SPIE Optics & Photonics, 13 August 2019

Modeling of Diffractive/Meta-Lenses using Fast Physical Optics

Frank Wyrowski[1], Site Zhang[2], Liangxin Yang[1], and Christian Hellmann[3]

[1] Applied Computational Optics Group, Friedrich-Schiller-Universität Jena

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Jena, Germany

LightTrans International

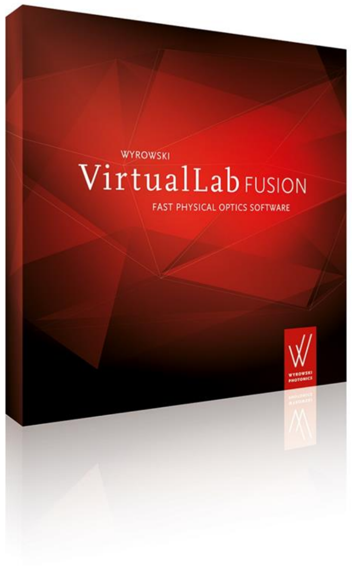
LightTrans
- Distribution of VirtualLab Fusion, together with distributors worldwide
- Technical support, seminars, and trainings
- Engineering projects

University of Jena

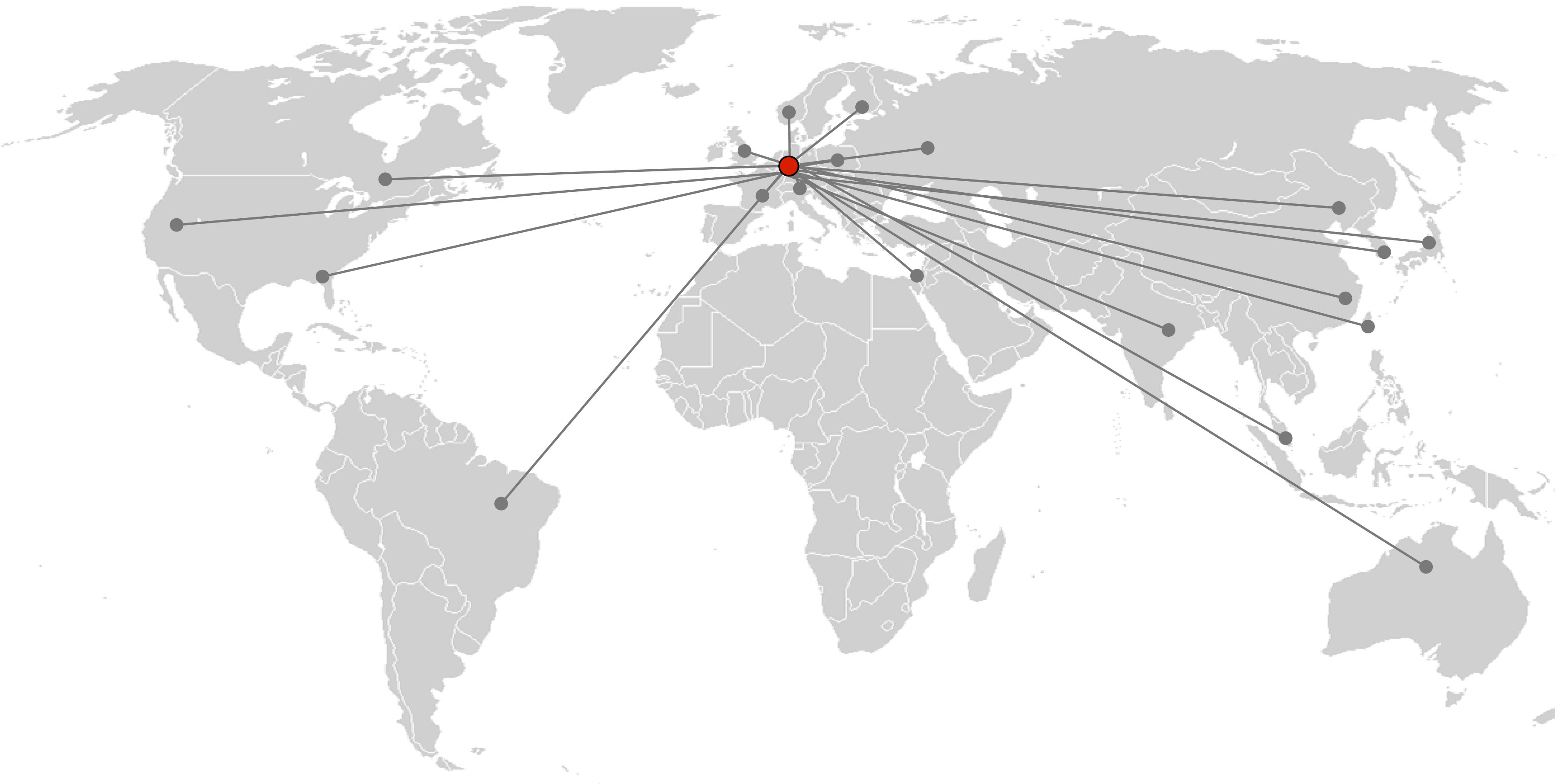
Applied Computational Optics Group R&D in optical modeling and design with emphasis on physical optics

Wyrowski Photonics

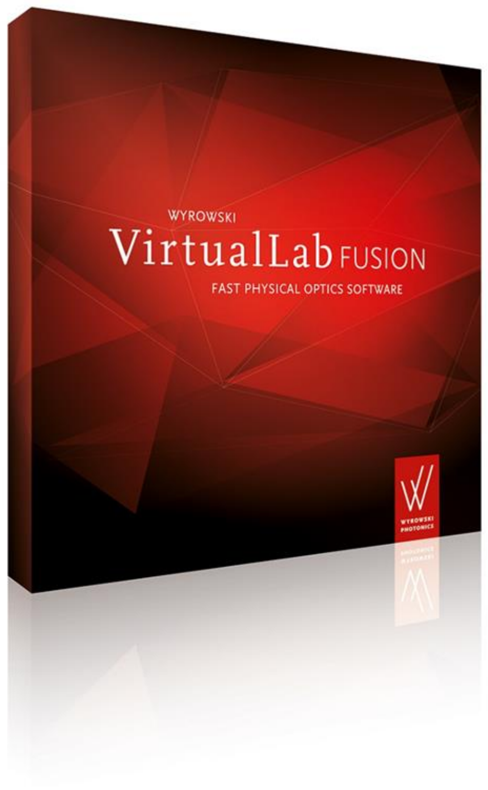
Wyrowski Photonics
Development of fast physical optics software VirtualLab Fusion



Optical Design Software and Services

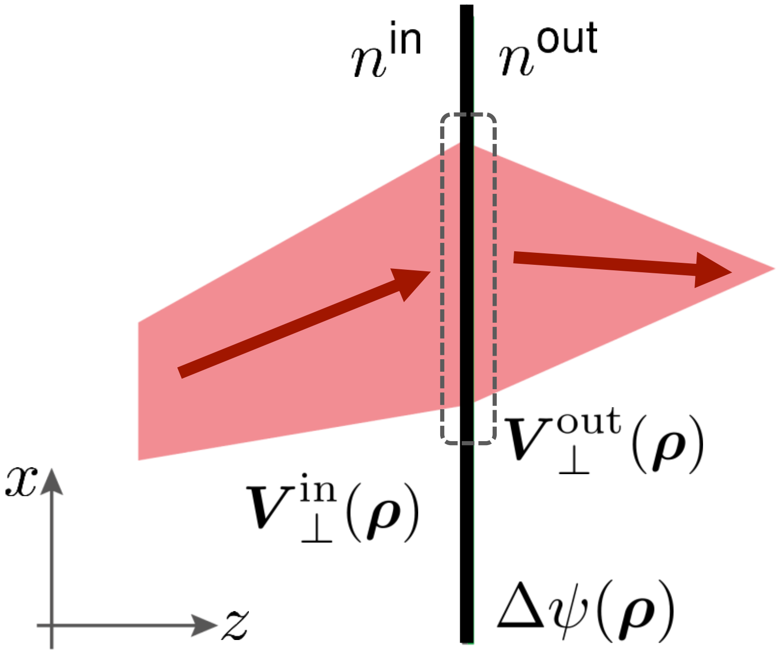


Booth #110



Emphasis of the common theoretical background for different types of microstructured layers

Unifying Approach – Wavefront Surface Response



$$\boldsymbol{V}_{\perp}^{\text{in}}(\boldsymbol{\rho}) = \boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \exp\big(\mathrm{i}\psi^{\text{in}}(\boldsymbol{\rho})\big)$$

with $\boldsymbol{\rho} := (x, y)$

- For plane surfaces we found $\nabla_{\perp}\psi^{\text{in}}(\boldsymbol{\rho}) = \nabla_{\perp}\psi^{\text{out}}(\boldsymbol{\rho})$ and in conclusion

$$\boldsymbol{U}_{\perp}^{\text{out}}(\boldsymbol{\rho}) \exp\big(\mathrm{i}\psi^{\text{in}}(\boldsymbol{\rho})\big) = \big\{\mathbf{B}(\boldsymbol{\rho};\psi^{\text{in}})\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})\big\} \exp\big(\mathrm{i}\psi^{\text{in}}(\boldsymbol{\rho})\big)\,.$$

- By introducing the wavefront surface response we assume an effect at the surface of the form

$$\boldsymbol{U}_{\perp}^{\text{out}}(\boldsymbol{\rho}) \exp\big(\mathrm{i}\psi^{\text{out}}(\boldsymbol{\rho})\big) = \big\{\mathbf{B}(\boldsymbol{\rho};\psi^{\text{in}})\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})\big\} \exp\big(\mathrm{i}\psi^{\text{out}}(\boldsymbol{\rho})\big)$$

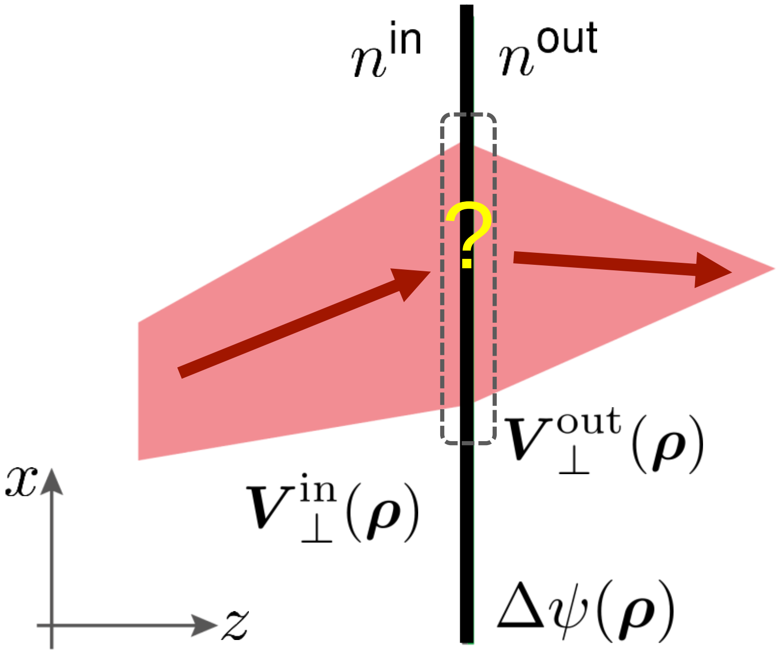
with

$$\psi^{\text{out}}(\boldsymbol{\rho}) = \psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})$$

and thus

$$\nabla_{\perp}\psi^{\text{out}}(\boldsymbol{\rho}) = \nabla_{\perp}\psi^{\text{in}}(\boldsymbol{\rho}) + \nabla_{\perp}\left(\Delta\psi(\boldsymbol{\rho})\right).$$

How to Realize Wavefront Surface Response (WSR)?



$$\boldsymbol{V}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho}) = \boldsymbol{U}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho}) \exp\big(\mathrm{i}\psi^{\mathrm{in}}(\boldsymbol{\rho})\big)$$

with $\boldsymbol{\rho} := (x, y)$

- From a physical-optics point of view the question arises, if there exists any manipulation of the structure of the surface, which provides an effect of the form:

$$\boldsymbol{U}_{\perp}^{\mathrm{out}}(\boldsymbol{\rho}) \exp\big(\mathrm{i}\psi^{\mathrm{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})\big) = \big\{\mathbf{B}(\boldsymbol{\rho}; \psi^{\mathrm{in}})\boldsymbol{U}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho})\big\} \exp\big(\mathrm{i}\psi^{\mathrm{in}}(\boldsymbol{\rho})\big)$$

- A detailed answer can only be given for a specific surface structure.
- By introducing microstructured layers onto the surface a wavefront surface response can be implemented:
  - Graded-index layer
  - Volume hologram layer
  - **Diffractive layer**
  - **Metamaterial layer**

Diffractive layer

… with application to lenses

Physical Optics Modeling: Diffractive Layer

- In general a wavefront surface response $\Delta\psi(\boldsymbol{\rho})$ leads to the equation $\nabla_\perp \psi^{\text{out}}(\boldsymbol{\rho}) = \nabla_\perp \psi^{\text{in}}(\boldsymbol{\rho}) + \nabla_\perp(\Delta\psi(\boldsymbol{\rho}))$ and because of the local plane wave assumption (homeomorphic zone) into

$$\boldsymbol{\kappa}^{\text{out}}(\boldsymbol{\rho}) = \boldsymbol{\kappa}^{\text{in}}(\boldsymbol{\rho}) + \boldsymbol{K}(\boldsymbol{\rho})$$

  with $\boldsymbol{K}(\boldsymbol{\rho}) \stackrel{\text{def}}{=} \nabla_\perp(\Delta\psi(\boldsymbol{\rho}))$.

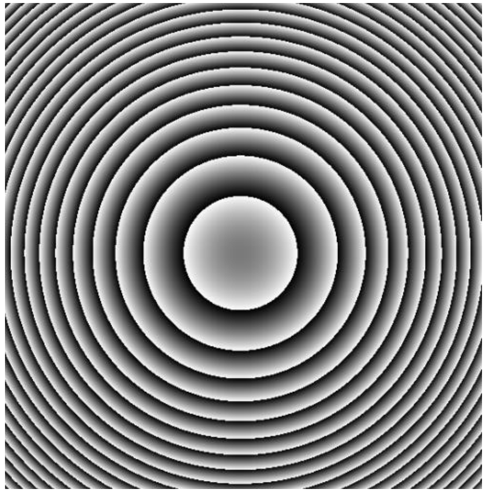
- This equation is directly related to a locally formulated grating equation

$$\boldsymbol{\kappa}^{\text{out}}(\boldsymbol{\rho}) = \boldsymbol{\kappa}^{\text{in}}(\boldsymbol{\rho}) + m\,(2\pi/d_x(\boldsymbol{\rho}), 2\pi/d_y(\boldsymbol{\rho}))$$

  with the local grating period $\boldsymbol{d}(\boldsymbol{\rho}) = (d_x(\boldsymbol{\rho}), d_y(\boldsymbol{\rho}))$.

