LightTrans

FRIEDRICH-SCHILLER-UNIVERSITÄT JENA

EOS Topical Meeting on Diffractive Optics, Jena, 17-09-2019

Physical-Optics Anatomy of the Gouy Phase Shift

Olga Baladron-Zorita[1,2], **Zongzhao Wang**[1,2] , **Christian Hellmann**[3] and **Frank Wyrowski**[1]

[1] Applied Computational Optics Group, Institute of Applied Physics, Friedrich Schiller University Jena, Jena, Germany

[2] LightTrans International UG, Jena, Germany

[3] Wyrowski Photonics GmbH, Jena, Germany

Context

On the Importance of Homeomorphic Operators in Physical and Geometrical Optics

Frank Wyrowski[1], **Olga Baladron-Zorita**[1,2] and **Zongzhao Wang**[1,2]
[1] Applied Computational Optics Group, Institute of Applied Physics, Friedrich Schiller University Jena, Jena, Germany
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Numerical Implementation of the Homeomorphic Fourier Transform and Its Application to Physical-Optics Modelling

Zongzhao Wang[1,2], **Olga Baladron-Zorita**[1,2] , **Christian Hellmann**[3] and **Frank Wyrowski**[1]
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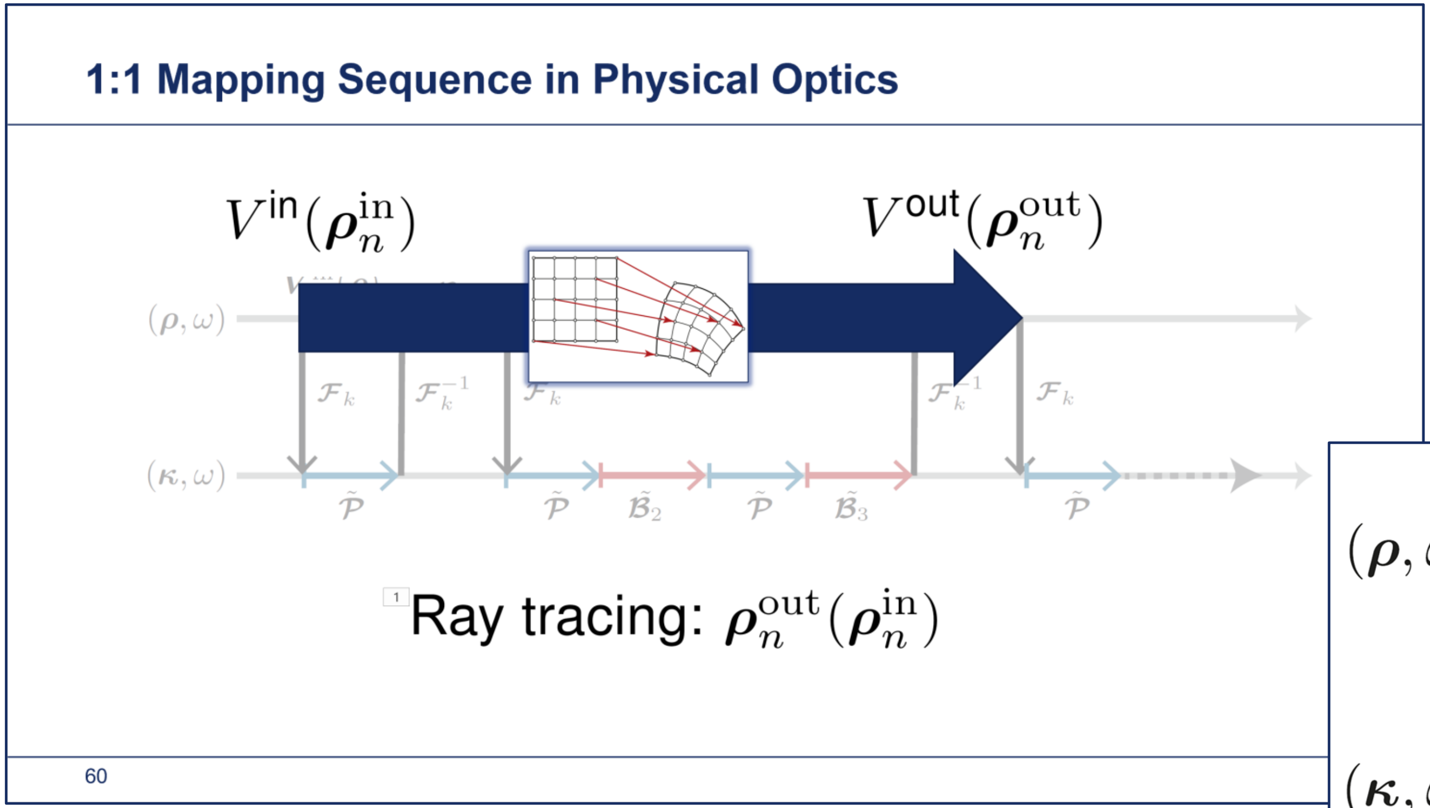
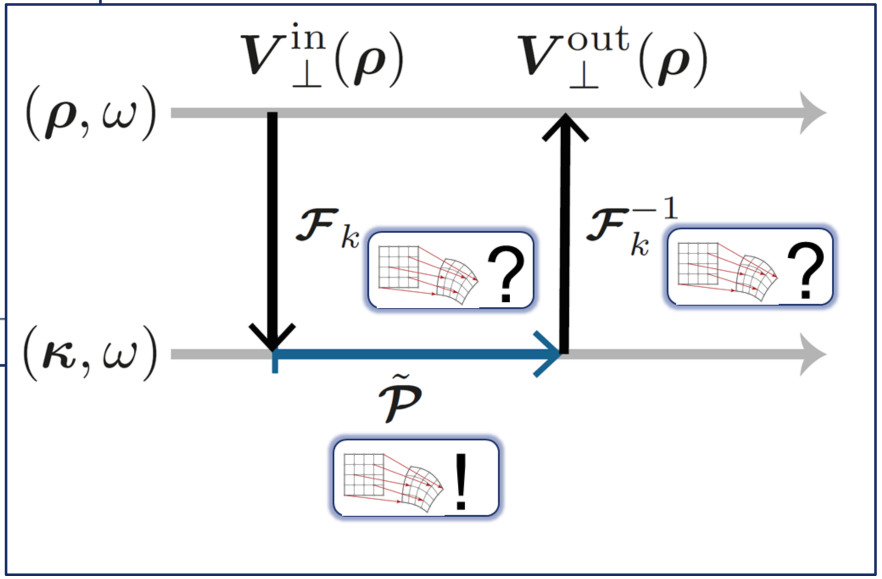
Context

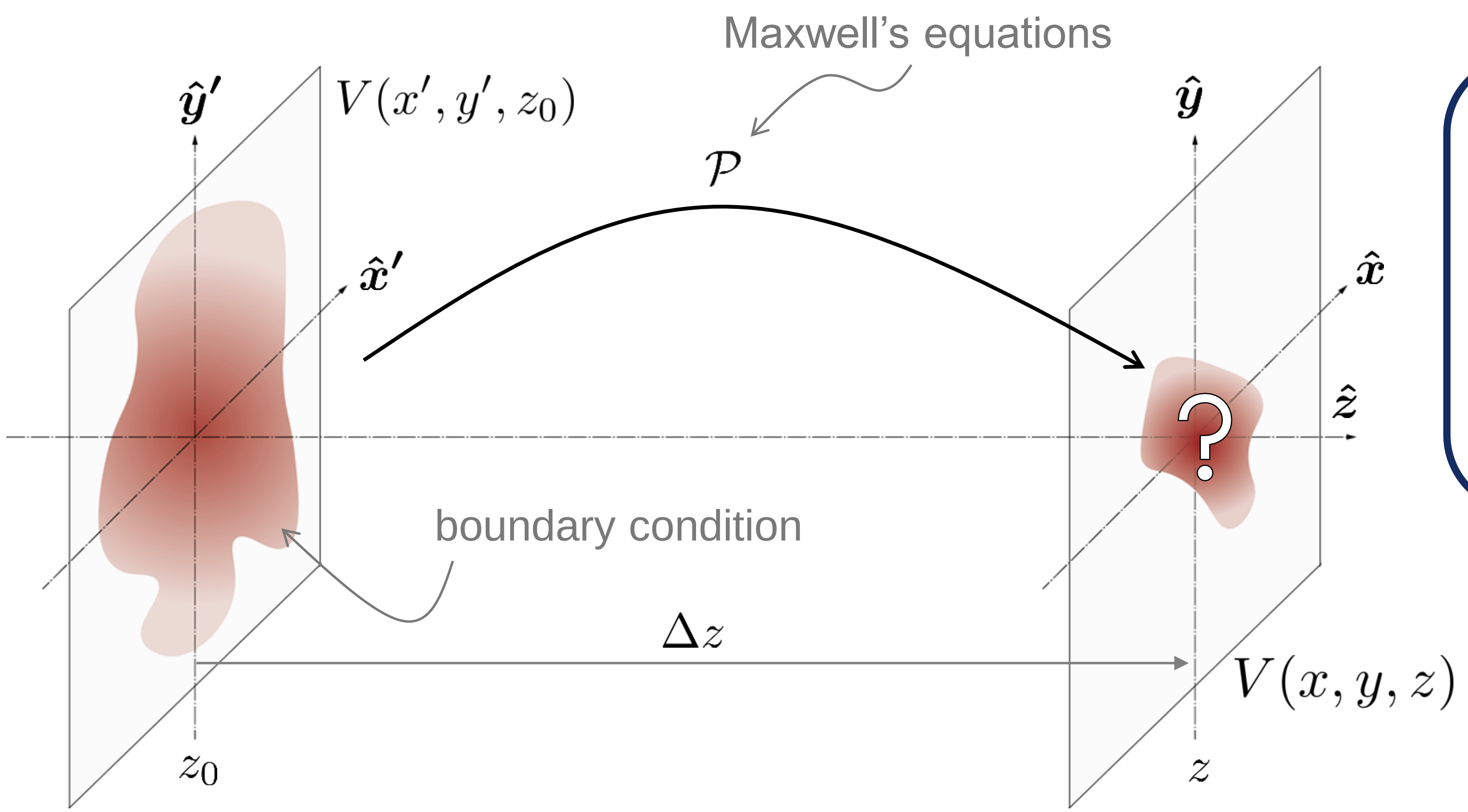


1:1 Mapping Sequence in Physical Optics
$V^{\text{in}}(\boldsymbol{\rho}_n^{\text{in}})$
$V^{\text{out}}(\boldsymbol{\rho}_n^{\text{out}})$
$(\boldsymbol{\rho}, \omega)$
$(\boldsymbol{\kappa}, \omega)$
$\mathcal{F}_k$
$\mathcal{F}_k^{-1}$
$\tilde{\mathcal{P}}$
$\tilde{\mathcal{B}}_2$
$\tilde{\mathcal{B}}_3$
Ray tracing: $\boldsymbol{\rho}_n^{\text{out}}(\boldsymbol{\rho}_n^{\text{in}})$
60



$\boldsymbol{V}_\perp^{\text{in}}(\boldsymbol{\rho})$
$\boldsymbol{V}_\perp^{\text{out}}(\boldsymbol{\rho})$
$(\boldsymbol{\rho}, \omega)$
$(\boldsymbol{\kappa}, \omega)$
$\mathcal{F}_k$
?
$\mathcal{F}_k^{-1}$
?
$\tilde{\mathcal{P}}$
!

Problem Statement: Free-Space Propagation in Physical Optics



$\{E_x, E_y, E_z, H_x, H_y, H_z\} \rightarrow V$

Free space:

- Infinitely extended medium
- Linear
- Homogeneous
- Isotropic
- Without free charges

$$\boldsymbol{\rho} \stackrel{\text{def}}{=} (x, y)$$

$$\boldsymbol{\kappa} \stackrel{\text{def}}{=} (k_x, k_y)$$

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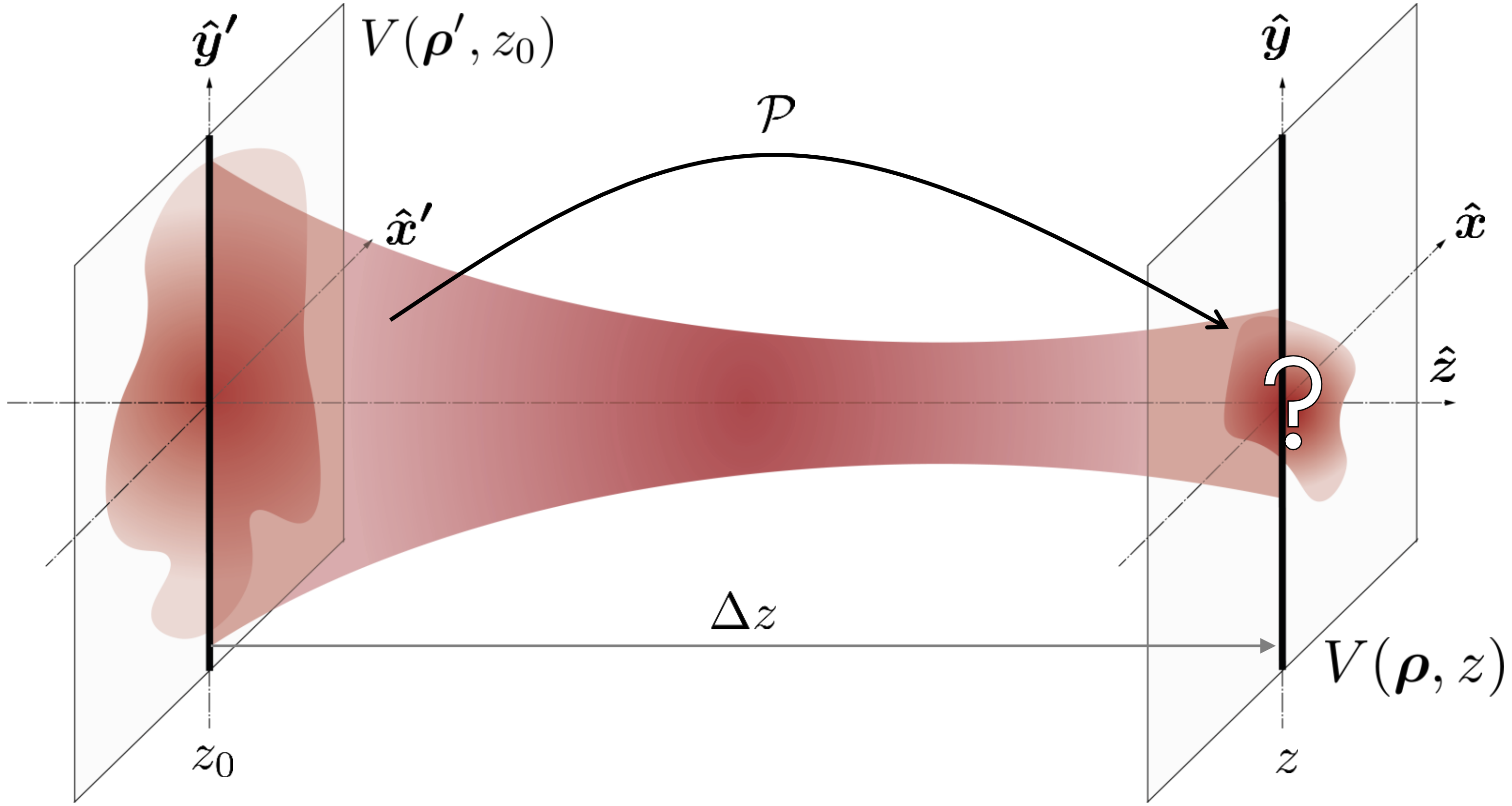

$\{E_x, E_y, E_z, H_x, H_y, H_z\} \to V$

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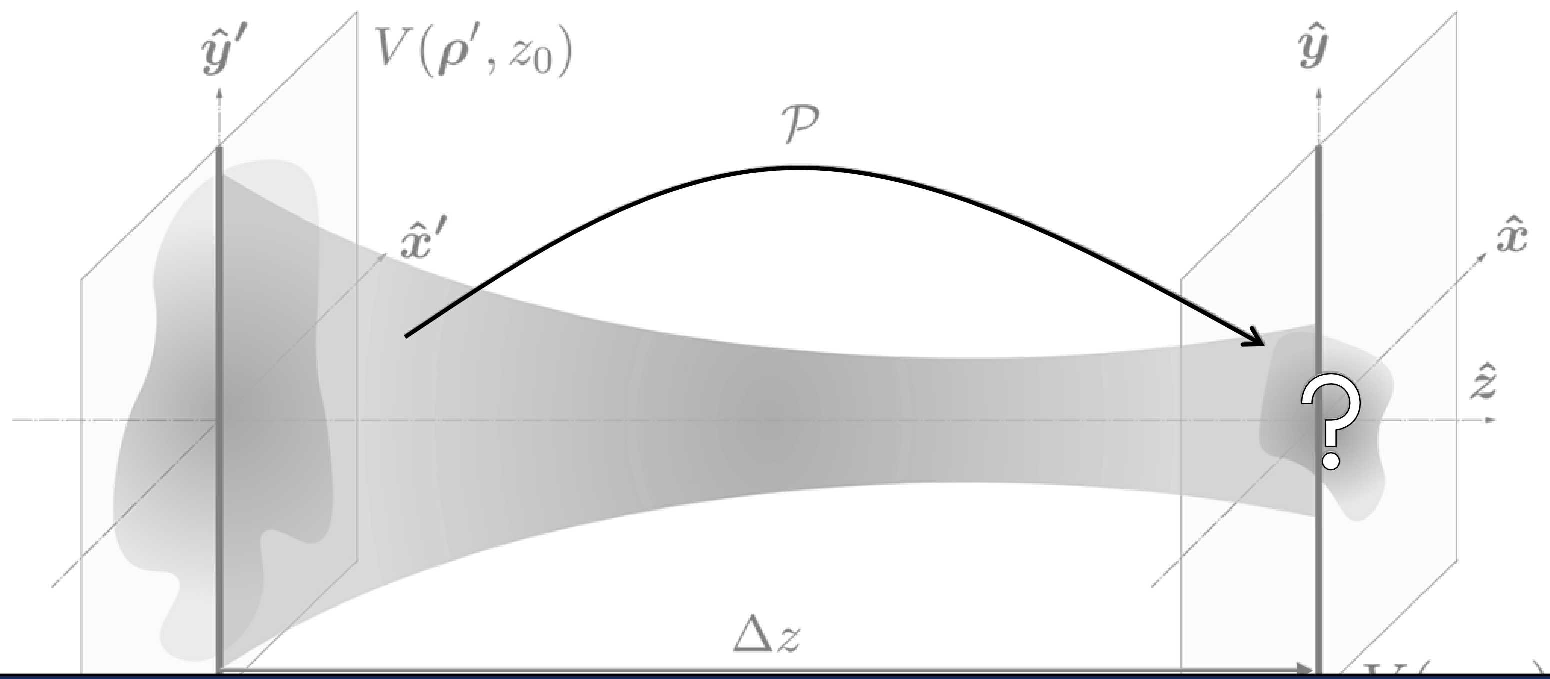

Free space:
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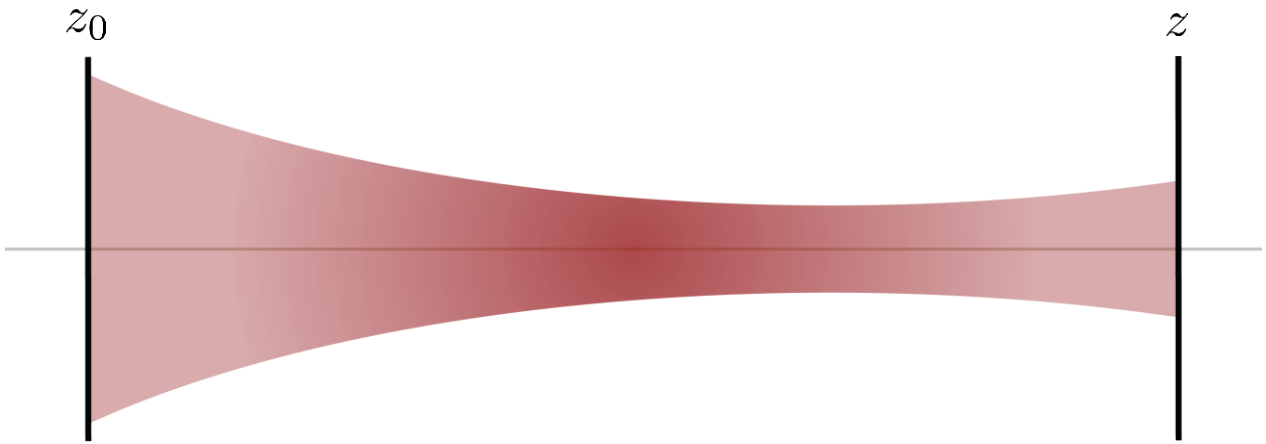
Fully vectorial analysis!

$\boldsymbol{\kappa} = (k_x, k_y)$

$\{E_x, E_y, E_z, H_x, H_y, H_z\} \rightarrow V$

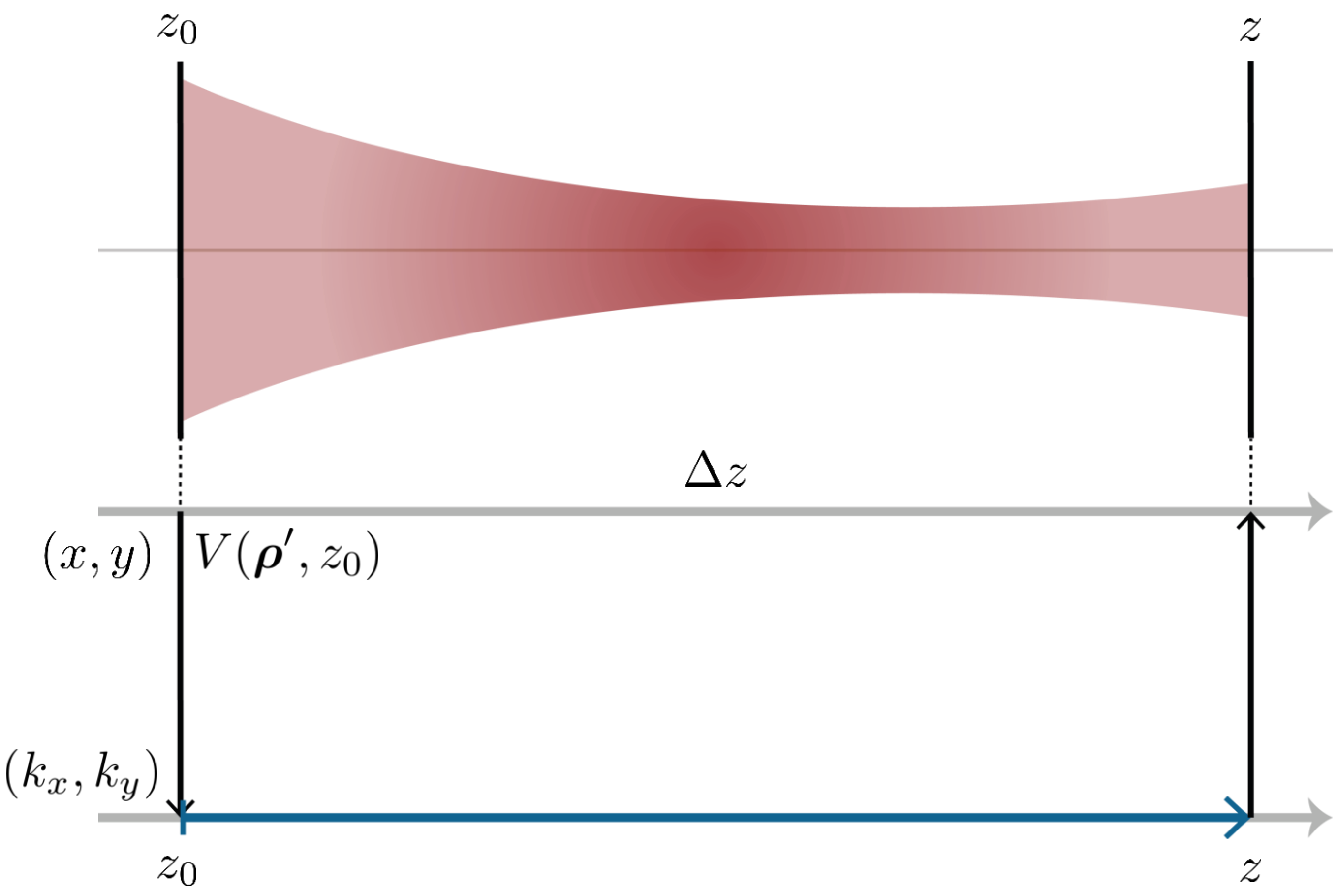
Free-Space Propagation: Spectrum of Plane Waves (SPW)



SPW propagation operator:

$$V(\boldsymbol{\rho}, z) =$$
$$= \mathcal{F}_{\kappa}^{-1}\{\mathcal{F}_{\kappa}\{V(\boldsymbol{\rho}', z_0)\} \cdot \exp[\mathrm{i}\check{k}_z(\kappa)\,\Delta z]\}$$

Free-Space Propagation: Spectrum of Plane Waves (SPW)



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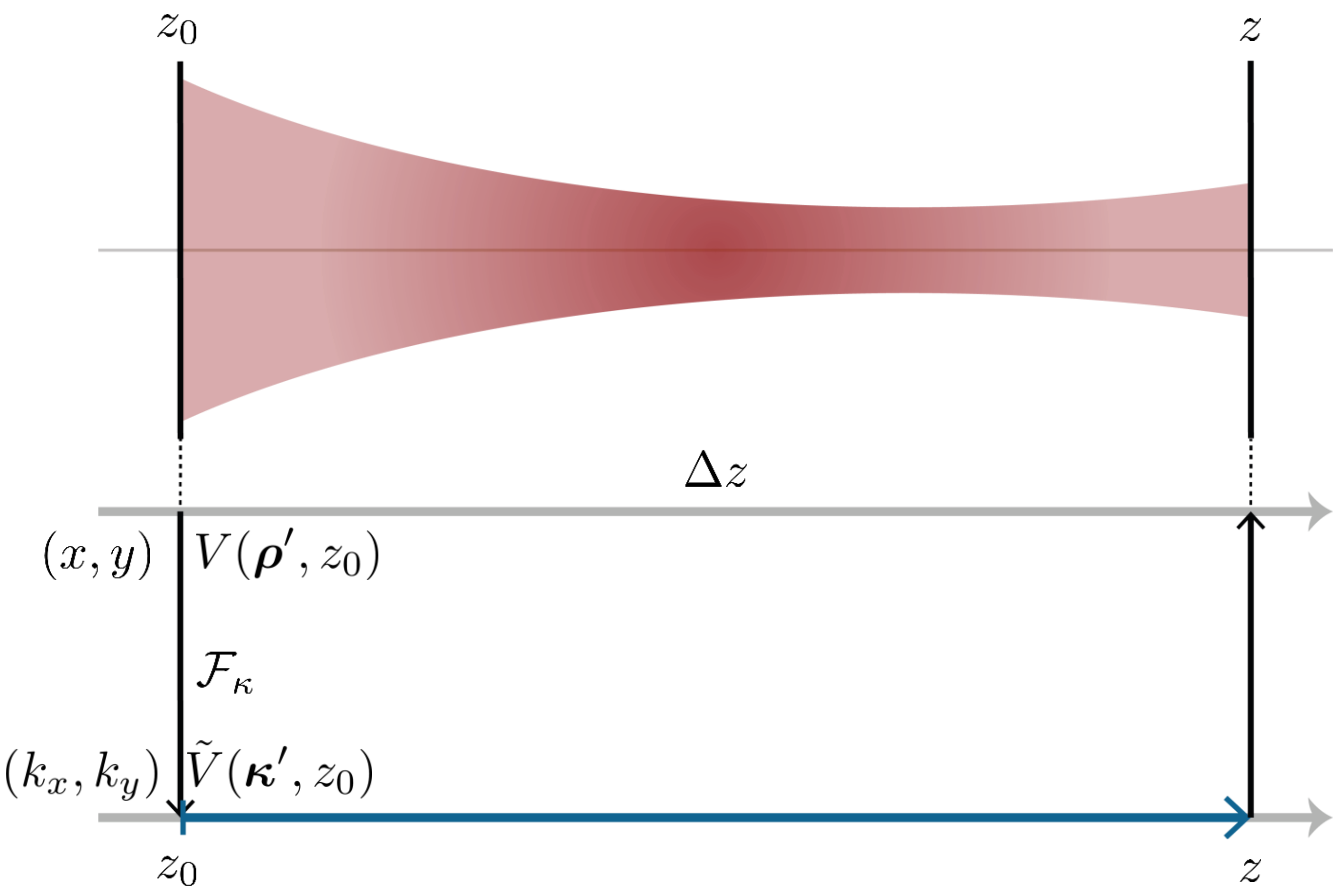

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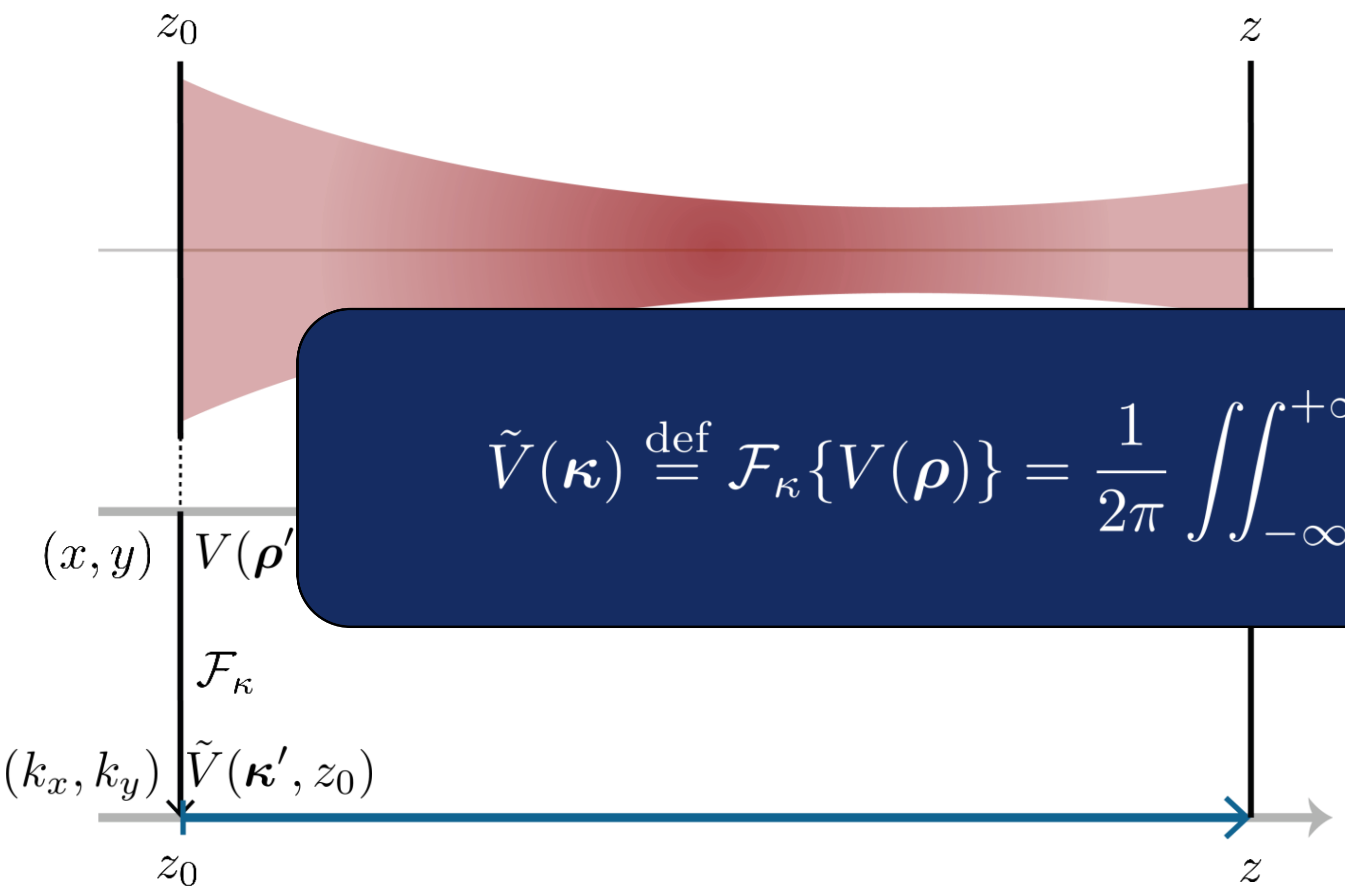

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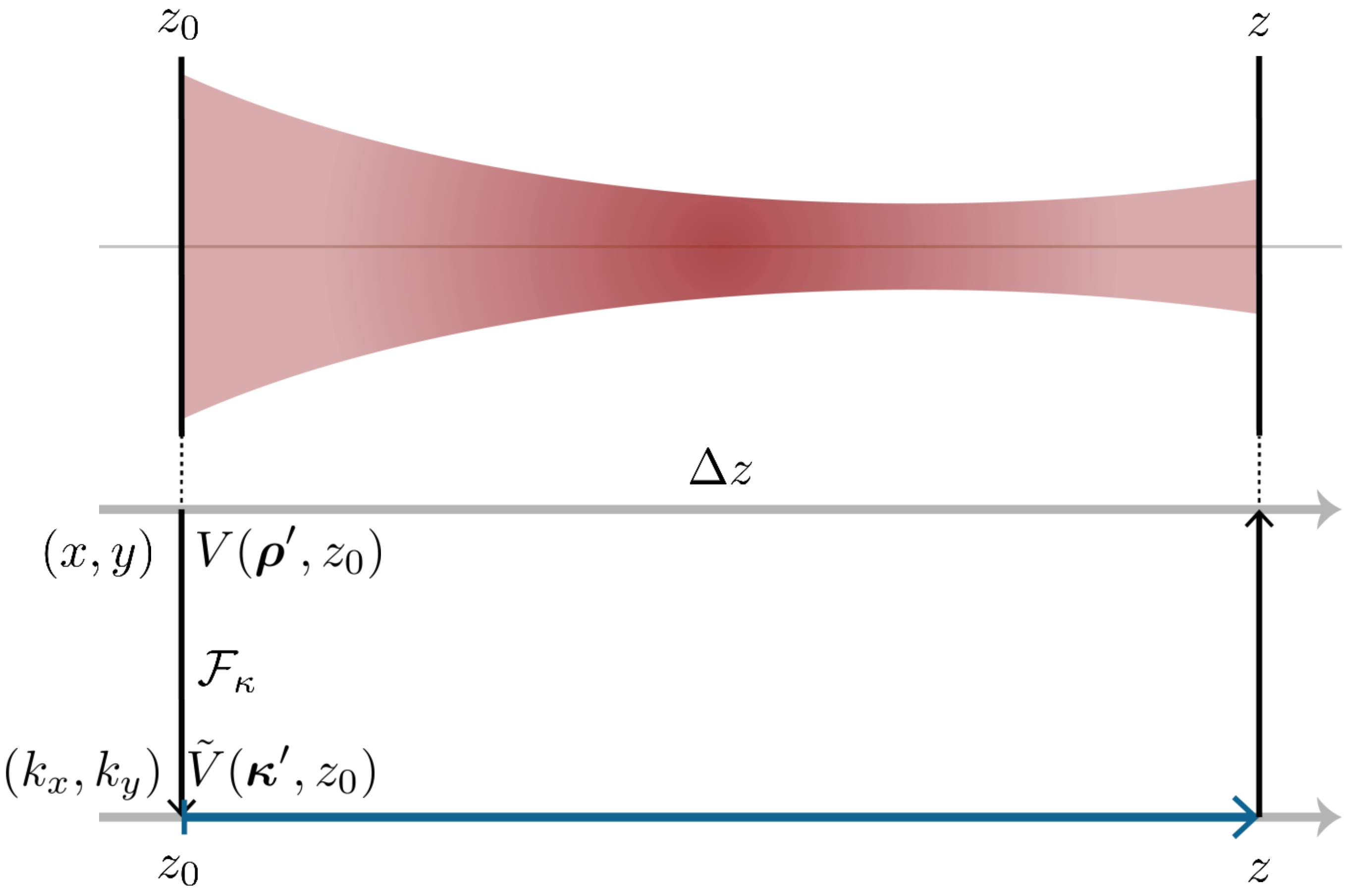
$$\tilde{V}(\boldsymbol{\kappa}) \stackrel{\text{def}}{=} \mathcal{F}_\kappa\{V(\boldsymbol{\rho})\} = \frac{1}{2\pi}\iint_{-\infty}^{+\infty} V(\boldsymbol{\rho}) \exp[-\mathrm{i}\boldsymbol{\kappa}\cdot\boldsymbol{\rho}]\,\mathrm{d}^2\rho$$

