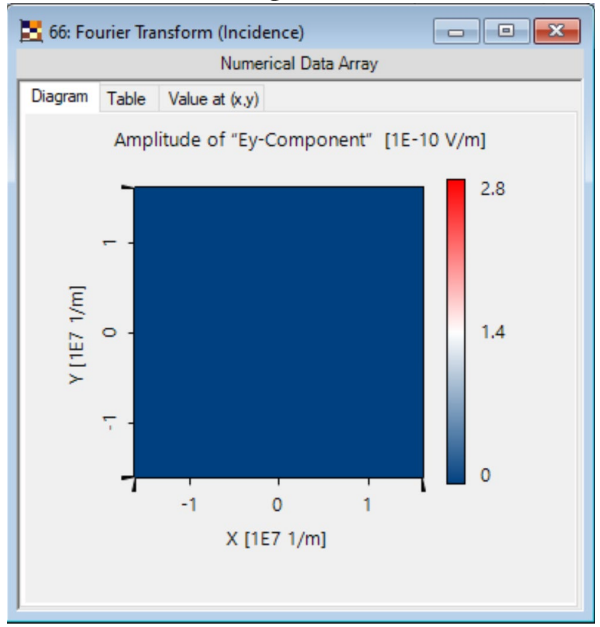


Electromagnetic Spherical Field: *k*-Domain

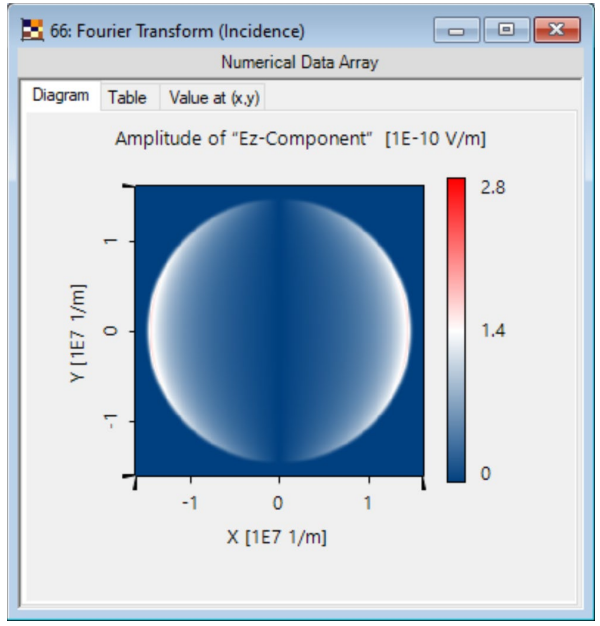
$|\tilde{E}_x(\boldsymbol{\kappa})|$

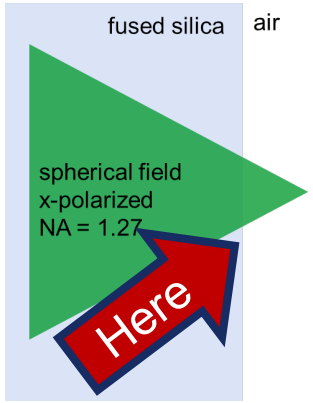


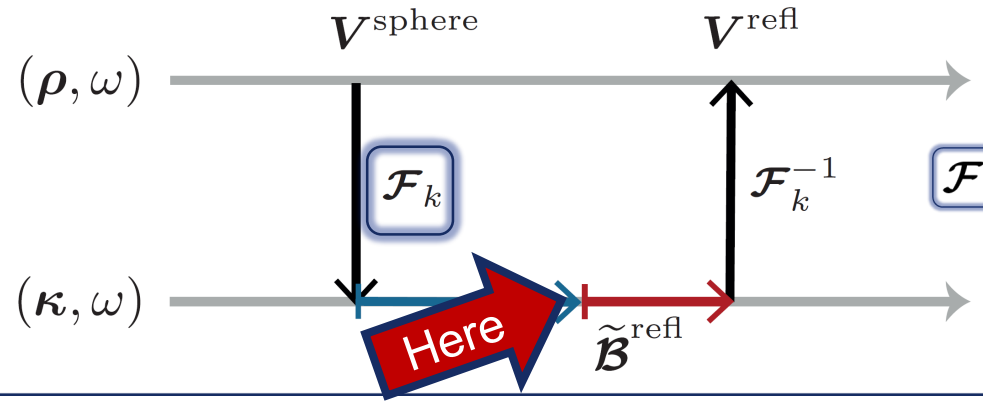
$|\tilde{E}_y(\boldsymbol{\kappa})|$



$|\tilde{E}_z(\boldsymbol{\kappa})|$

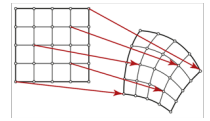






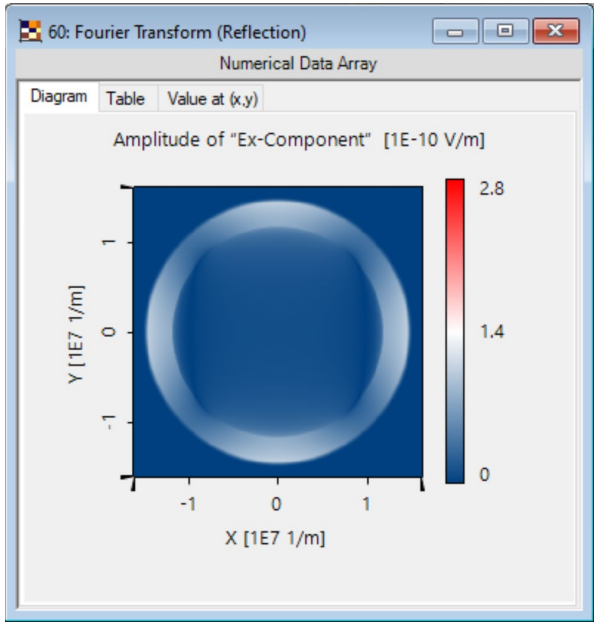
$$\tilde{\mathcal{P}}_{||} = \times \exp(\mathrm{i}k_z\Delta z)$$