- That leads to the basic principle of a diffractive layer via:

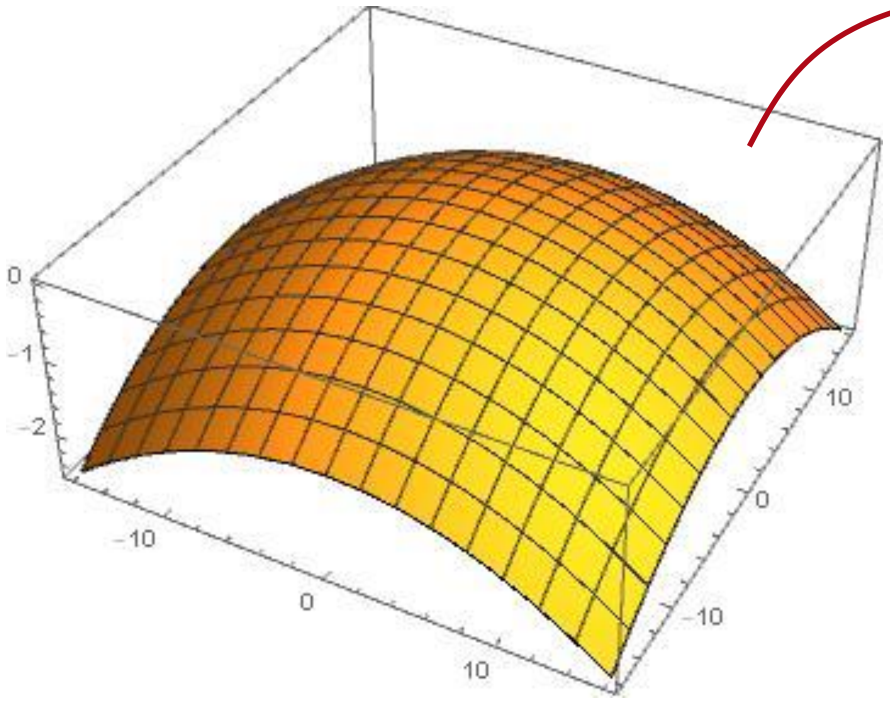
$$\boldsymbol{d}(\boldsymbol{\rho}) = 2\pi\left((\partial\psi(\boldsymbol{\rho})/\partial x)^{-1}, (\partial\psi(\boldsymbol{\rho})/\partial y)^{-1}\right)$$

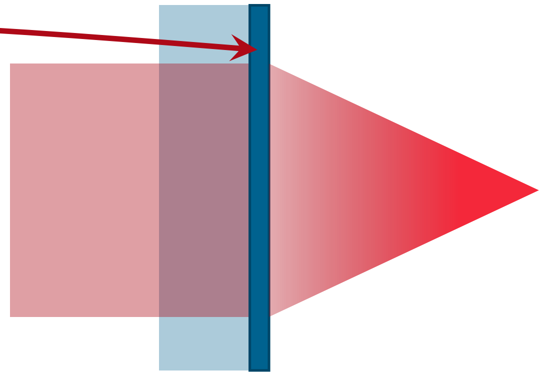


Wavefront Surface Response of Focusing Lens

- In order to transform a plane incident field into a spherical con-vernegt one the wavefront surface response should be:

$$\Delta\psi(\boldsymbol{\rho}) = k_0 n \left(f - \sqrt{\|\boldsymbol{\rho}\|^2 + f^2}\right)$$

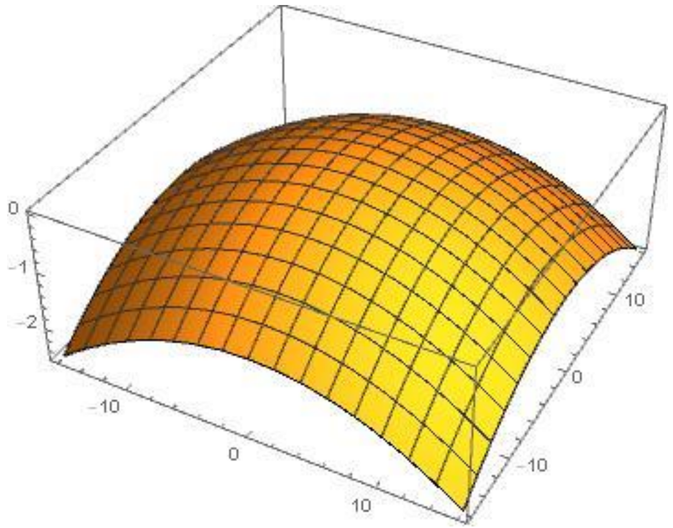


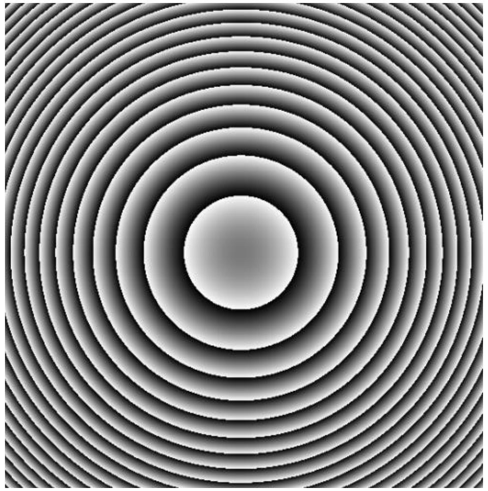


Structure Design

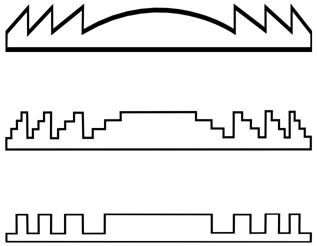
- Wrap the WSR: $(\Delta\psi(\boldsymbol{\rho}))^{\text{DOE}} = \mod_{p2\pi}\left\{k_0 n\left(f - \sqrt{\|\boldsymbol{\rho}\|^2 + f^2}\right)\right\}$ with $p \in \mathbb{N}$.
- For $p = 1$ local radial period follows with $d(\rho) = 2\pi/\Delta\psi'(\rho)$.
- Structure design by inverse Thin Element Approximation (TEA): The height profile h^{DOE} is given by:

$$h^{\text{DOE}}(\rho) = \frac{\lambda}{2\pi\Delta n}\Delta\psi(\rho)^{\text{DOE}}$$

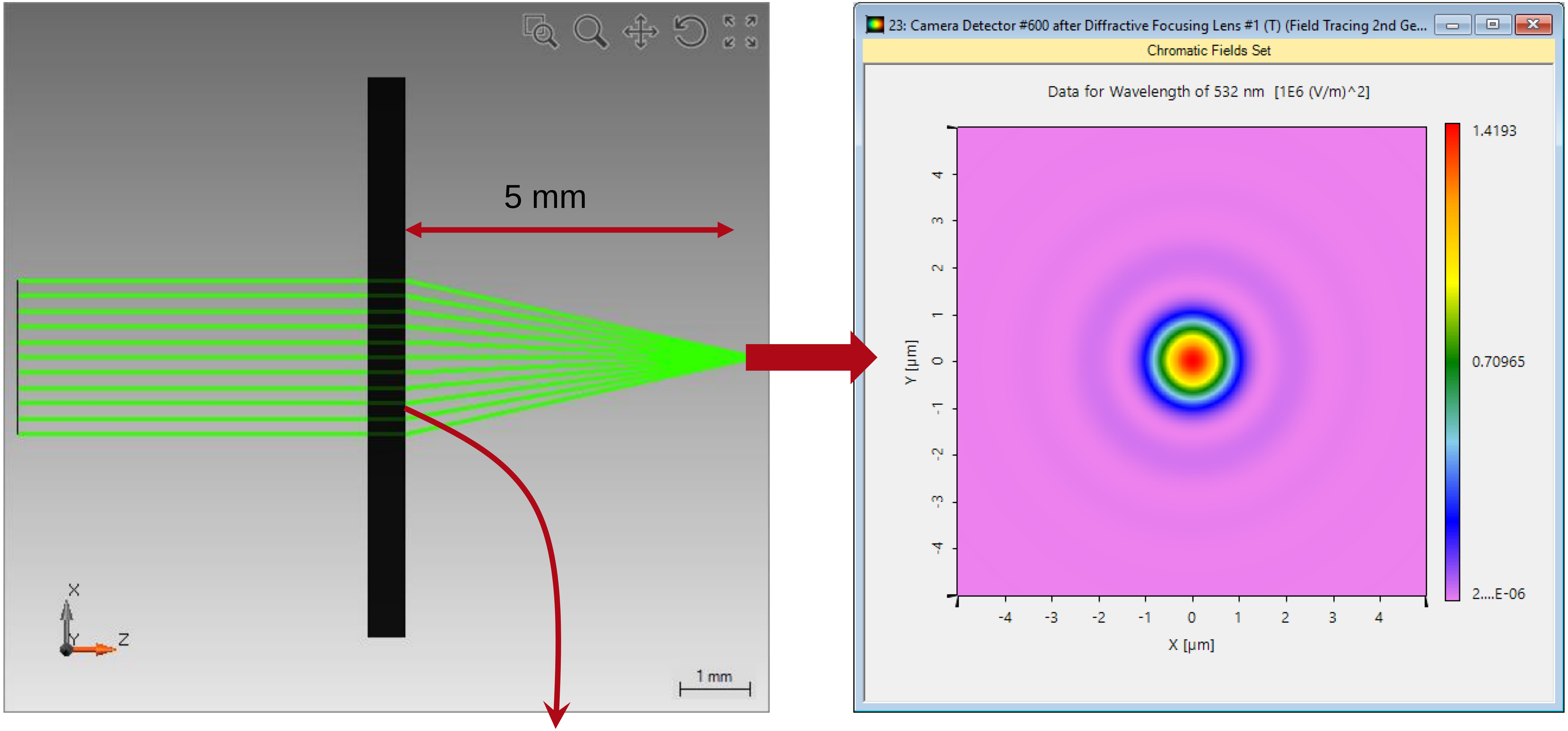




Profile quantization

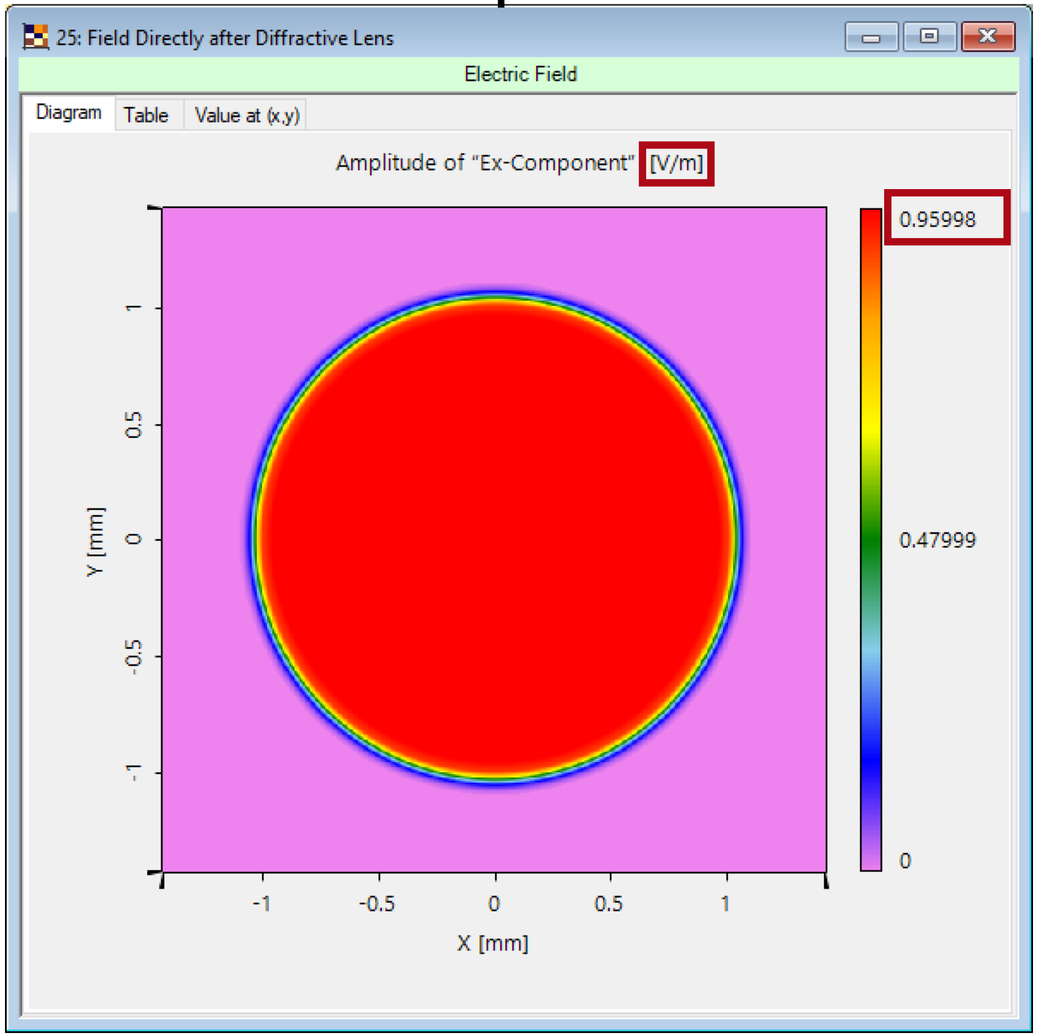


Focusing Diffractive Lens with NA=0.2: Ray and Field Tracing



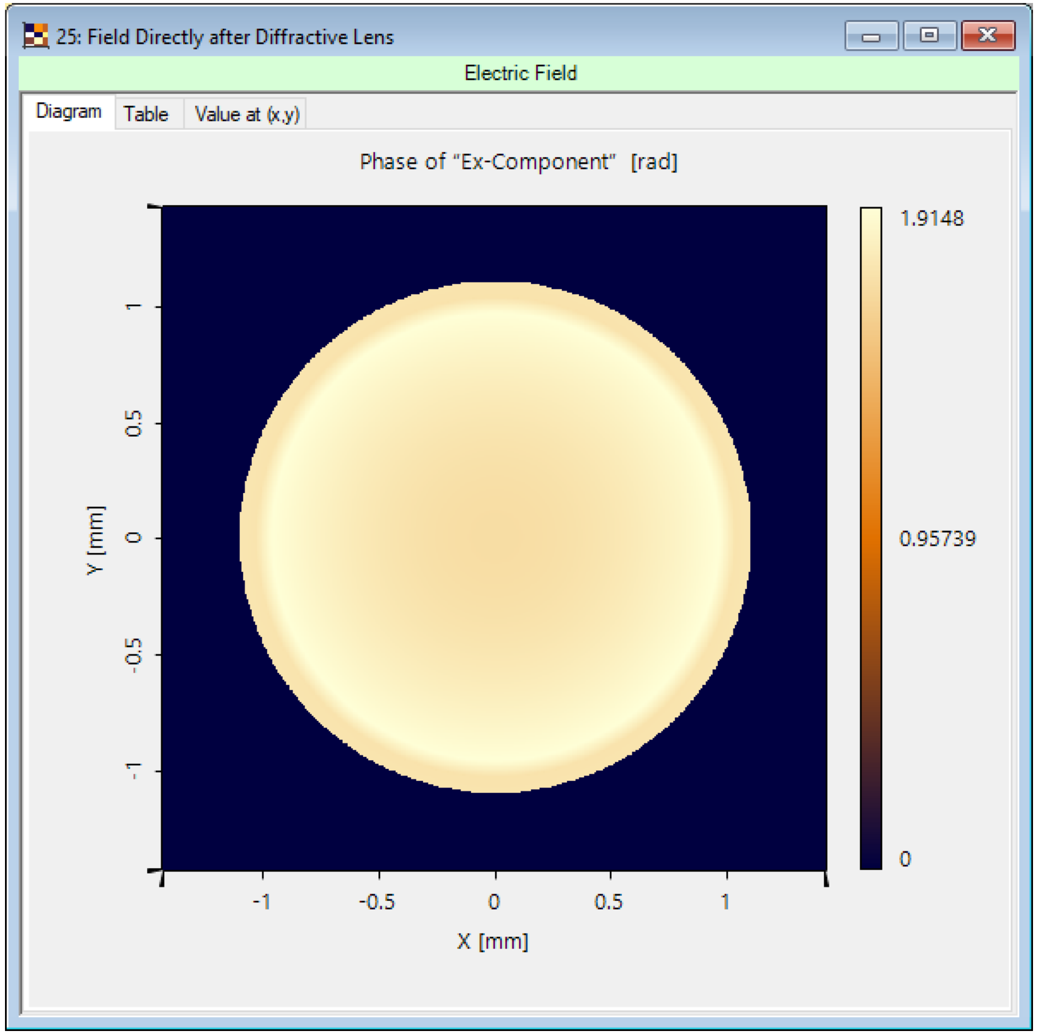
Focusing Diffractive Lens with NA=0.2: Ray and Field Tracing