Free-Space Propagation: Spectrum of Plane Waves (SPW)



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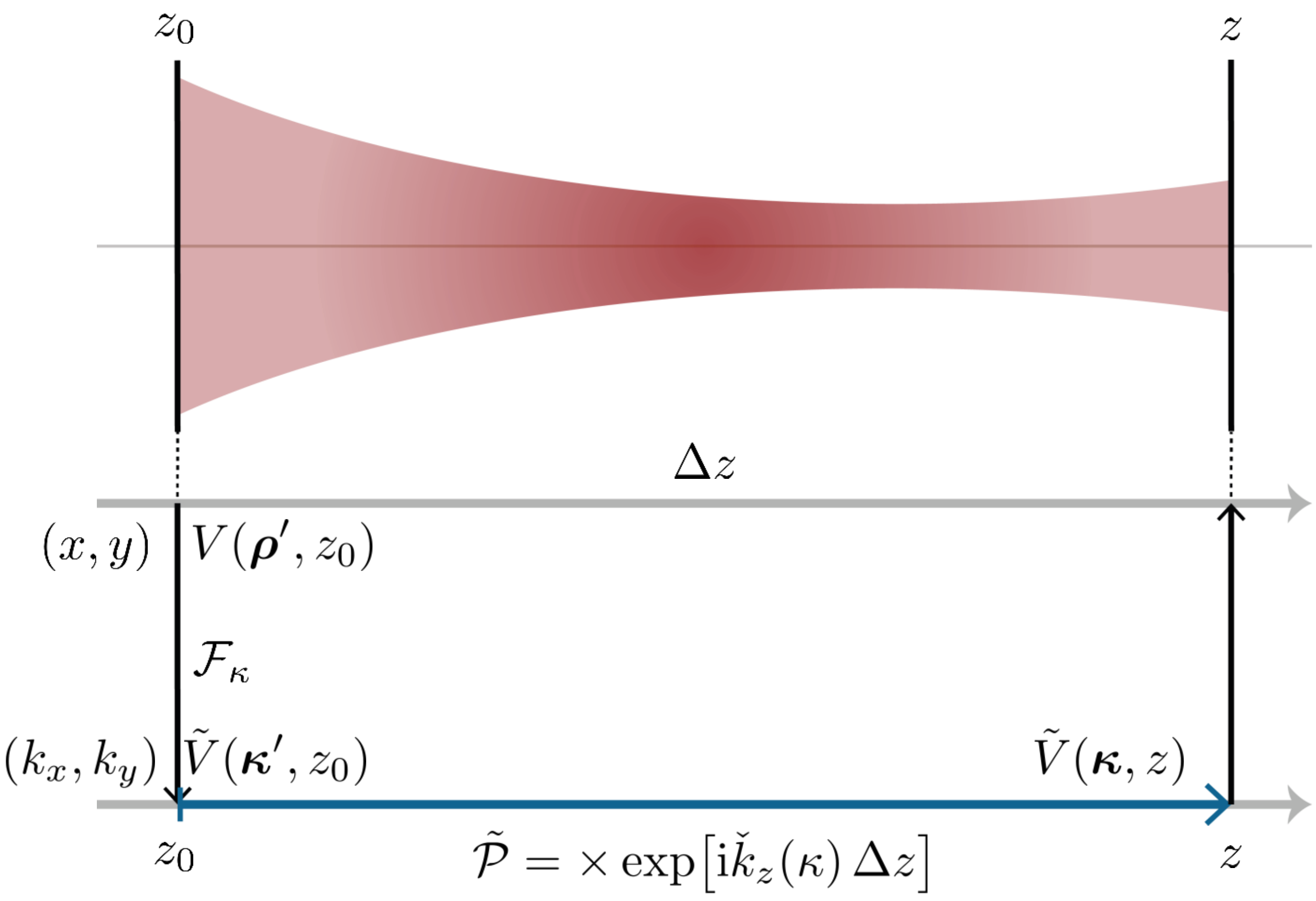

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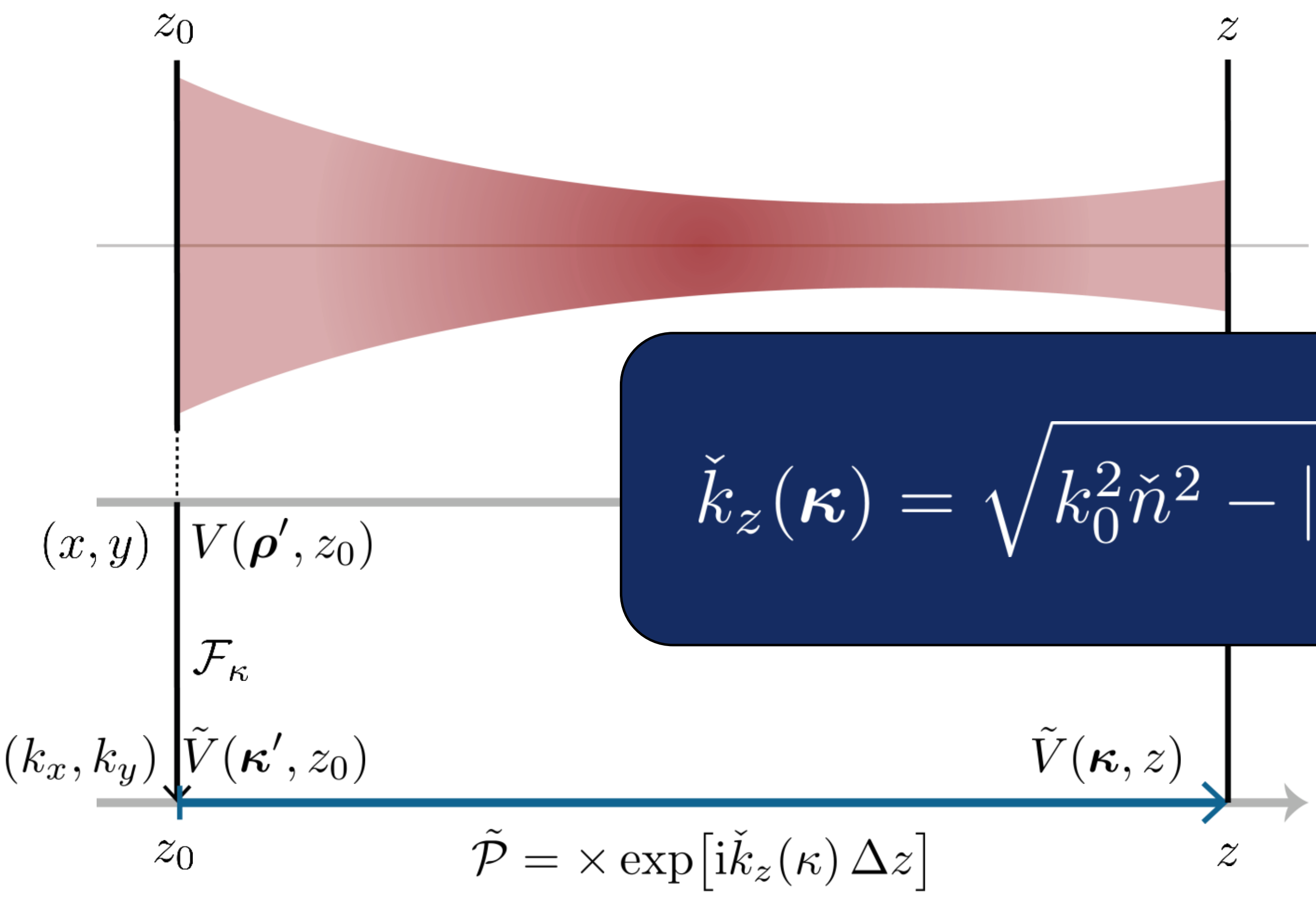

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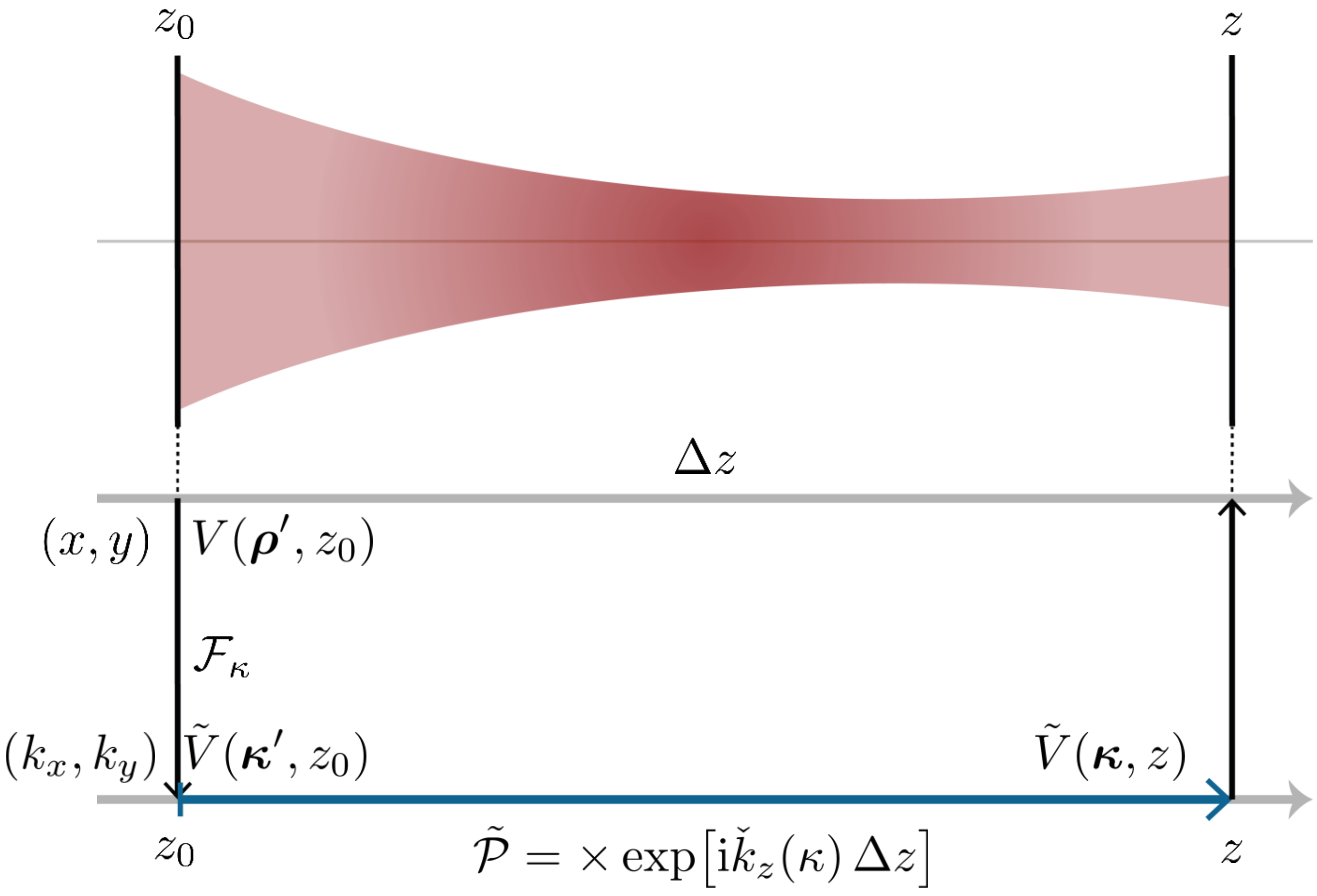
$$\check{k}_z(\boldsymbol{\kappa}) = \sqrt{k_0^2\check{n}^2 - ||\boldsymbol{\kappa}||^2} \in \mathbb{C}$$

Free-Space Propagation: Spectrum of Plane Waves (SPW)



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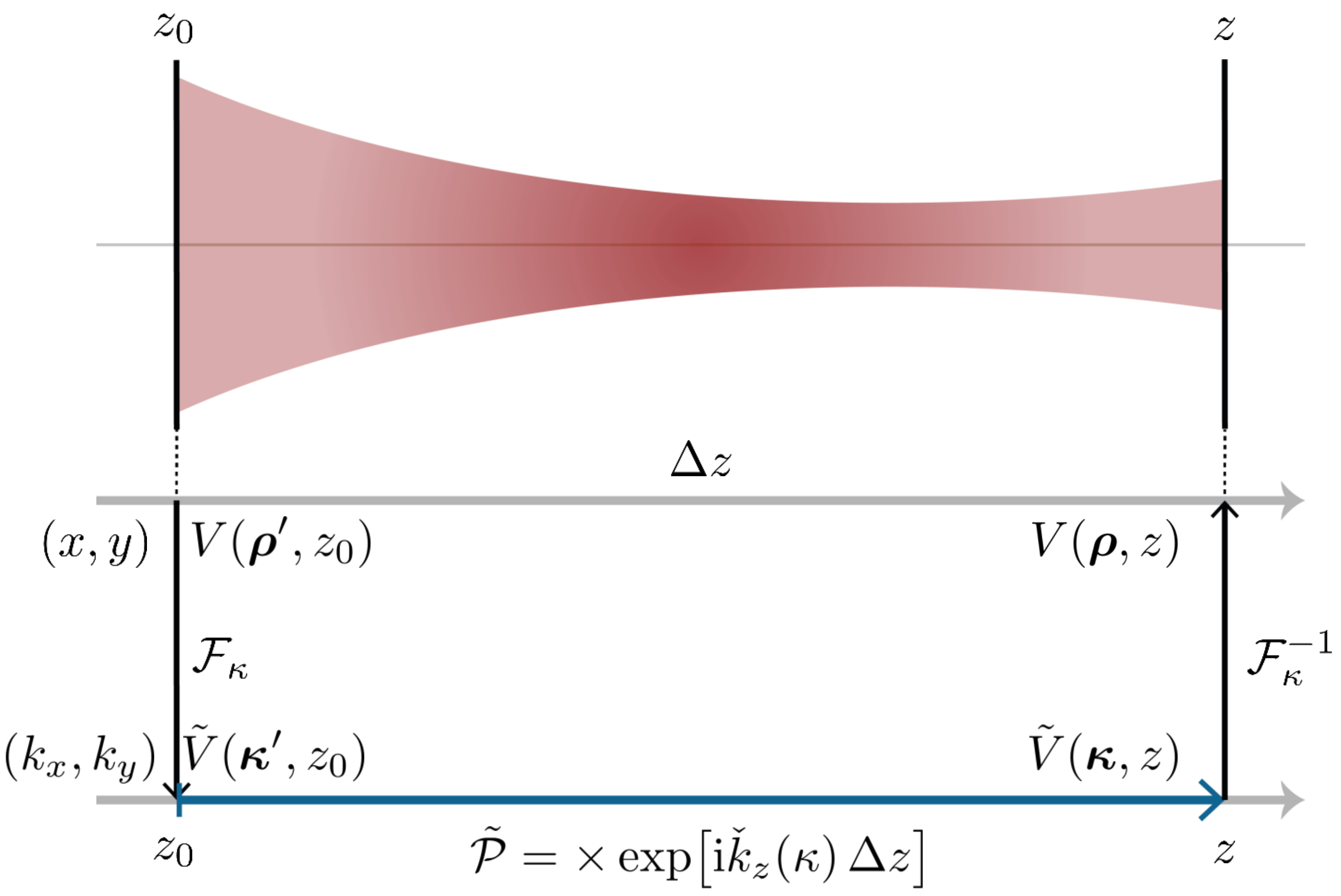

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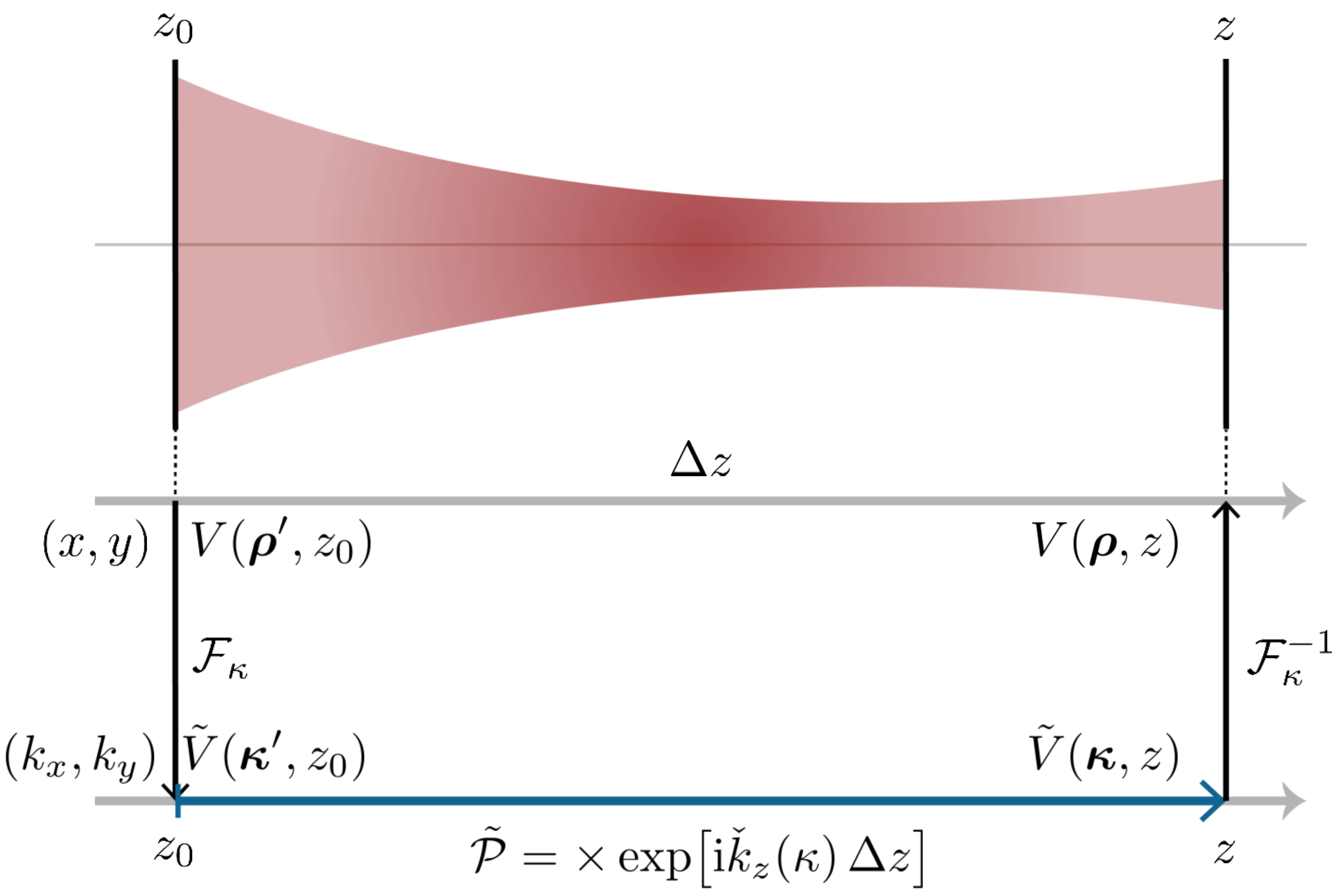

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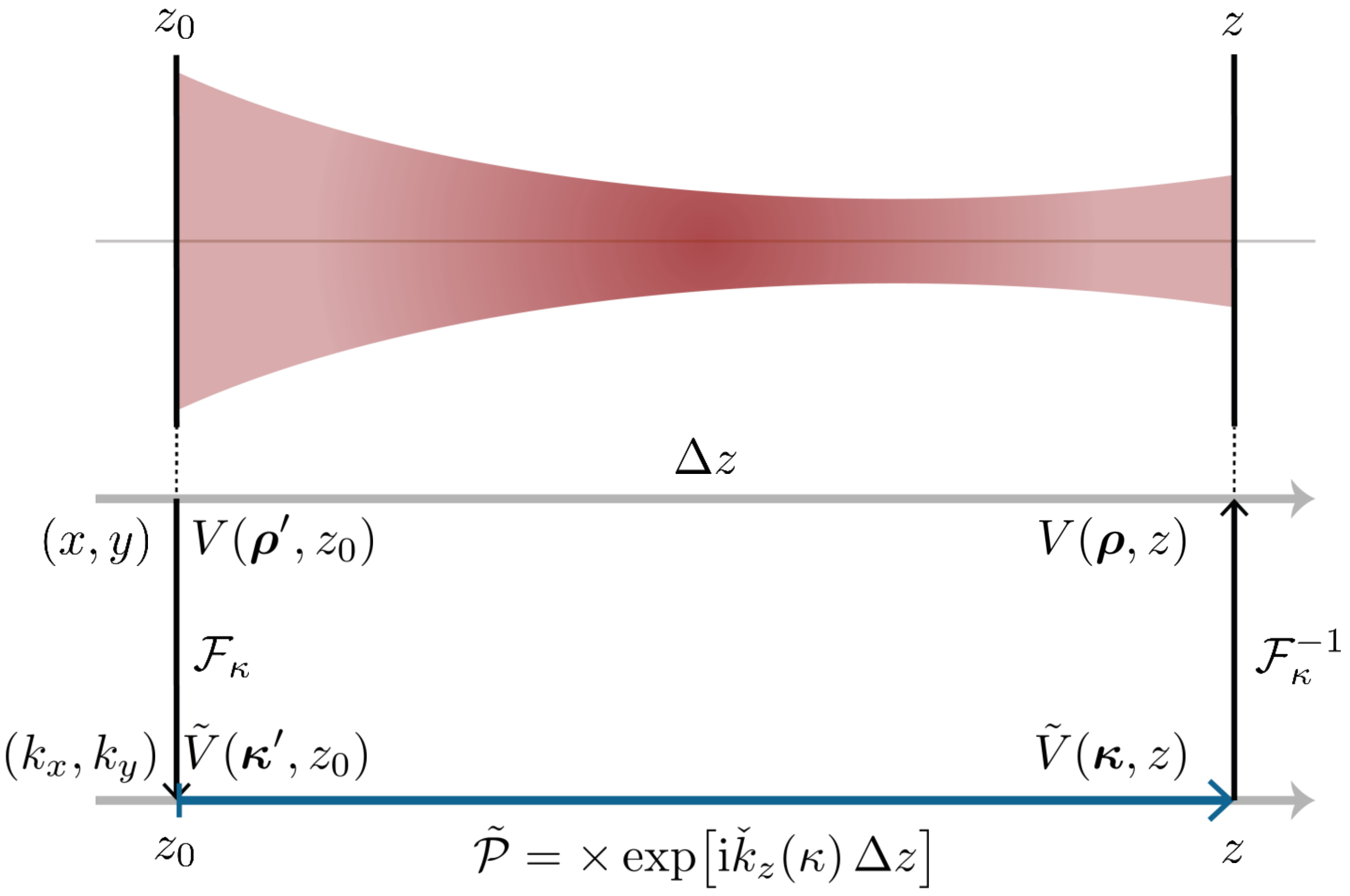

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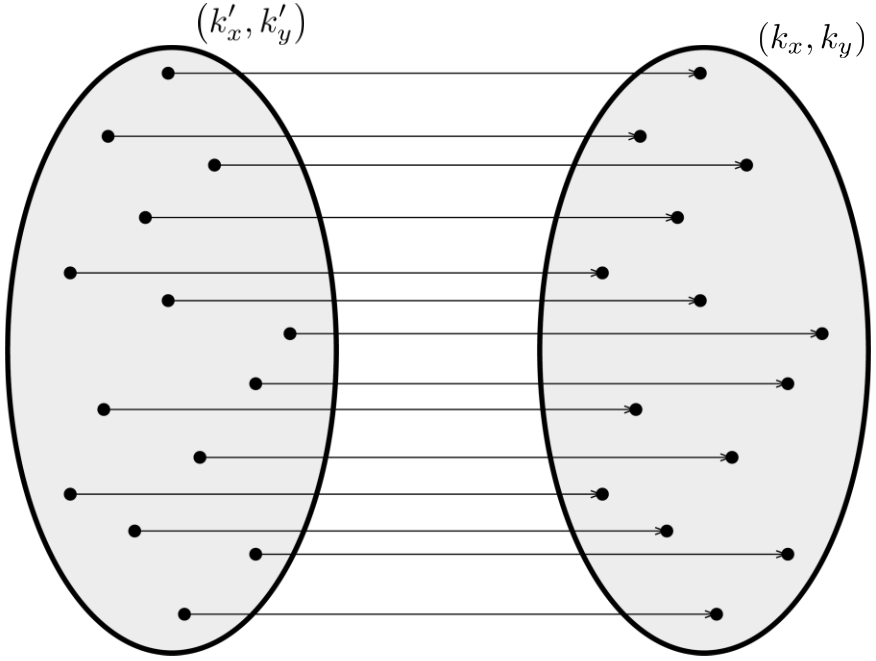

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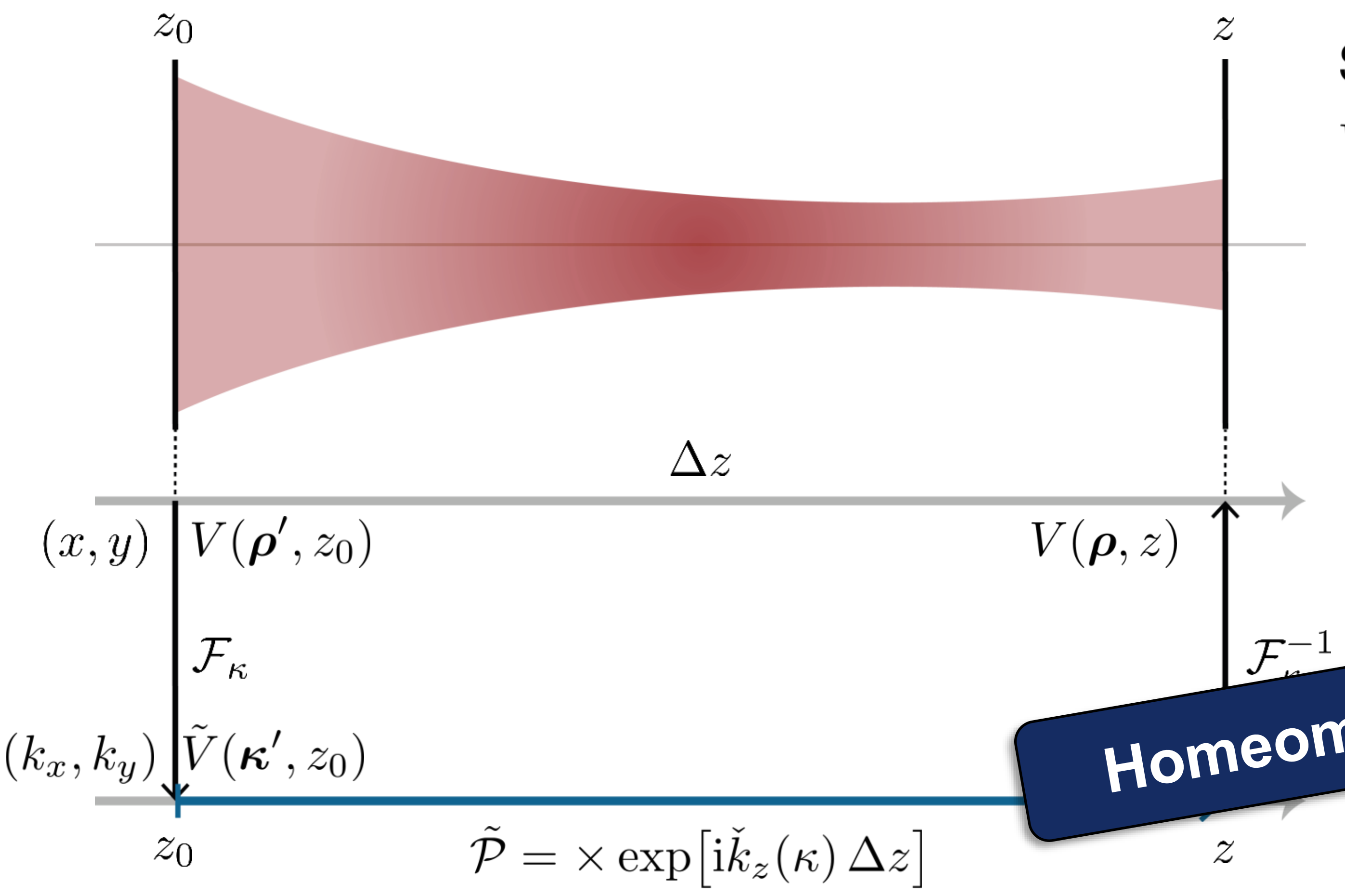

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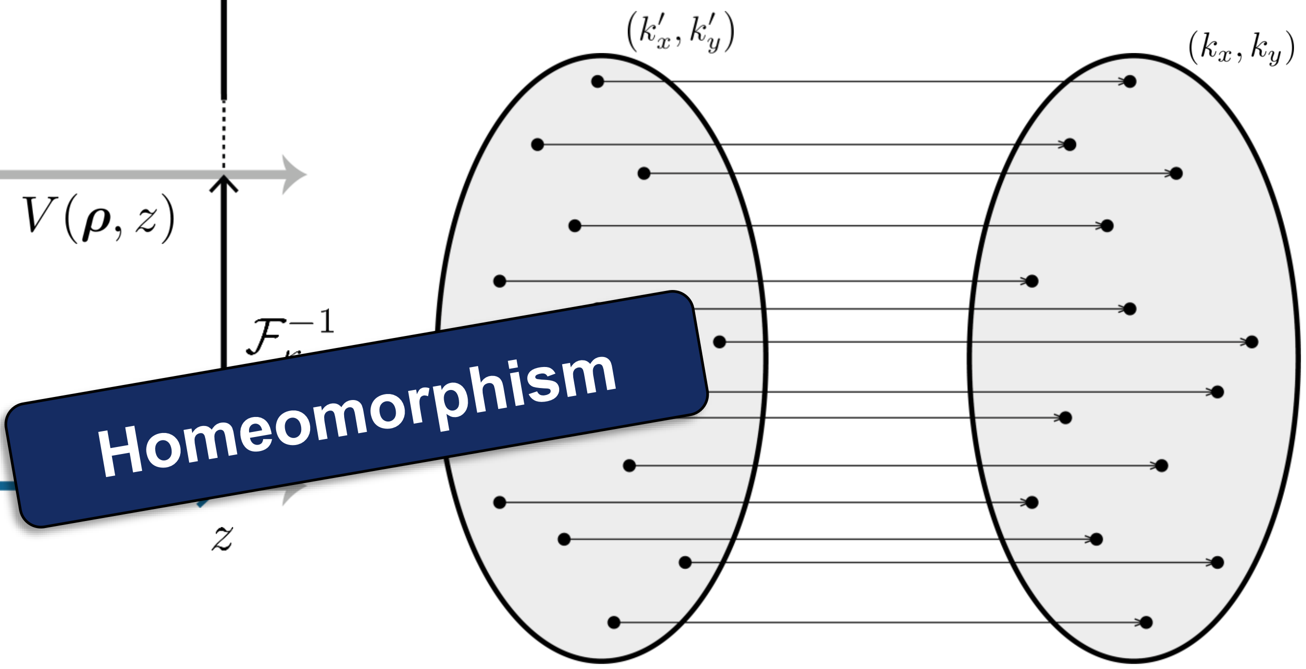