$\mathcal{F}$ Homeomorphic Fourier Transform

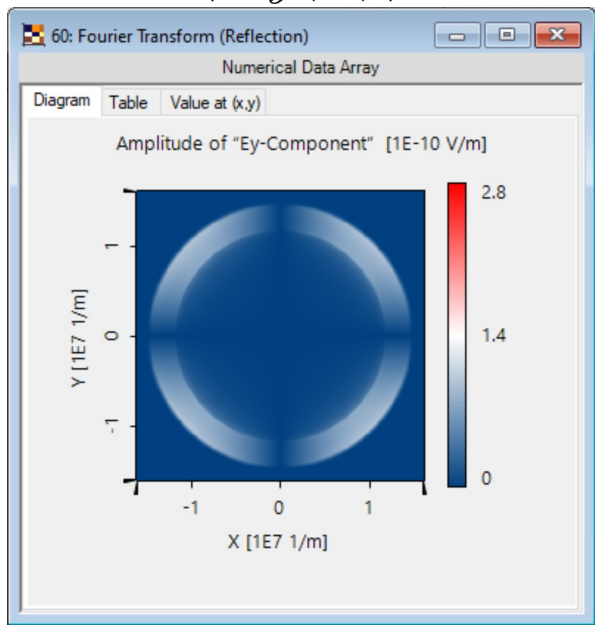


Electromagnetic Spherical Field: *k*-Domain

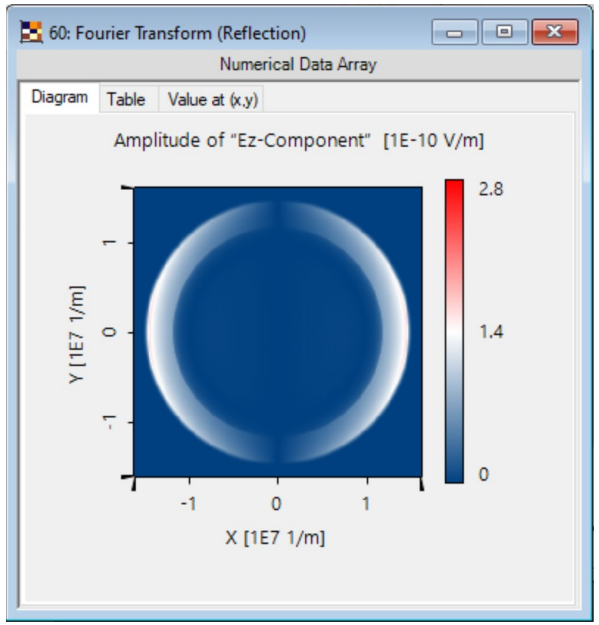
$|\tilde{E}_x(\boldsymbol{\kappa})|$

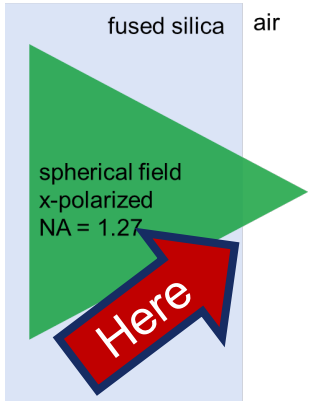


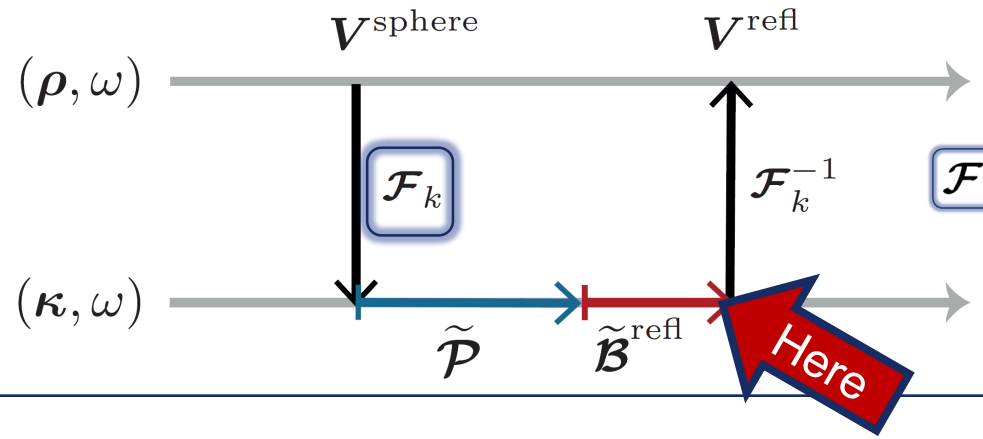
$|\tilde{E}_y(\boldsymbol{\kappa})|$