Amplitude



Ex

Phase



Focusing Diffractive Lens with NA=0.2: Ray and Field Tracing

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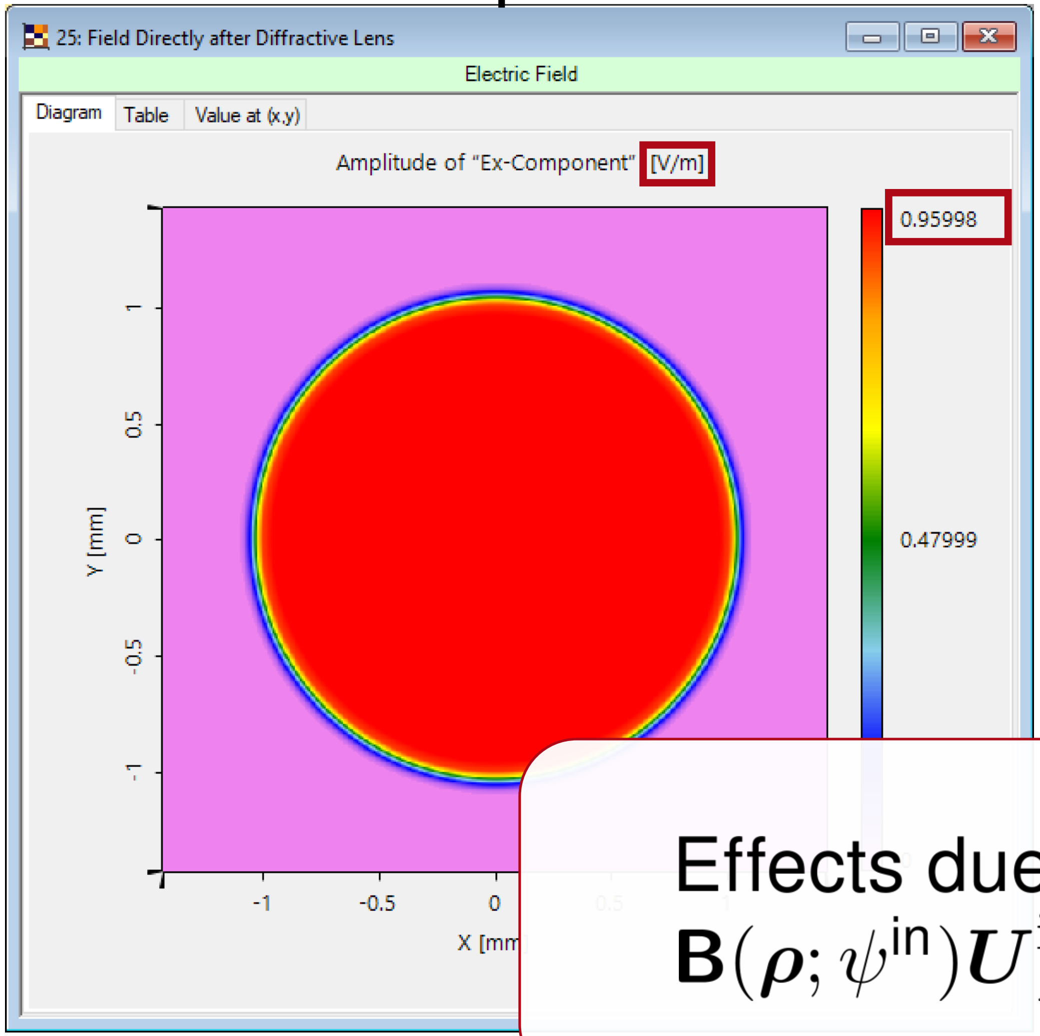
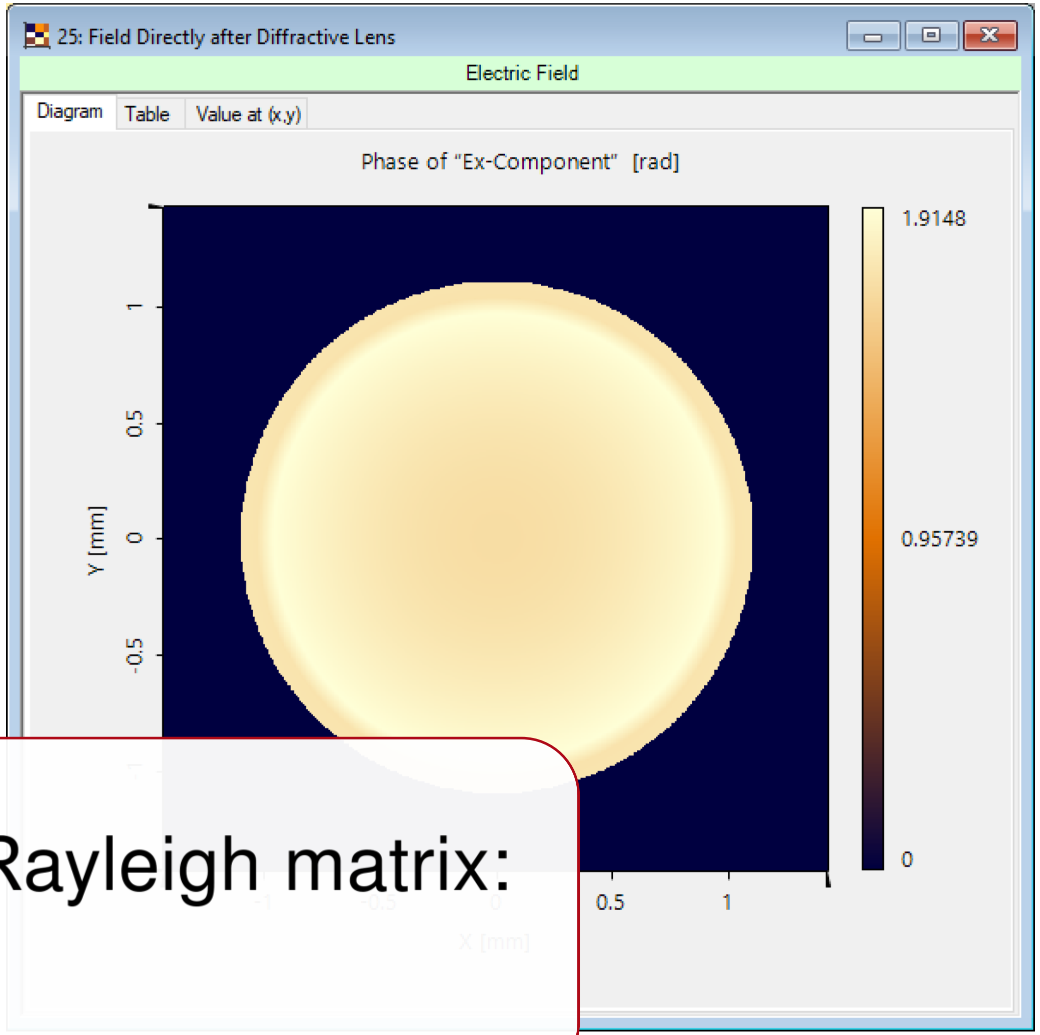


Ex

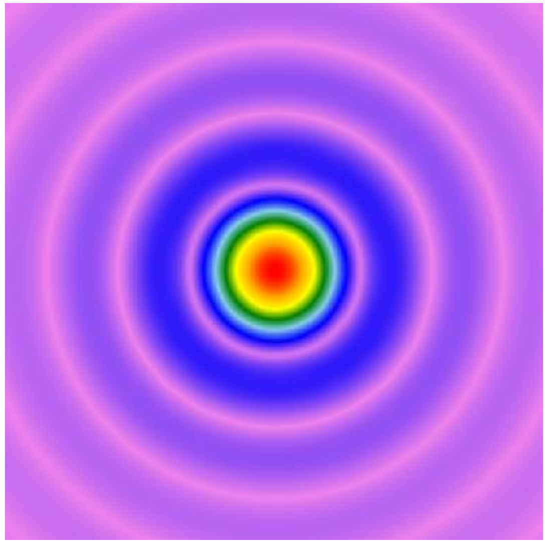
Phase



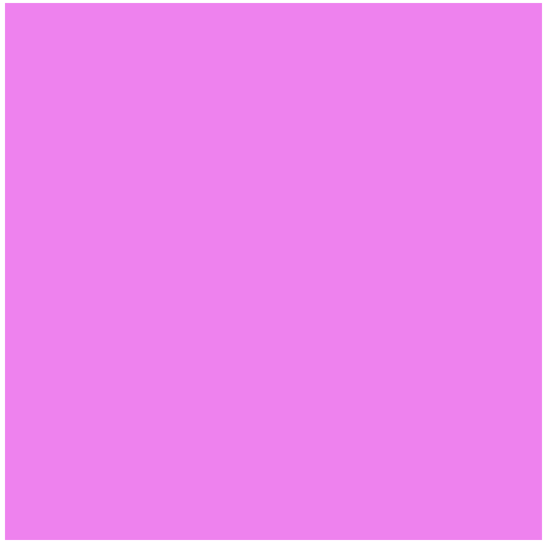
Effects due to Rayleigh matrix:

$\mathbf{B}(\boldsymbol{\rho};\psi^{\text{in}})\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})$

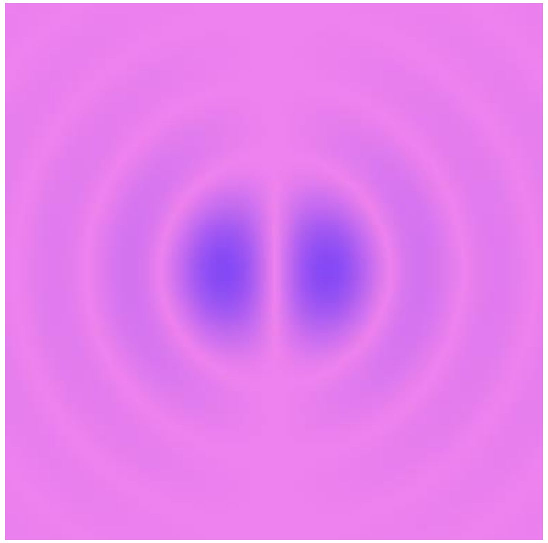
Focusing Diffractive Lens with NA=0.2: Ray and Field Tracing



Amplitude Ex



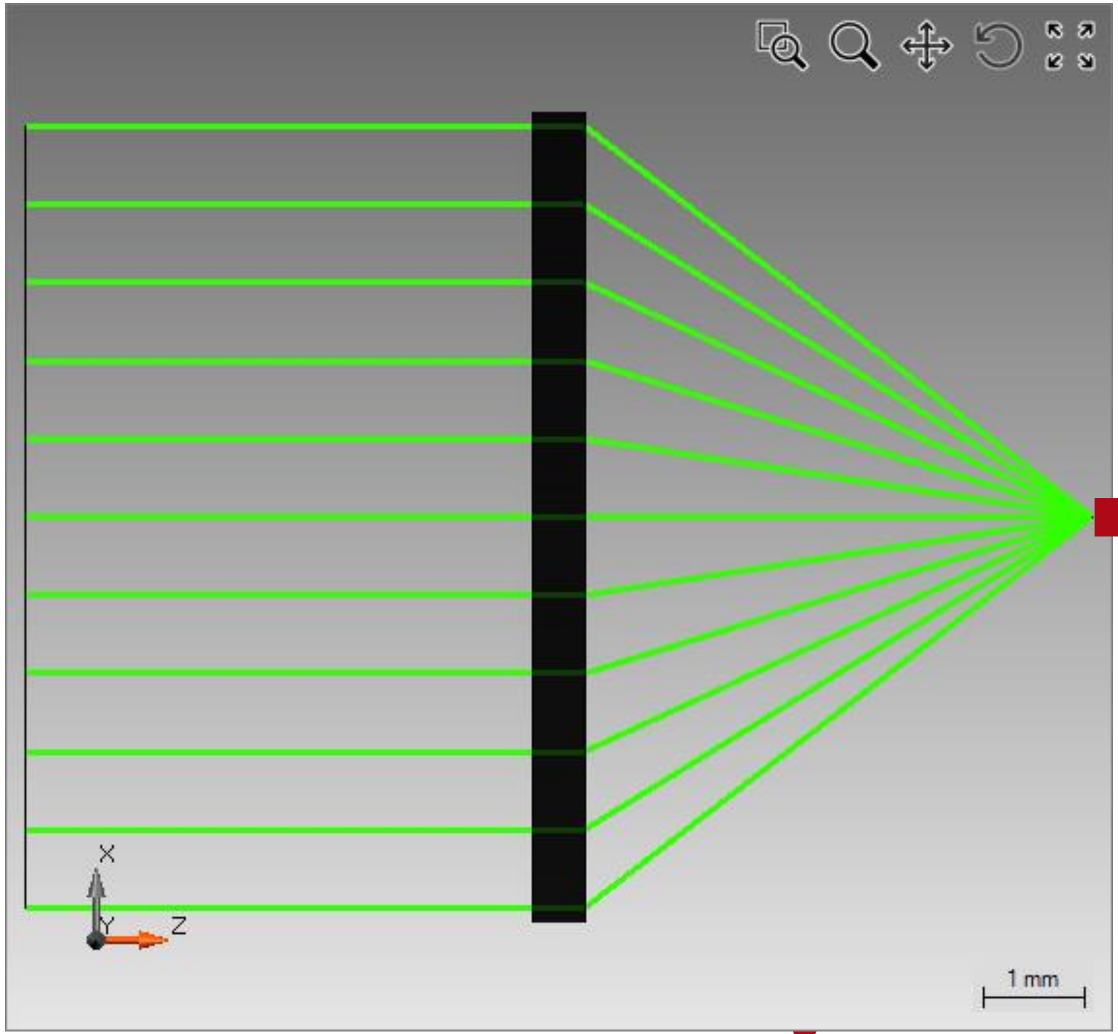
Amplitude Ey

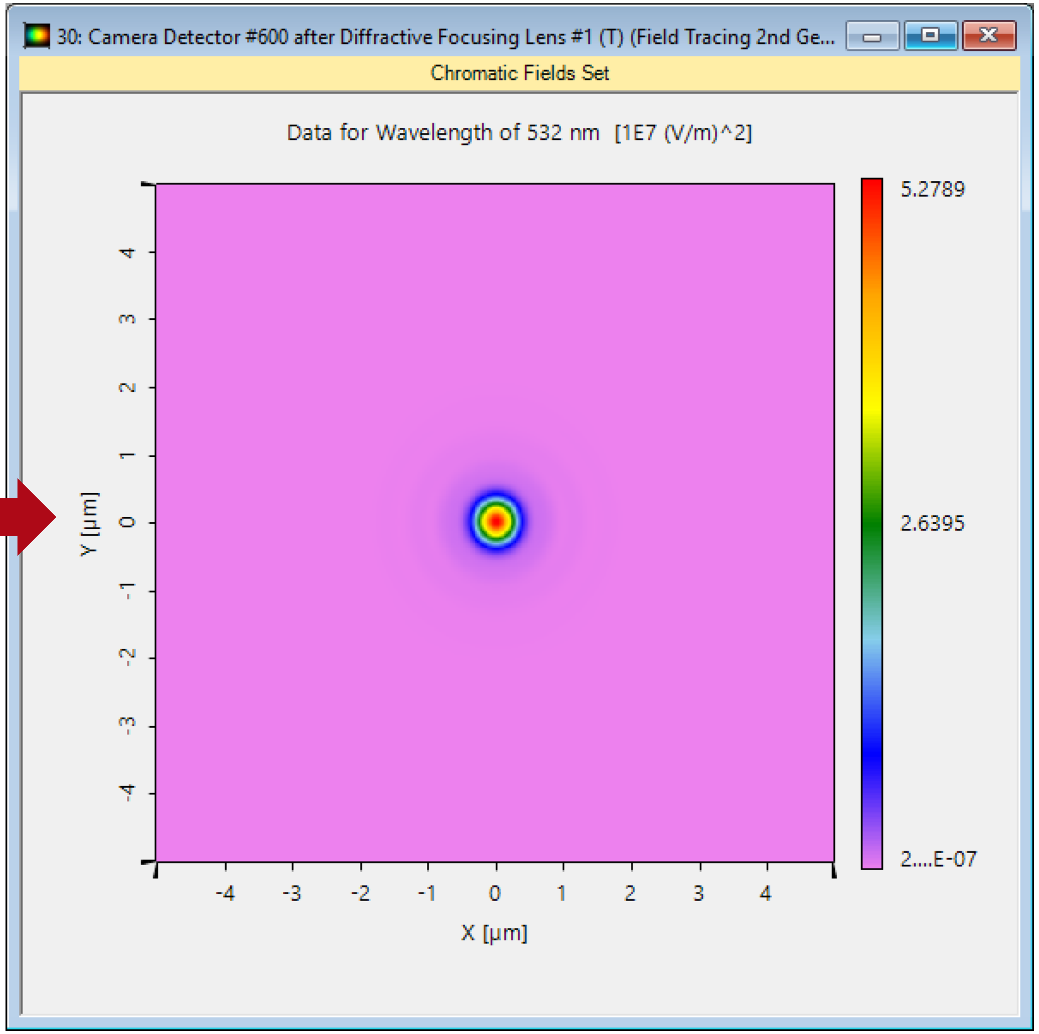


Amplitude Ez

- Amplitudes in Focus (Same scaling!)