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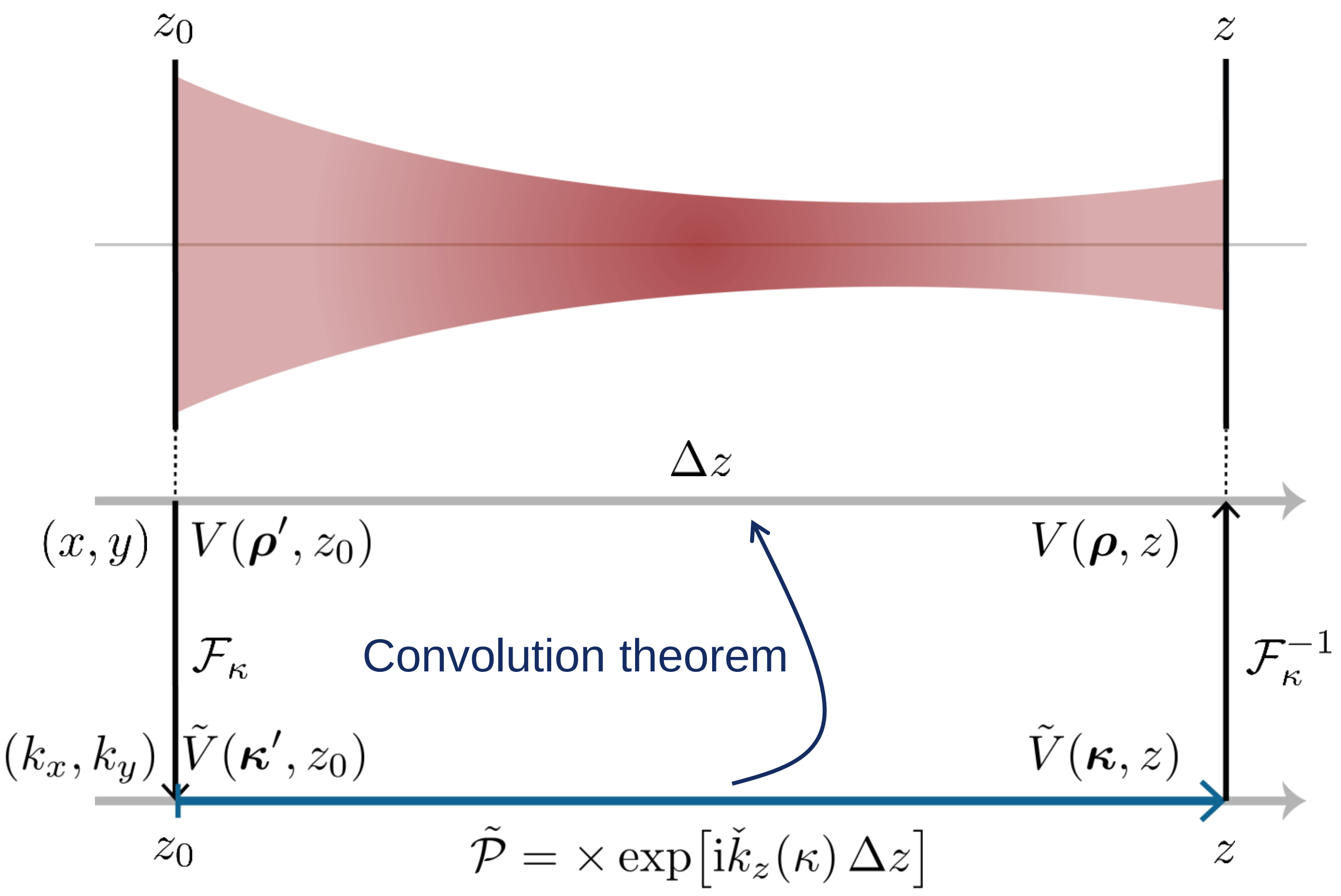

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Convolution theorem:

$$\mathcal{F}_\kappa^{-1}\left\{\tilde{f}_1 \cdot \tilde{f}_2\right\} = f_1 \circledast f_2$$

SPW and the Convolution Theorem

SPW propagation operator:

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Convolution theorem:

$$\mathcal{F}_{\kappa}^{-1}\left\{\tilde{f}_1 \cdot \tilde{f}_2\right\} = f_1 \circledast f_2$$

$$V^{\mathrm{I}}(\boldsymbol{\rho}, z) = -\frac{1}{2\pi}\iint_{-\infty}^{+\infty} V(\boldsymbol{\rho}', z_0)\,\frac{\exp(\mathrm{i}k_0\check{n})}{R}\left(\mathrm{i}k_0\check{n} - \frac{1}{R}\right)\frac{\Delta z}{R}\mathrm{d}^2\rho'$$

$$V^{\mathrm{II}}(\boldsymbol{\rho}, z) = -\frac{1}{2\pi}\iint_{-\infty}^{+\infty} \frac{\partial}{\partial z}V(\boldsymbol{\rho}', z_0)\,\frac{\exp(\mathrm{i}k_0\check{n}R)}{R}\mathrm{d}^2\rho'$$

SPW and the Convolution Theorem

SPW propagation operator:

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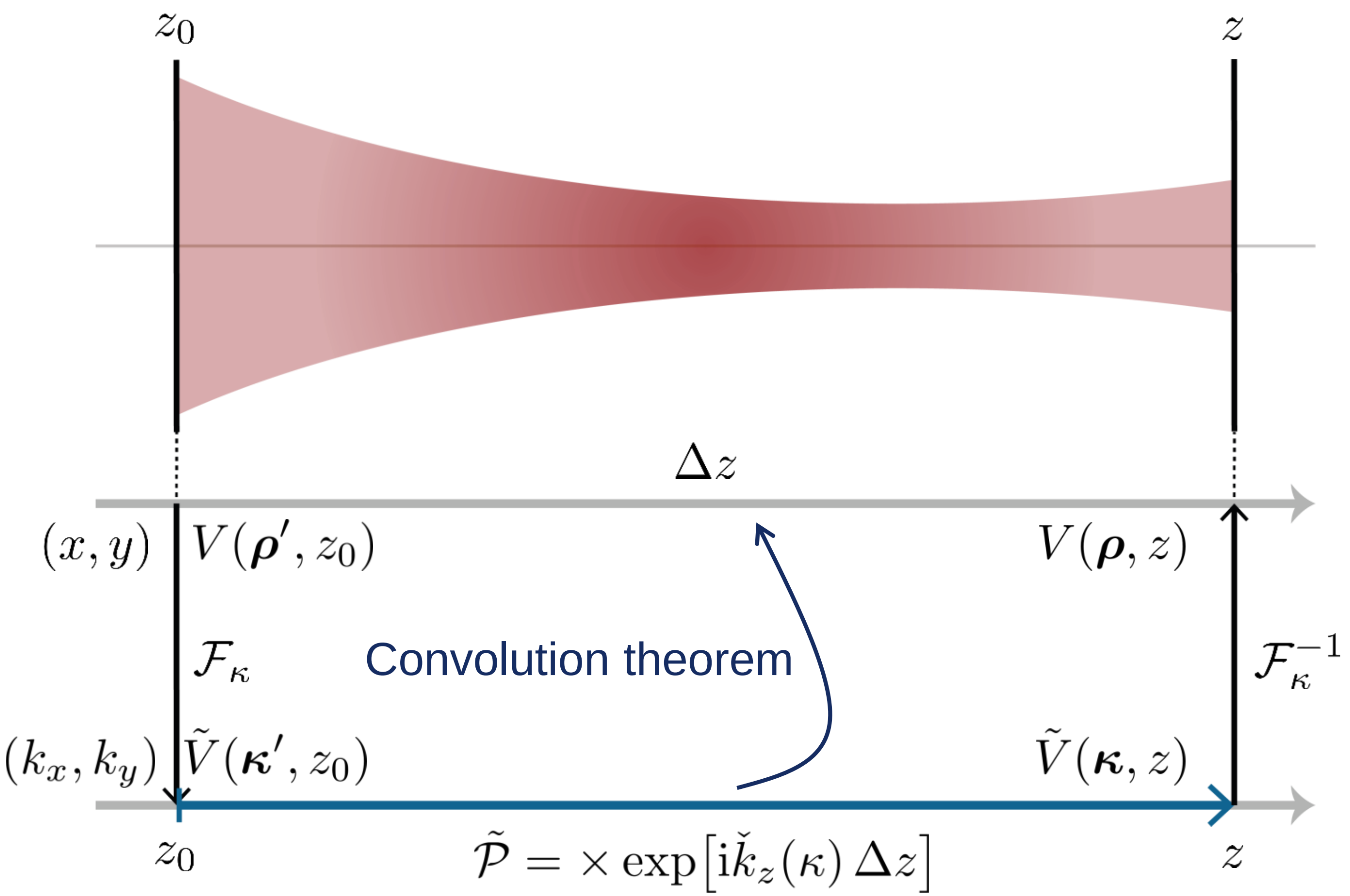
Spectrum of Plane Waves (SPW)

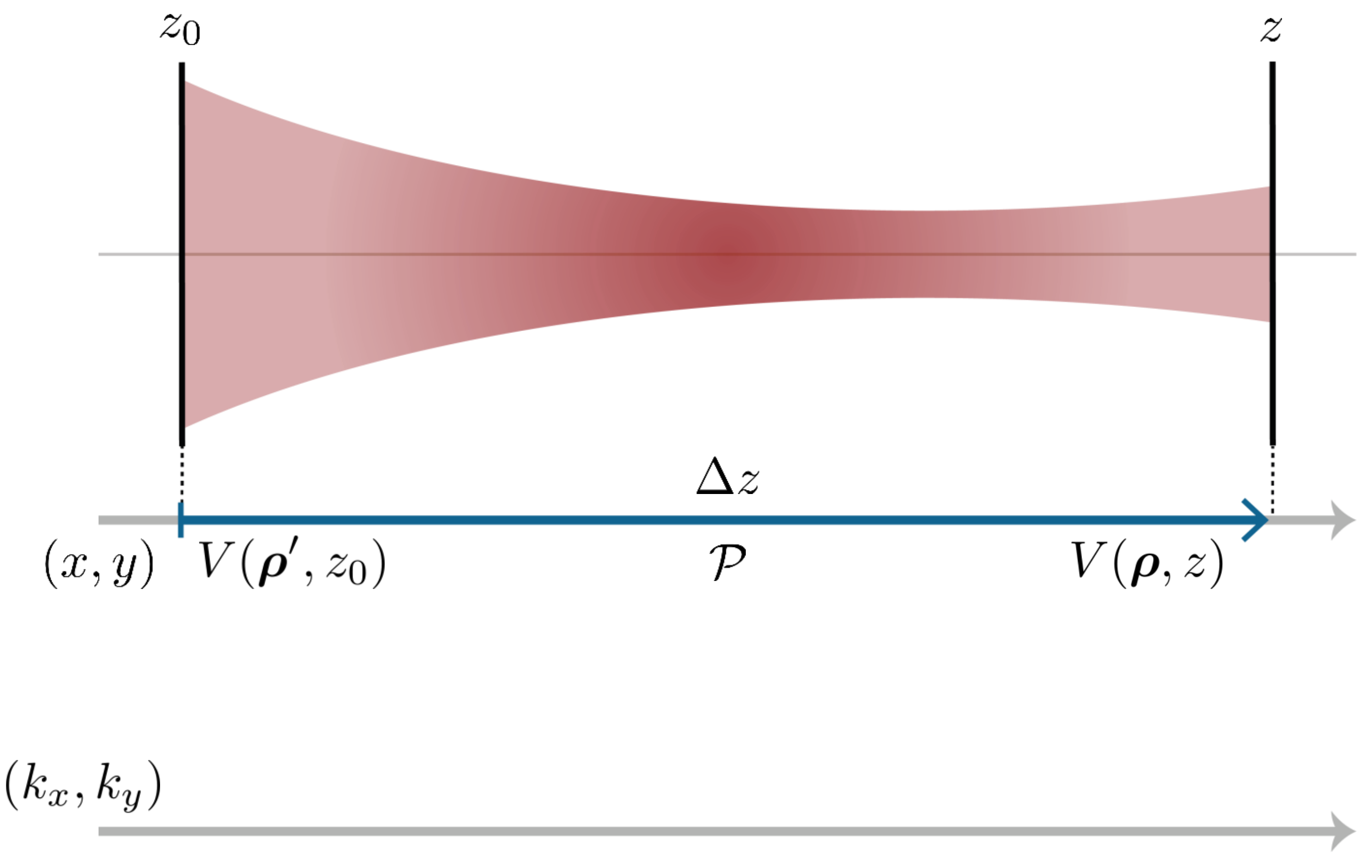
↓

Rayleigh-Sommerfeld Diffraction Integral

Free-Space Propagation: Rayleigh-Sommerfeld Integral



Free-Space Propagation: Rayleigh-Sommerfeld Integral



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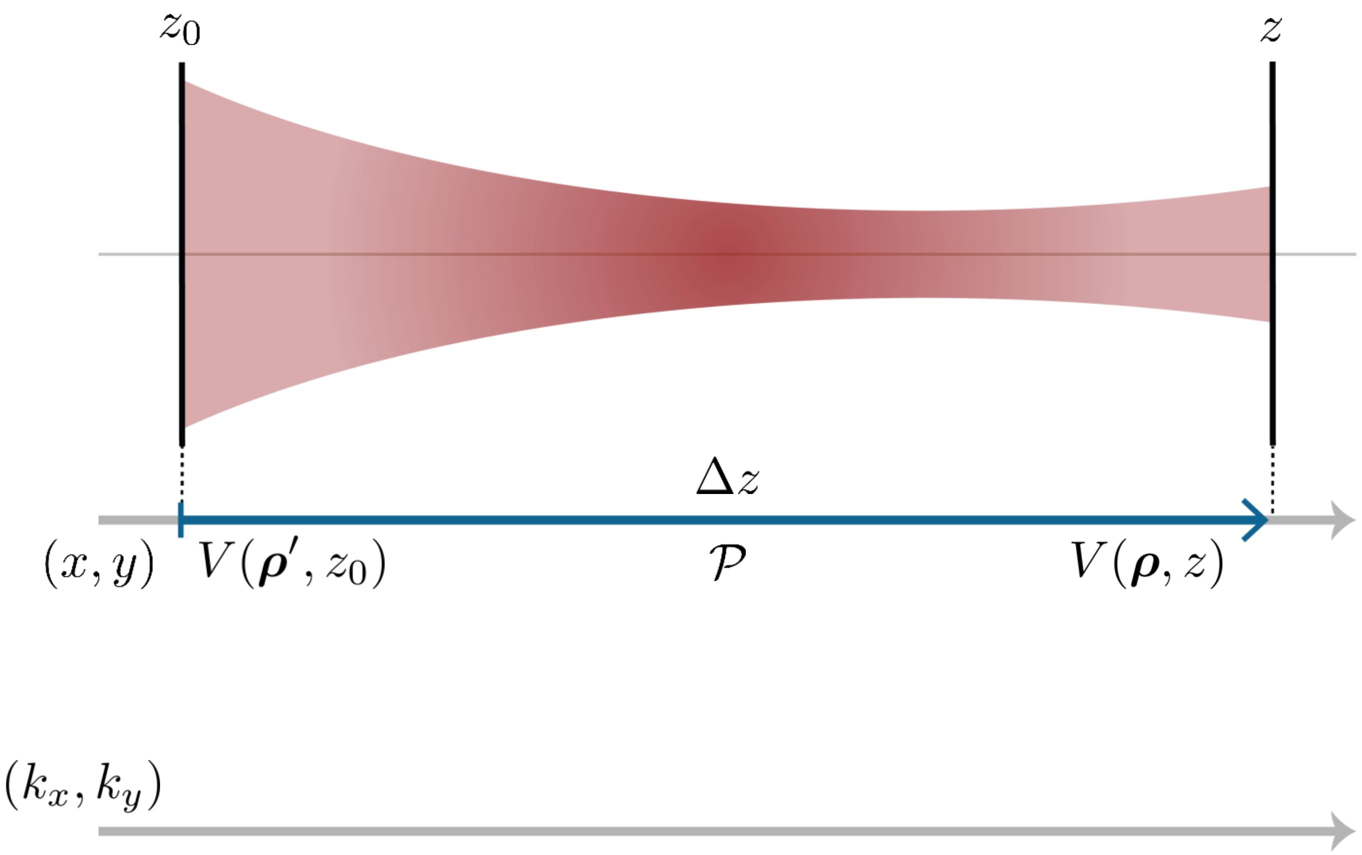
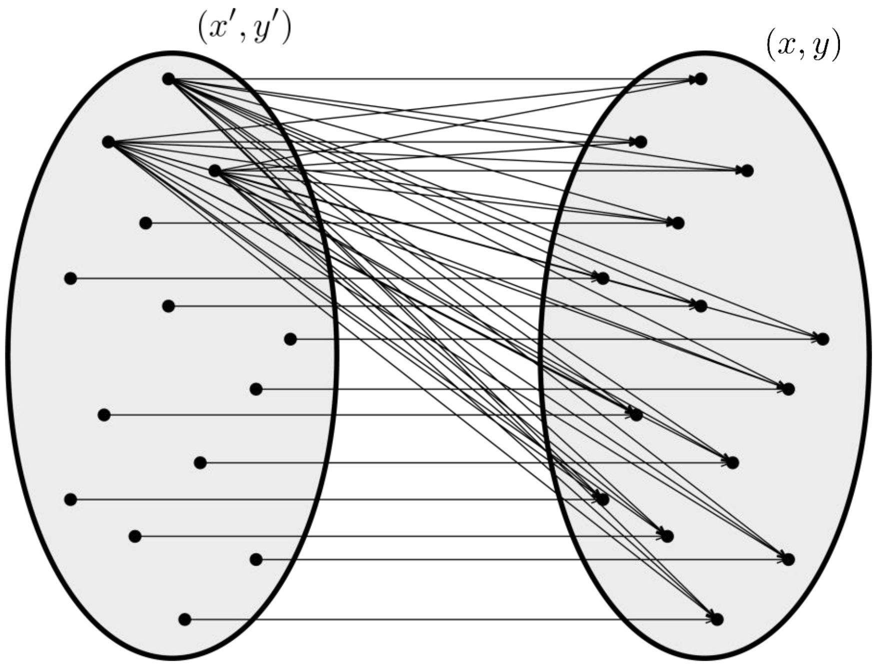
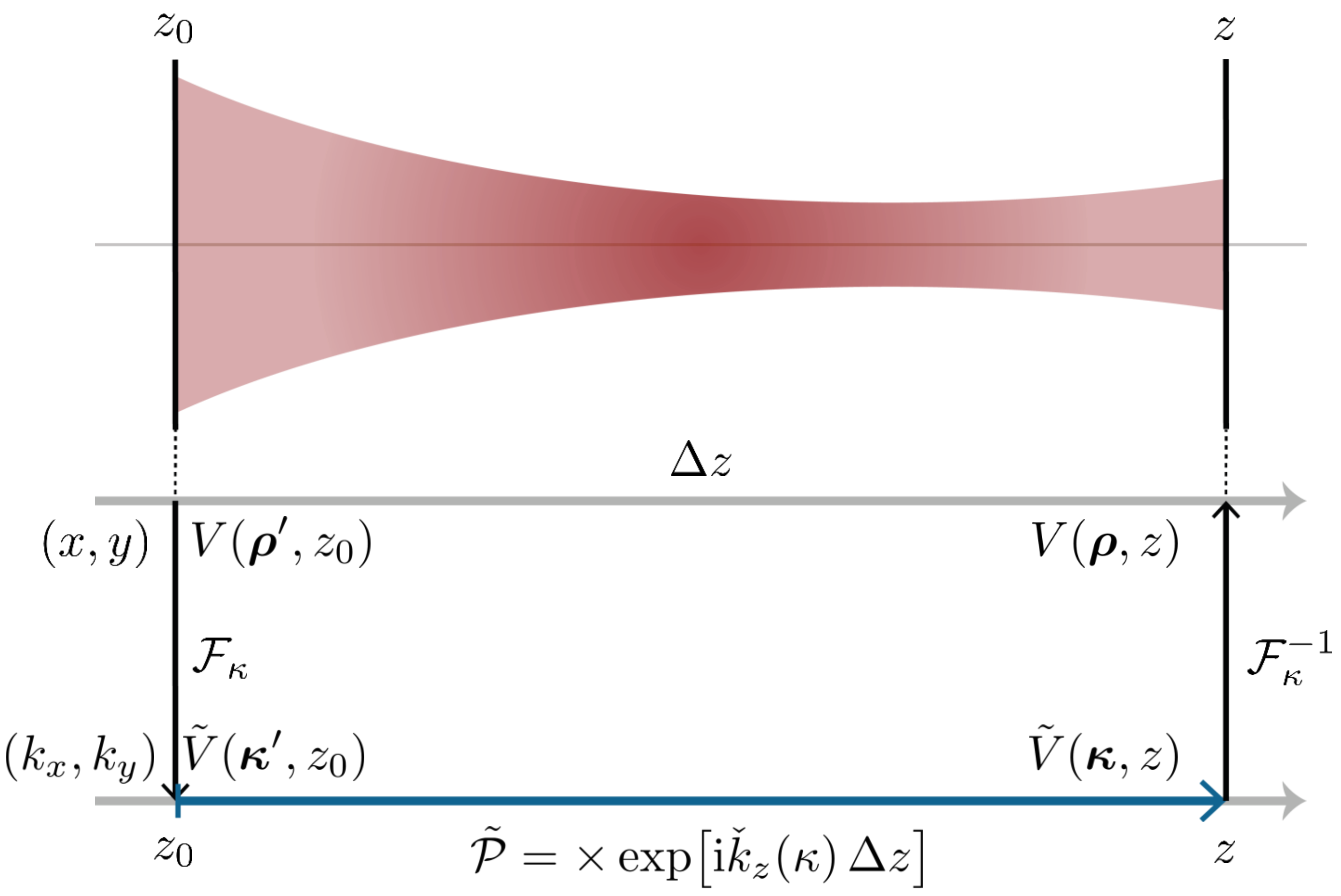
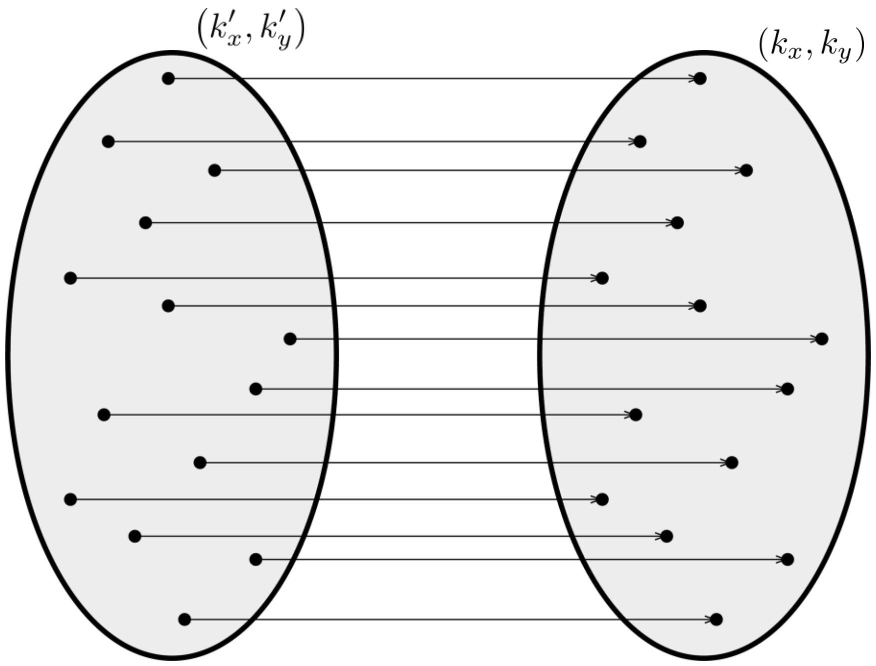


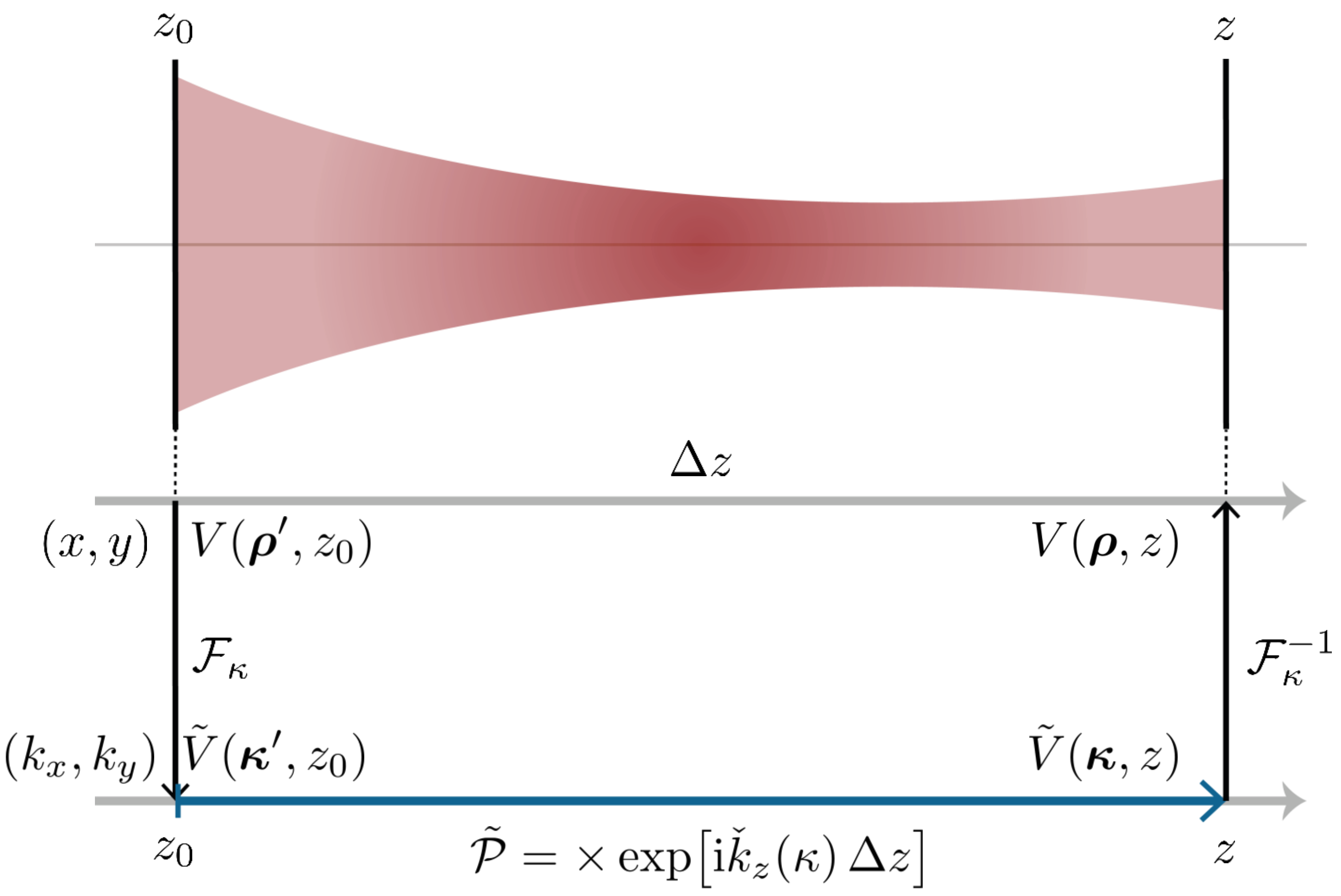


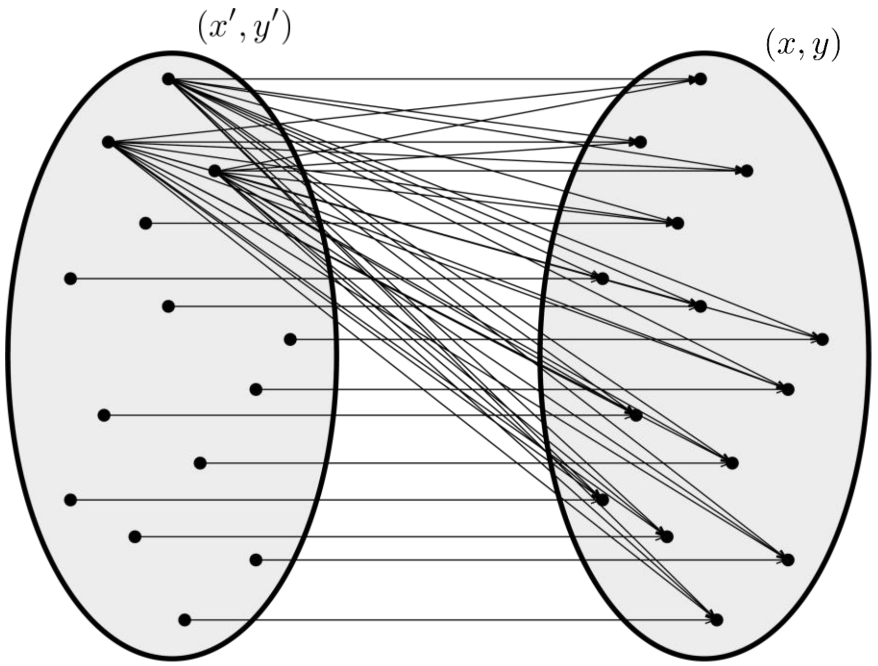
Homeomorphism from κ^{in} to κ^{out}…



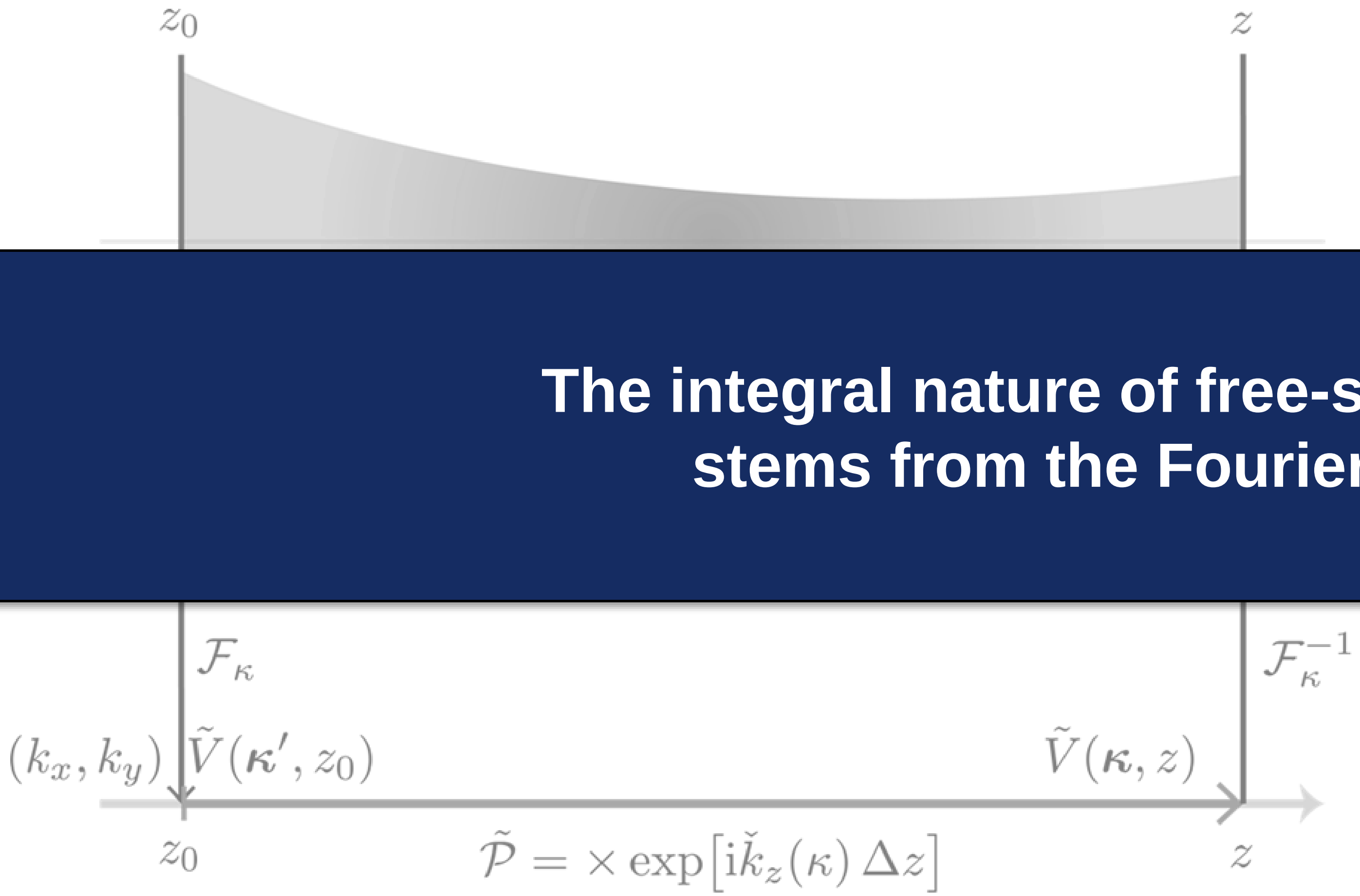


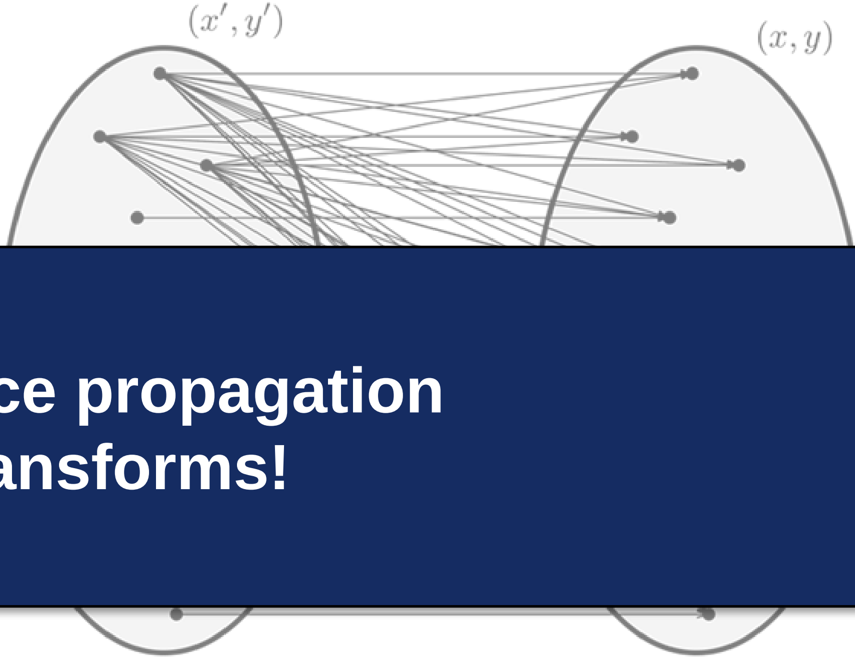
… but not from ρ^{in} to ρ^{out}





… but not from ρ^in^ to ρ^out^





The integral nature of free-space propagation stems from the Fourier transforms!