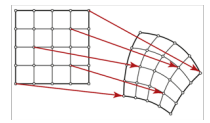
$|\tilde{E}_z(\boldsymbol{\kappa})|$





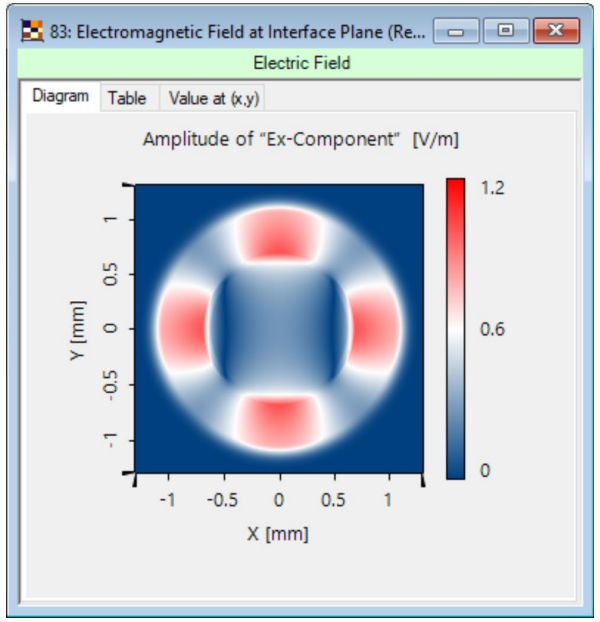


$\mathcal{F}$ Homeomorphic Fourier Transform

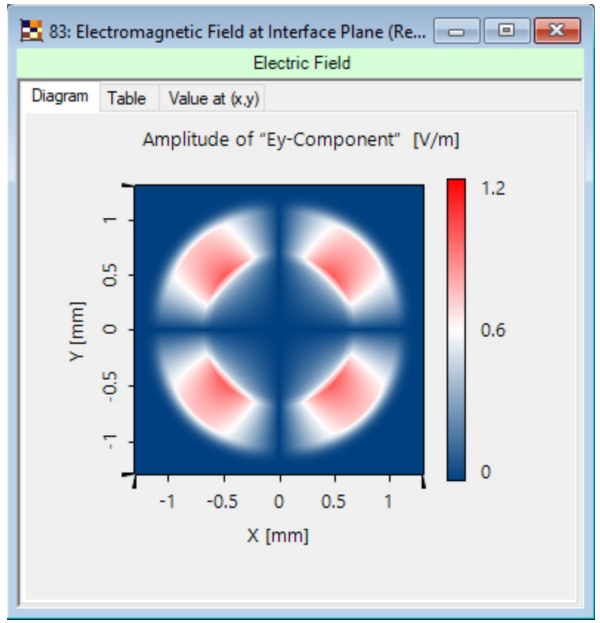


Electromagnetic Spherical Field: *k*-Domain

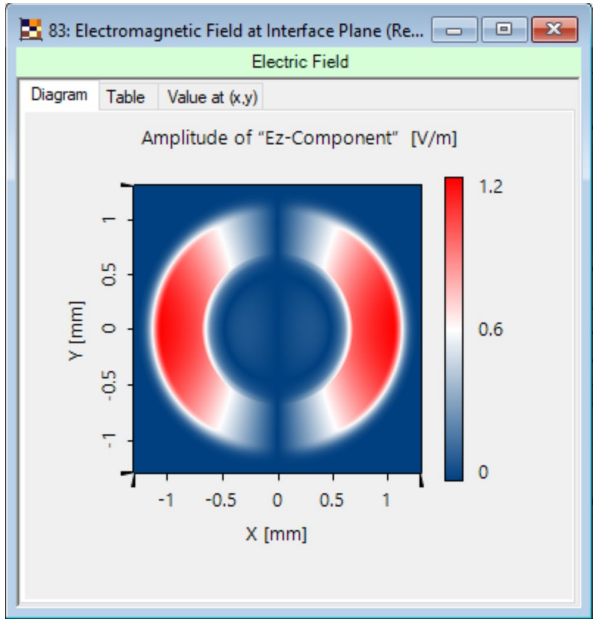
$|E_x(\boldsymbol{\rho})|$