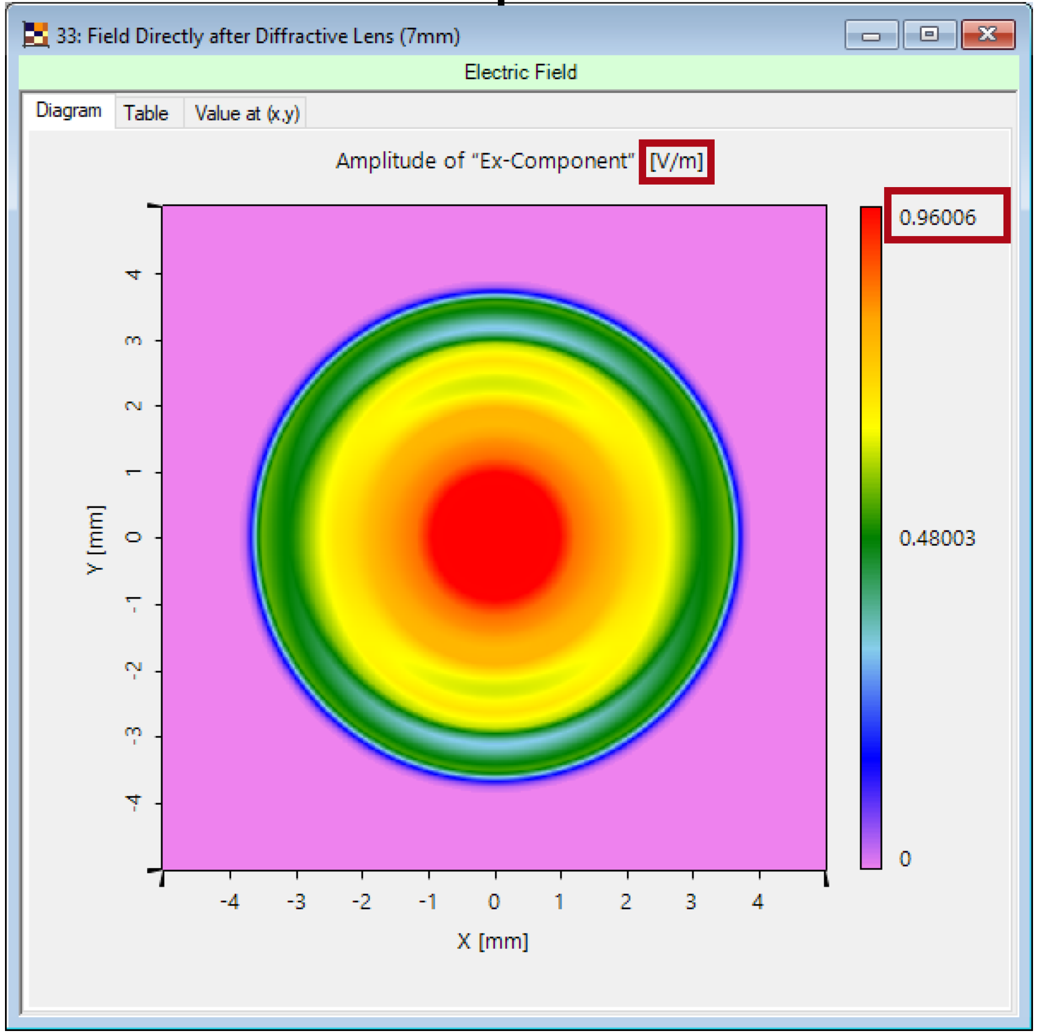
Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing





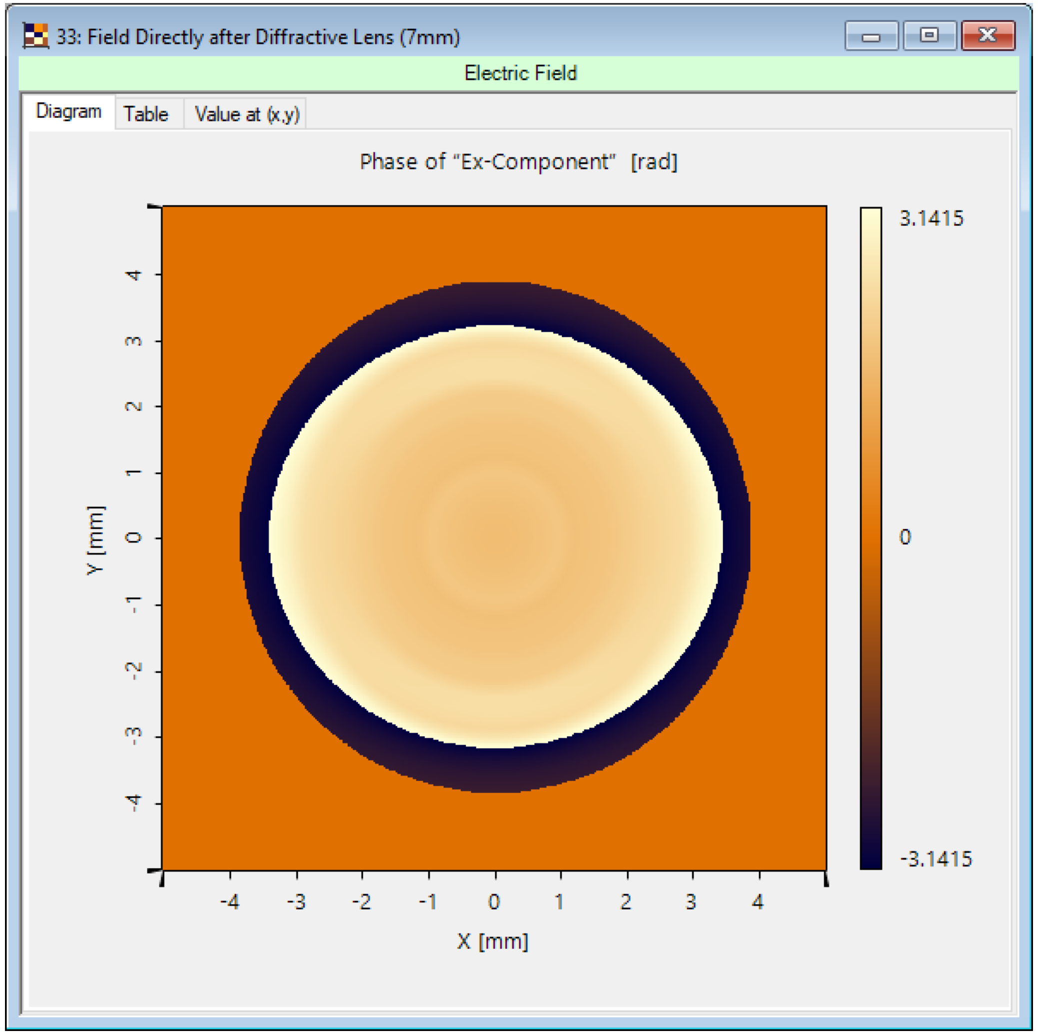
Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing

Amplitude



Ex

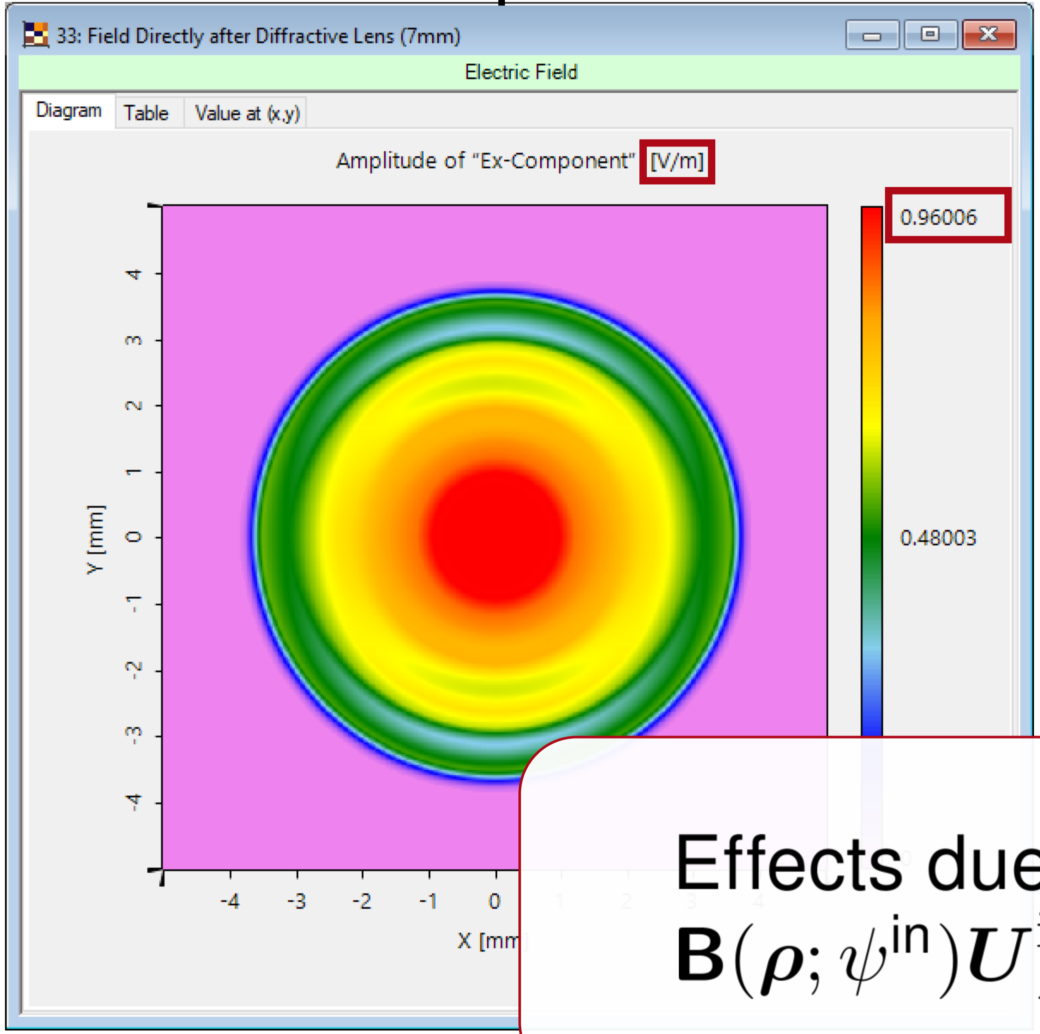
Phase



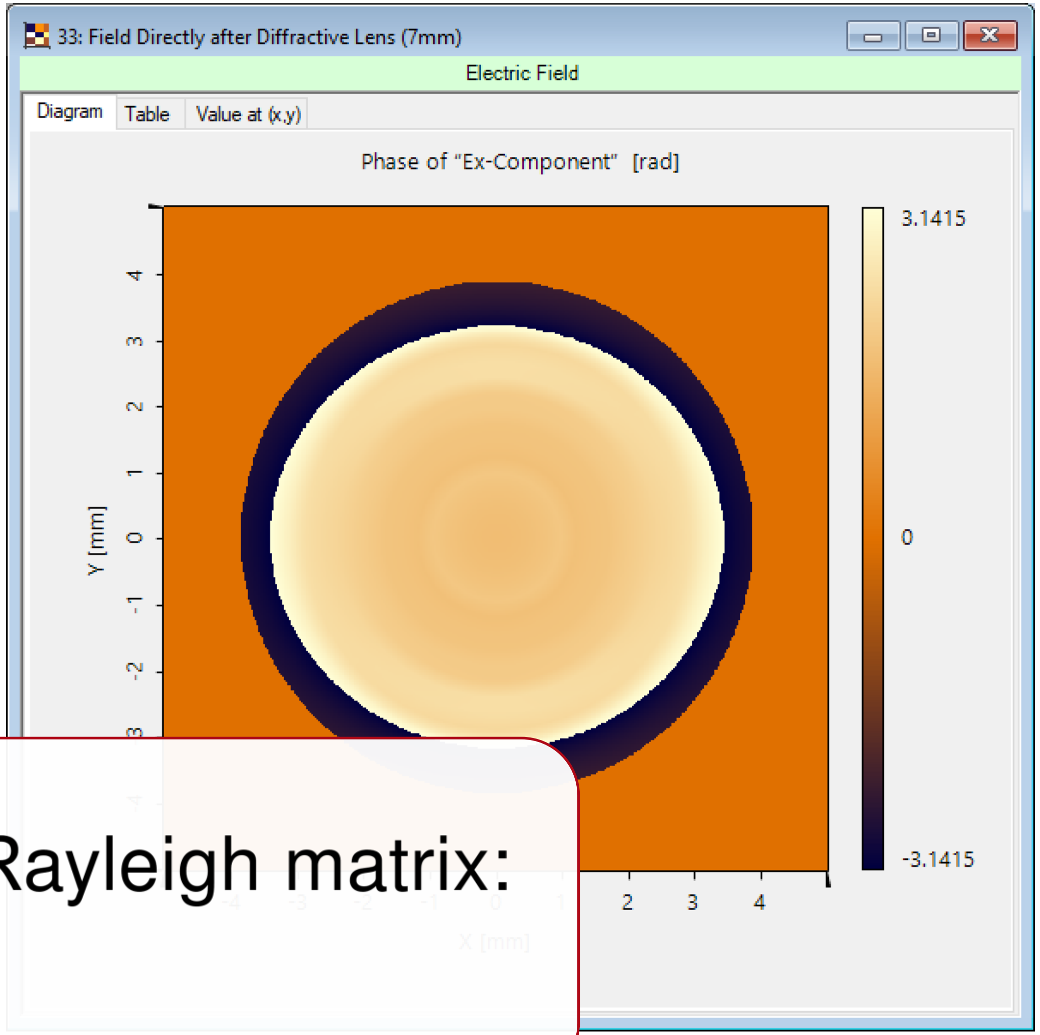
Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing

Amplitude

Phase



Ex



Effects due to Rayleigh matrix:
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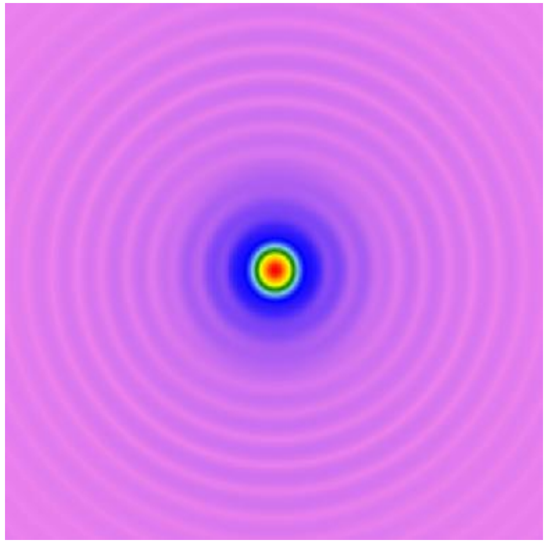
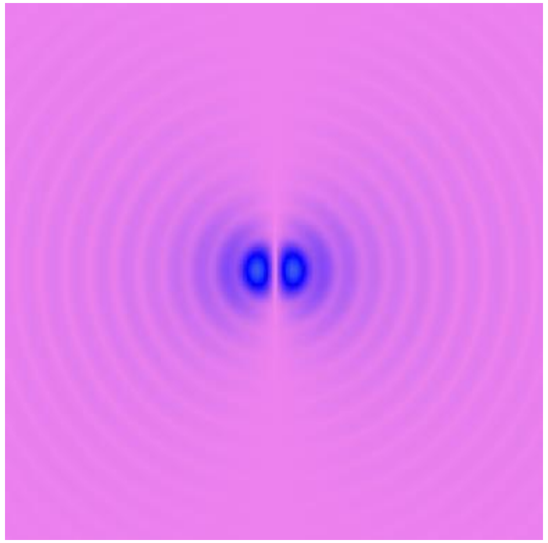
Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing



Amplitude Ex

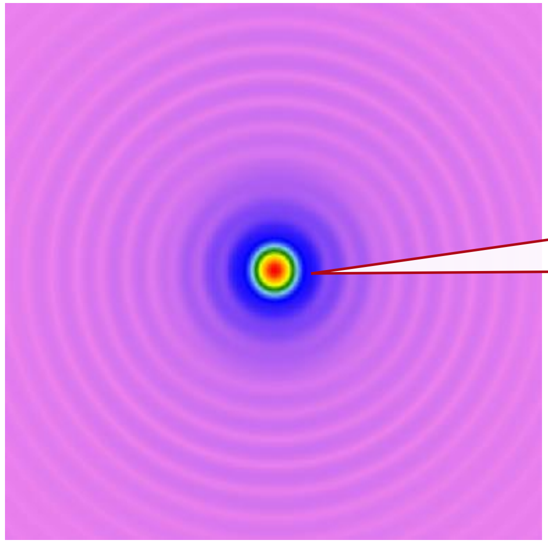
Amplitude Ey



Amplitude Ez

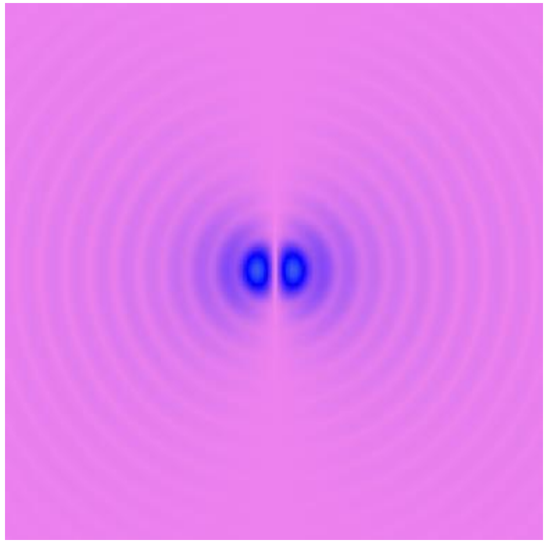
- Amplitudes in Focus (Same scaling!)

Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing



Amplitude Ex

Amplitude Ey



Amplitude Ez

Vectorial grating effects reduce spot quality for high NA

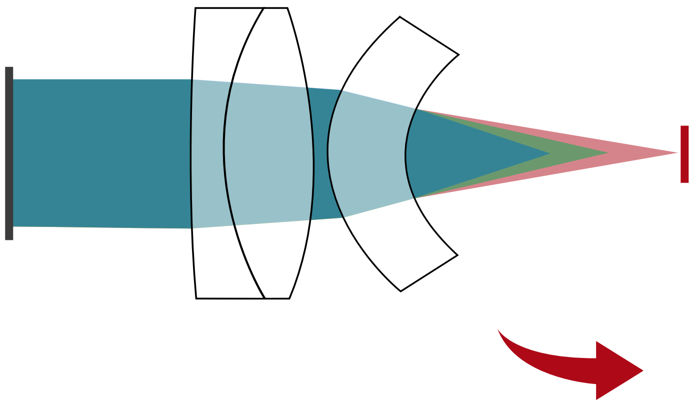
- Amplitudes in Focus (Same scaling!)

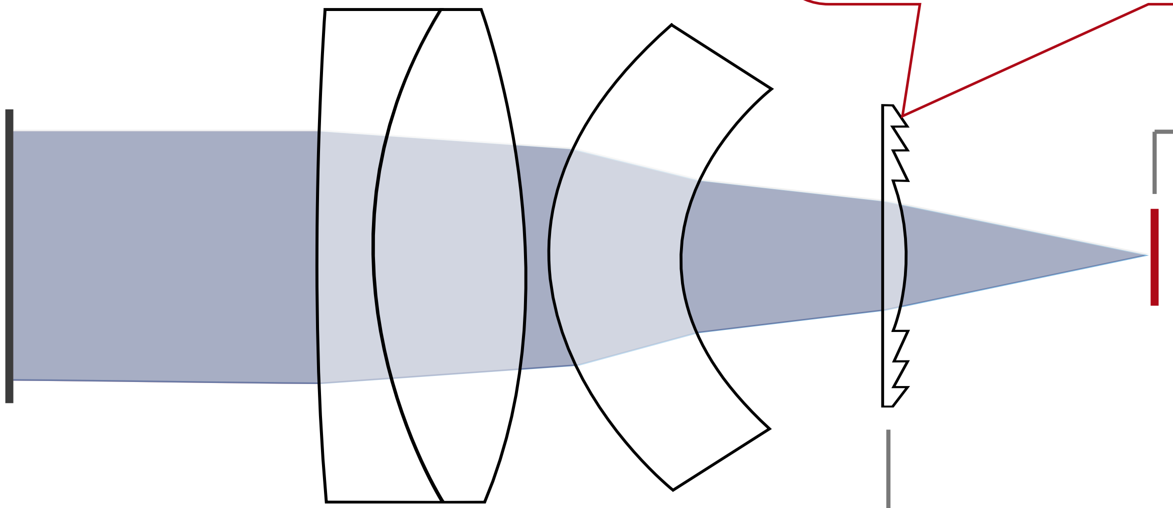
Combination of OpticStudio® and VirtualLab Fusion

Complementary workflows

Color Aberration Correction by DOE

Optimization of Binary Surface = Wavefront Surface Response in OpticStudio®





detectors

- point spread function
- modulation transfer function (MTF)

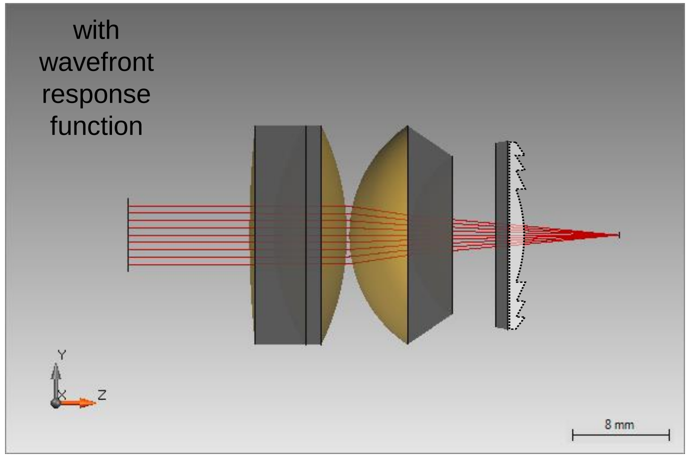
plane wave

- wavelength (486, 587, 656)nm
- field of view along y-axis (0; 20)°
- linearly polarized along x-axis
- aperture 5mm×5mm

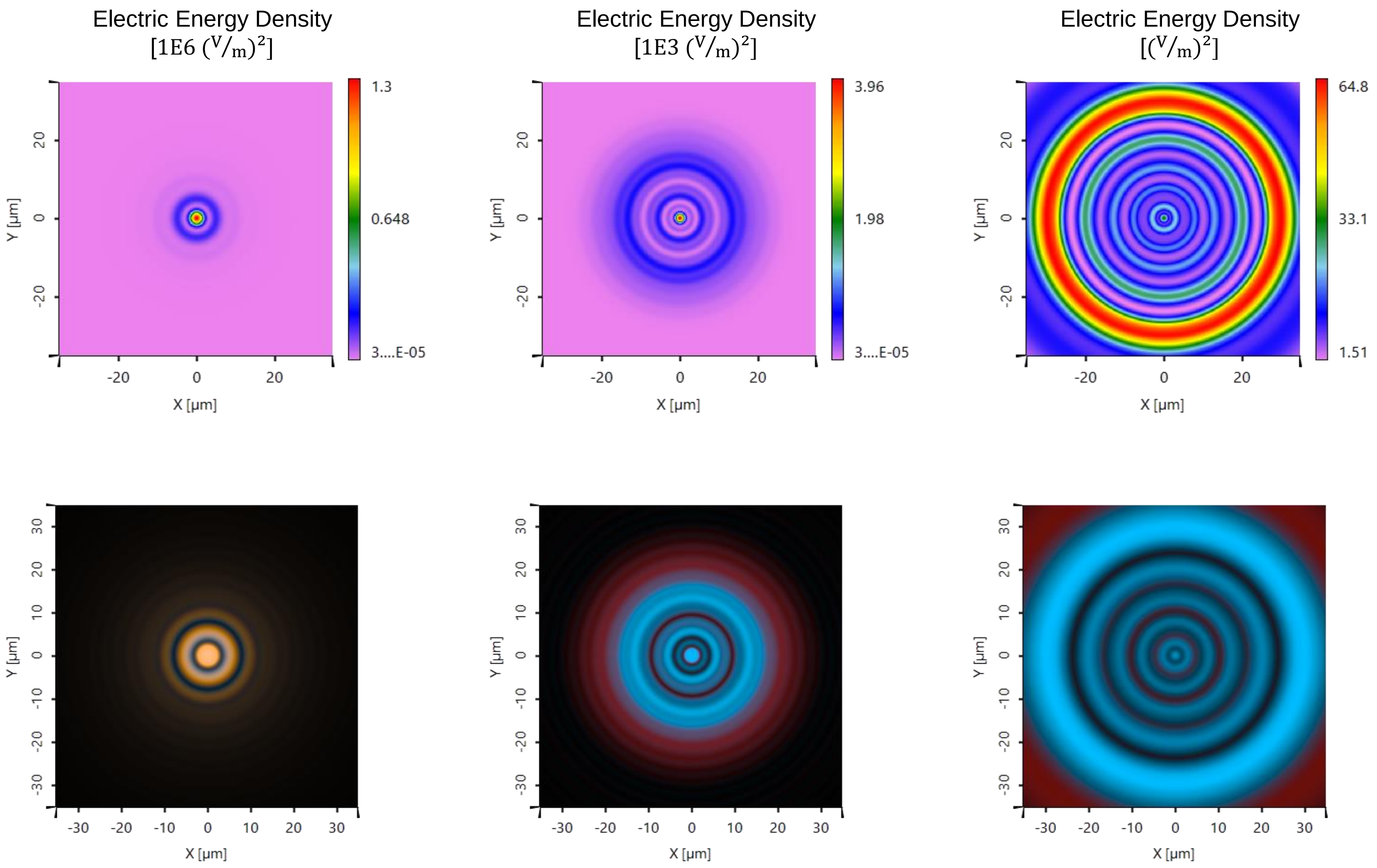
lens solution

- wavefront response function
- diffractive lens
- meta lens

Inclusion of Higher Orders: On-Axis

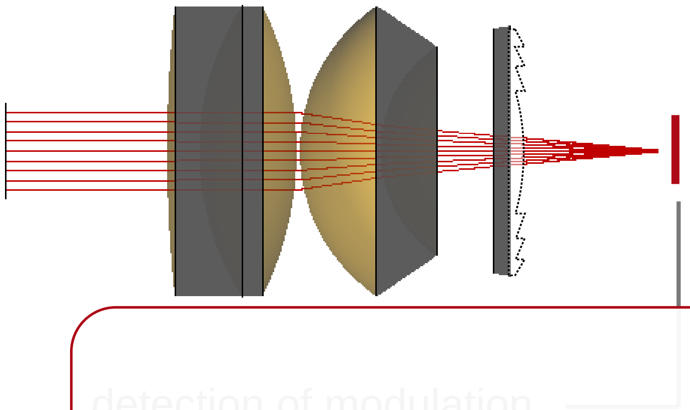


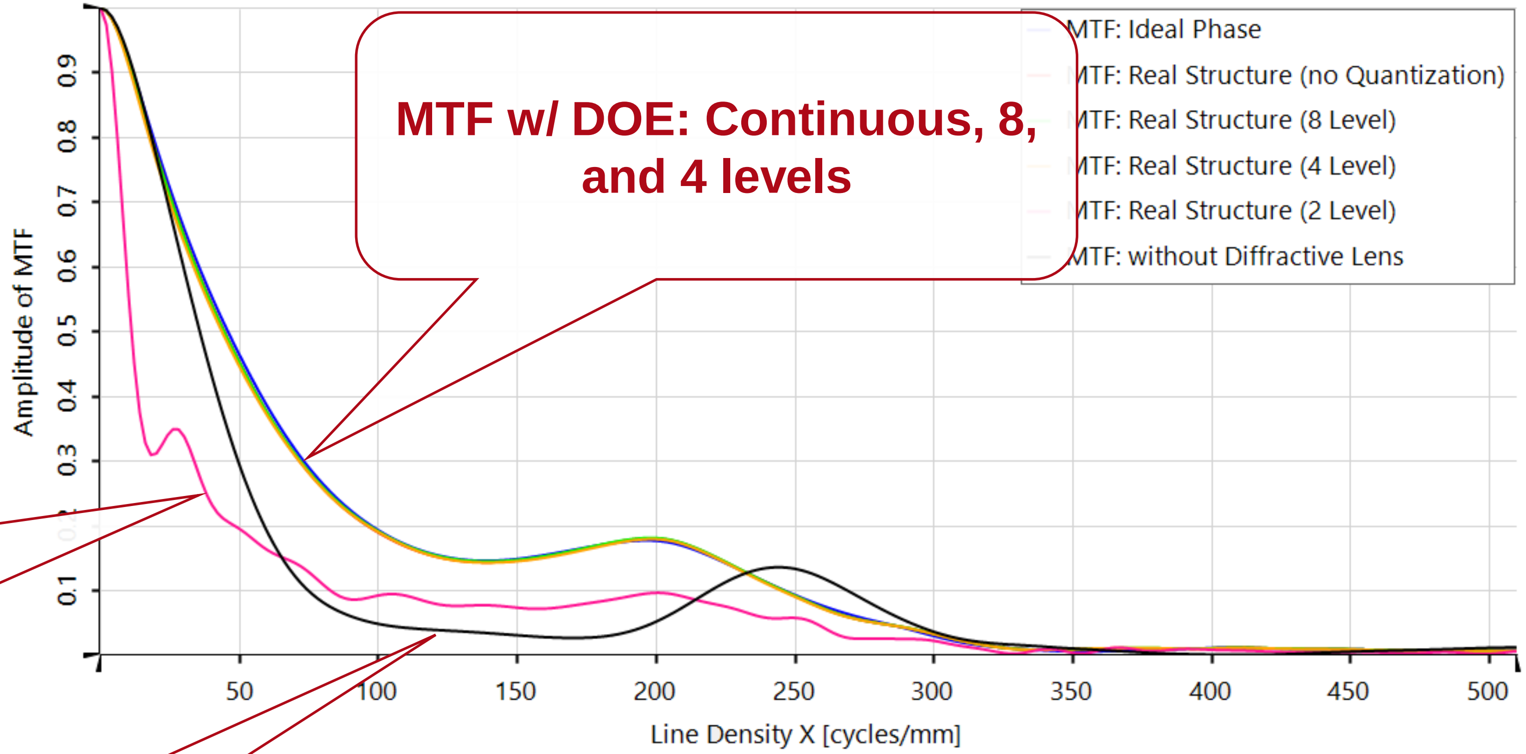
simulation time per order ~seconds



+1st diffraction order 0th diffraction order -1st diffraction order

Results: MTF for Various Diffractive Lens Structures





MTF w/ DOE: Continuous, 8, and 4 levels

MTF w/ DOE: Binary

MTF w/o DOE

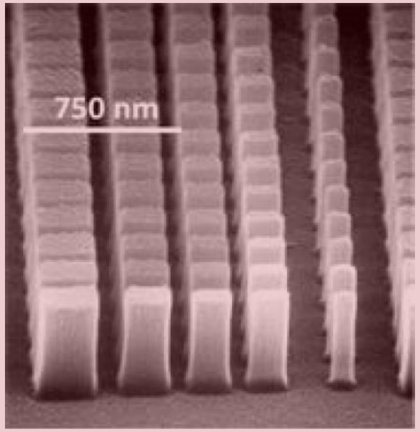
Metasurfaces

Realization of wavefront surface responses by nanostructured layers

Physical Effects for Realizing Metasurfaces

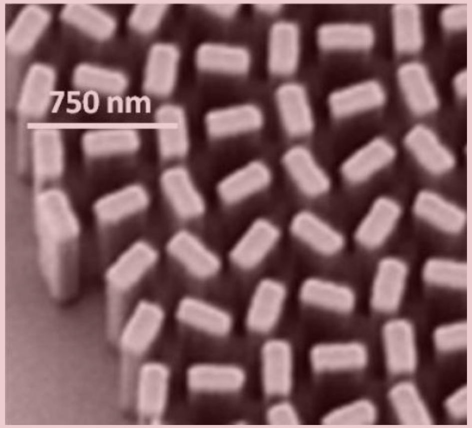
- Propagation phase delay
  - Centrosymmetric (polarization insensitive)