The Homeomorphic Fourier Transform: A Reminder

$$V(\boldsymbol{\rho}) = U(\boldsymbol{\rho})\exp[\mathrm{i}\psi(\boldsymbol{\rho})]$$

$$\boldsymbol{\nabla}_\perp\psi(\boldsymbol{\rho}) = \boldsymbol{\kappa} \to \boldsymbol{\rho}(\boldsymbol{\kappa})$$

$$\tilde{V}(\boldsymbol{\kappa}) \approx \alpha[\boldsymbol{\rho}(\boldsymbol{\kappa})]\,U[\boldsymbol{\rho}(\boldsymbol{\kappa})]\exp(\mathrm{i}\{\psi[\boldsymbol{\rho}(\boldsymbol{\kappa})] - \boldsymbol{\kappa}\cdot\boldsymbol{\rho}(\boldsymbol{\kappa})\})$$

$$\alpha(\boldsymbol{\rho}) = \sigma(\boldsymbol{\rho})\sqrt{\frac{1}{\left|\psi_{xy}^2(\boldsymbol{\rho}) - \psi_{xx}(\boldsymbol{\rho})\,\psi_{yy}(\boldsymbol{\rho})\right|}}$$

$$\mathcal{F}_{\kappa}^{\text{hom}}\{V(\boldsymbol{\rho})\}$$

$$\mathcal{F}_{\kappa}^{\text{hom},-1}\left\{\tilde{V}(\boldsymbol{\kappa})\right\}$$

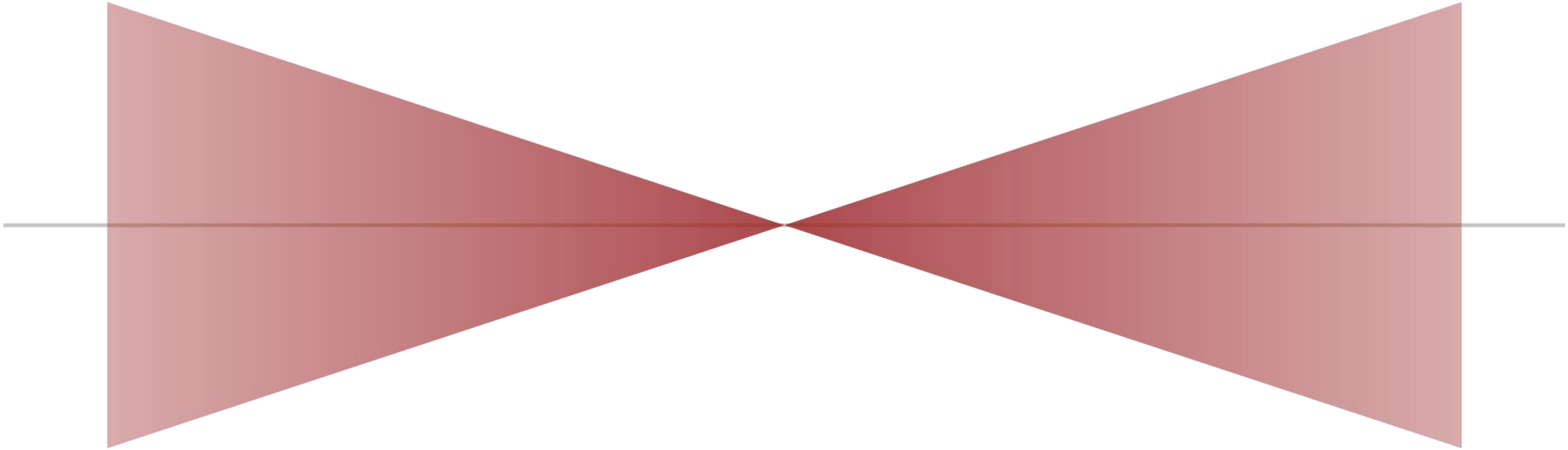
$$\tilde{V}(\boldsymbol{\kappa}) = \tilde{A}(\boldsymbol{\kappa})\exp\left[\mathrm{i}\tilde{\psi}(\boldsymbol{\kappa})\right]$$

$$\tilde{\boldsymbol{\nabla}}_\perp\tilde{\psi}(\boldsymbol{\kappa}) = -\boldsymbol{\rho} \to \boldsymbol{\kappa}(\boldsymbol{\rho})$$

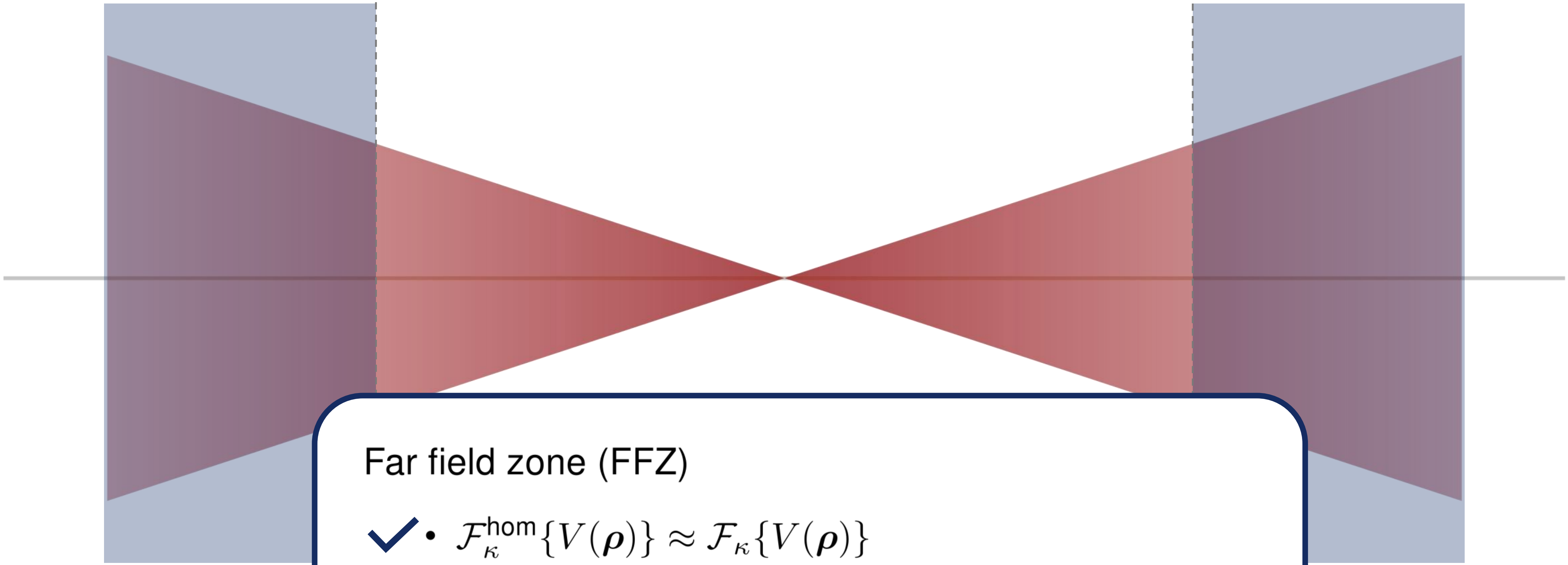
$$V(\boldsymbol{\rho}) \approx \tilde{\alpha}[\boldsymbol{\kappa}(\boldsymbol{\rho})]\,\tilde{A}[\boldsymbol{\kappa}(\boldsymbol{\rho})]\exp\left(\mathrm{i}\left\{\tilde{\psi}[\boldsymbol{\kappa}(\boldsymbol{\rho})] + \boldsymbol{\kappa}(\boldsymbol{\rho})\cdot\boldsymbol{\rho}\right\}\right)$$

$$\tilde{\alpha}(\boldsymbol{\kappa}) = \tilde{\sigma}(\boldsymbol{\kappa})\sqrt{\frac{1}{\left|\tilde{\psi}_{k_xk_y}^2(\boldsymbol{\kappa}) - \tilde{\psi}_{k_xk_x}(\boldsymbol{\kappa})\,\tilde{\psi}_{k_yk_y}(\boldsymbol{\kappa})\right|}}$$

Far, Homeomorphic & Diffractive Field Zones (FFZ, HFZ, DFZ)



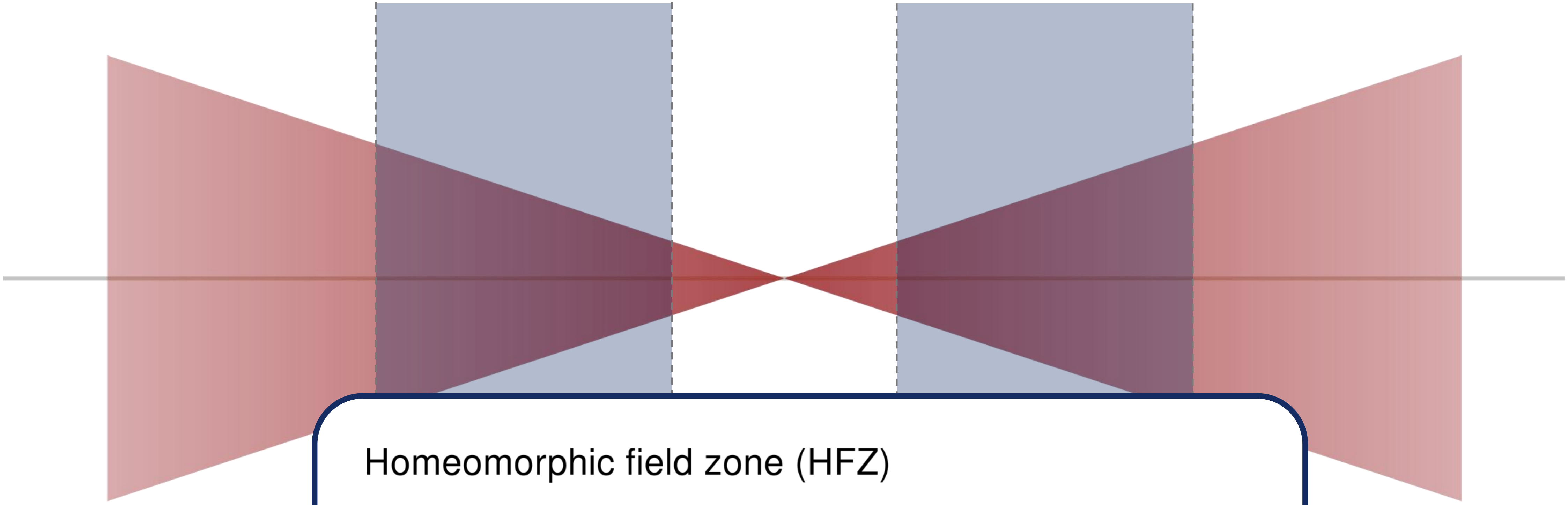
Far, Homeomorphic & Diffractive Field Zones (FFZ, HFZ, DFZ)



Far field zone (FFZ)

- ✓ $\mathcal{F}_{\kappa}^{\text{hom}}\{V(\boldsymbol{\rho})\} \approx \mathcal{F}_{\kappa}\{V(\boldsymbol{\rho})\}$
- ✓ $\mathcal{F}_{\kappa}^{\text{hom}}\{V(\boldsymbol{\rho})\} \approx \mathcal{F}_{\kappa}\{V(\boldsymbol{\rho})\}$, even if $\psi(\boldsymbol{\rho}) = \psi^{\text{sph}}(\boldsymbol{\rho})$

Far, Homeomorphic & Diffractive Field Zones (FFZ, HFZ, DFZ)

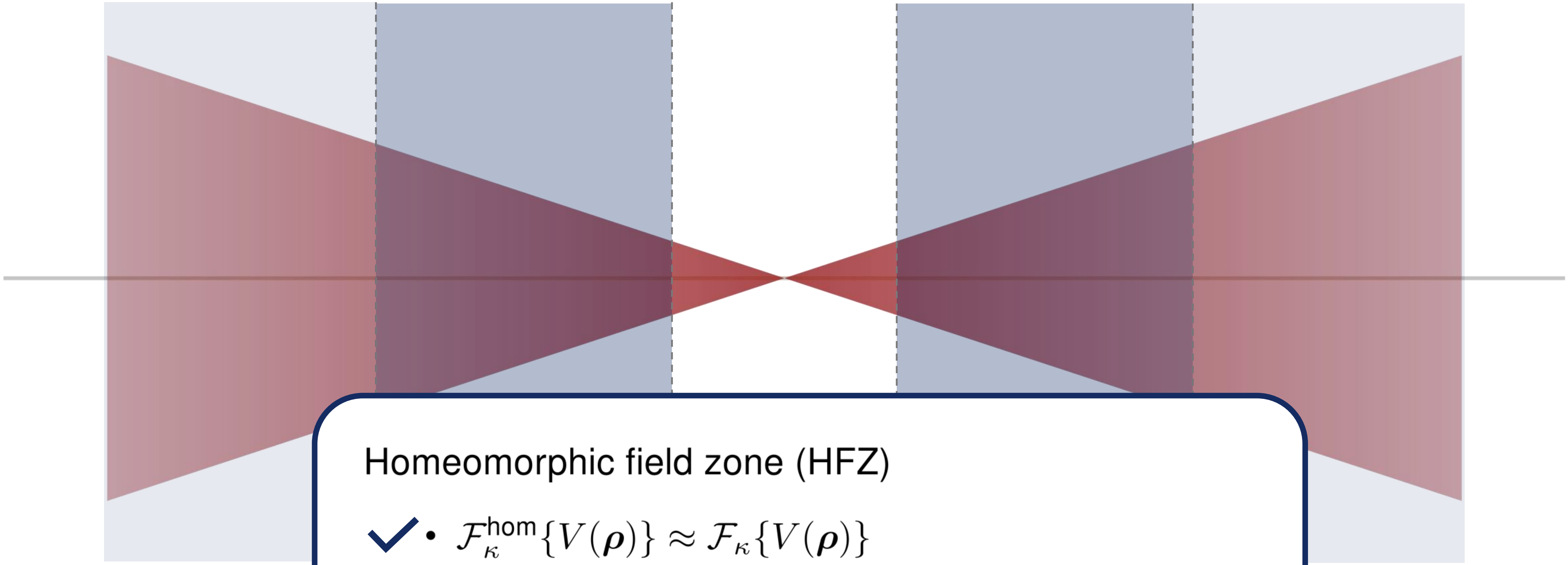


Homeomorphic field zone (HFZ)

✓ • $\mathcal{F}_{\kappa}^{\text{hom}}\{V(\boldsymbol{\rho})\} \approx \mathcal{F}_{\kappa}\{V(\boldsymbol{\rho})\}$

✗ • $\cancel{\mathcal{F}_{\kappa}^{\text{hom}}\{V(\boldsymbol{\rho})\} \approx \mathcal{F}_{\kappa}\{V(\boldsymbol{\rho})\}}$~~, even if~~ $\cancel{\psi(\boldsymbol{\rho}) = \psi^{\text{spn}}(\boldsymbol{\rho})}$

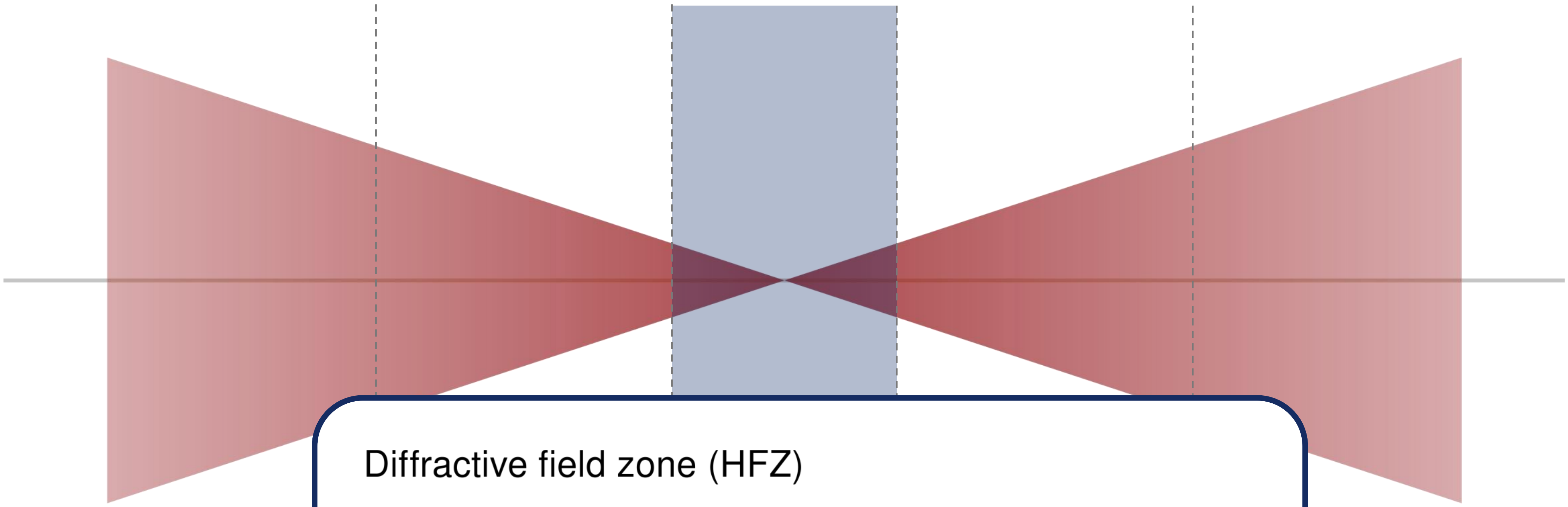
Far, Homeomorphic & Diffractive Field Zones (FFZ, HFZ, DFZ)



Homeomorphic field zone (HFZ)

- ✓ $\mathcal{F}_\kappa^{\text{hom}}\{V(\boldsymbol{\rho})\} \approx \mathcal{F}_\kappa\{V(\boldsymbol{\rho})\}$
- ✗ $\cancel{\mathcal{F}_\kappa^{\text{hom}}\{V(\boldsymbol{\rho})\} \approx \mathcal{F}_\kappa\{V(\boldsymbol{\rho})\}, \text{ even if } \psi(\boldsymbol{\rho}) = \psi^{\text{sph}}(\boldsymbol{\rho})}$

Far, Homeomorphic & Diffractive Field Zones (FFZ, HFZ, DFZ)



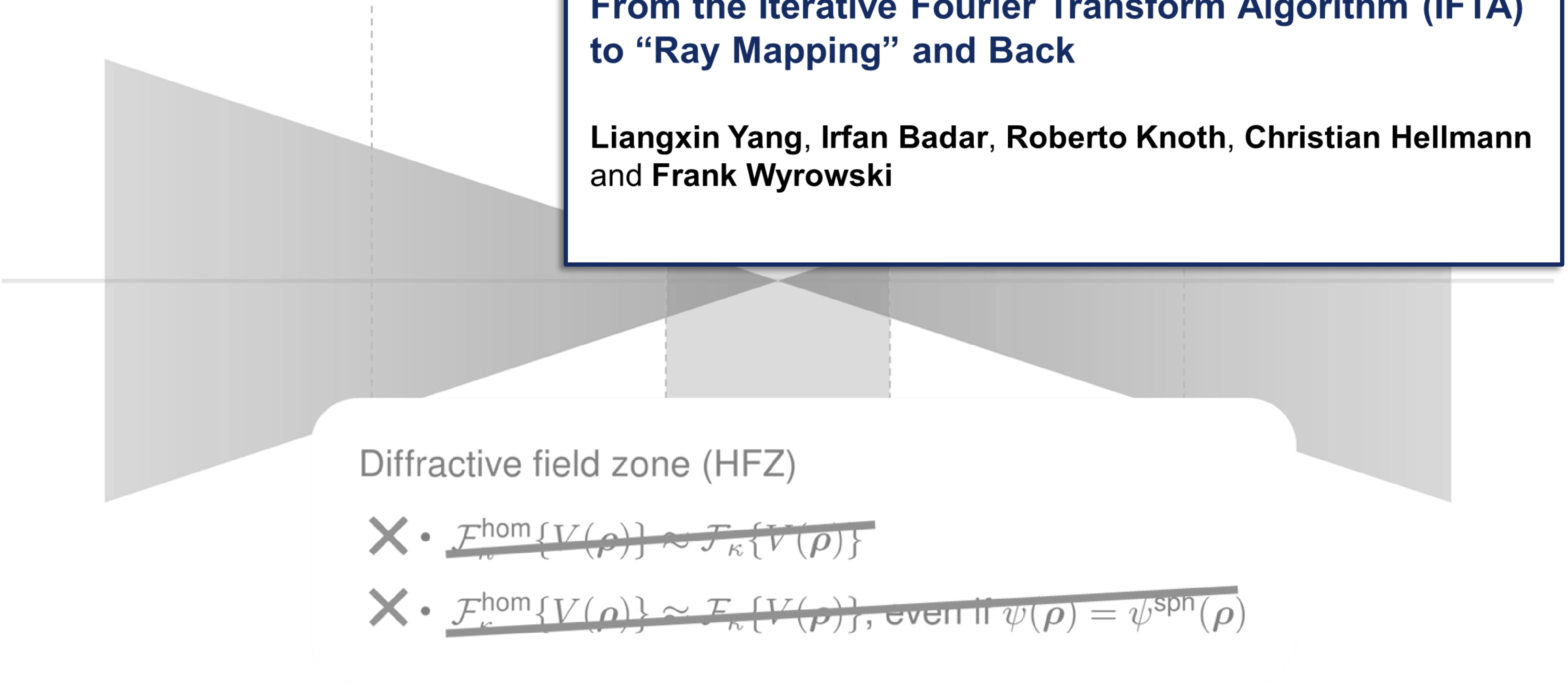
Diffractive field zone (HFZ)

- ✗ $\cancel{\mathcal{F}_\kappa^{\text{hom}}\{V(\boldsymbol{\rho})\} \approx \mathcal{F}_\kappa\{V(\boldsymbol{\rho})\}}$
- ✗ ~~$\mathcal{F}_\kappa^{\text{hom}}\{V(\boldsymbol{\rho})\} \approx \mathcal{F}_\kappa\{V(\boldsymbol{\rho})\}$, even if $\psi(\boldsymbol{\rho}) = \psi^{\text{spn}}(\boldsymbol{\rho})$~~

Far, Homeomorphic & Diffractive Field Zones (FFZ, HFZ, DFZ)

From the Iterative Fourier Transform Algorithm (IFTA) to “Ray Mapping” and Back

Liangxin Yang, **Irfan Badar**, **Roberto Knoth**, **Christian Hellmann** and **Frank Wyrowski**

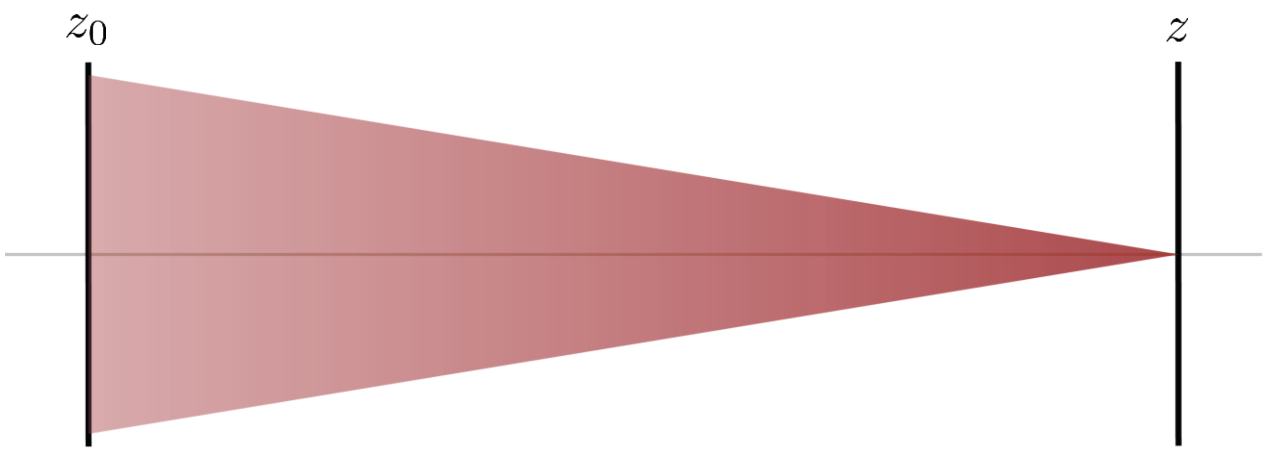


The Homeomorphic Fourier Transform in Free-Space Propagation

Coordinate system: $z = 0$ always assumed at focus!

The Homeomorphic Fourier Transform in Free-Space Propagation

From Homeomorphic Field Zone to Diffractive Field Zone



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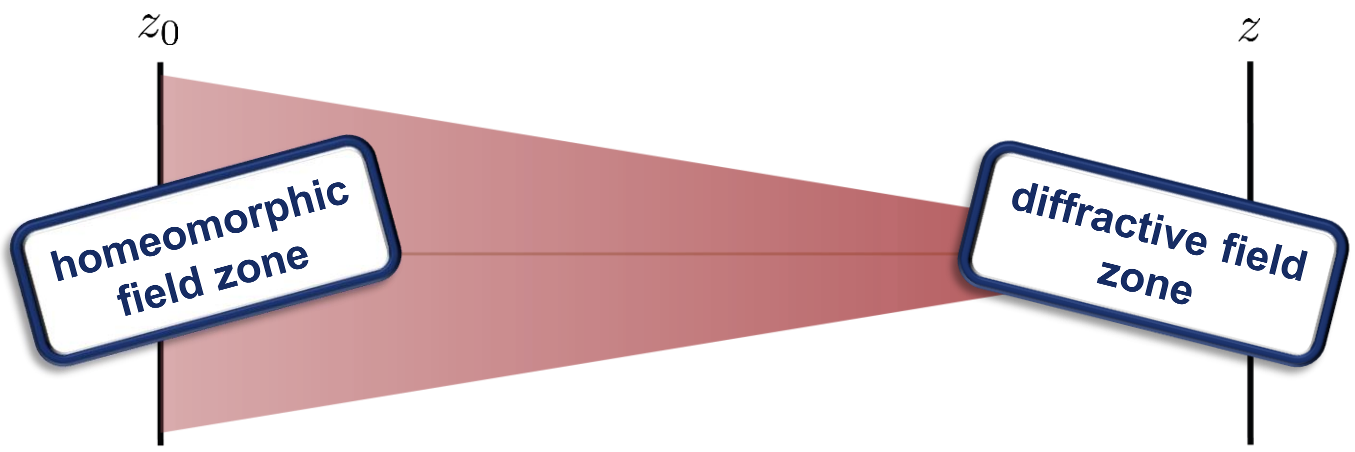

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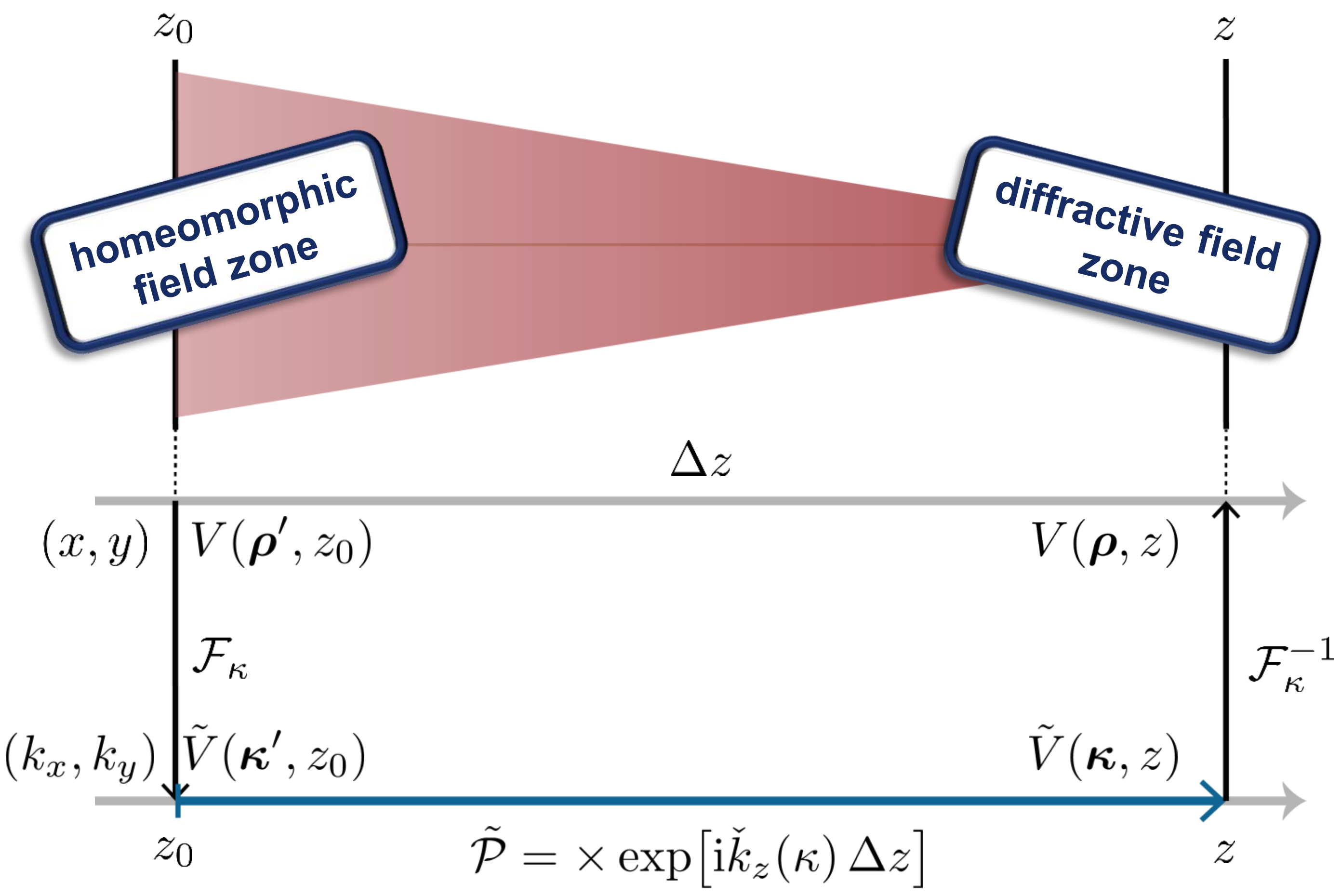

SPW propagation operator:

$$V(\boldsymbol{\rho}, z) =$$
$$= \mathcal{F}_\kappa^{-1}\left\{\mathcal{F}_\kappa\{V(\boldsymbol{\rho}', z_0)\} \cdot \exp\left[\mathrm{i}\check{k}_z(\kappa)\,\Delta z\right]\right\}$$

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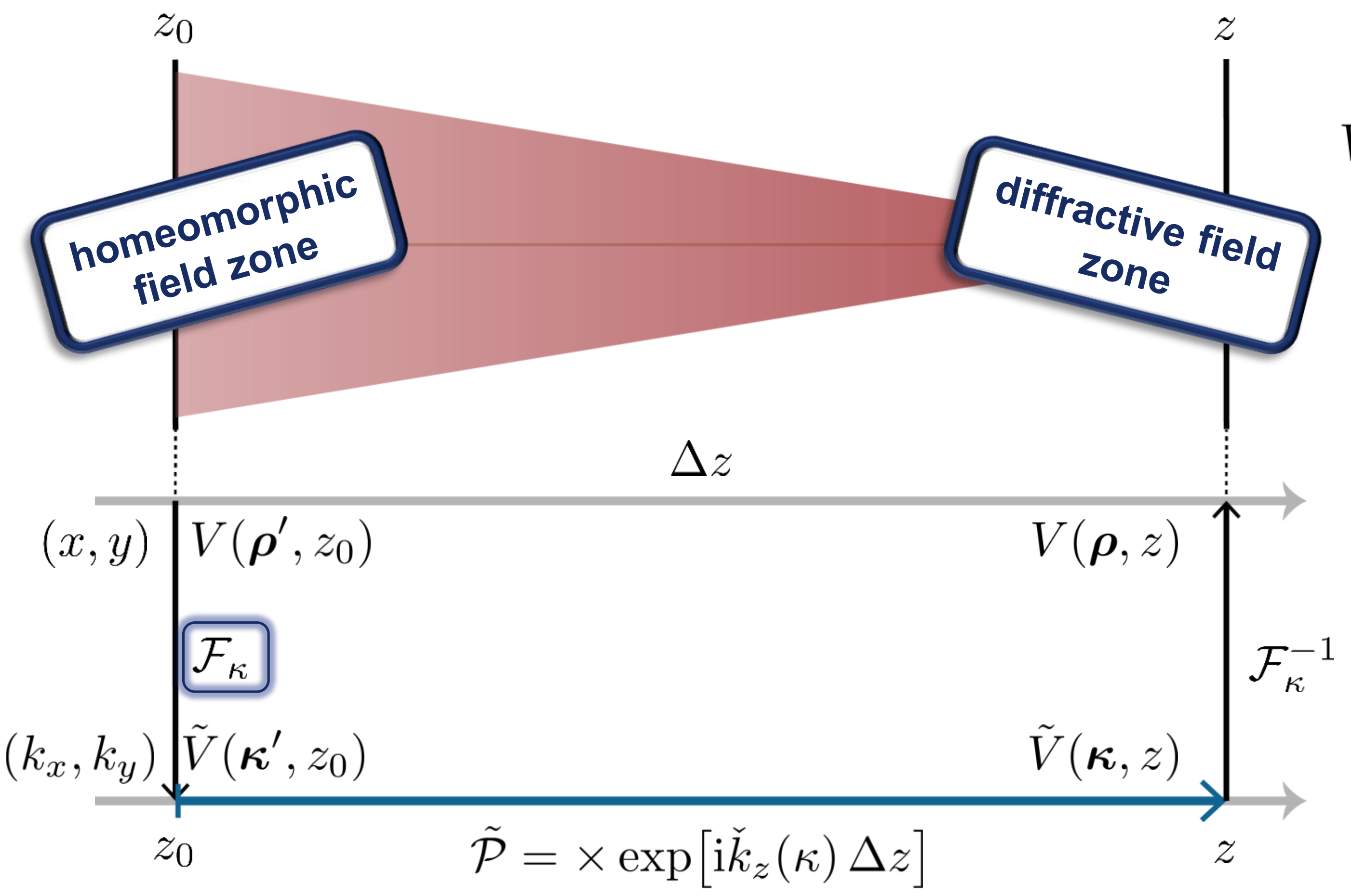

$$
\begin{aligned}
&V(\boldsymbol{\rho}, z) = \\
&= \mathcal{F}_\kappa^{-1}\left\{\boxed{\mathcal{F}_\kappa}\left\{V(\boldsymbol{\rho}', z_0)\right\} \cdot \exp\left[\mathrm{i}\check{k}_z(\kappa)\,\Delta z\right]\right\}
\end{aligned}
$$

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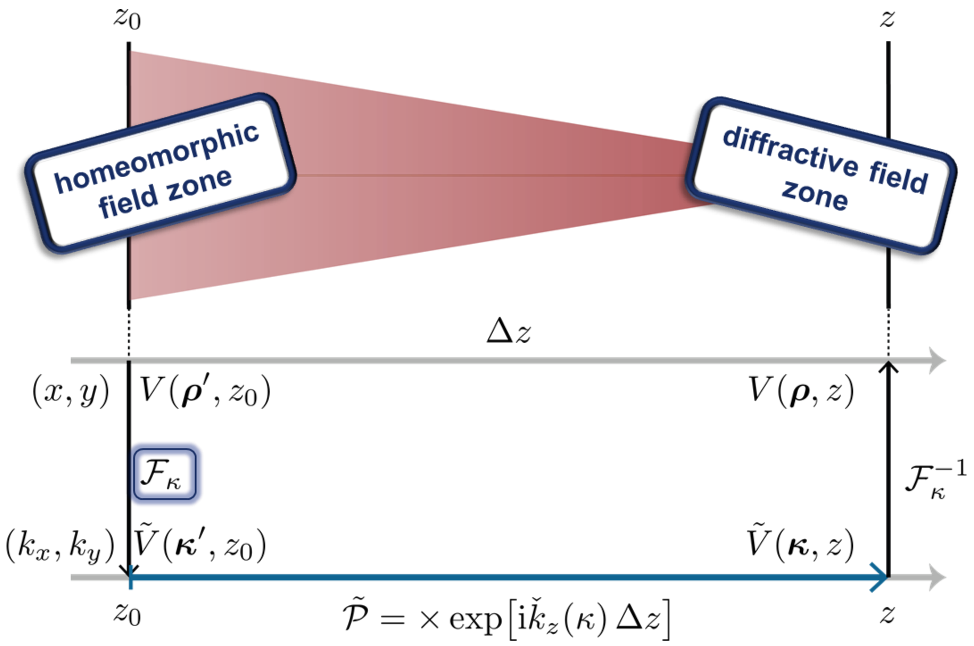

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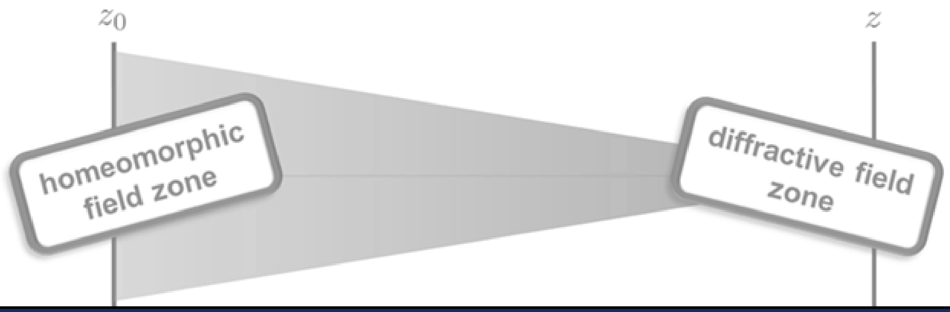
$$V(\boldsymbol{\rho}, z) = \frac{1}{2\pi}\iint_{-\infty}^{+\infty} \alpha[\boldsymbol{\rho}'(\boldsymbol{\kappa})]\, U[\boldsymbol{\rho}'(\boldsymbol{\kappa})] \exp(\mathrm{i}\{\psi[\boldsymbol{\rho}'(\boldsymbol{\kappa})] - \boldsymbol{\kappa}\cdot\boldsymbol{\rho}'(\boldsymbol{\kappa})\}) \times$$
$$\times \exp\left[\mathrm{i}\check{k}_z(\boldsymbol{\kappa})\,\Delta z\right] \exp(\mathrm{i}\boldsymbol{\kappa}\cdot\boldsymbol{\rho})\, \mathrm{d}^2\kappa$$

From Homeomorphic Field Zone to Diffractive Field Zone



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Analytic solution for $\psi^{\mathrm{sph}}(\boldsymbol{\rho}')$!