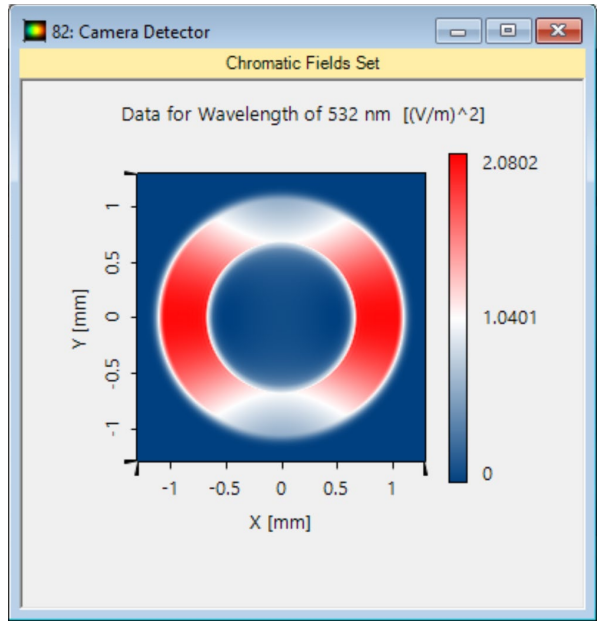
$|E_y(\boldsymbol{\rho})|$

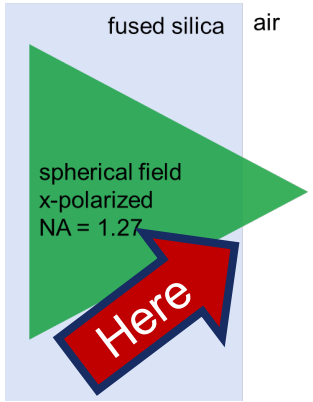


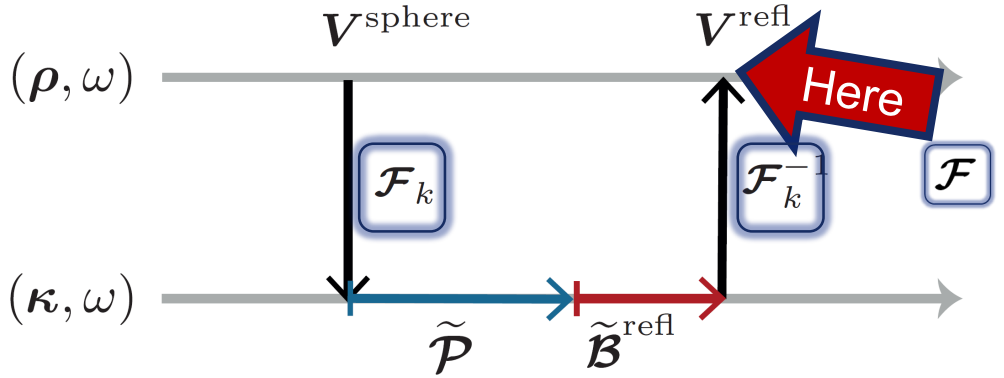
$|E_z(\boldsymbol{\rho})|$



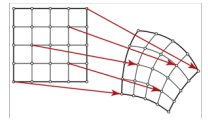
$|E_x(\boldsymbol{\rho})|^2 + |E_y(\boldsymbol{\rho})|^2 + |E_z(\boldsymbol{\rho})|^2$