P. Lalanne *et al*., J. Opt. Soc. Am. A **16**, 1143-1156 (1999).

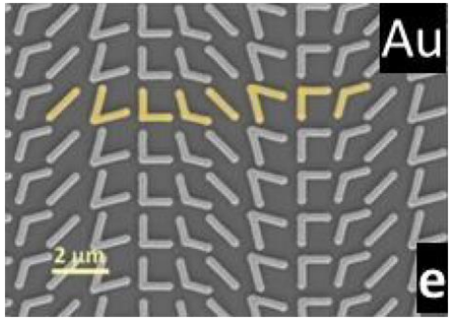


  - Rotationally asymmetric (form birefringence)

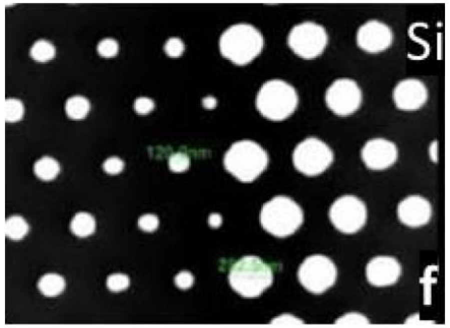
M. Khorasaninejad *et al*., Science **352**, 1190-1194 (2016).



- Resonance phase delay



N. Yu *et al*., Science **334**, 333–337 (2011).



Y. F. Yu *et al*., Laser Photonics Rev. **9**, 412-418 (2015).

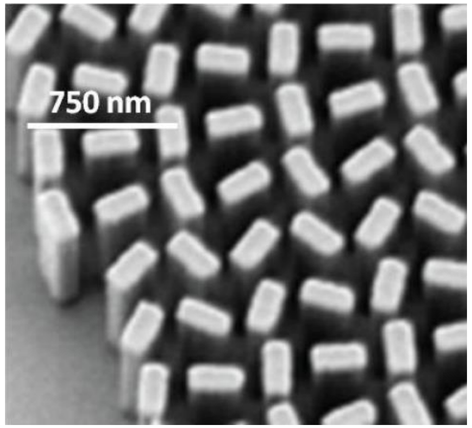
Physical Optics Modeling: Metasurface Layer

- In general a real surface layer structure does not just realize the desired wavefront response but additional effects and fields:

$$\boldsymbol{V}_\perp^{\mathrm{out}}(\boldsymbol{\rho}) = \left\{\mathbf{B}(\boldsymbol{\rho};\psi^{\mathsf{in}})\boldsymbol{U}_\perp^{\mathrm{in}}(\boldsymbol{\rho})\right\}\exp\left(\mathrm{i}\psi^{\mathsf{in}}(\boldsymbol{\rho})+\Delta\psi(\boldsymbol{\rho})\right)+\boldsymbol{V}_\perp^{\mathrm{res}}(\boldsymbol{\rho})$$

- For nanofin-based metalayers the typical result can be written as:

$$\begin{aligned}\boldsymbol{V}_\perp^{\mathrm{out}}(\boldsymbol{\rho}) = &\left\{\mathbf{B}^{+}(\boldsymbol{\rho};\psi^{\mathsf{in}})\boldsymbol{U}_\perp^{\mathrm{in}}(\boldsymbol{\rho})\right\}\exp\left(\mathrm{i}\psi^{\mathsf{in}}(\boldsymbol{\rho})+\Delta\psi(\boldsymbol{\rho})\right)\\ &+\left\{\mathbf{B}^{-}(\boldsymbol{\rho};\psi^{\mathsf{in}})\boldsymbol{U}_\perp^{\mathrm{in}}(\boldsymbol{\rho})\right\}\exp\left(\mathrm{i}\psi^{\mathsf{in}}(\boldsymbol{\rho})-\Delta\psi(\boldsymbol{\rho})\right)\end{aligned}$$



M. Khorasaninejad *et al.*,
Science **352**, 1190-1194 (2016).

Physical Optics Modeling: Metasurface Layer

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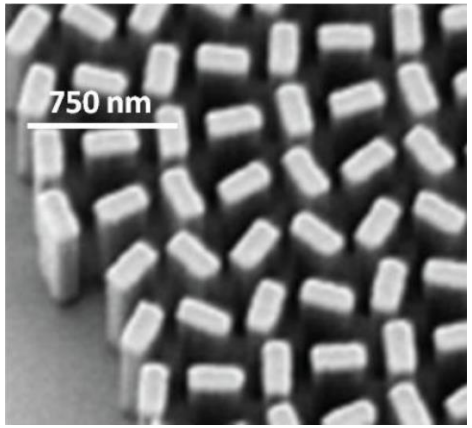
Calculation by Fourier modal method (FMM), a.k.a. RCWA

- For nanofin-based metalayers the typical result can be written as:

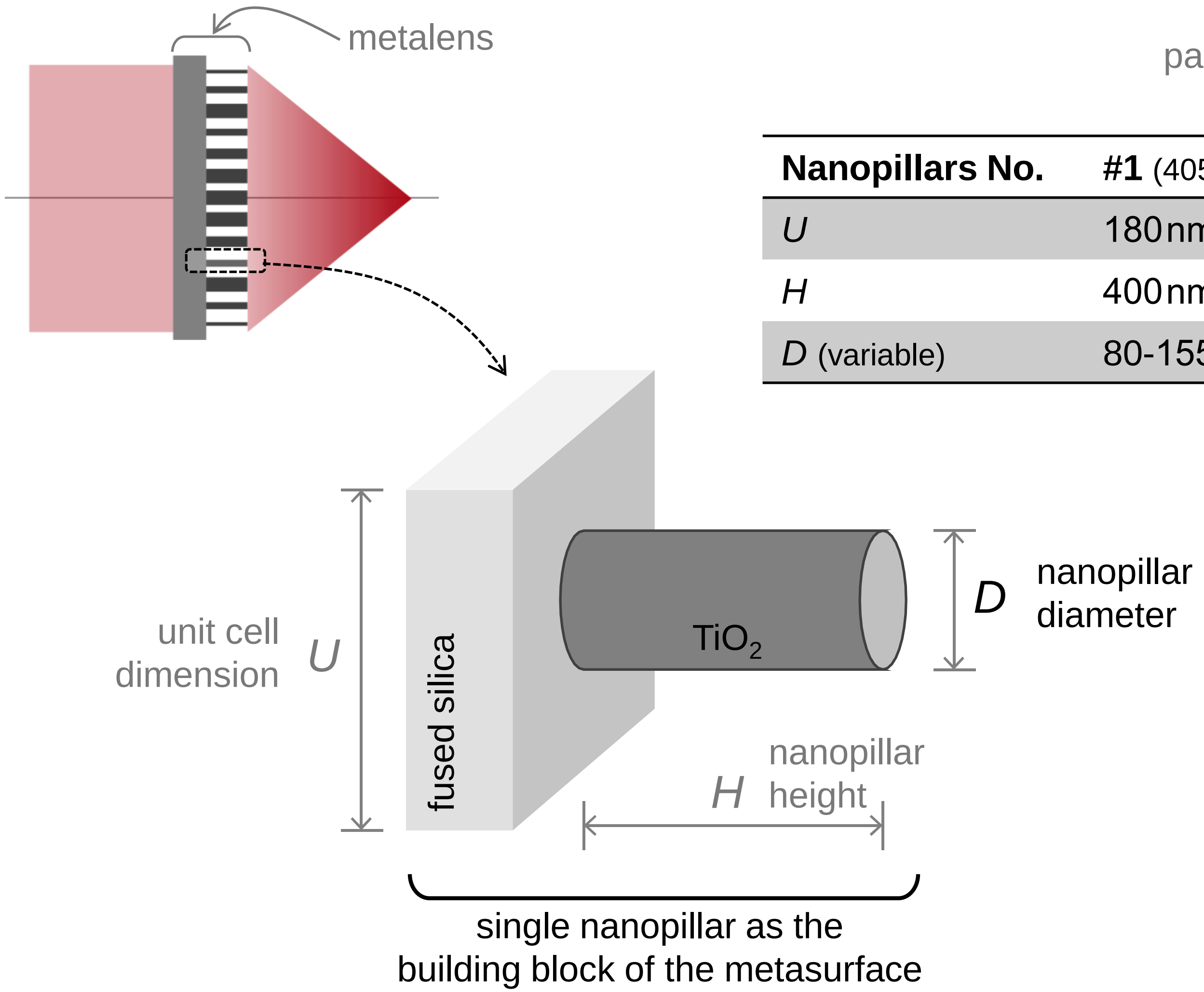
$$\begin{aligned}\boldsymbol{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) &= \left\{\mathbf{B}^{+}(\boldsymbol{\rho};\psi^{\text{in}})\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})\right\}\exp\left(\mathrm{i}\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})\right) \\ &+ \left\{\mathbf{B}^{-}(\boldsymbol{\rho};\psi^{\text{in}})\boldsymbol{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})\right\}\exp\left(\mathrm{i}\psi^{\text{in}}(\boldsymbol{\rho}) - \Delta\psi(\boldsymbol{\rho})\right)\end{aligned}$$



M. Khorasaninejad *et al.*, Science **352**, 1190-1194 (2016).

Rigorous Analysis of Nanopillar Metasurface Building Block

Modeling Task



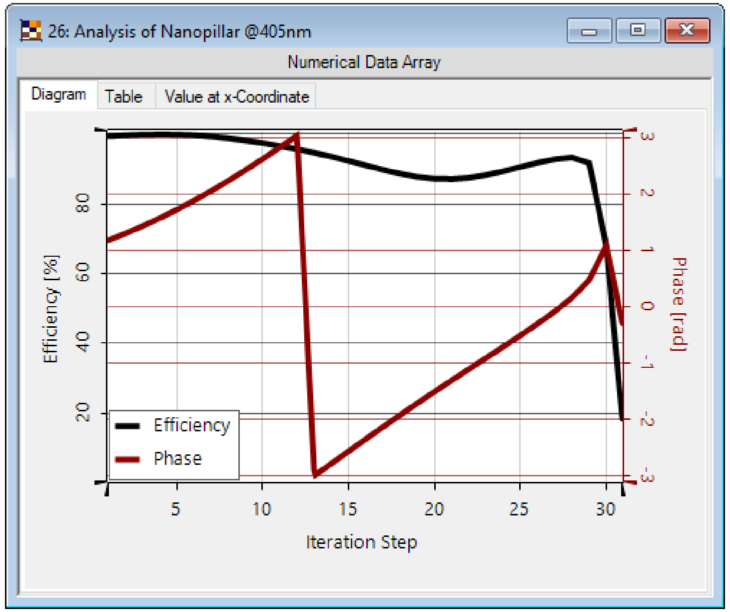
parameters from M. Khorasaninejad,
Nano Lett. 2016, 16, 7229-7234

Nanopillars No.	**#1** (405 nm)	**#2** (532 nm)	**#3** (660 nm)
U	180 nm	250 nm	350 nm
H	400 nm	600 nm	600 nm
D (variable)	80-155 nm	100-220 nm	100-320 nm

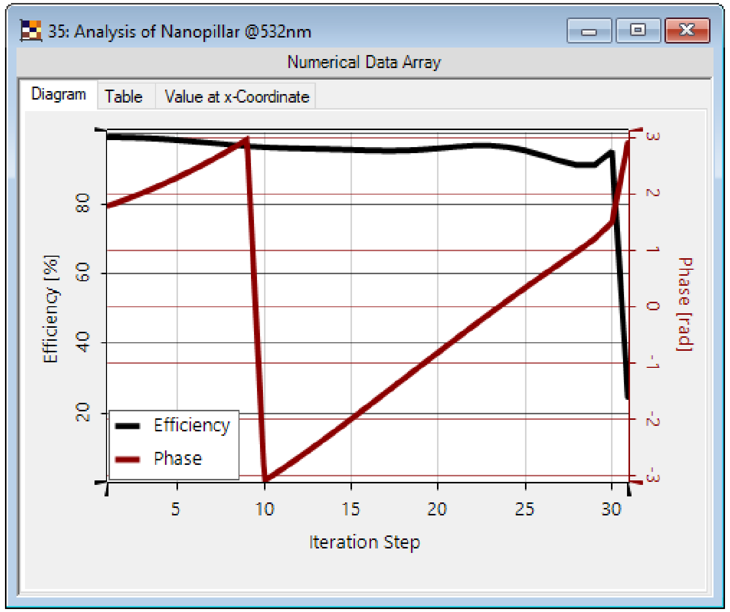
By varying the nanopillar diameter, the metasurface building block is supposed to have phase modulation covering 2π. How to evaluate such nanopillar structure rigorously?