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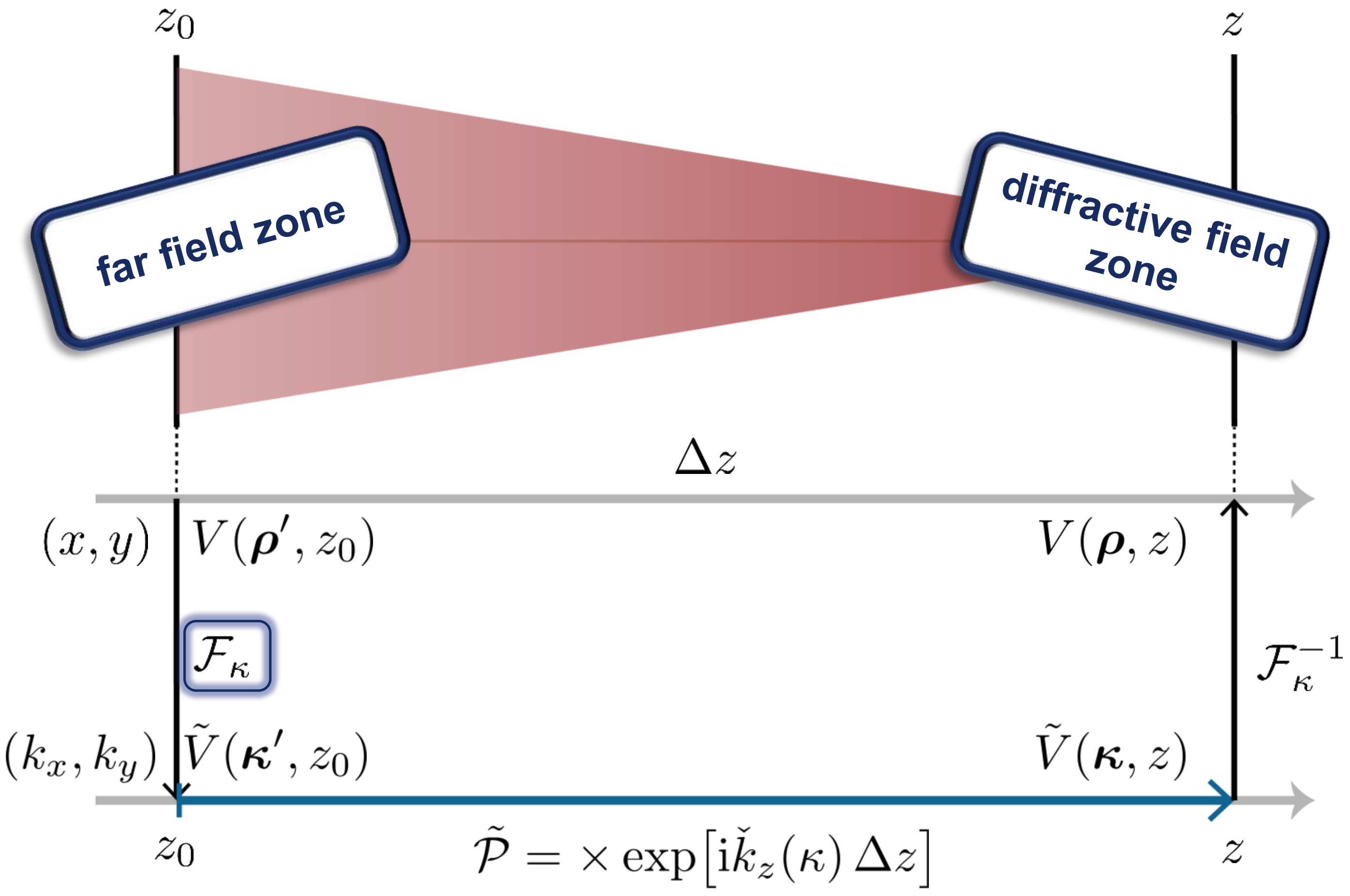
From Far Field Zone to Diffractive Field Zone



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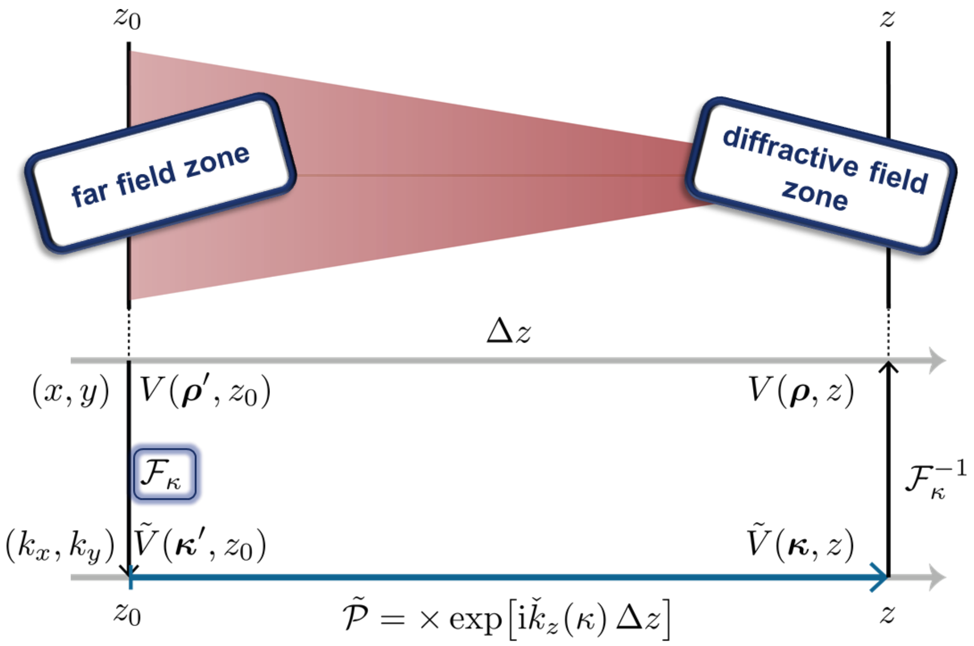

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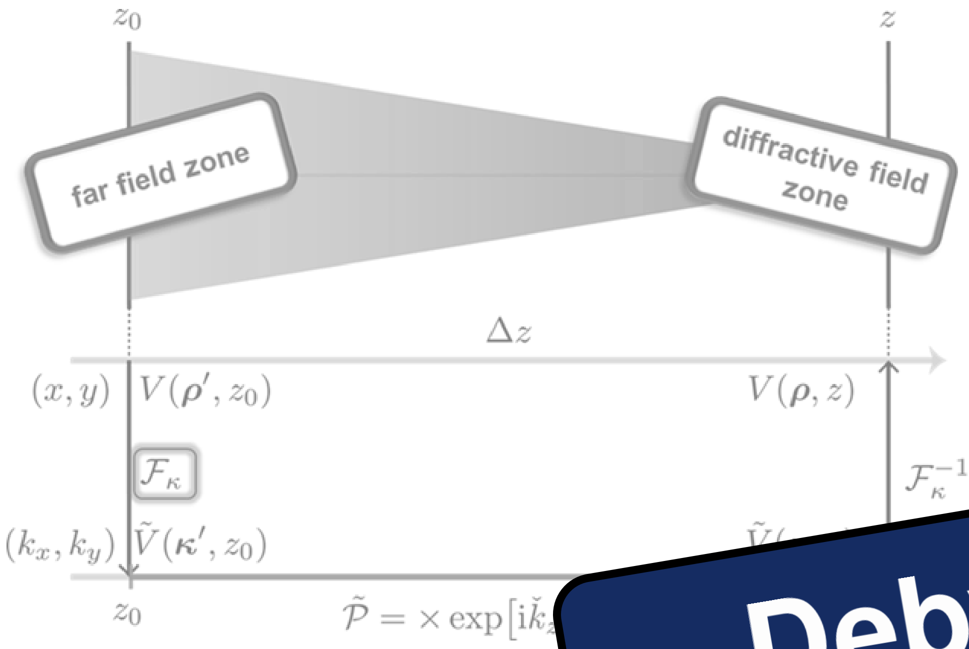
with ψ^{sph}!

$$V(\boldsymbol{\rho}, z) = \frac{1}{2\pi} \iint_{-\infty}^{+\infty} t\left[\frac{\boldsymbol{\kappa}}{\check{k}_z(\boldsymbol{\kappa})} R\right] \frac{-\mathrm{i}}{\check{k}_z(\boldsymbol{\kappa})} \exp[\mathrm{i}\boldsymbol{\kappa} \cdot \boldsymbol{\rho}]\, \mathrm{d}^2\kappa$$

From Far Field Zone to Diffractive Field Zone



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with ψ^{sph}!

Debye integral

$$V(\boldsymbol{\rho}, z) = \frac{1}{2\pi}\iint_{-\infty}^{+\infty} t\left[\frac{\boldsymbol{\kappa}}{\check{k}_z(\boldsymbol{\kappa})}R\right]\frac{-\mathrm{i}}{\check{k}_z(\boldsymbol{\kappa})}\exp[\mathrm{i}\boldsymbol{\kappa}\cdot\boldsymbol{\rho}]\,\mathrm{d}^2\kappa$$

Debye, Ann. d. Phys., 1909

Assumptions of Debye Integral

1. Homeomorphic Fourier transform at input plane.
2. Spherical wavefront at input plane.
3. Propagation **exactly** to the focal point.

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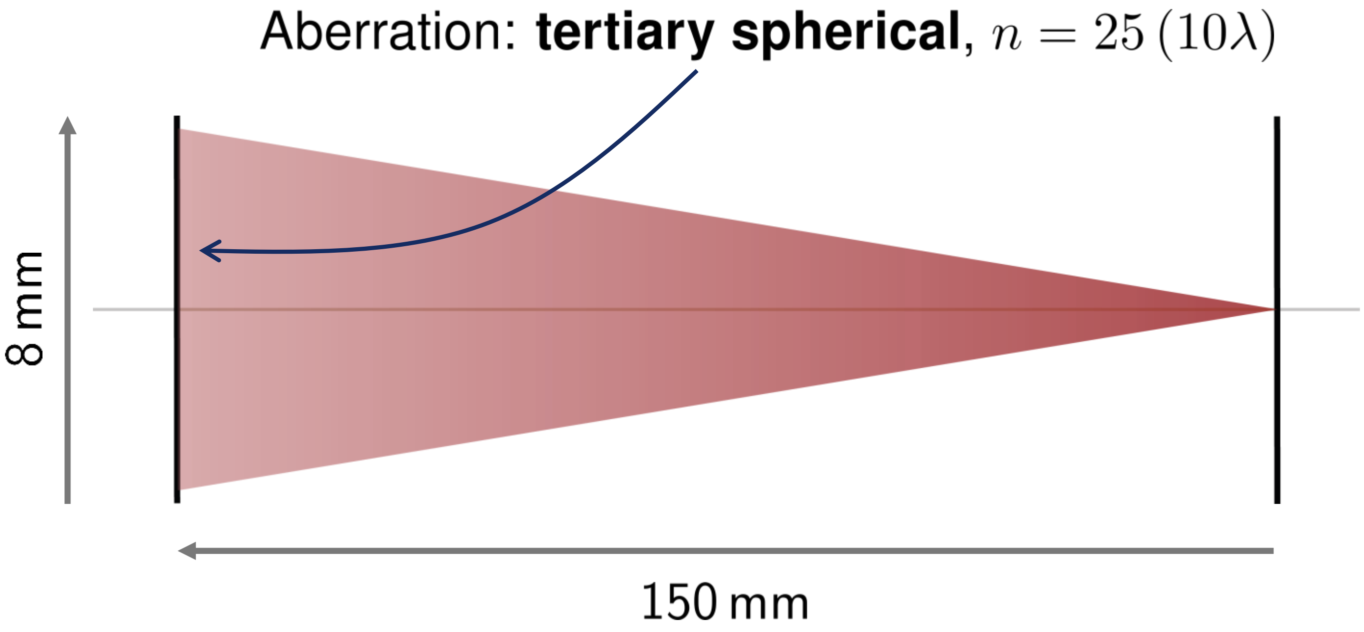
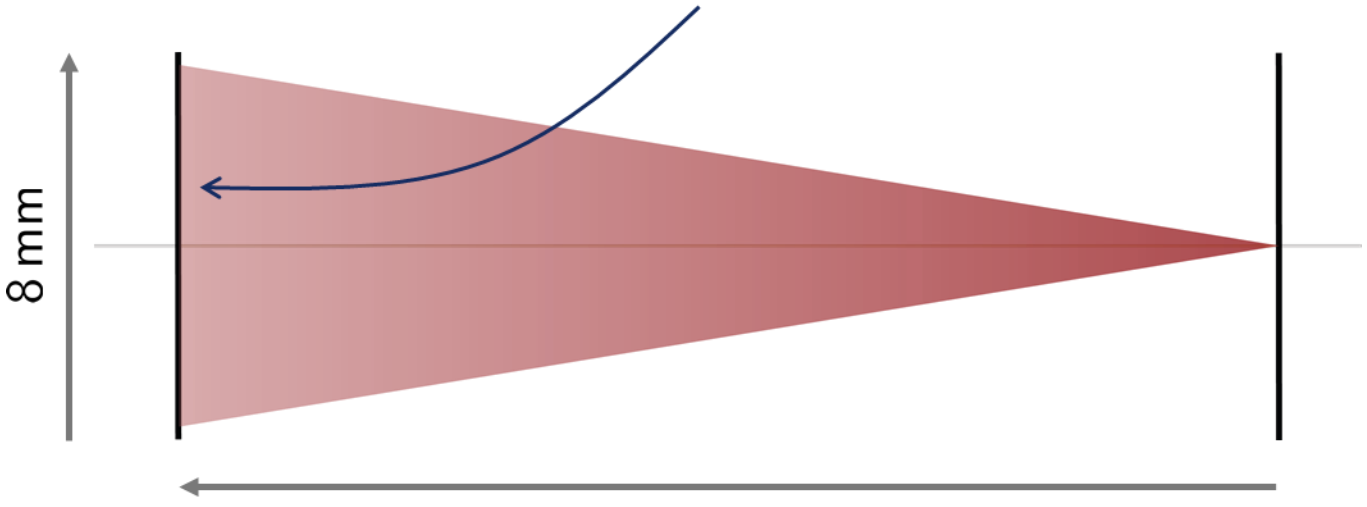
Analysis of the Effect of Wavefront Generalisation

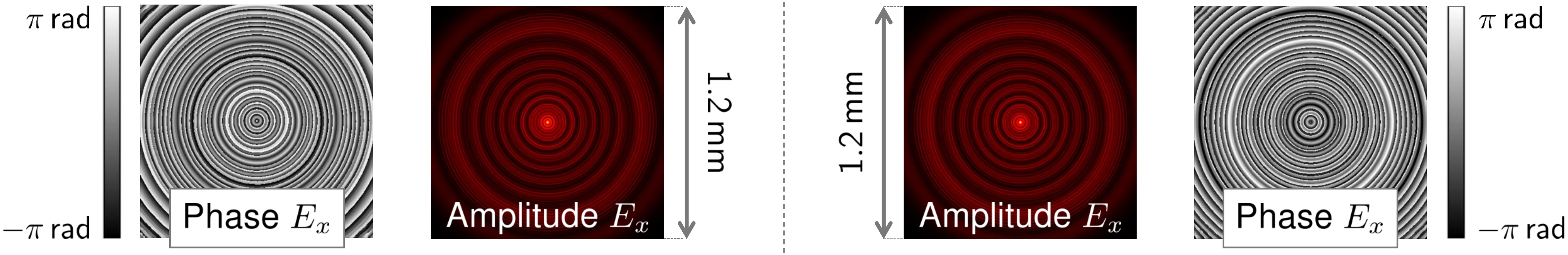


Analysis of the Effect of Wavefront Generalisation



Debye integral

With general wavefront in HFT



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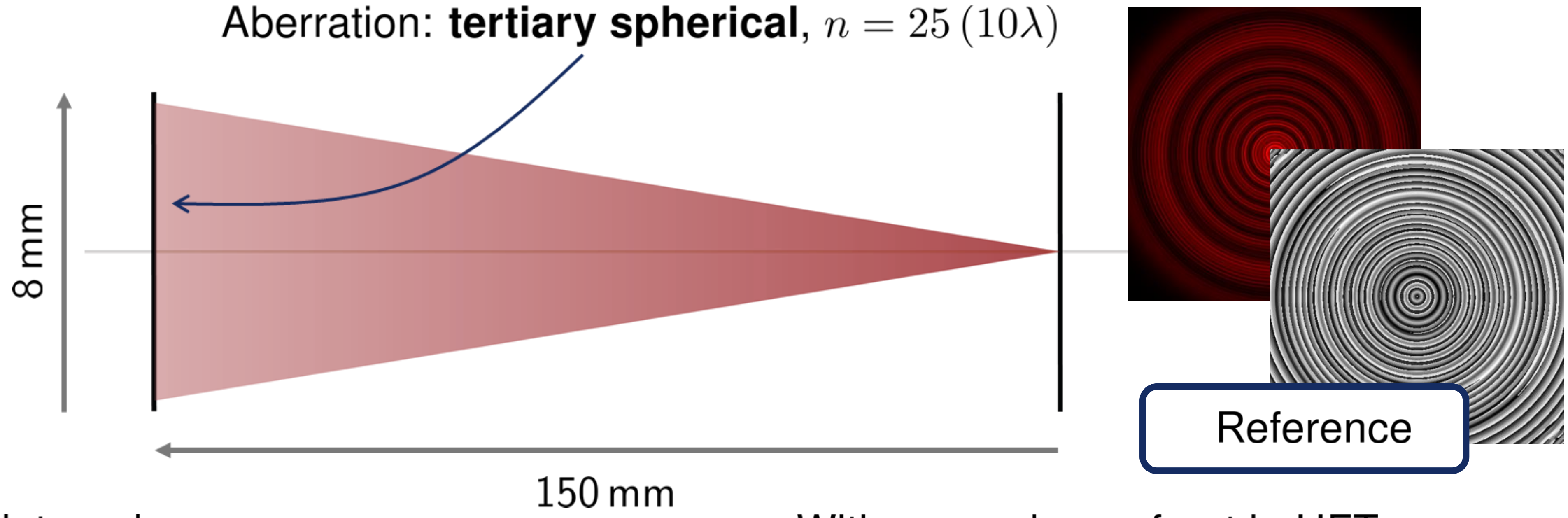
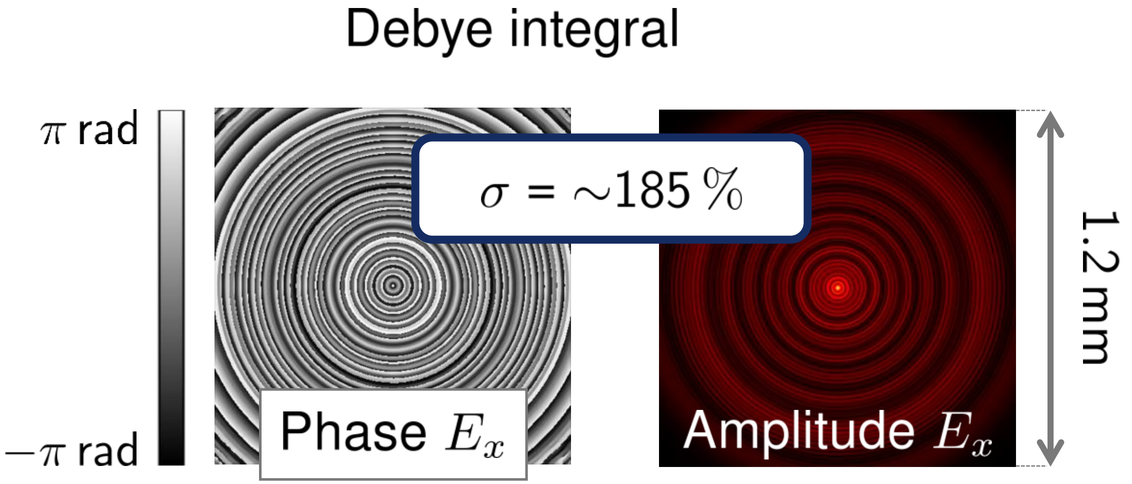
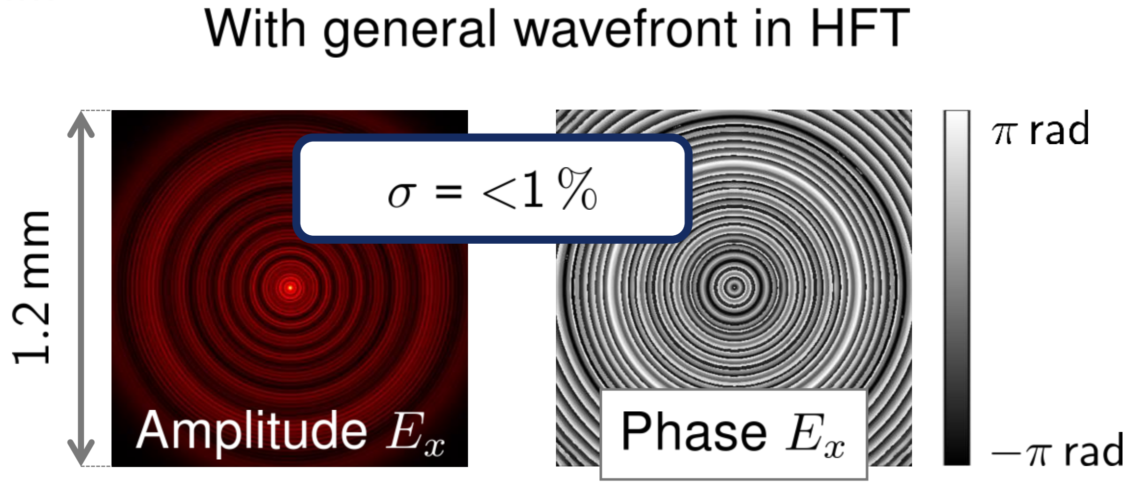
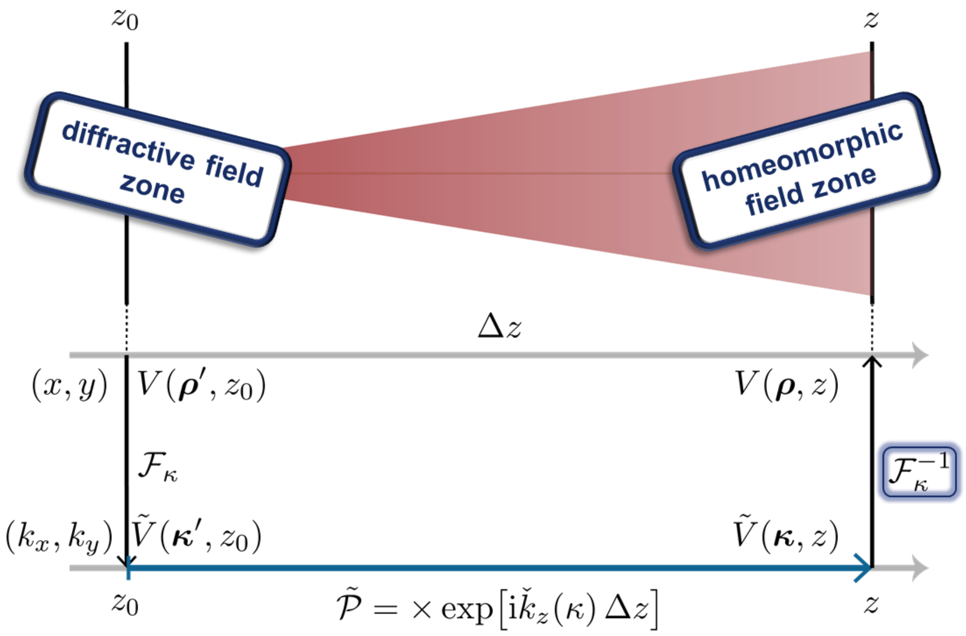






From Diffractive Field Zone to Homeomorphic Field Zone



$$V(\boldsymbol{\rho}, z) =$$
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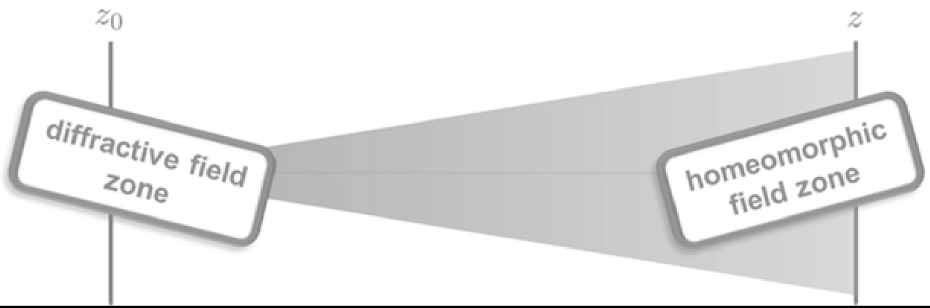
$$V(\boldsymbol{\rho}, z) = \mathcal{F}_\kappa^{-1,\mathrm{hom}}\left\{\tilde{V}(\boldsymbol{\kappa}, z)\right\} = \tilde{\alpha}[\boldsymbol{\kappa}(\boldsymbol{\rho})]\,\tilde{A}[\boldsymbol{\kappa}(\boldsymbol{\rho})]\exp\left(\mathrm{i}\left\{\tilde{\psi}[\boldsymbol{\kappa}(\boldsymbol{\rho})] + \boldsymbol{\kappa}(\boldsymbol{\rho}) \cdot \boldsymbol{\rho}\right\}\right)$$

From Diffractive Field Zone to Homeomorphic Field Zone



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Analytic solution for $\psi^{\mathrm{sph}}(\boldsymbol{\rho}')$!

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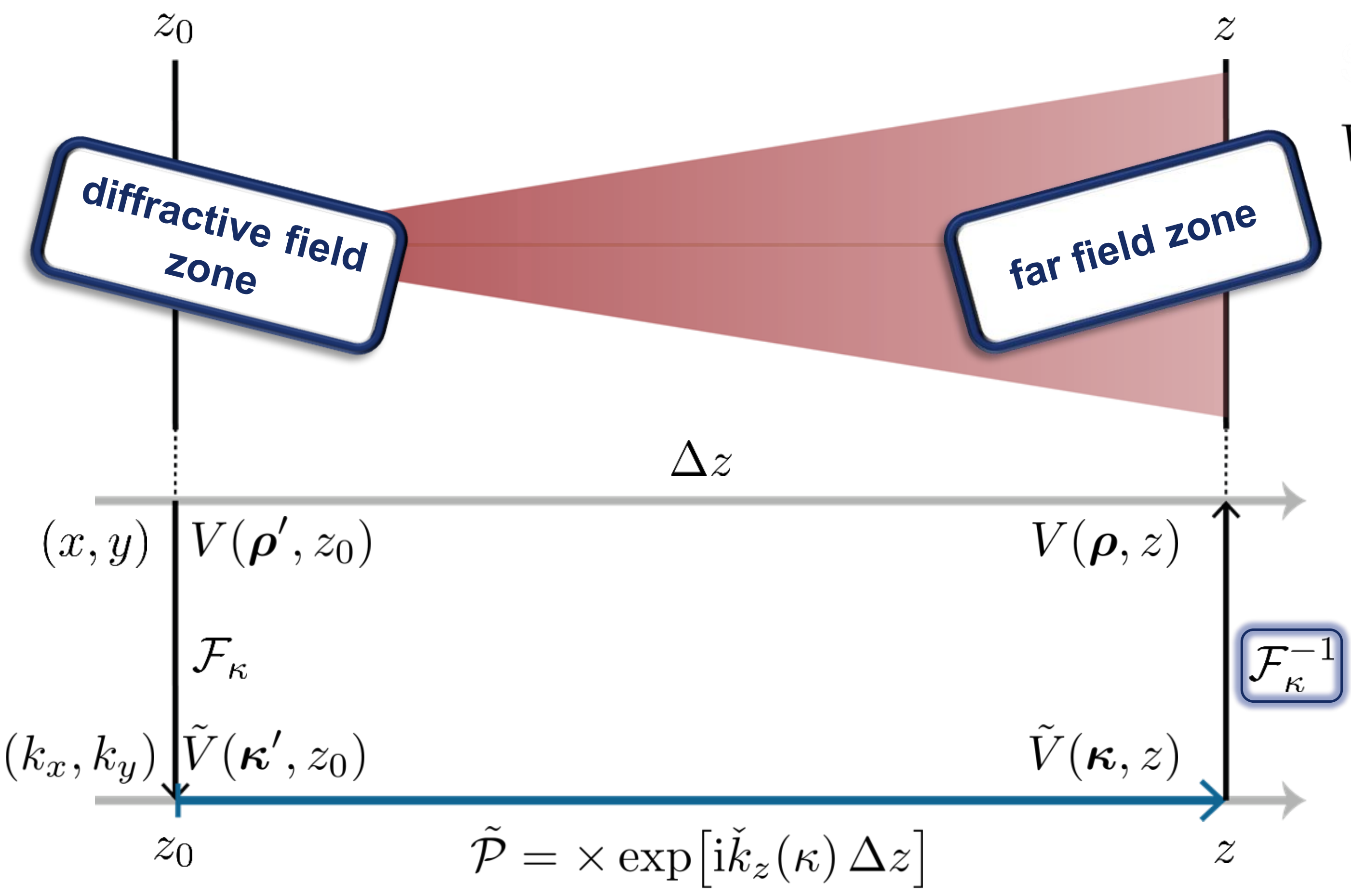
From Diffractive Field Zone to Far Field Zone



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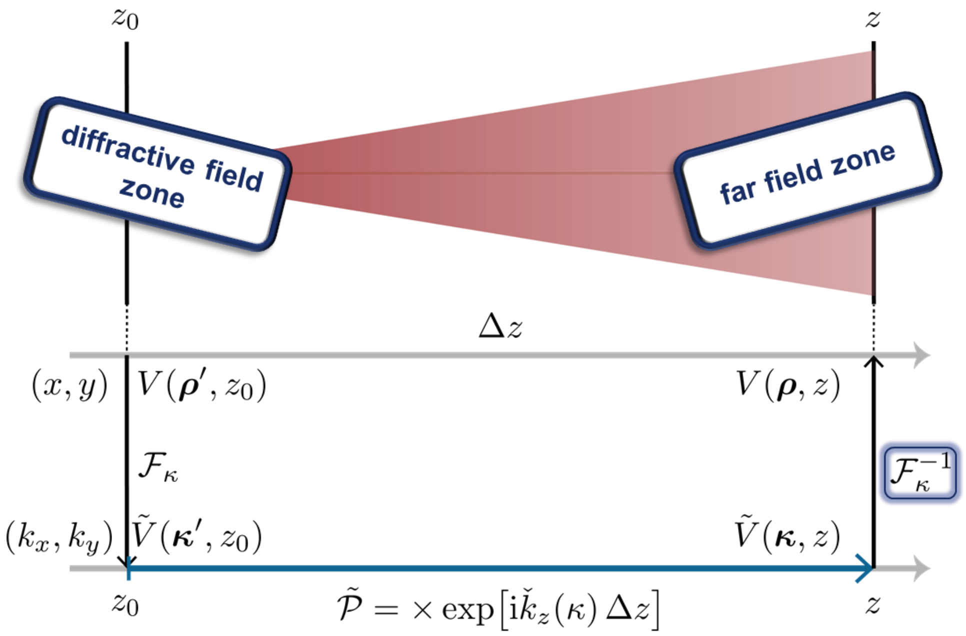