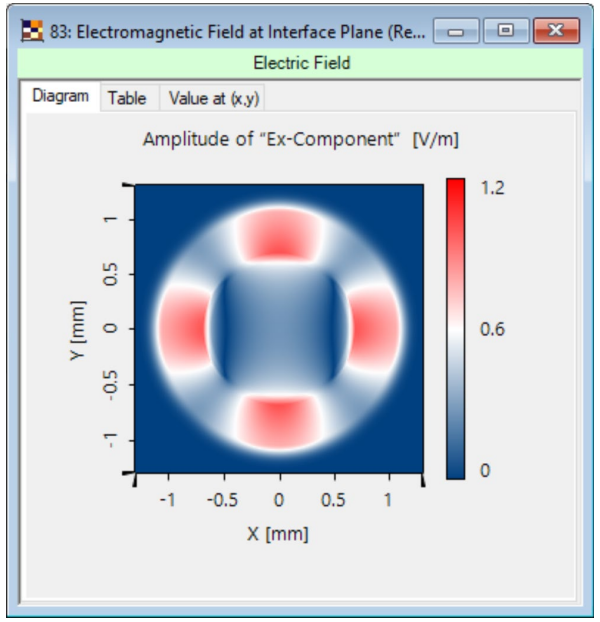


$\mathcal{F}$ Homeomorphic Fourier Transform

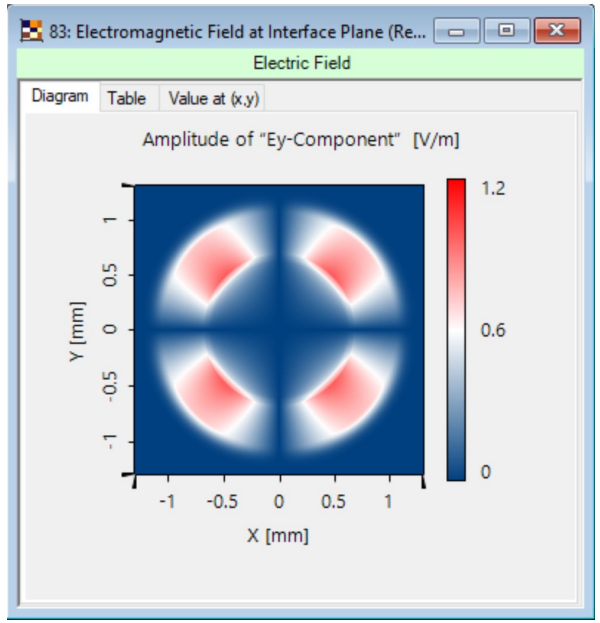


Electromagnetic Spherical Field: *k*-Domain

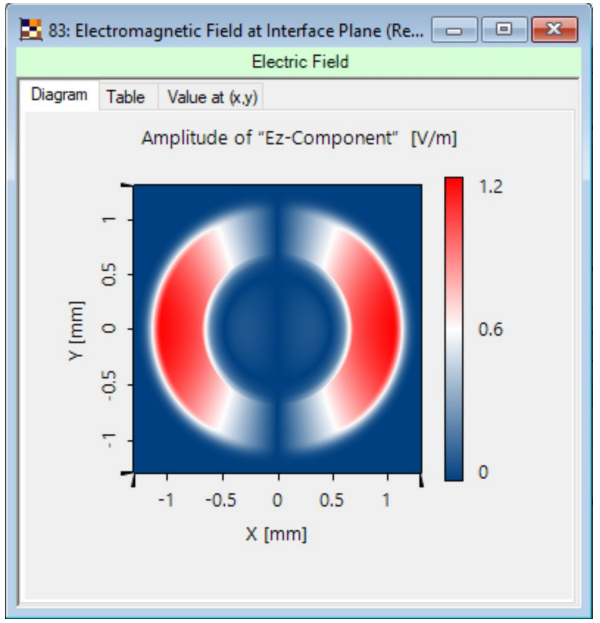
$|E_x(\boldsymbol{\rho})|$



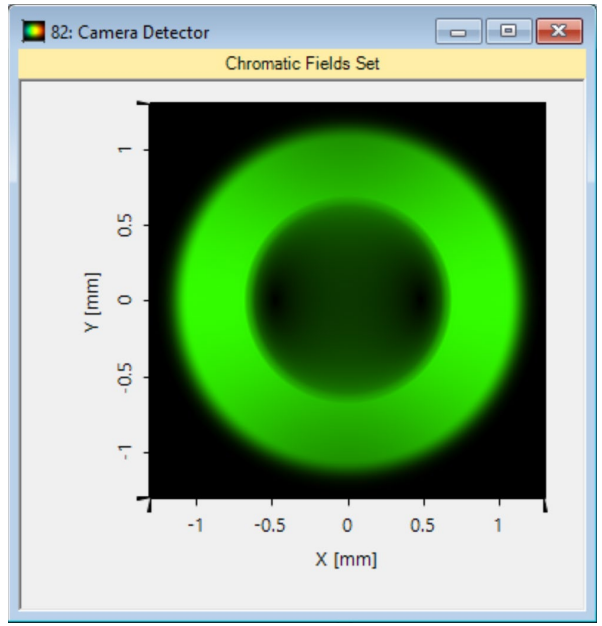
$|E_y(\boldsymbol{\rho})|$

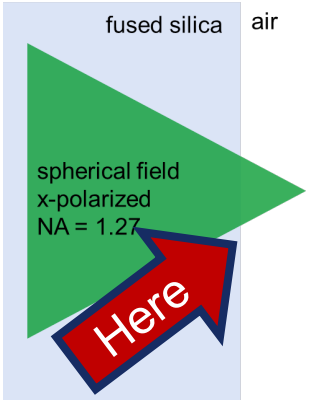


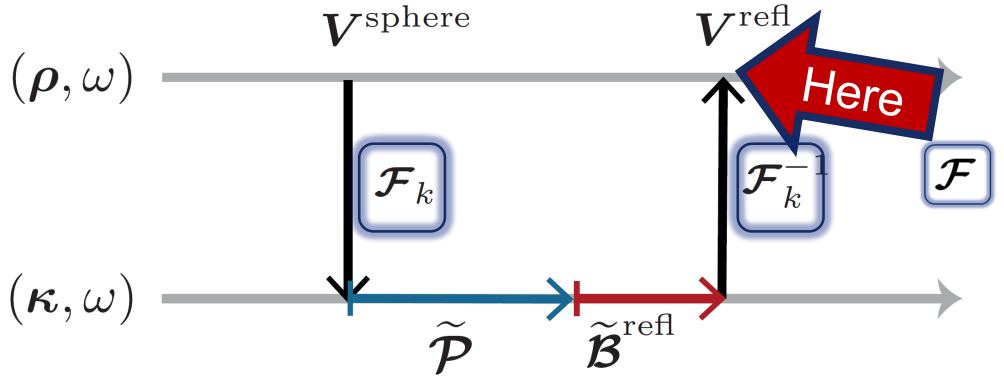
$|E_z(\boldsymbol{\rho})|$



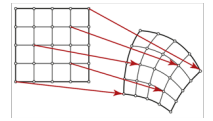
$|E_x(\boldsymbol{\rho})|^2 + |E_y(\boldsymbol{\rho})|^2 + |E_z(\boldsymbol{\rho})|^2$







$\mathcal{F}$ Homeomorphic Fourier Transform



Conclusion

- Field tracing: Connecting field solvers.
- Evaluation of all relevant lightpaths for a non-sequential connection.
- Simulation by operator sequence per lightpath.
- Switching between domains and exploiting homeomorphic operations enable fast physical optics.
- Homeomorphic Fourier transform essential mathematical concept.
- Physical interpretation: Homeomorphic physical-optics operator sequences reveal the “geometric” part of electromagnetic theory.

Autumn Release 2019

Thank You!