Nanopillar Analysis vs. Pillar Diameter

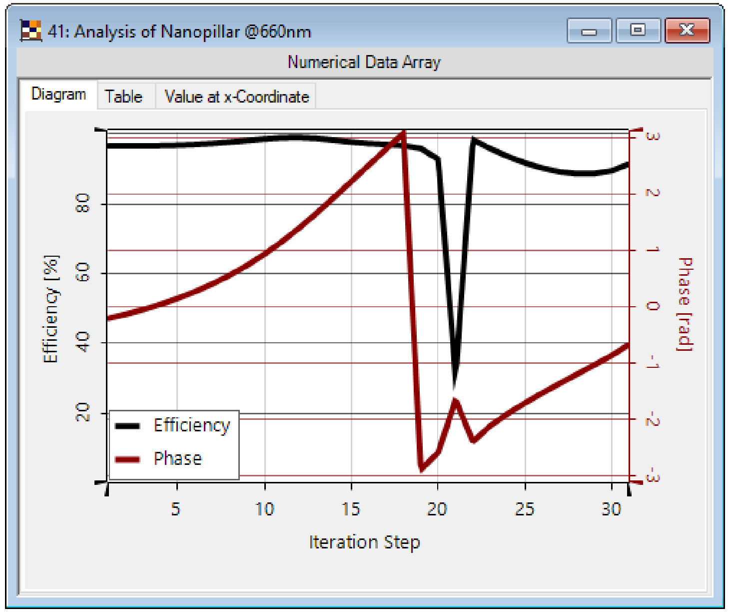
nanopillar #1



nanopillar #2



nanopillar #3



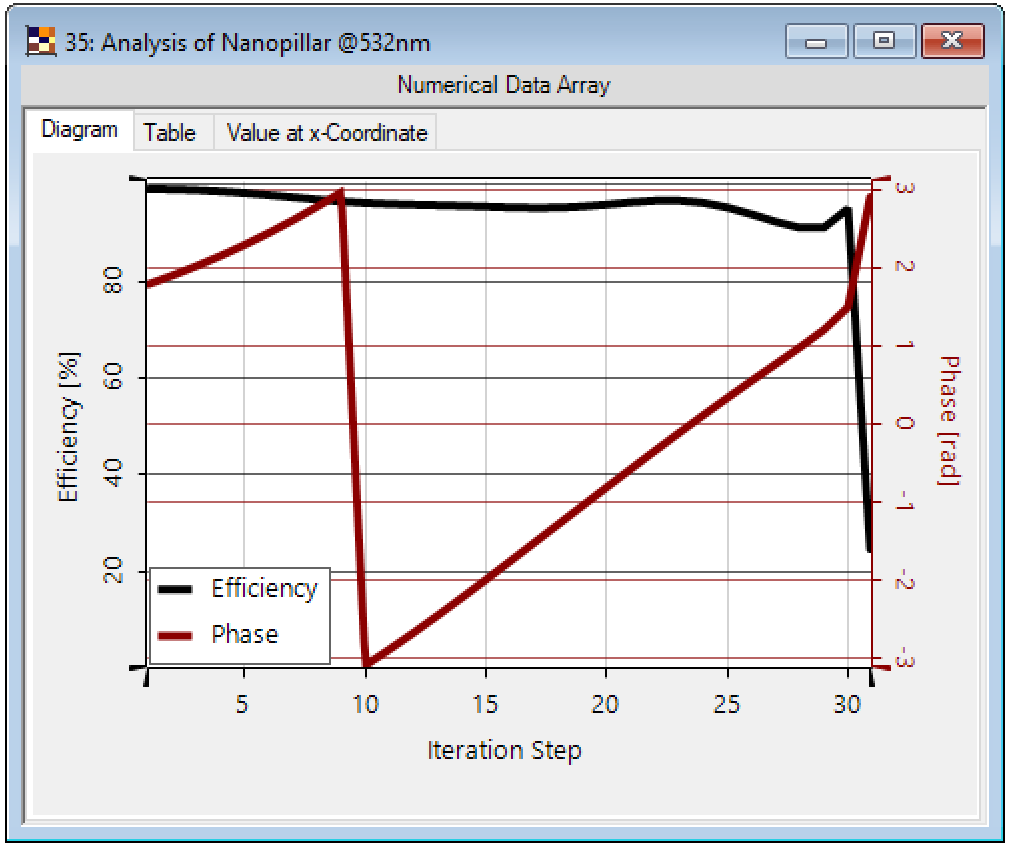
Nanopillars No.	**#1** (405 nm)	**#2** (532 nm)	**#3** (660 nm)
U	180 nm	250 nm	350 nm
H	400 nm	600 nm	600 nm
D (variable)	80-155 nm	100-220 nm	100-320 nm

Nanopillar Analysis vs. Pillar Diameter

- The phase modulation covers 2π range, and it changes almost linearly with pillar diameter, which enables convenient phase control.
- The transmission efficiency remains above 90% for varying pillar diameter over the design range.

Nanopillars No.	#1 (405nm)	#2 (532nm)	#3 (660nm)
U	180nm	250nm	350nm
H	400nm	600nm	600nm
D (variable)	80-155nm	100-220nm	100-320nm

nanopillar #2

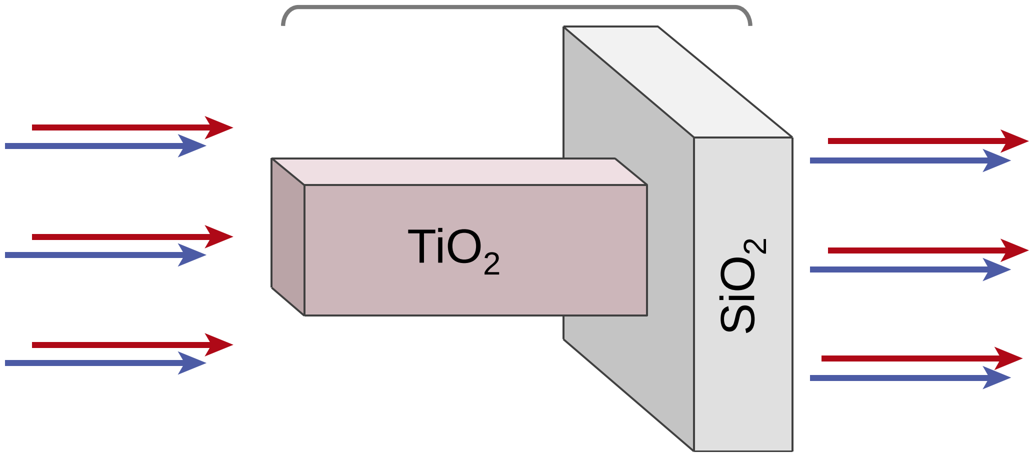


Form Birefringence Analysis

How to analyze the form birefringence for the metasurface building block – nanofin structure?

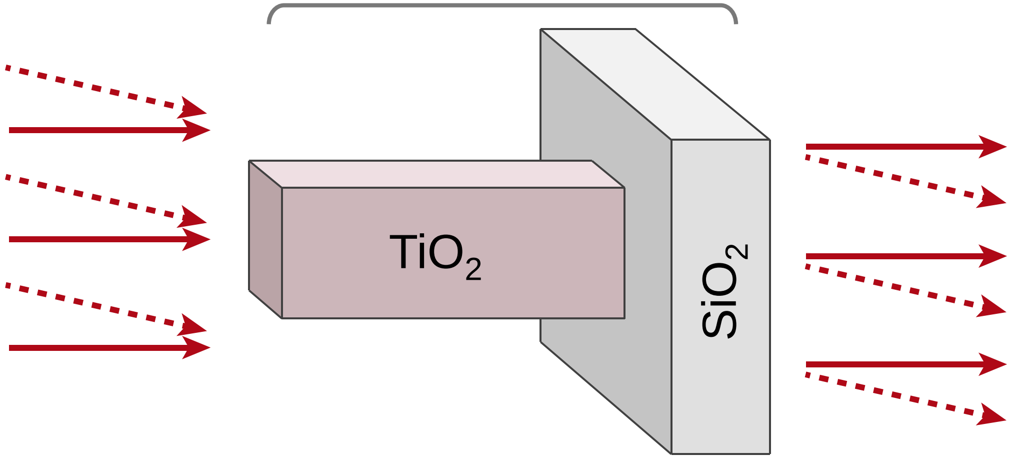
plane wave
- normal incidence
- wavelengths 405, 532, 660 nm

$$\begin{pmatrix} b_{xx}(\omega) & b_{xy}(\omega) \\ b_{yx}(\omega) & b_{yx}(\omega) \end{pmatrix} = ?$$

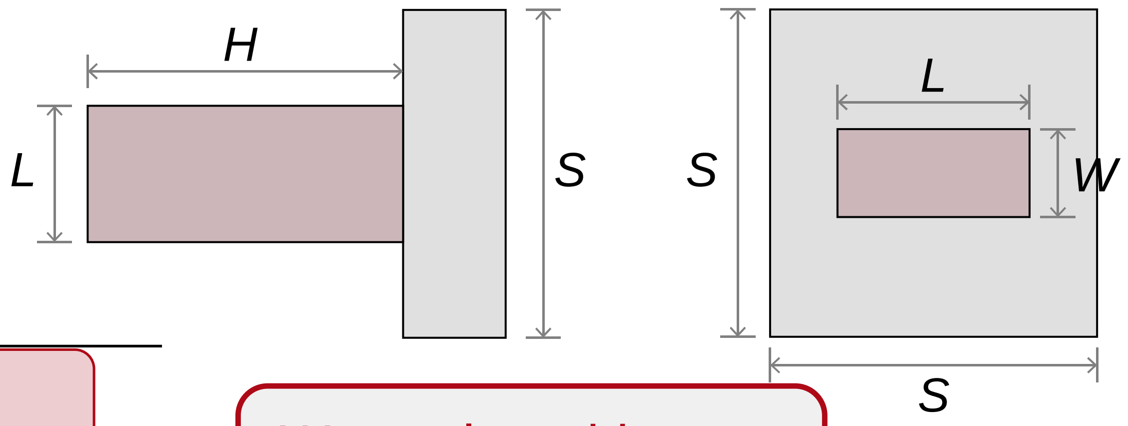


plane wave
- wavelength @405 nm
- incidence angle [-25°, 25°]

$$\begin{pmatrix} b_{xx}(\alpha) & b_{xy}(\alpha) \\ b_{yx}(\alpha) & b_{yx}(\alpha) \end{pmatrix} = ?$$



Nanofin Structural Designs



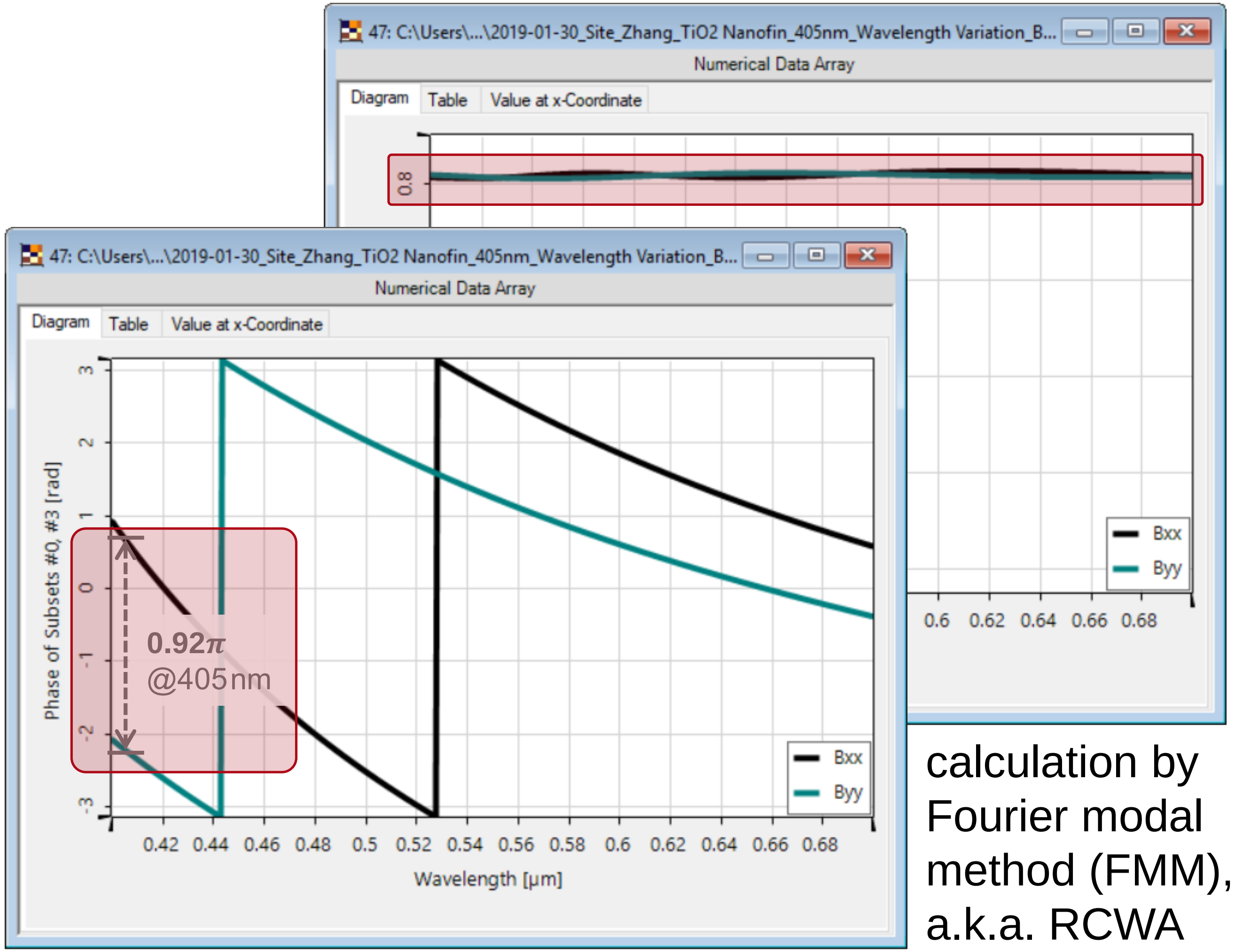
structural parameters

Case	660 nm Design	532 nm Design	405 nm Design
refractive index	n = 2.36	n = 2.43	n = 2.63
W	85 nm	95 nm	40 nm
L	410 nm	250 nm	150 nm
H	600 nm	600 nm	600 nm
S	430 nm	325 nm	200 nm

We analyze this case as an example.

M. Khorasaninejad, W. T. Chen, R. C. Devlin, J. Oh, A. Y. Zhu, F. Capasso, *Science* **2016**, 352 (6290), 1190-1194

Spectral Analysis for Nanofin – 405nm Design



calculation by Fourier modal method (FMM), a.k.a. RCWA

- B-matrix for metasurface building block

$$\begin{pmatrix} b_{xx} & b_{xy} \\ b_{yx} & b_{yy} \end{pmatrix}.$$

- Ideally, it should function as … i.e.