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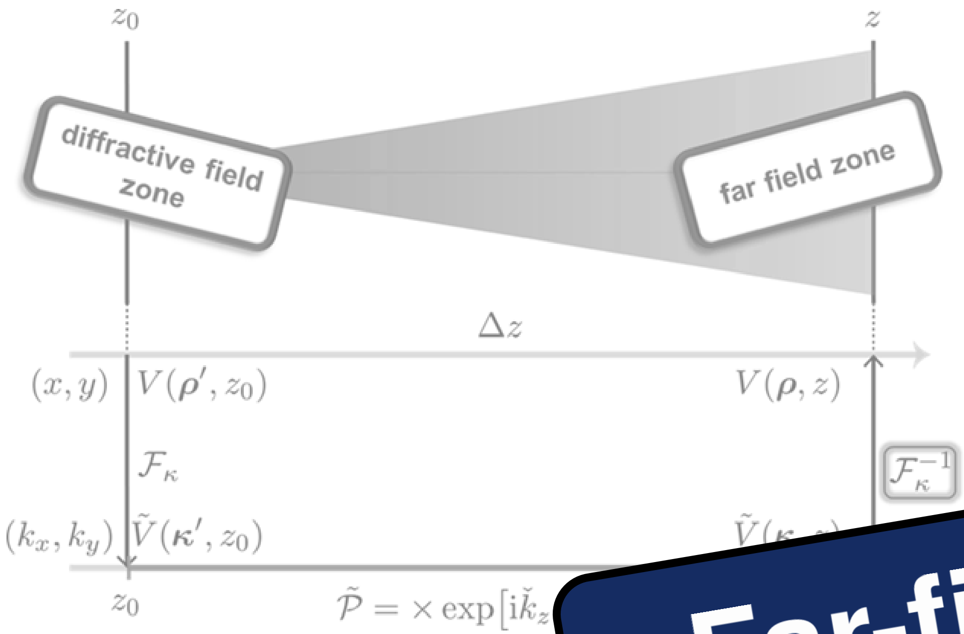
with ψ^{sph}!

$$V(\boldsymbol{\rho}) = -\mathrm{i}\frac{k_0\check{n}\Delta z \exp\{\mathrm{i}[\mathrm{sign}(\Delta z)\, k_0\check{n}\check{r}(\boldsymbol{\rho})]\}}{2\pi\check{r}^2(\boldsymbol{\rho})} \iint_{\infty}^{+\infty} V(\boldsymbol{\rho}', z_0) \exp\left(-\mathrm{i}\frac{k_0\check{n}\boldsymbol{\rho}' \cdot \boldsymbol{\rho}}{\check{r}(\boldsymbol{\rho})}\right) \mathrm{d}^2\rho'$$

From Diffractive Field Zone to Far Field Zone



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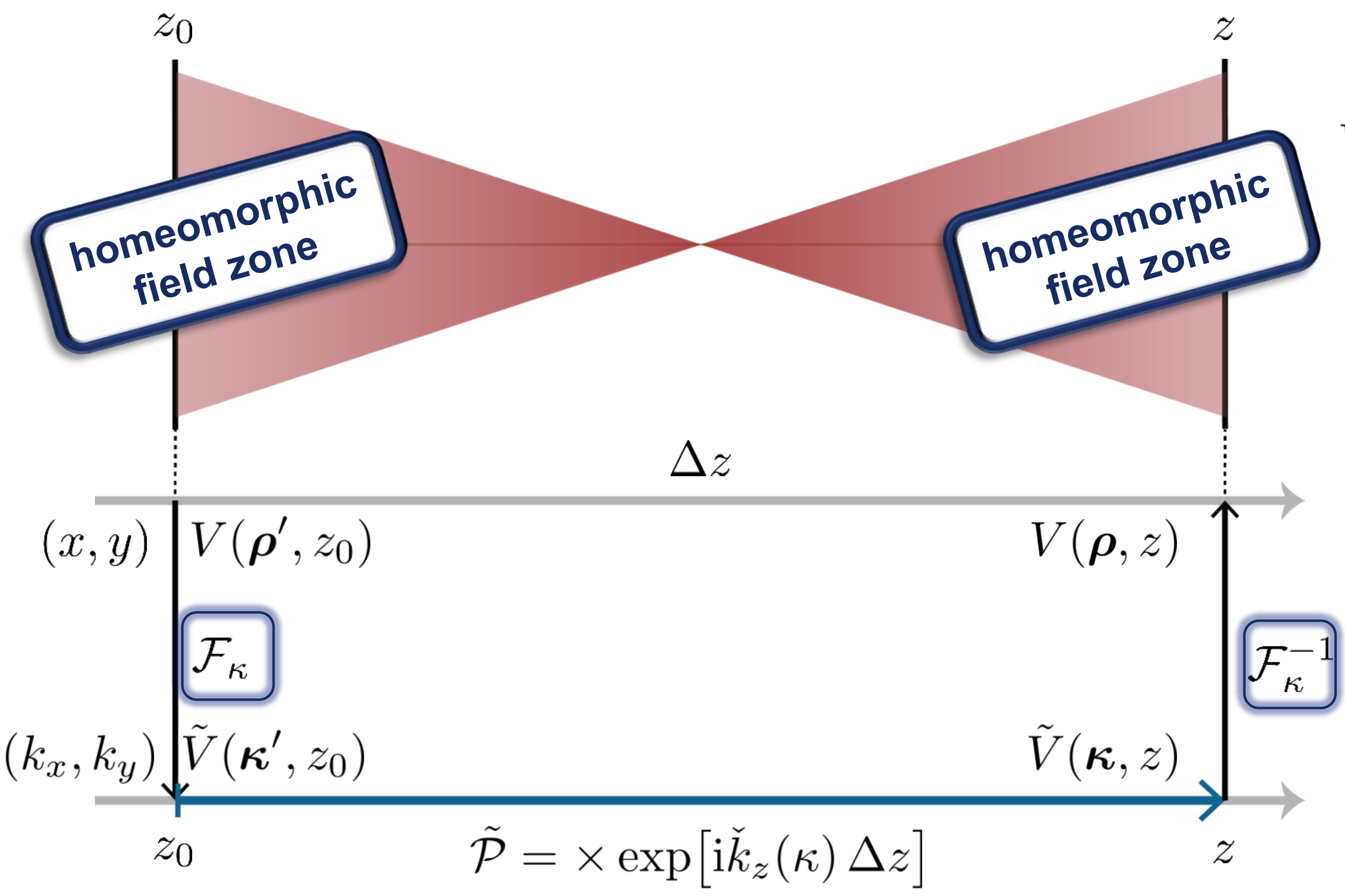
Far-field integral

$$\boxed{V(\boldsymbol{\rho}) = -\mathrm{i}\frac{k_0\breve{n}\Delta z \exp\{\mathrm{i}[\mathrm{sign}(\Delta z)\, k_0\breve{n}\breve{r}(\boldsymbol{\rho})]\}}{2\pi\breve{r}^2(\boldsymbol{\rho})} \iint_{\infty}^{+\infty} V(\boldsymbol{\rho}', z_0) \exp\left(-\mathrm{i}\frac{k_0\breve{n}\boldsymbol{\rho}'\cdot\boldsymbol{\rho}}{\breve{r}(\boldsymbol{\rho})}\right) \mathrm{d}^2\rho'}$$

From Homeomorphic Field Zone to Homeomorphic Field Zone



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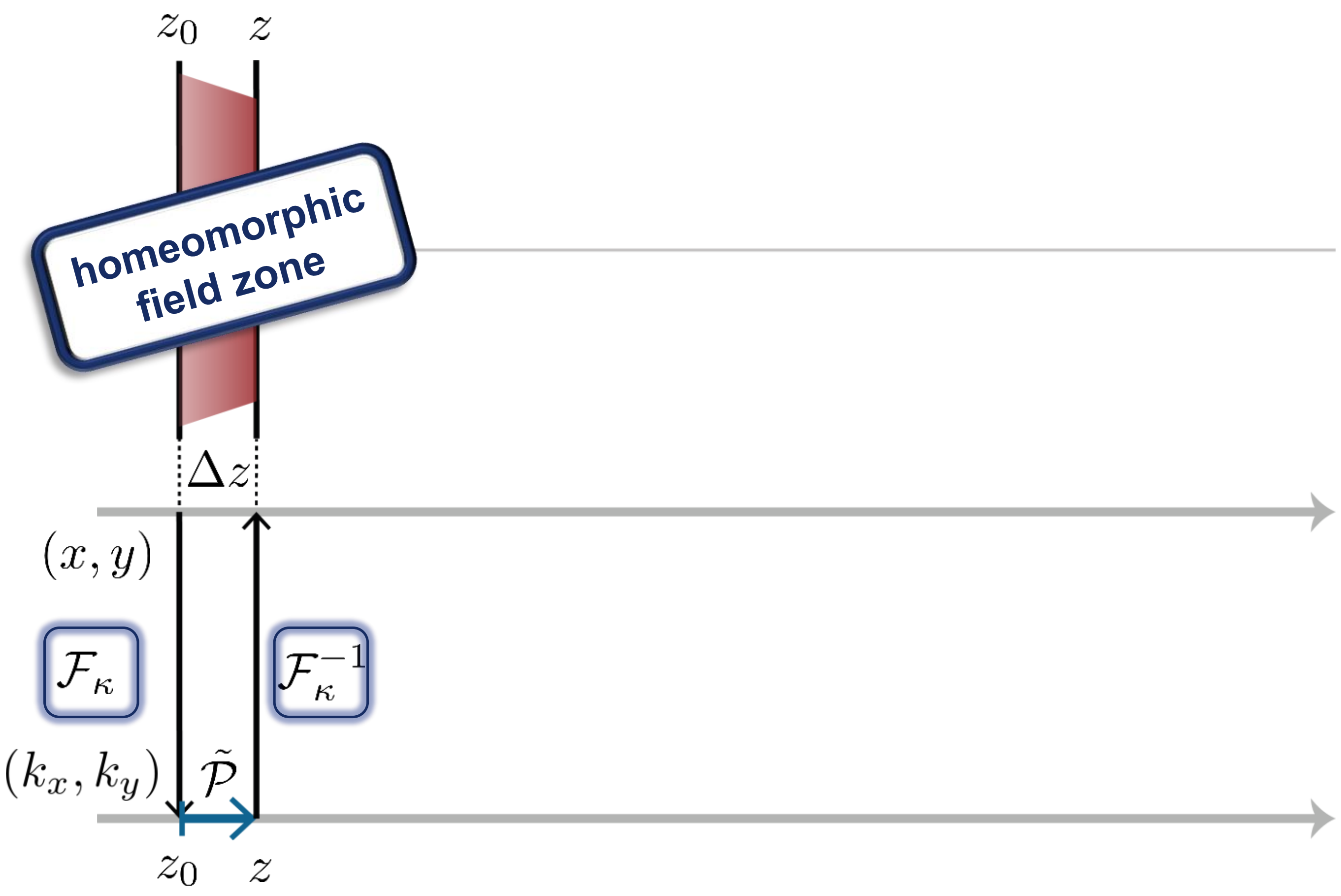

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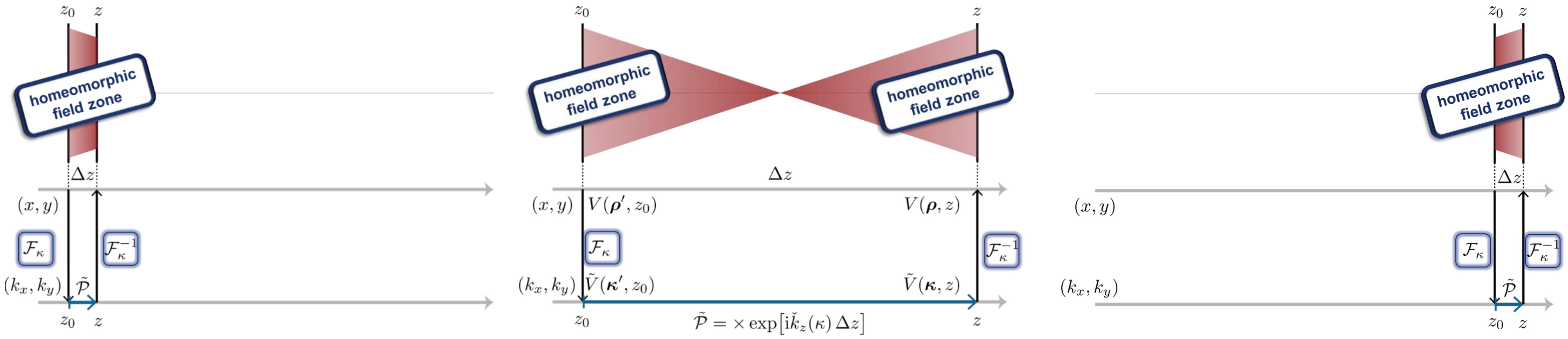
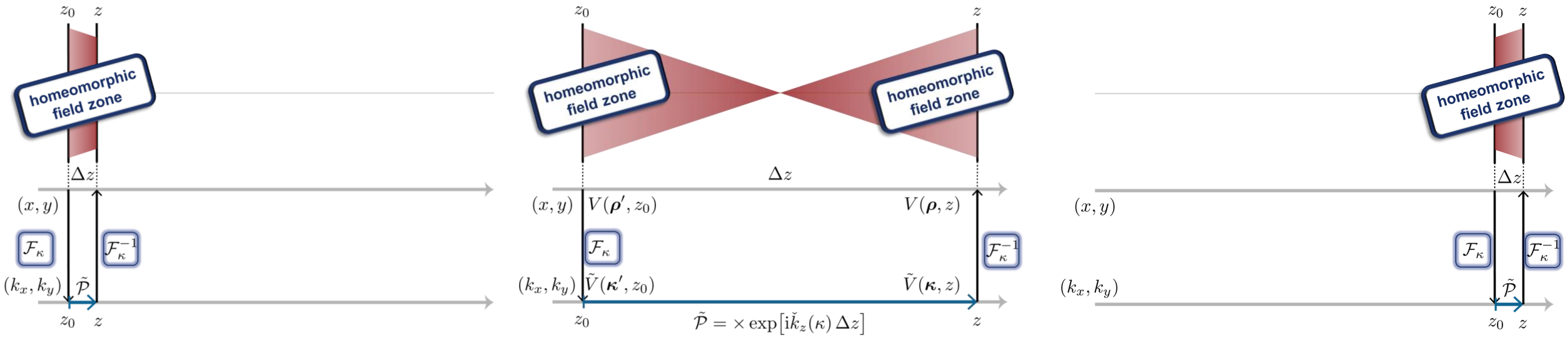
From Homeomorphic Field Zone to Homeomorphic Field Zone

$$\boldsymbol{\rho}' = \boldsymbol{\rho}'(\boldsymbol{\rho})$$



From Homeomorphic Field Zone to Homeomorphic Field Zone

$$\boldsymbol{\rho}(\boldsymbol{\kappa}) + \frac{\boldsymbol{\kappa}}{\check{k}_z(\boldsymbol{\kappa})}\Delta z = \boldsymbol{\rho}'(\boldsymbol{\kappa})$$



From Homeomorphic Field Zone to Homeomorphic Field Zone

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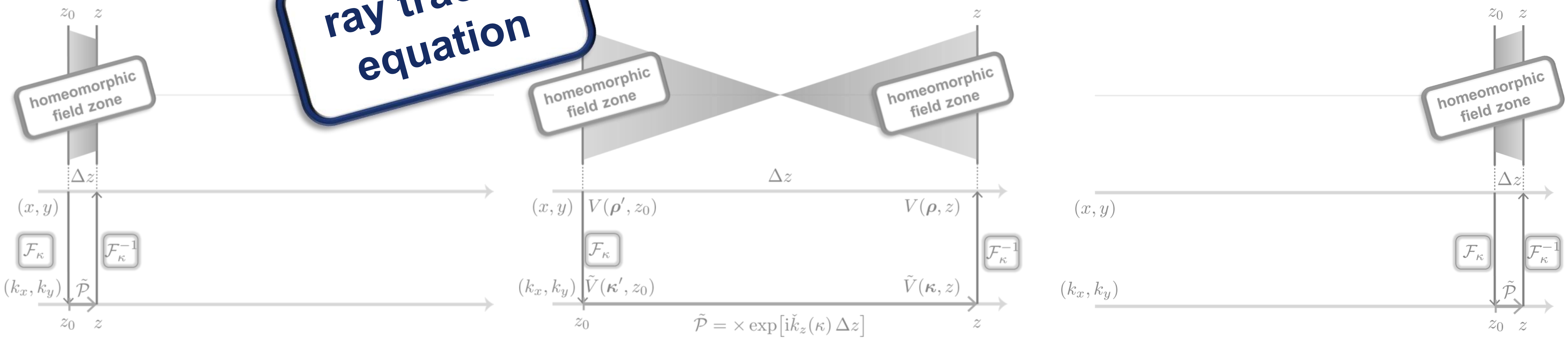
ray tracing equation



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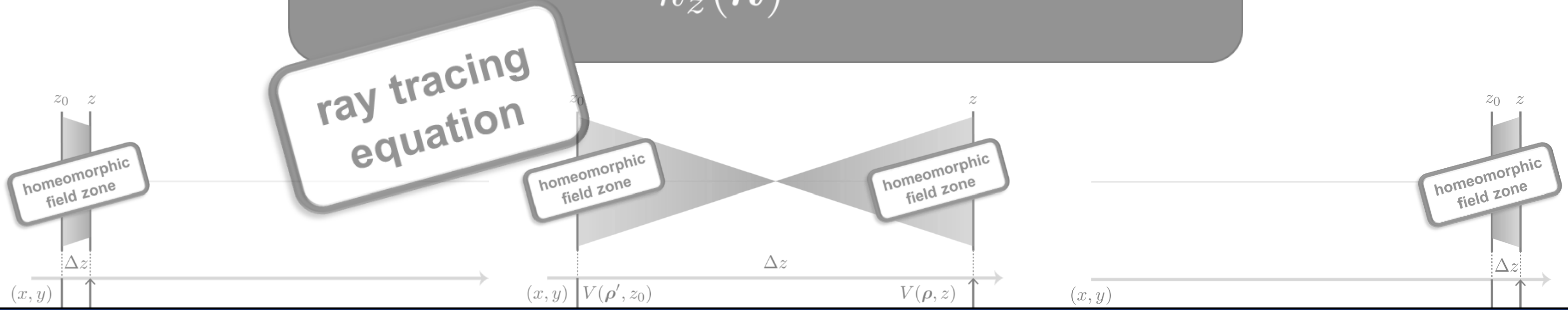
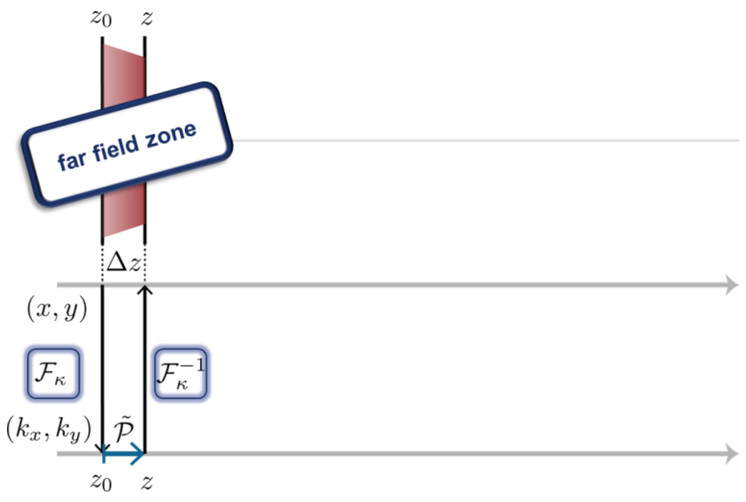
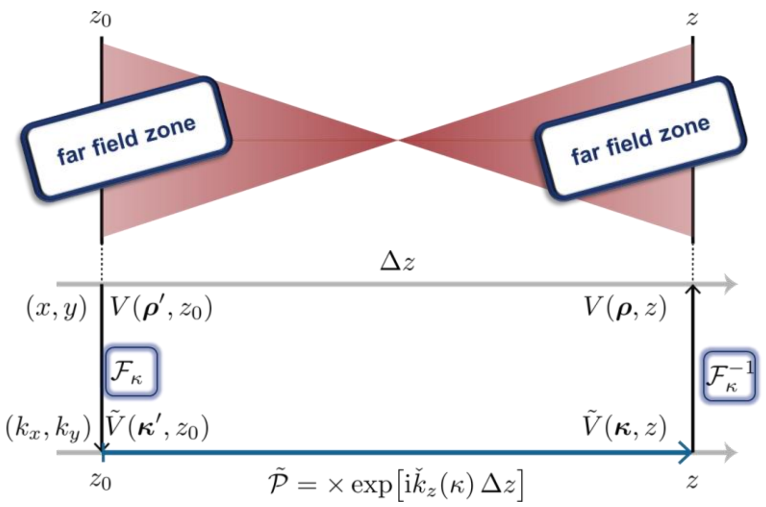


Analytic solution for $\psi^{\mathrm{sph}}(\boldsymbol{\rho}')$!

From Far-Field Zone to Far-Field Zone

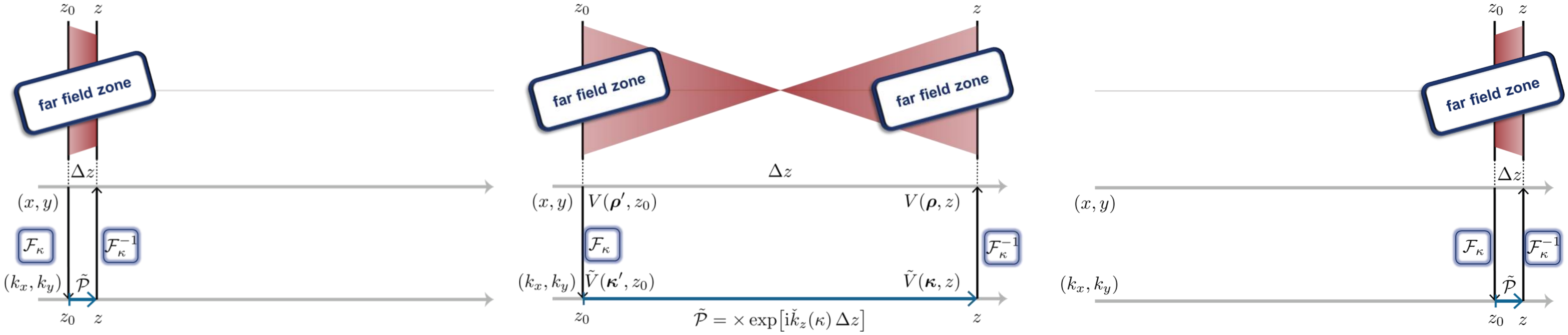
$$V(\boldsymbol{\rho}, z) = \frac{R}{R+\Delta z} U\left(\frac{R}{R+\Delta z}\boldsymbol{\rho}, z_0\right) \exp[\mathrm{i}\,\mathrm{sign}(R+\Delta z)\, k_0 \check{n} \check{r}(\boldsymbol{\rho}, z)]$$





From Far-Field Zone to Far-Field Zone

$$V(\boldsymbol{\rho}, z) = \underbrace{\frac{R}{R+\Delta z}}_{\text{Energy conservation}} \overbrace{U\left(\frac{R}{R+\Delta z}\boldsymbol{\rho}, z_0\right)}^{\text{Mapping of amplitude values}} \underbrace{\exp[\mathrm{i}\,\mathrm{sign}(R+\Delta z)\, k_0 \check{n} \check{r}(\boldsymbol{\rho}, z)]}_{\exp[\mathrm{i}\psi^{\mathsf{sph}}(\boldsymbol{\rho}'(\boldsymbol{\rho}))+k_0 \check{n}\mathrm{OPL}]}$$



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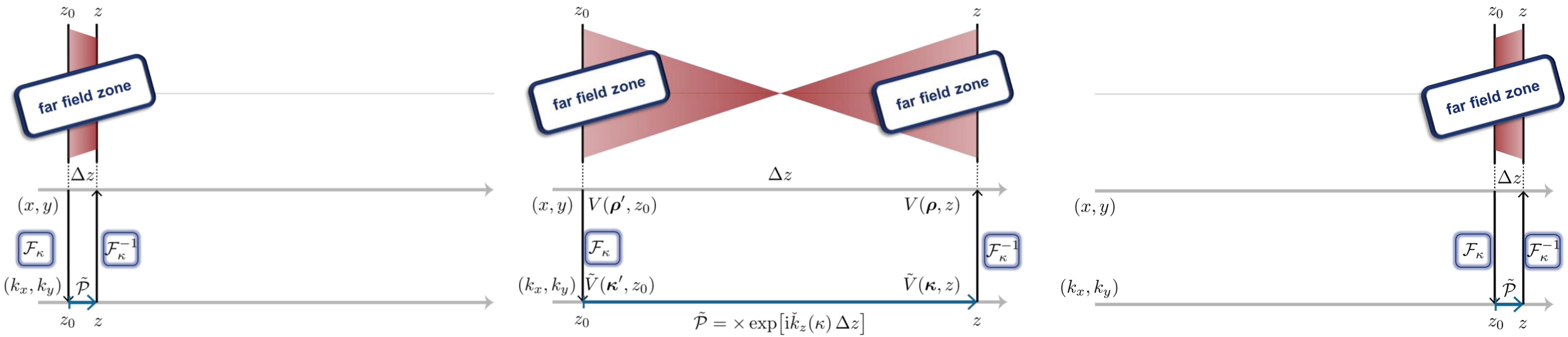

$R < 0$, $\Delta z < |R|$

$R < 0$, $\Delta z > |R|$

$R > 0$, $\Delta z > 0$

From Far-Field Zone to Far-Field Zone

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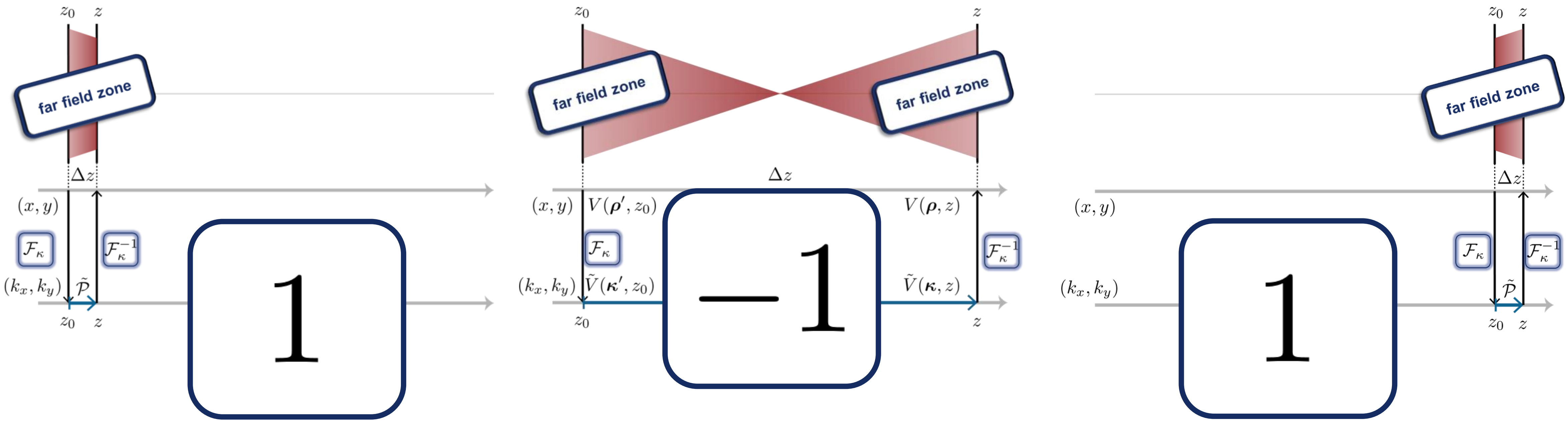

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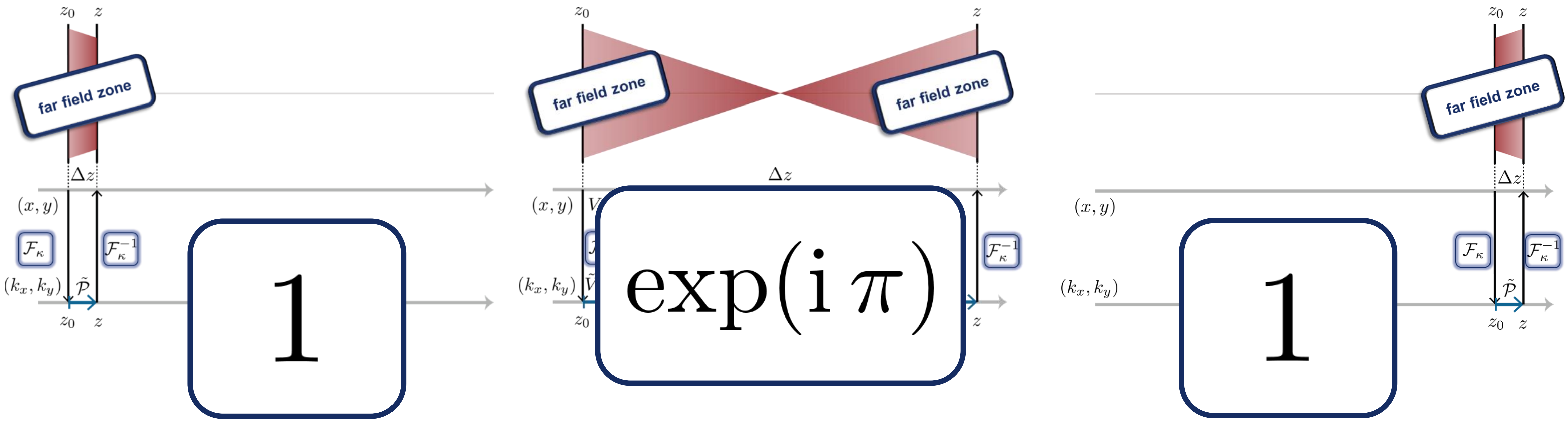

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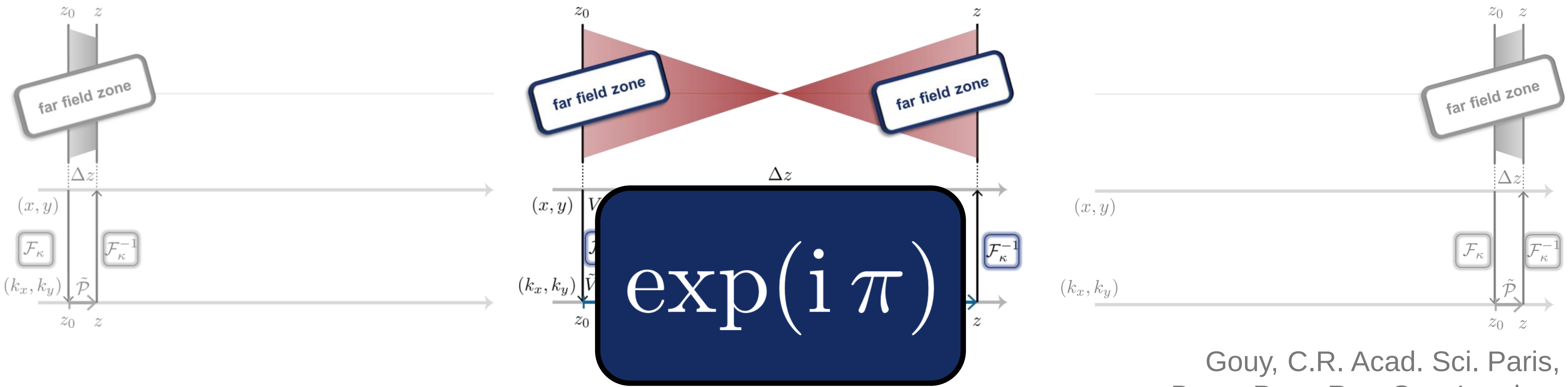

Gouy, C.R. Acad. Sci. Paris, 1890
Berry, Proc. Roy. Soc. London, 1984
…
Baladron-Zorita et al, J. Opt. Soc. Am. A, 2019

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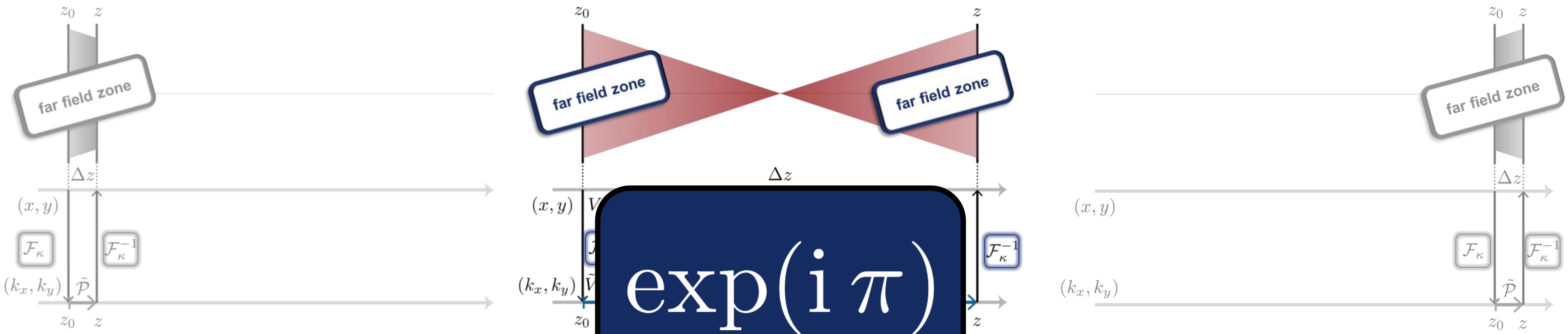

$$\exp(\mathrm{i}\,\pi)$$

Conclusion also valid for HFZ to HFZ propagation!