Almost half-wave plate effect

$$b_{yx} \approx 0,$$

$$\boxed{b_{xx} \approx -b_{yy}.}$$

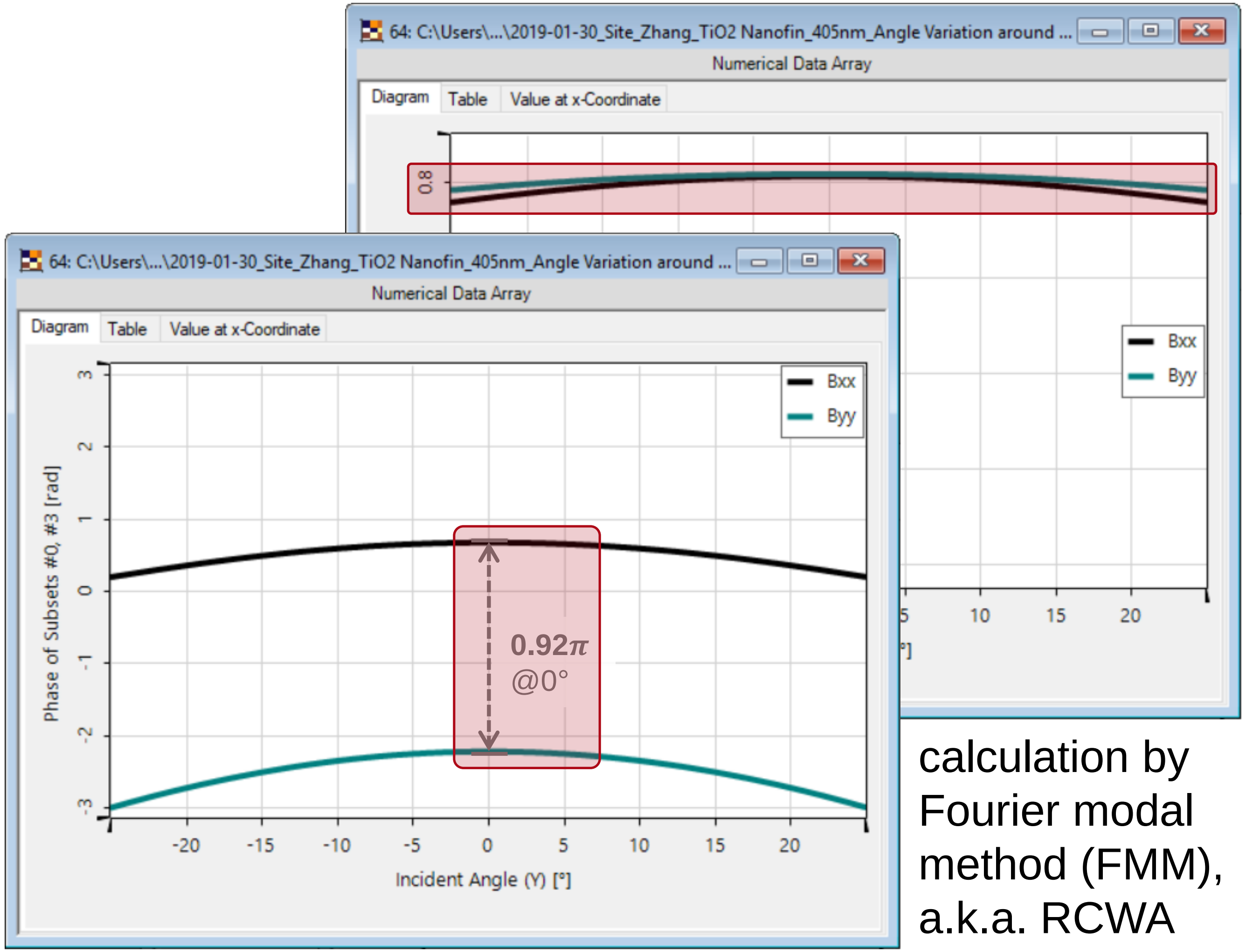
Angular Analysis for Nanofin – 405nm Design

- B-matrix for metasurface building block

$$\begin{pmatrix} b_{xx} & b_{xy} \\ b_{yx} & b_{yy} \end{pmatrix}.$$

- Ideally, it should function as a half-wave plate, i.e.

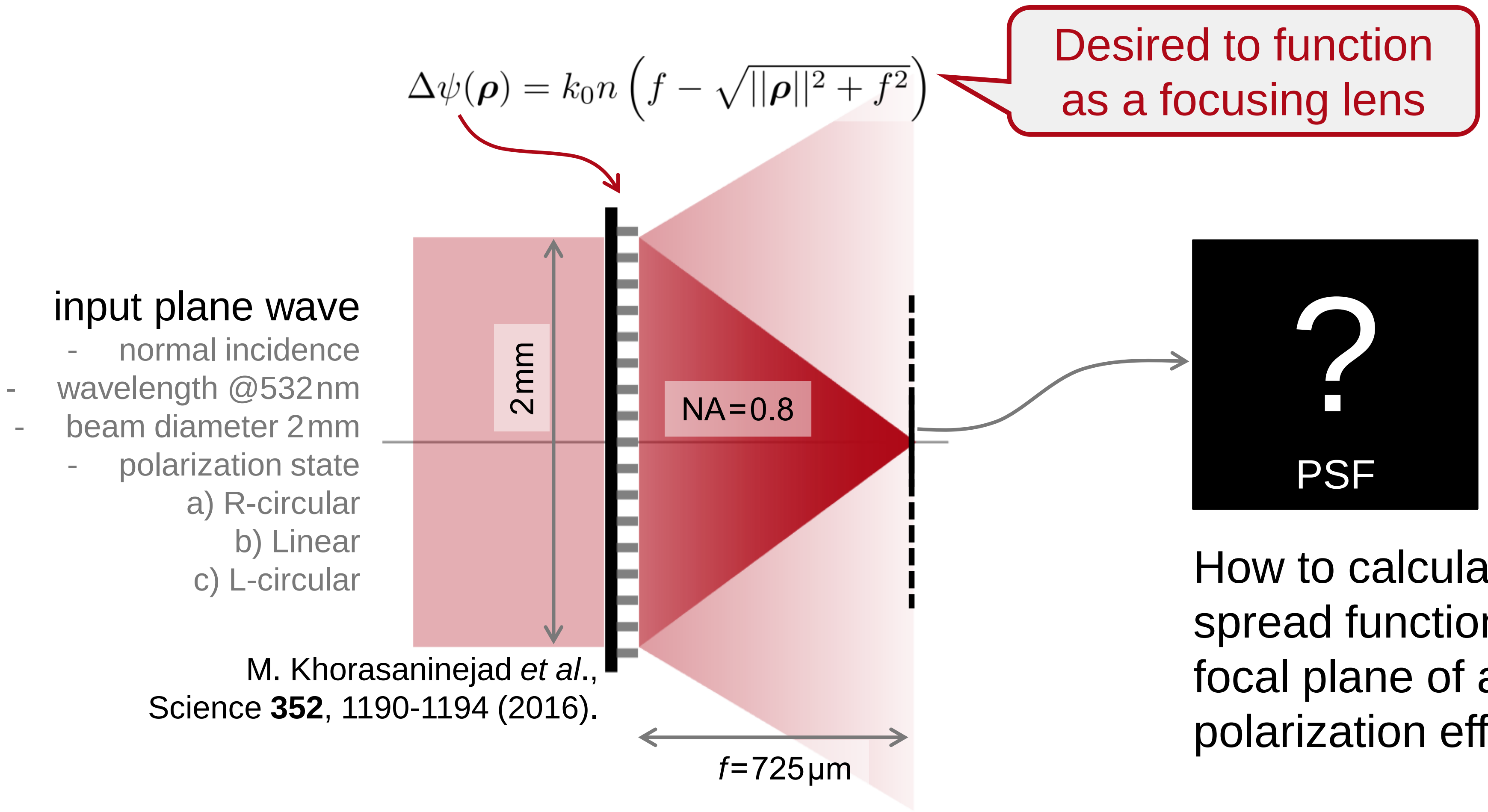
$$b_{xy} \approx 0\,,$$
$$b_{yx} \approx 0\,,$$
$$\boxed{b_{xx} \approx -b_{yy}\,.}$$



calculation by Fourier modal method (FMM), a.k.a. RCWA

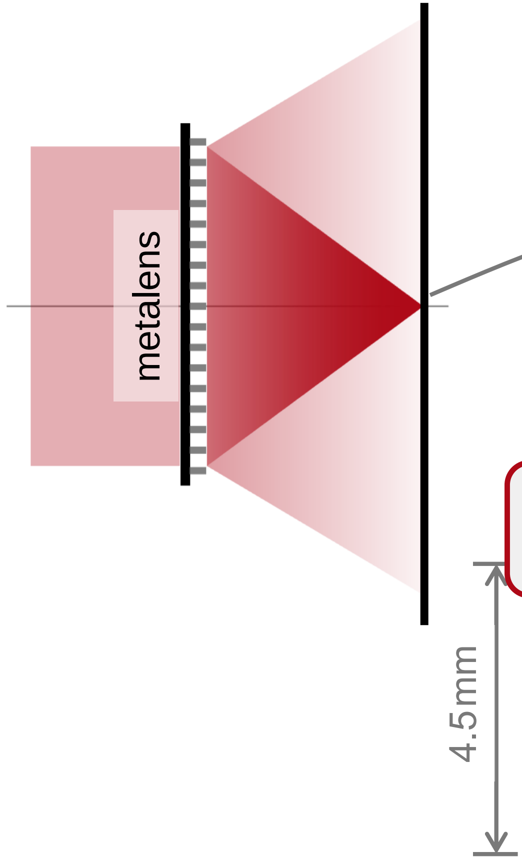
Modeling of A Polarization-Dependent Metalens

High-NA Metalens Simulation

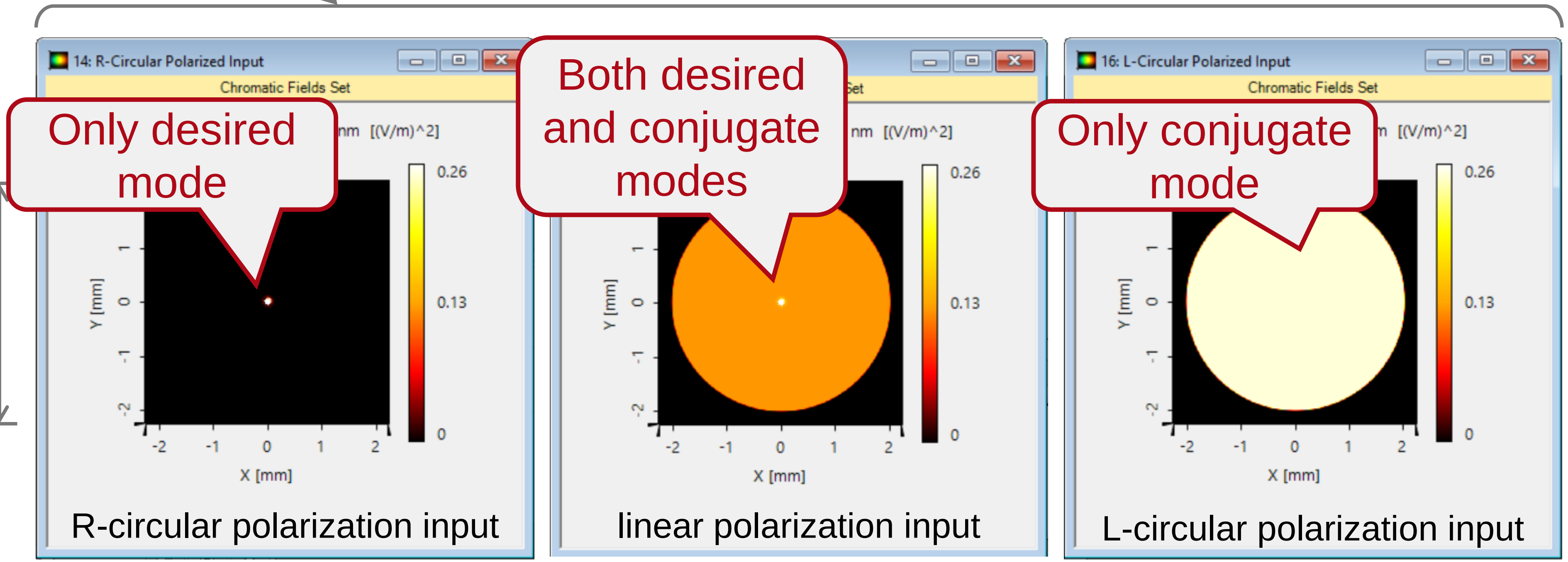


How to calculate the point spread function (PSF) at the focal plane of a metalens, with polarization effects considered?

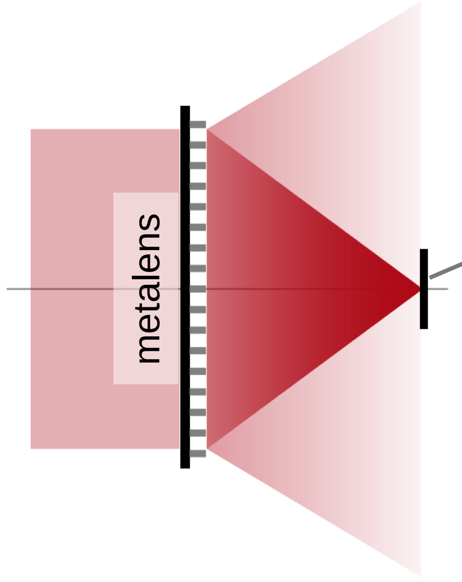
High-NA Metalens Simulation



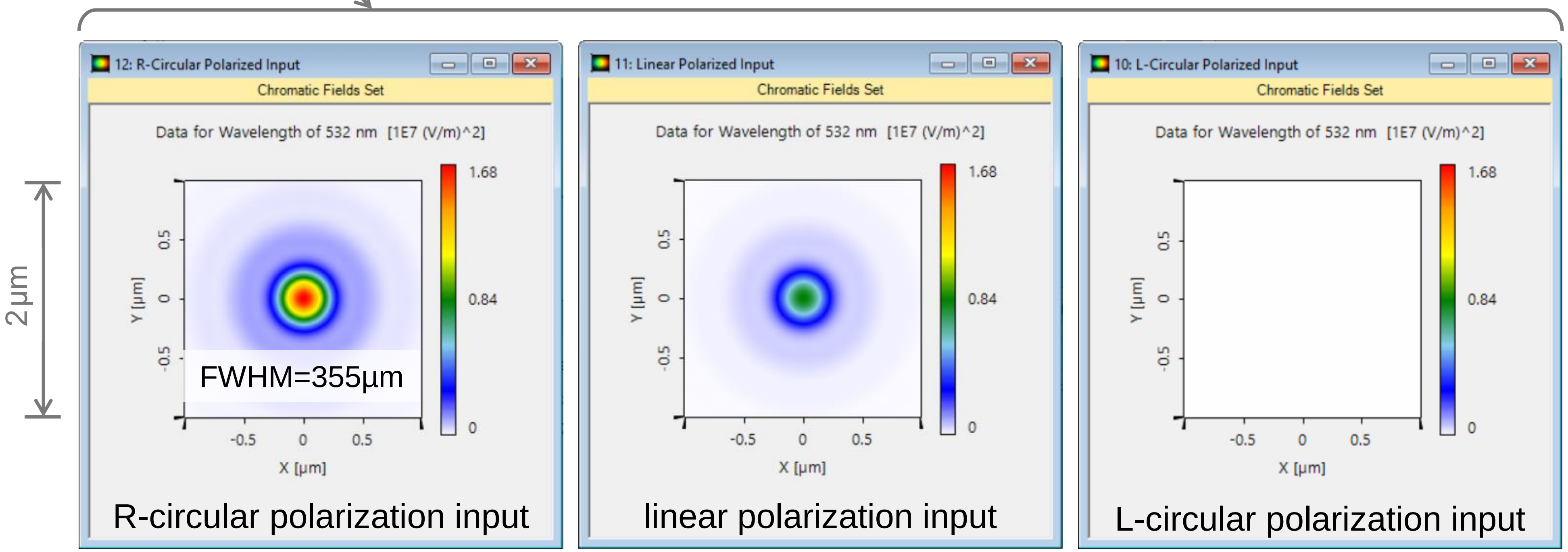
$$\boldsymbol{V}_{\perp}^{\mathrm{out}}(\boldsymbol{\rho}) = \left\{\mathbf{B}^{+}(\boldsymbol{\rho};\psi^{\mathrm{in}})\boldsymbol{U}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho})\right\}\exp\left(\mathrm{i}\psi^{\mathrm{in}}(\boldsymbol{\rho})+\Delta\psi(\boldsymbol{\rho})\right) + \left\{\mathbf{B}^{-}(\boldsymbol{\rho};\psi^{\mathrm{in}})\boldsymbol{U}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho})\right\}\exp\left(\mathrm{i}\psi^{\mathrm{in}}(\boldsymbol{\rho})-\Delta\psi(\boldsymbol{\rho})\right)$$



High-NA Metalens Simulation



$$\boldsymbol{V}_{\perp}^{\mathrm{out}}(\boldsymbol{\rho}) = \left\{\mathbf{B}^{+}(\boldsymbol{\rho};\psi^{\mathrm{in}})\boldsymbol{U}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho})\right\}\exp\!\left(\mathrm{i}\psi^{\mathrm{in}}(\boldsymbol{\rho})+\Delta\psi(\boldsymbol{\rho})\right) + \left\{\mathbf{B}^{-}(\boldsymbol{\rho};\psi^{\mathrm{in}})\boldsymbol{U}_{\perp}^{\mathrm{in}}(\boldsymbol{\rho})\right\}\exp\!\left(\mathrm{i}\psi^{\mathrm{in}}(\boldsymbol{\rho})-\Delta\psi(\boldsymbol{\rho})\right)$$



VirtualLab Fusion – Connecting Field Solvers

Booth #110