Gouy, C.R. Acad. Sci. Paris, 1890
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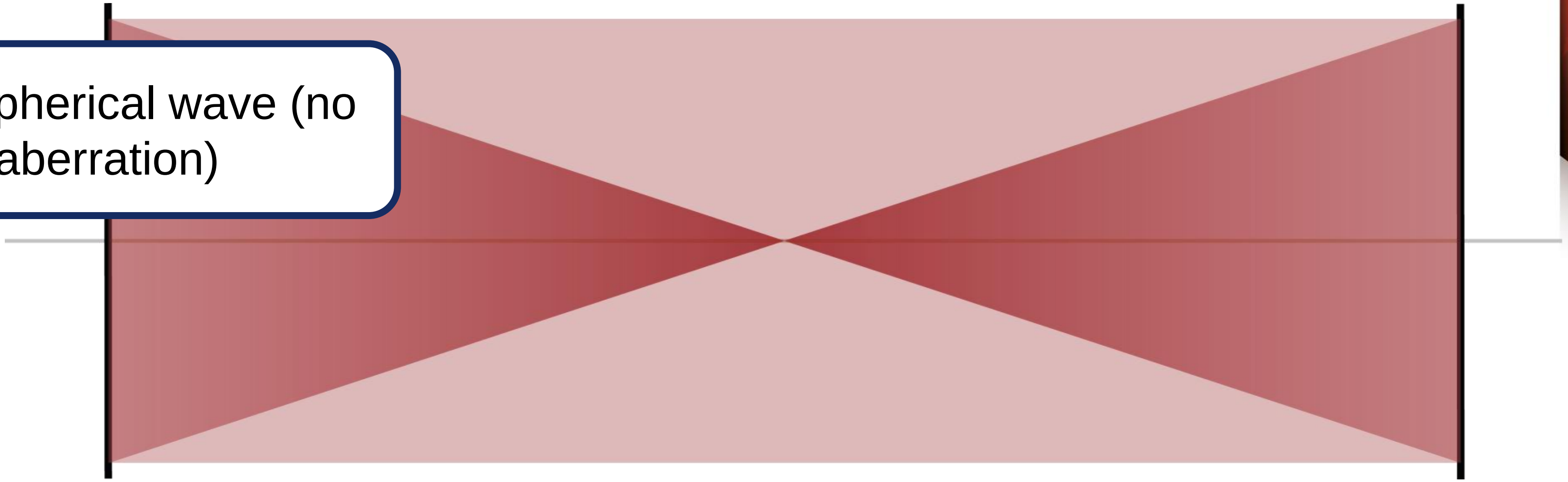
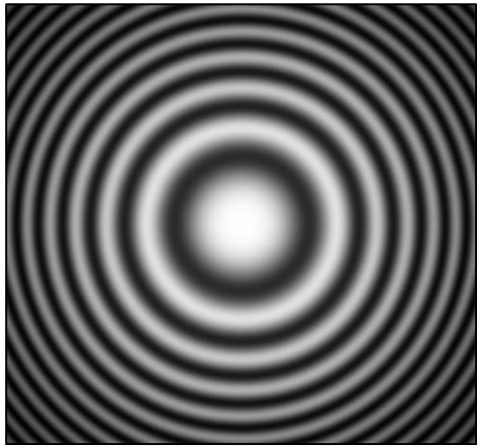
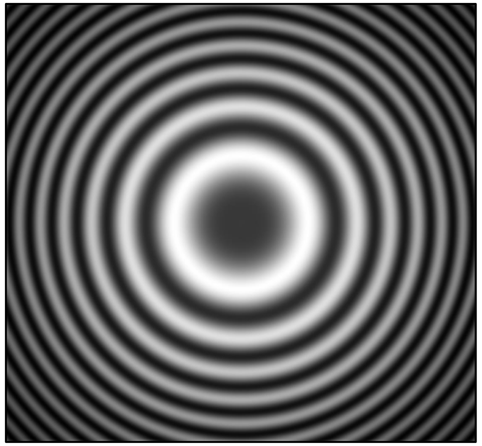
Gouy Phase: Simulation

Ideal spherical wave (no aberration)







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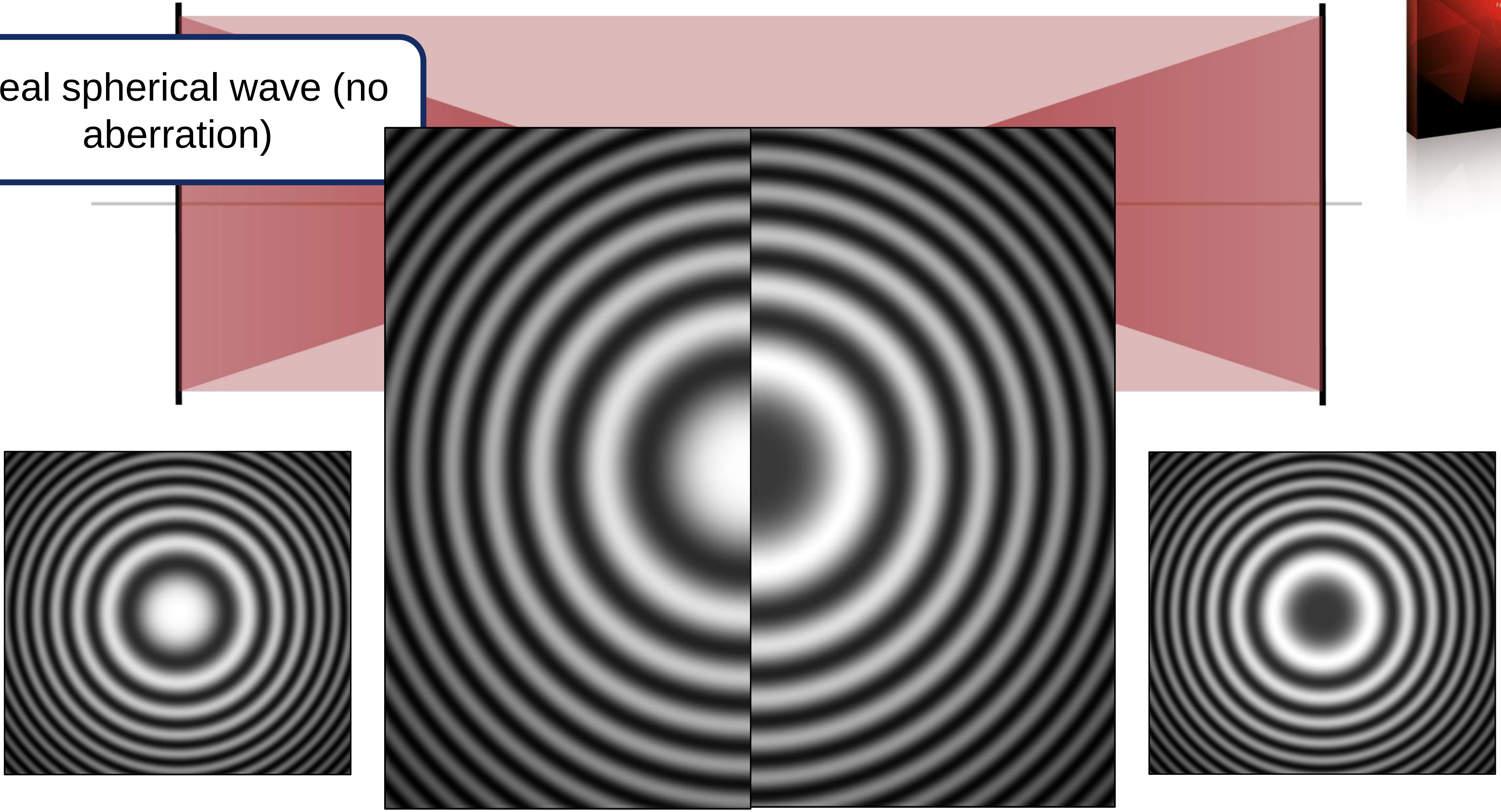
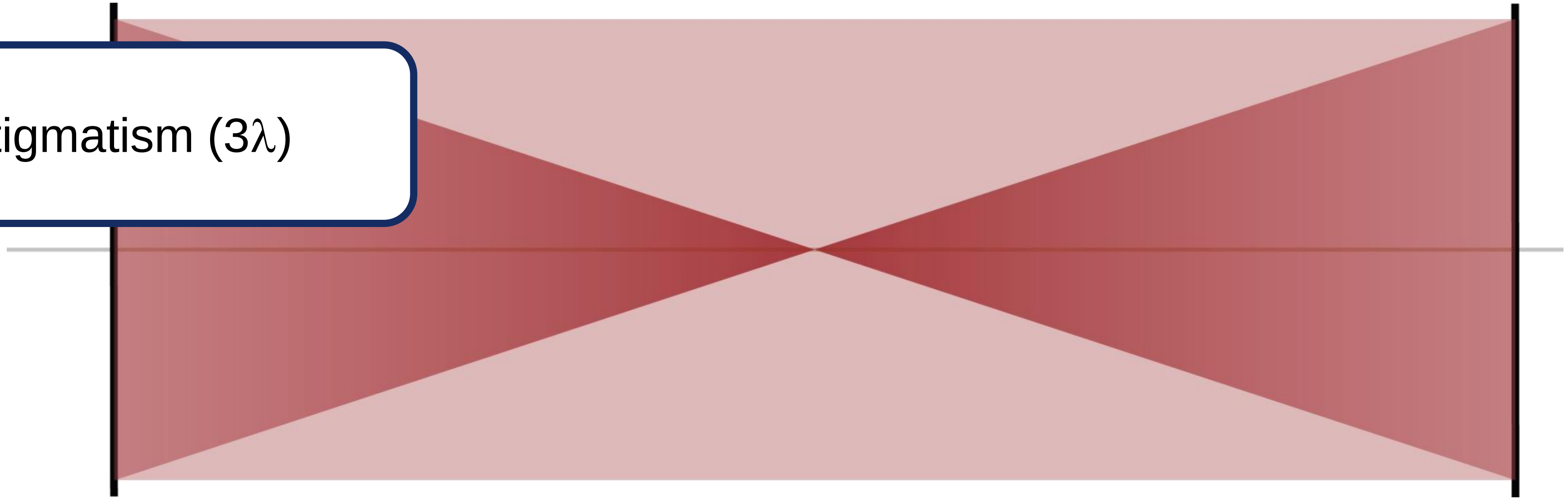
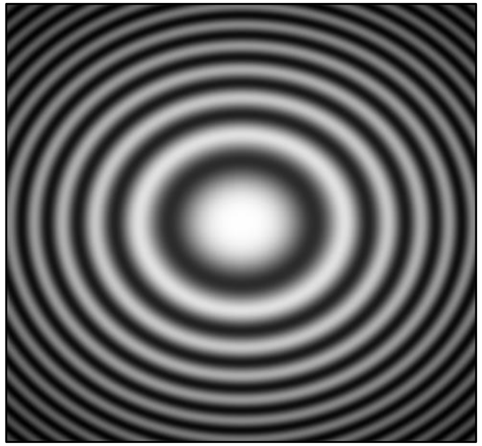
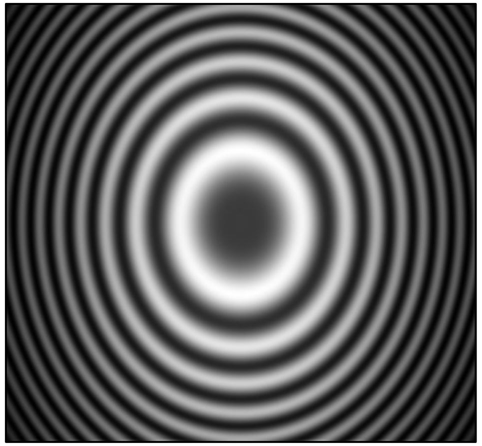


Gouy Phase: Simulation

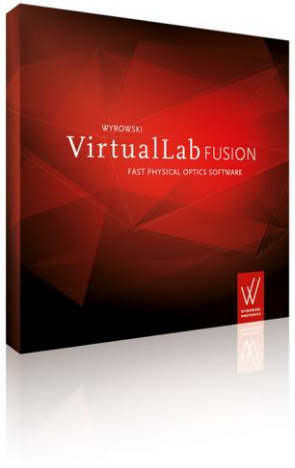
Astigmatism (3λ)



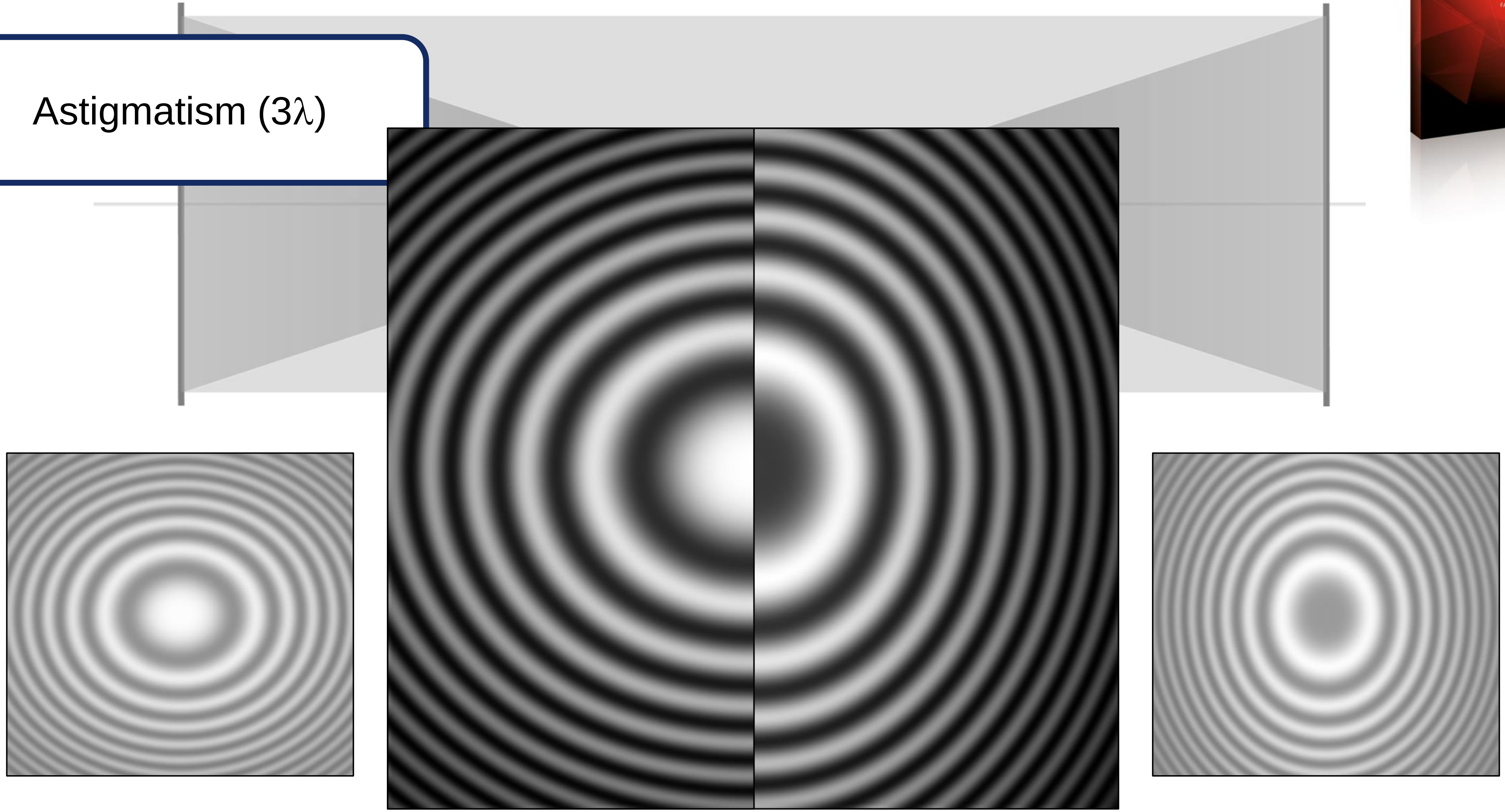




Gouy Phase: Simulation

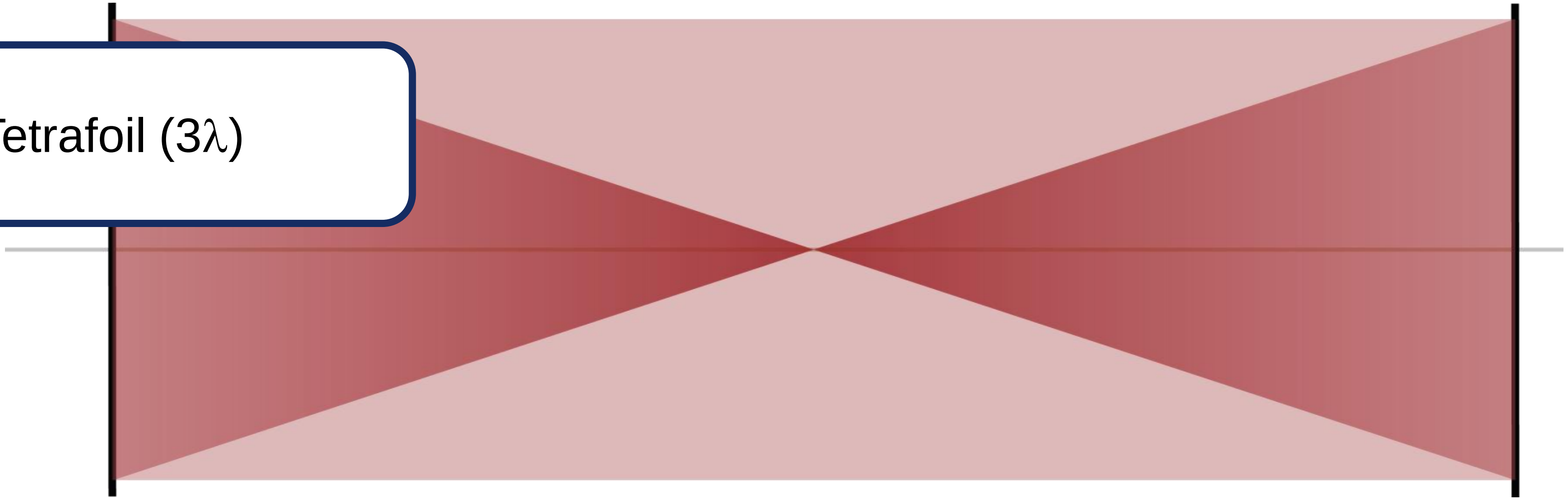


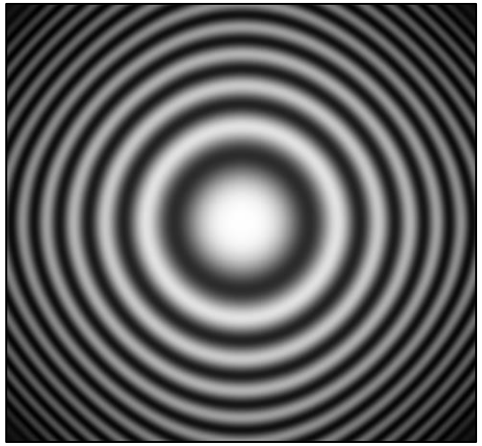
Astigmatism (3λ)

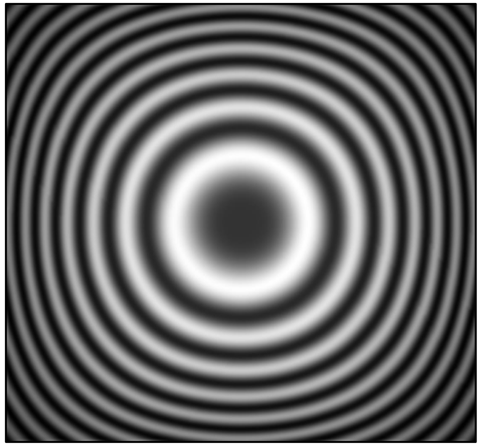


Gouy Phase: Simulation

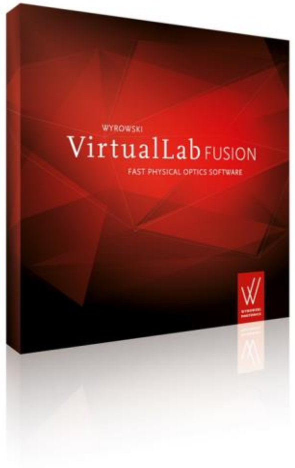
Tetrafoil (3λ)

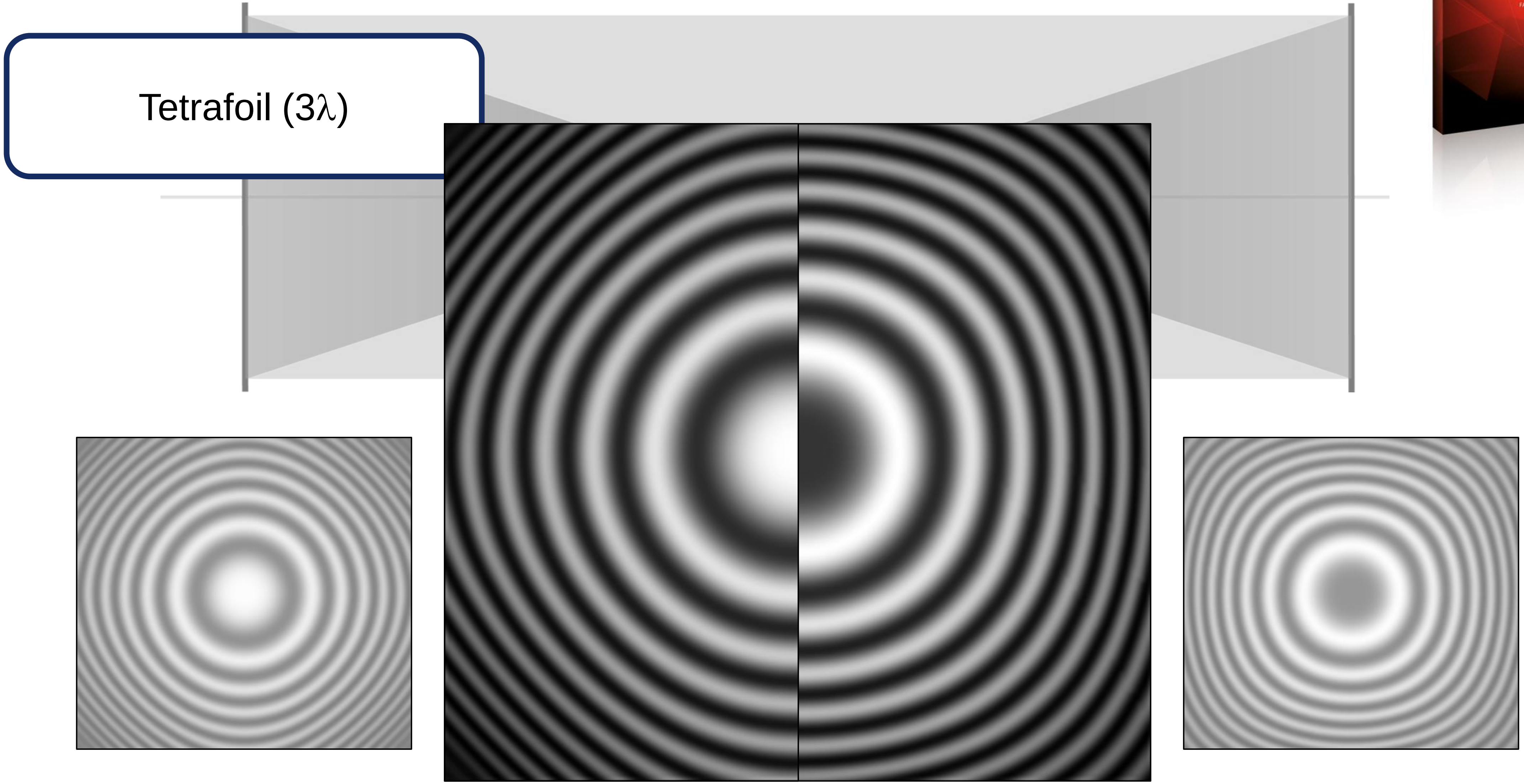






Gouy Phase: Simulation





The HFT and the Gouy Phase in Beams with OAM

Gouy phase for fundamental Gaussian beam: $\exp\left[\mathrm{i}\ \arctan\left(\frac{z}{z_\mathsf{R}}\right)\right]$

Gouy phase for Laguerre-Gaussian beam: $(2p+\ell+1)\exp\left[\mathrm{i}\ \arctan\left(\frac{z}{z_\mathsf{R}}\right)\right]$

$p \rightarrow$ radial order

$\ell \rightarrow$ azimuthal order

$z_\mathsf{R} \rightarrow$ Rayleigh length

The HFT and the Gouy Phase in Beams with OAM

Gouy phase for fundamental Gaussian beam: $$\exp\left[\mathrm{i}\ \arctan\left(\frac{z}{z_\mathrm{R}}\right)\right]$$

Gouy phase for Laguerre-Gaussian beam: $$(2p+\ell+1)\exp\left[\mathrm{i}\ \arctan\left(\frac{z}{z_\mathrm{R}}\right)\right]$$

Test: can the HFT propagation formula predict this effect?

The HFT and the Gouy Phase in Beams with OAM

$$V(\boldsymbol{\rho}, z) = \frac{R}{R + \Delta z} U\left(\frac{R}{R + \Delta z}\boldsymbol{\rho}, z_0\right) \exp[\mathrm{i}\,\mathrm{sign}(R + \Delta z)\, k_0 \check{n}\check{r}(\boldsymbol{\rho}, z)]$$

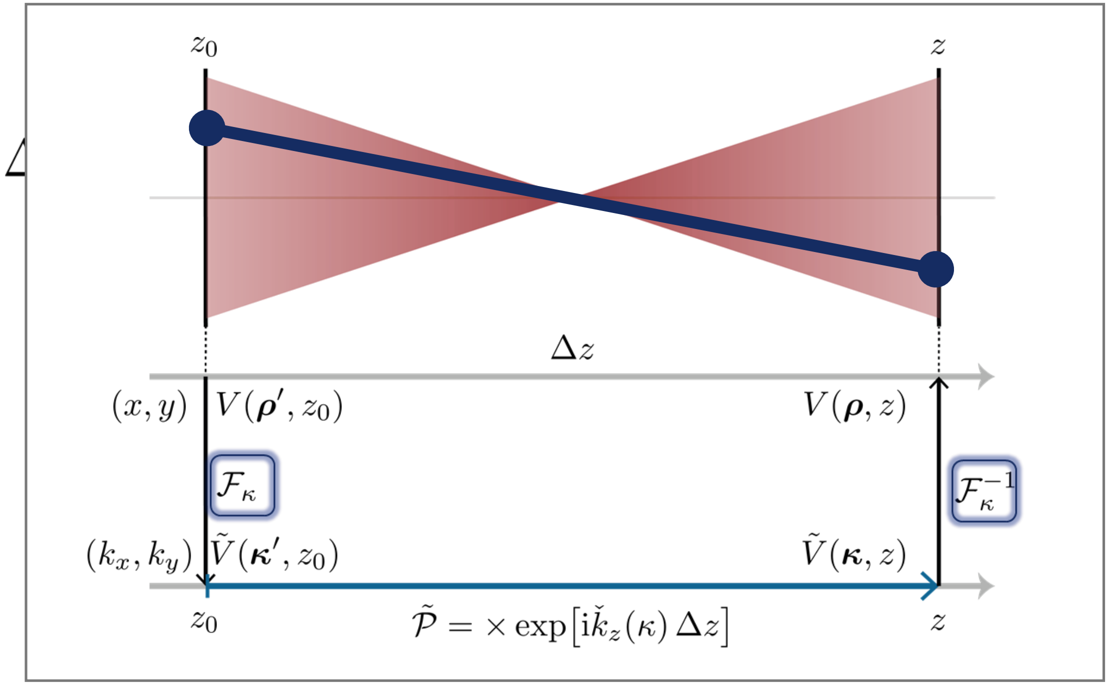
Mapping: $\boldsymbol{\rho}' \to -\boldsymbol{\rho}$

Mapping in polar coordinates: $\left\{ \begin{array}{rcl} \rho & = & \rho' \\ \varphi & = & \varphi' + \pi \end{array} \right.$

Gouy phase for Laguerre-Gaussian beam: $(2p + \ell + 1) \exp\left[\mathrm{i}\ \arctan\left(\frac{z}{z_{\mathrm{R}}}\right)\right]$

The HFT and the Gouy Phase in Beams with OAM

$$V(\boldsymbol{\rho}, z) = \frac{R}{R+\Delta z} U\left(\frac{R}{R+\Delta z}\boldsymbol{\rho}, z_0\right) \exp[\mathrm{i}\,\mathrm{sign}(R+\Delta$$

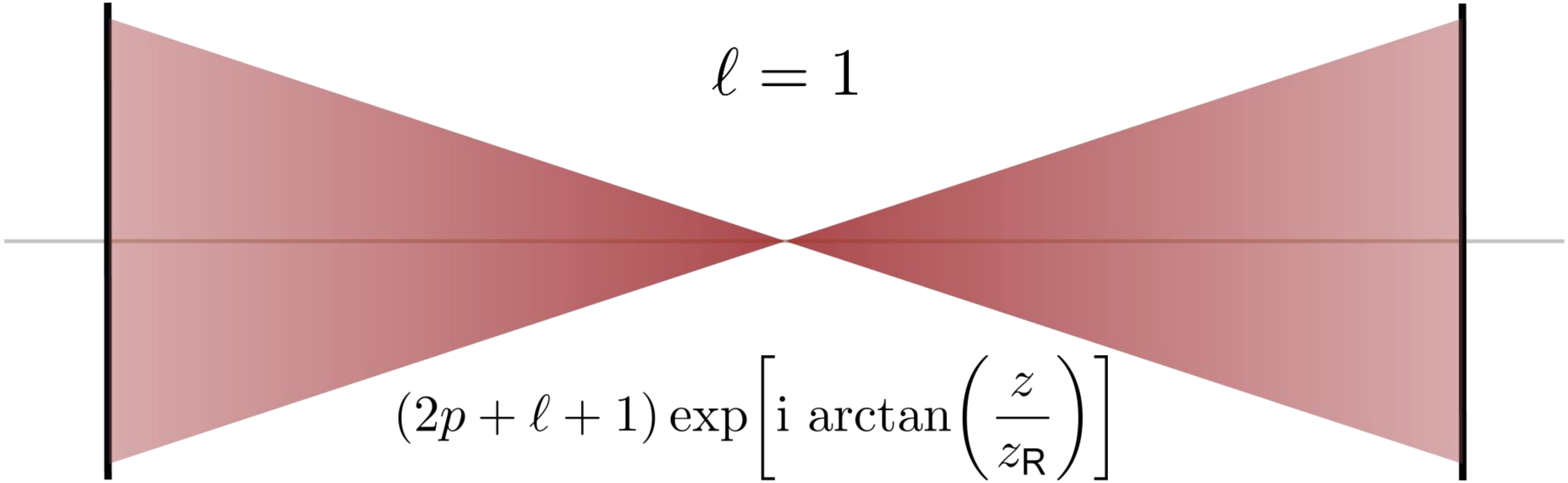


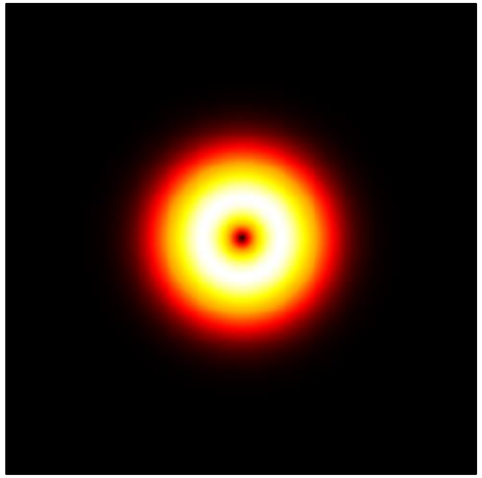
Mapping: $\boldsymbol{\rho}' \to -\boldsymbol{\rho}$

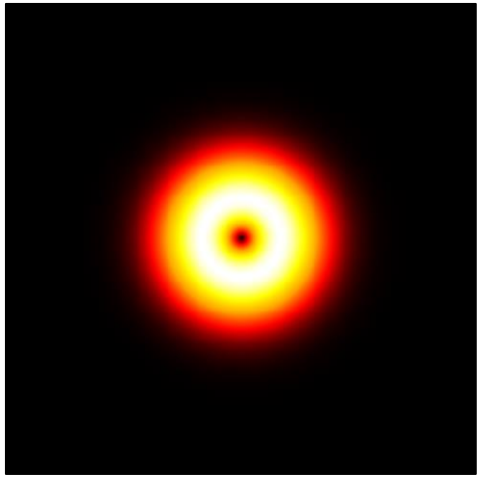
Mapping in polar coordinates: $\begin{cases} \rho &= \rho' \\ \varphi &= \varphi' + \pi \end{cases}$

Gouy phase for Laguerre-Gaussian beam: $(2p + \ell + 1) \exp\left[\mathrm{i}\ \arctan\left(\frac{z}{z_\mathrm{R}}\right)\right]$

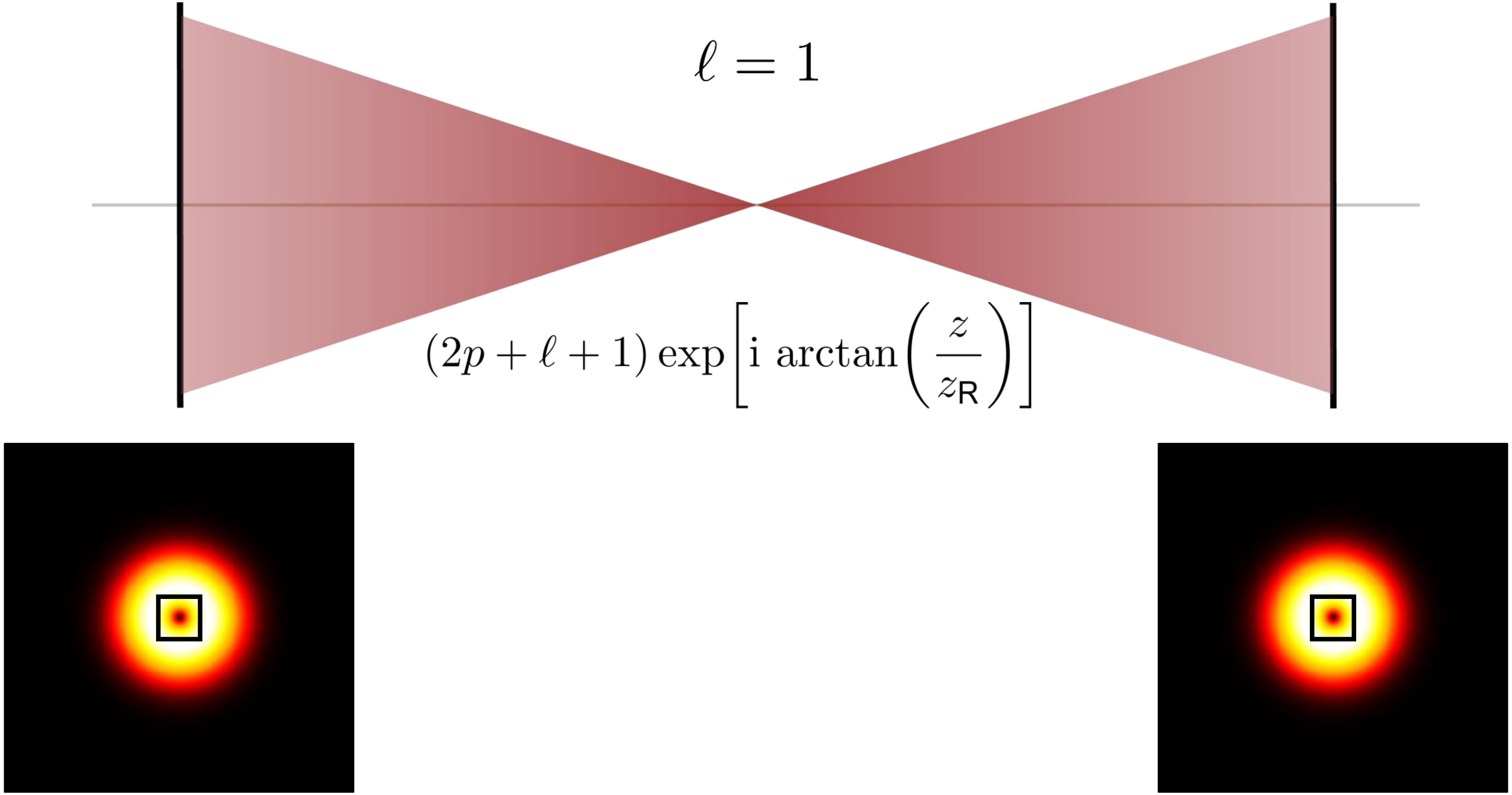
The HFT and the Gouy Phase in Beams with OAM: Simulation



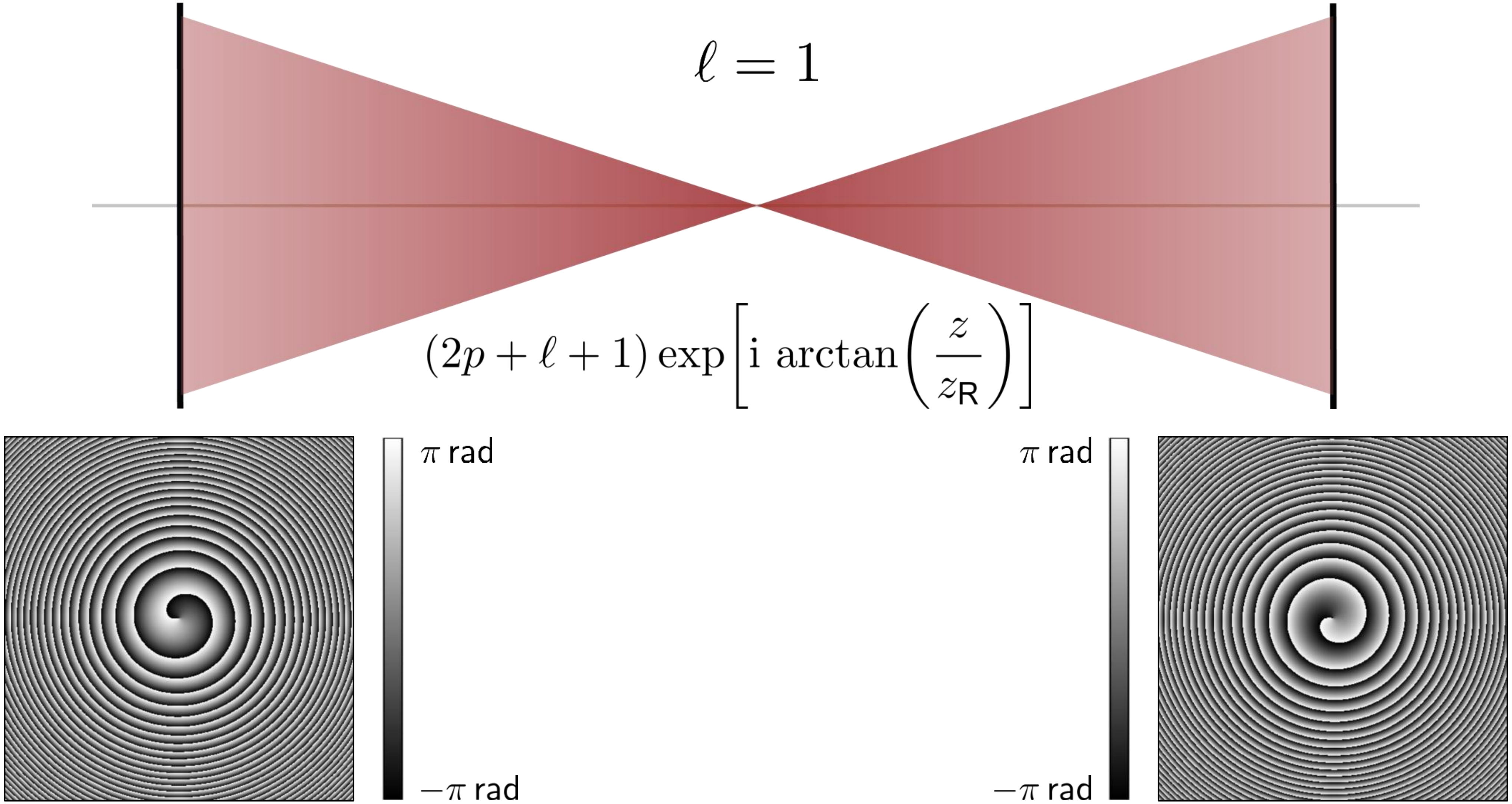




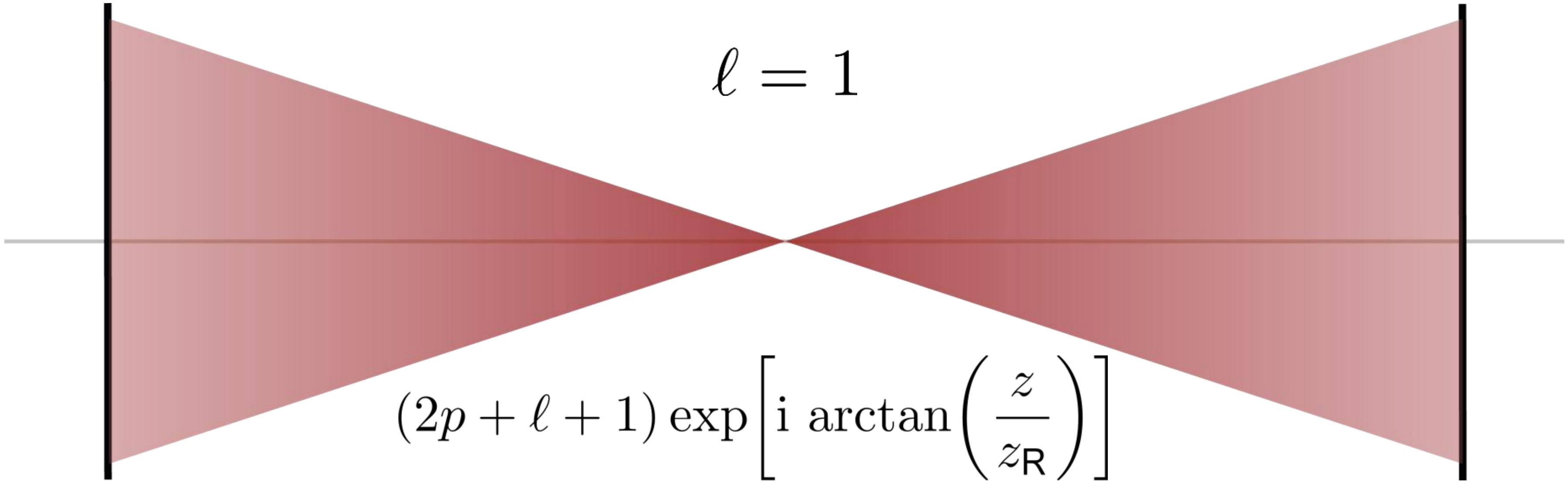
The HFT and the Gouy Phase in Beams with OAM: Simulation

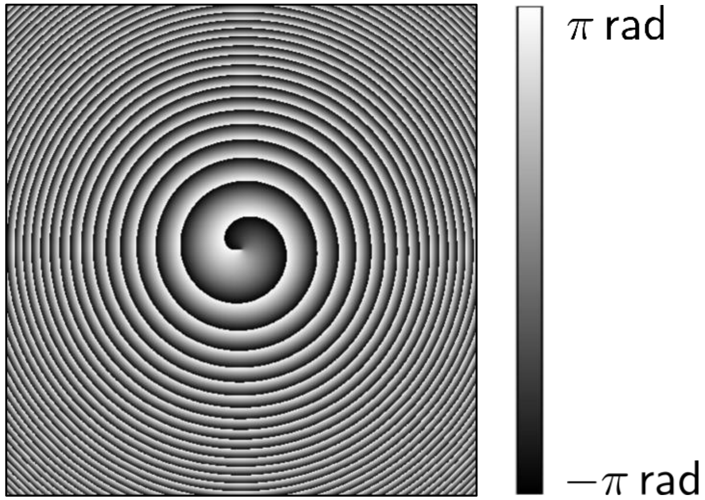


The HFT and the Gouy Phase in Beams with OAM: Simulation

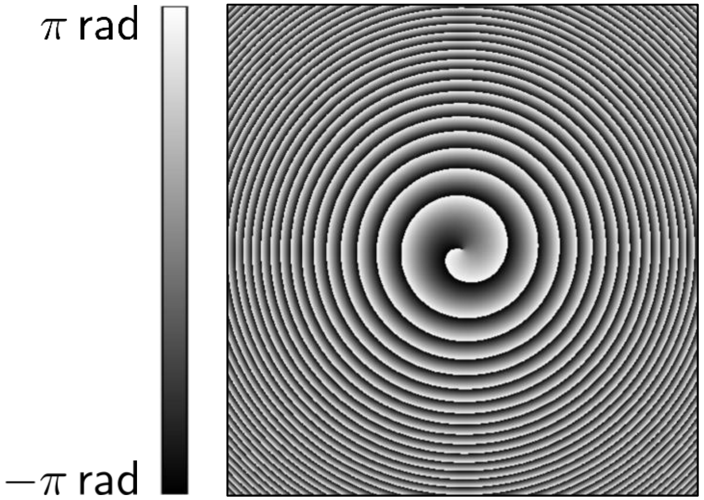


The HFT and the Gouy Phase in Beams with OAM: Simulation

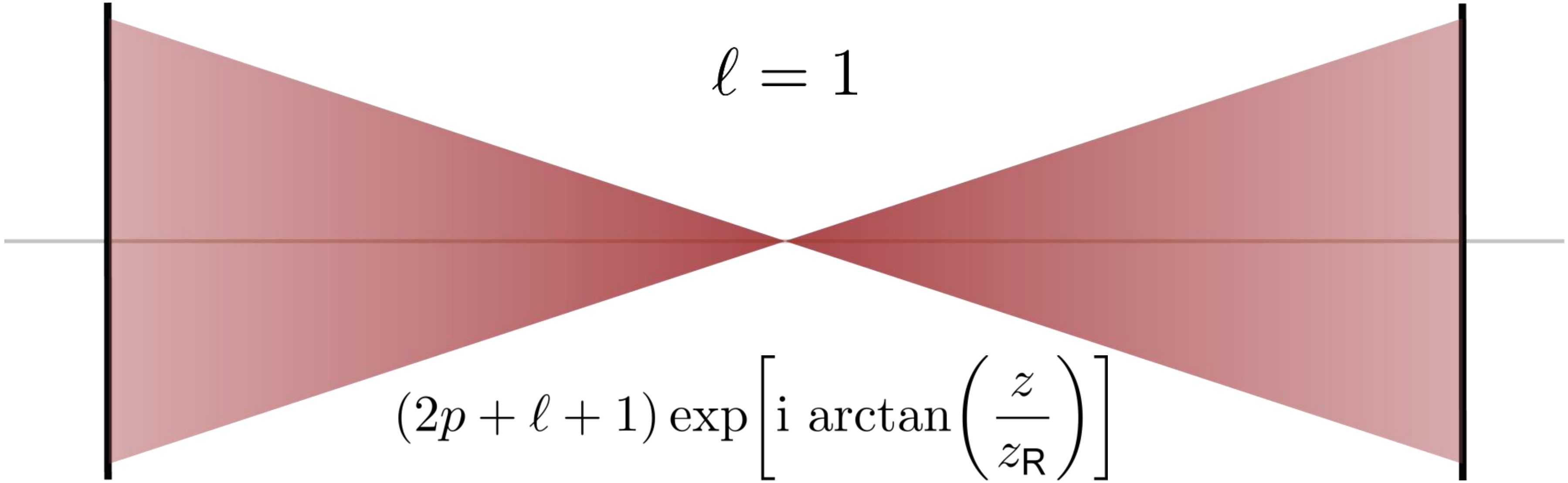


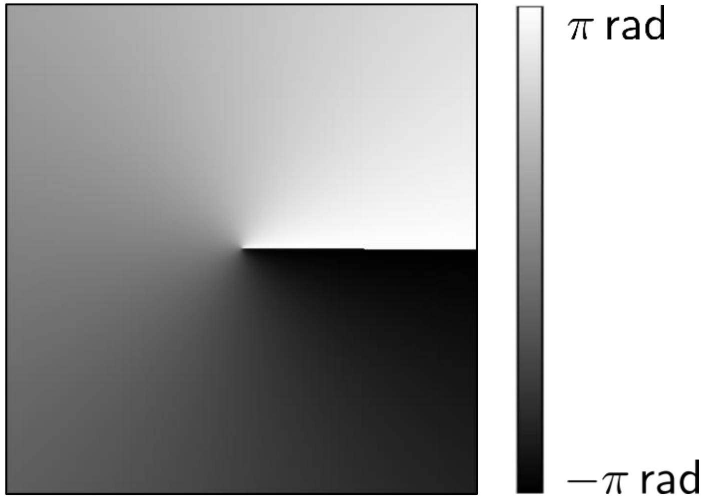


$$\boxed{\exp\left(\psi^{\mathsf{sph}}\right)}$$

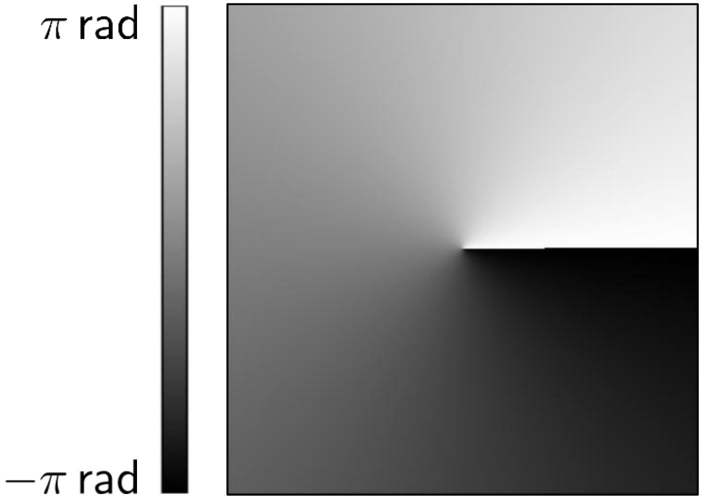


The HFT and the Gouy Phase in Beams with OAM: Simulation

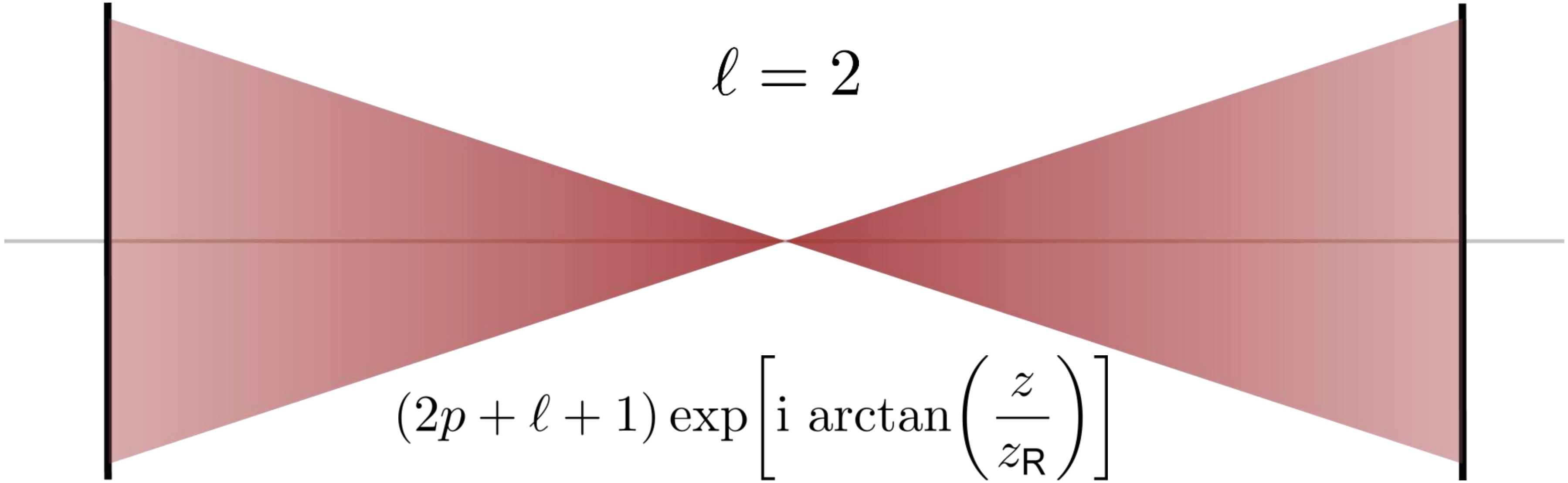




$$\cancel{\exp(\psi^{\text{sph}})}$$

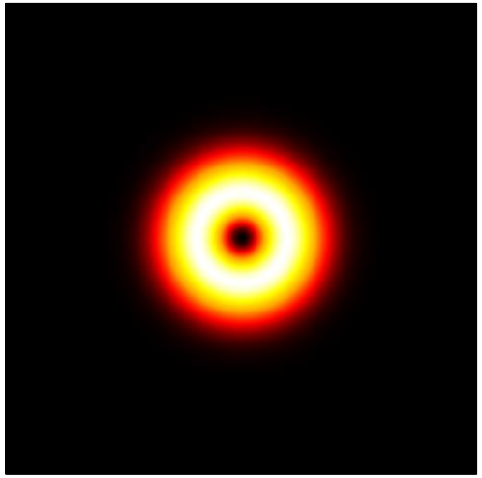


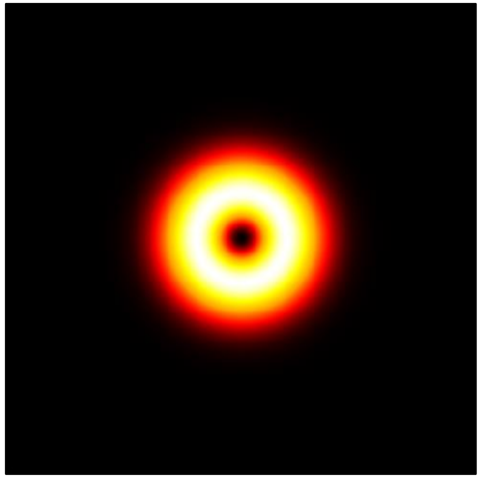
The HFT and the Gouy Phase in Beams with OAM: Simulation



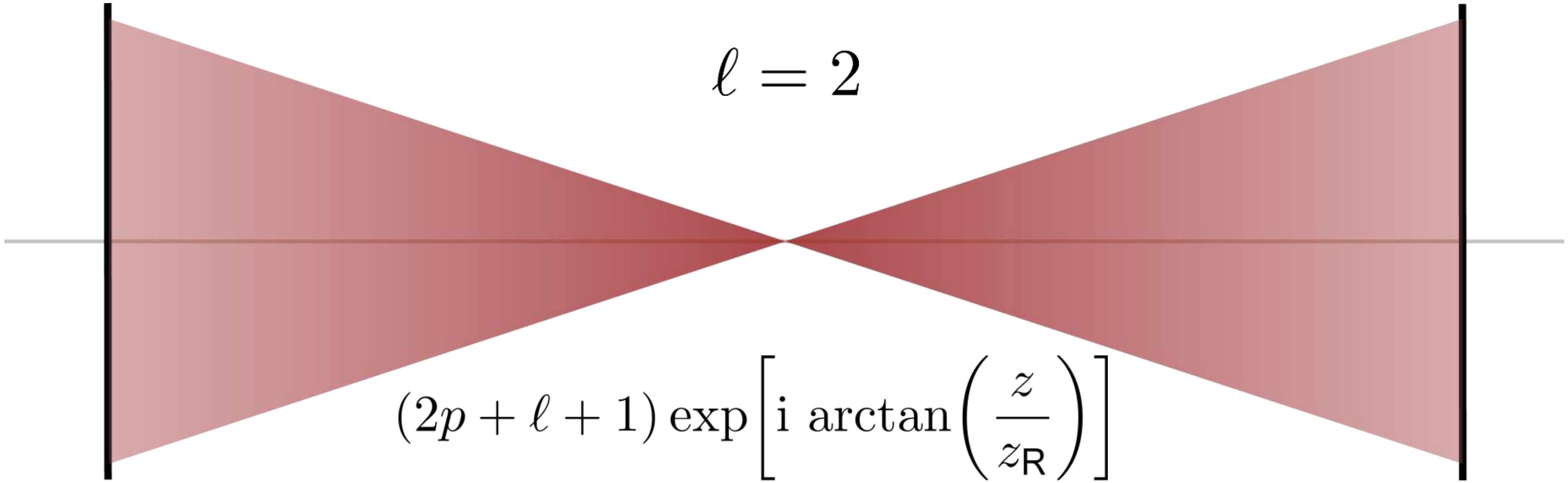
$$\ell = 2$$

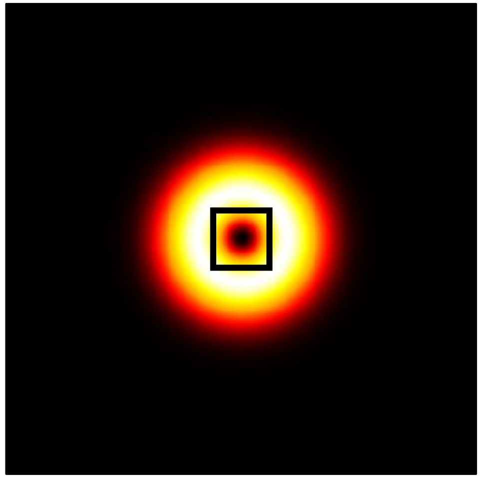
$$(2p + \ell + 1)\exp\left[\mathrm{i}\,\arctan\left(\frac{z}{z_{\mathrm{R}}}\right)\right]$$

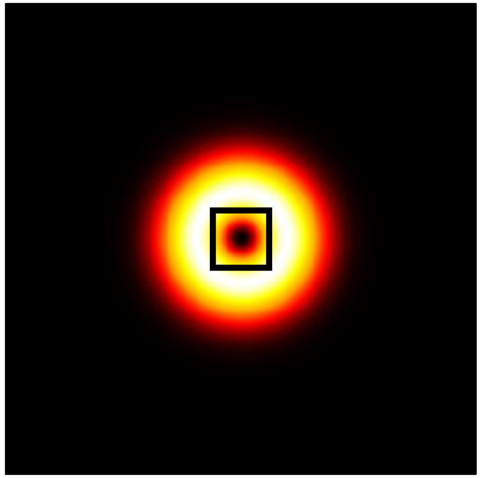




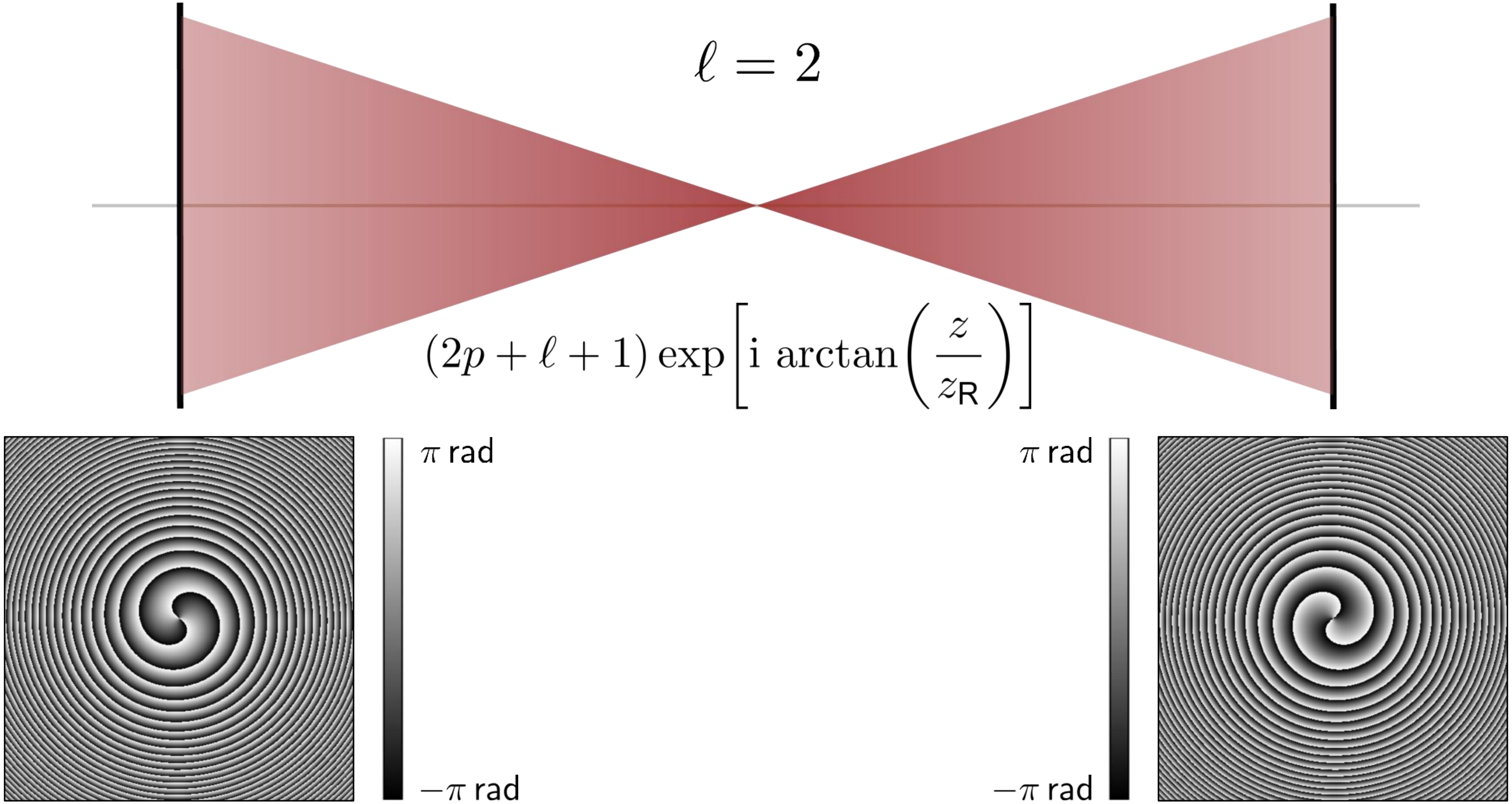
The HFT and the Gouy Phase in Beams with OAM: Simulation



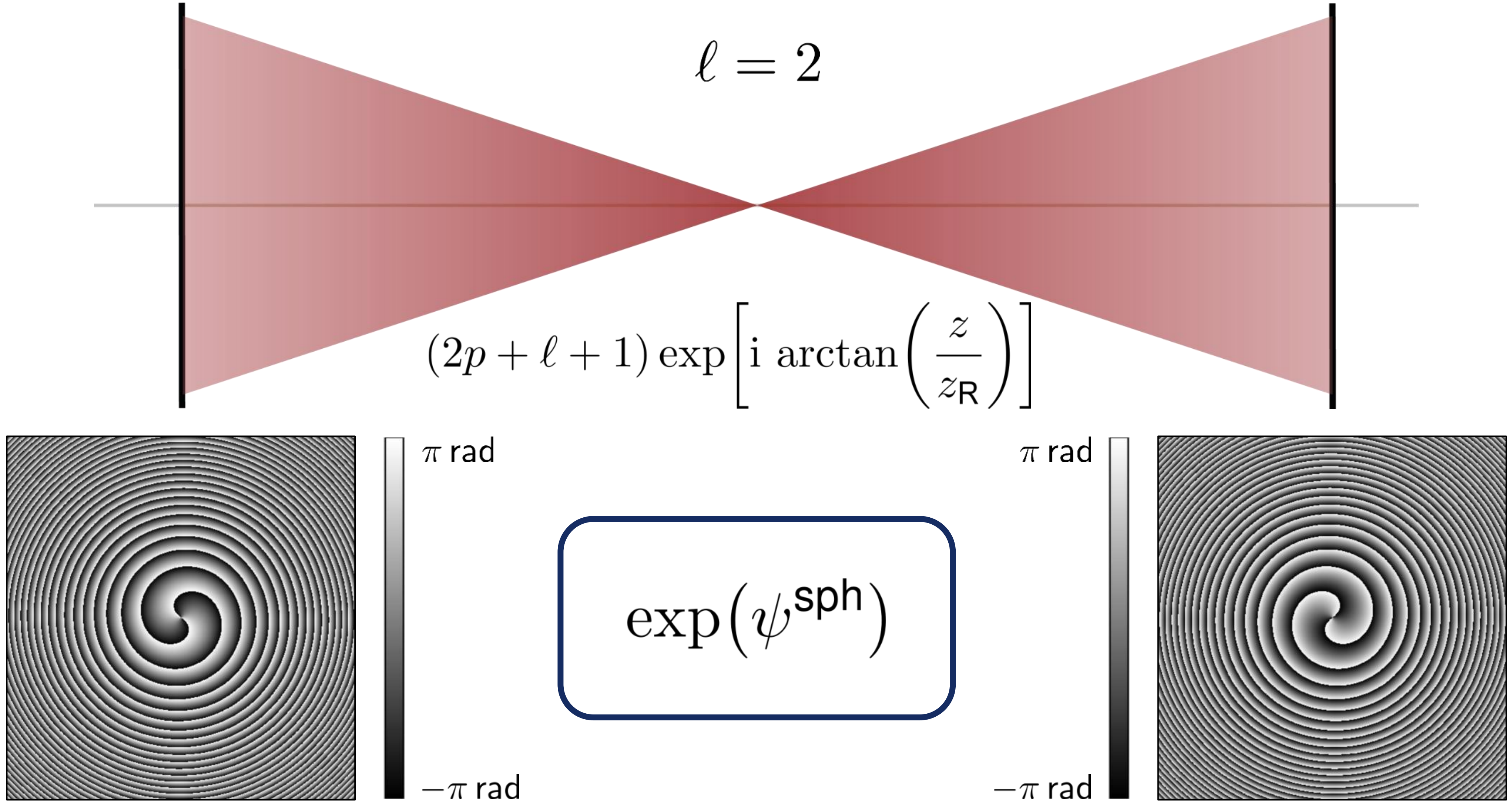




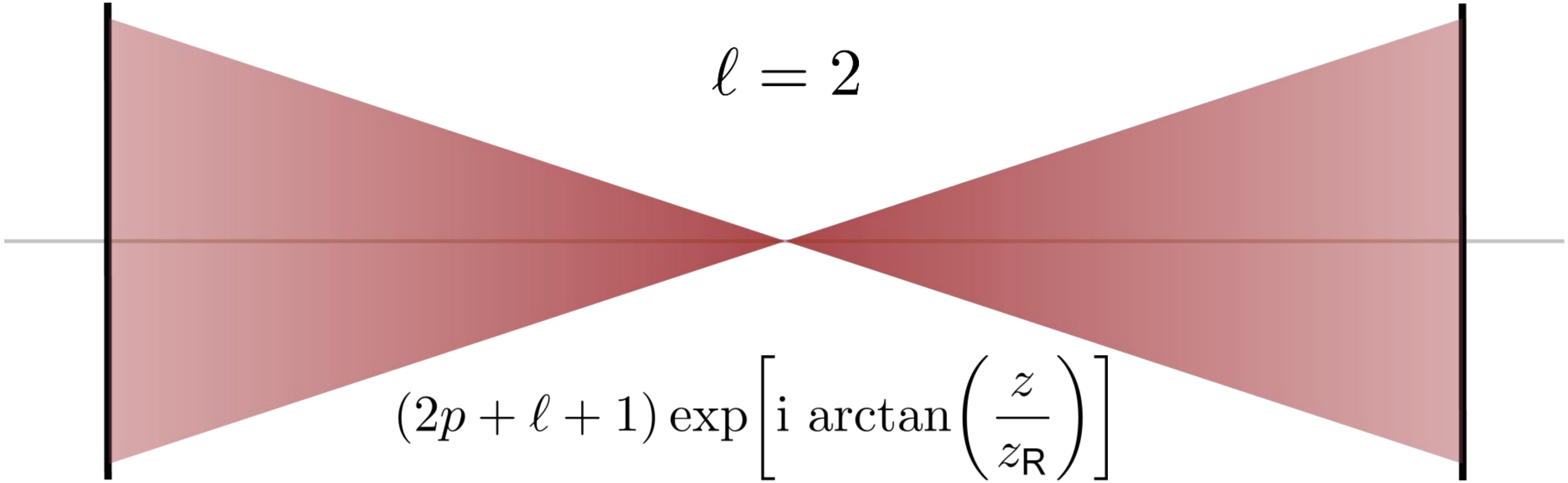
The HFT and the Gouy Phase in Beams with OAM: Simulation

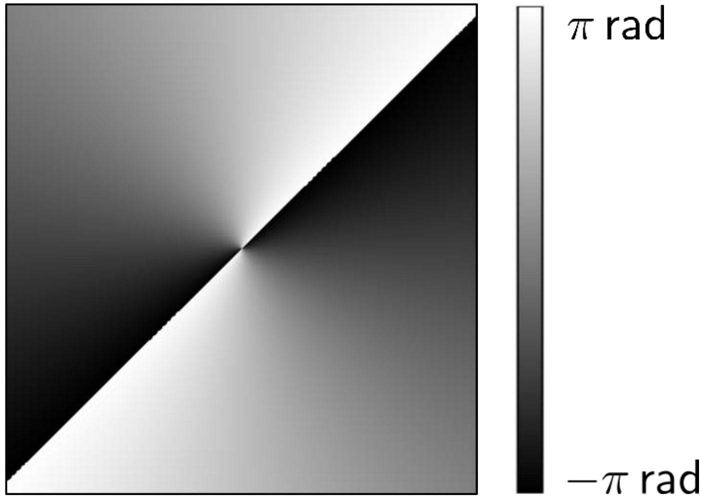


The HFT and the Gouy Phase in Beams with OAM: Simulation

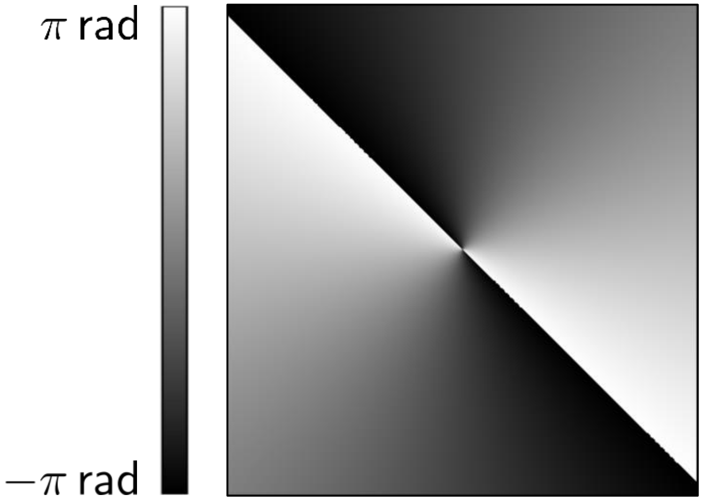


The HFT and the Gouy Phase in Beams with OAM: Simulation





$$\cancel{\exp(\psi^{\mathrm{sph}})}$$



Thank you!

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