Modeling of Diffractive/Meta-Lenses using Fast Physical Optics

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Jena, Germany
LightTrans International

- Distribution of VirtualLab Fusion, together with distributors worldwide
- Technical support, seminars, and trainings
- Engineering projects
University of Jena

Applied Computational Optics Group R&D in optical modeling and design with emphasis on physical optics
Wyrowski Photonics

Development of fast physical optics software VirtualLab Fusion
Optical Design Software and Services

Booth #110
Emphasis of the common theoretical background for different types of microstructured layers
Unifying Approach – Wavefront Surface Response

- For plane surfaces we found $\nabla_\perp \psi^{\text{in}}(\rho) = \nabla_\perp \psi^{\text{out}}(\rho)$ and in conclusion

$$U^{\text{out}}_\perp(\rho) \exp(i\psi^{\text{in}}(\rho)) = \{B(\rho; \psi^{\text{in}})U^{\text{in}}_\perp(\rho)\} \exp(i\psi^{\text{in}}(\rho)).$$

- By introducing the wavefront surface response we assume an effect at the surface of the form

$$U^{\text{out}}_\perp(\rho) \exp(i\psi^{\text{out}}(\rho)) = \{B(\rho; \psi^{\text{in}})U^{\text{in}}_\perp(\rho)\} \exp(i\psi^{\text{out}}(\rho))$$

with

$$\psi^{\text{out}}(\rho) = \psi^{\text{in}}(\rho) + \Delta \psi(\rho)$$

and thus

$$\nabla_\perp \psi^{\text{out}}(\rho) = \nabla_\perp \psi^{\text{in}}(\rho) + \nabla_\perp (\Delta \psi(\rho)).$$
How to Realize Wavefront Surface Response (WSR)?

- From a physical-optics point of view the question arises, if there exists any manipulation of the structure of the surface, which provides an effect of the form:

\[ U_{\perp}^{\text{out}}(\rho) \exp(i\psi_{\text{in}}(\rho) + \Delta \psi(\rho)) = \left\{ B(\rho; \psi_{\text{in}})U_{\perp}^{\text{in}}(\rho) \right\} \exp(i\psi_{\text{in}}(\rho)) \]

- A detailed answer can only be given for a specific surface structure.

- By introducing microstructured layers onto the surface a wavefront surface response can be implemented:
  - Graded-index layer
  - Volume hologram layer
  - Diffractive layer
  - Metamaterial layer

\[ V_{\perp}^{\text{in}}(\rho) = U_{\perp}^{\text{in}}(\rho) \exp(i\psi_{\text{in}}(\rho)) \]

with \( \rho := (x, y) \)
Diffractive layer

… with application to lenses
Physical Optics Modeling: Diffractive Layer

• In general a wavefront surface response $\Delta \psi(\rho)$ leads to the equation $\nabla_\perp \psi^{\text{out}}(\rho) = \nabla_\perp \psi^{\text{in}}(\rho) + \nabla_\perp (\Delta \psi(\rho))$ and because of the local plane wave assumption (homeomorphic zone) into

$$\kappa^{\text{out}}(\rho) = \kappa^{\text{in}}(\rho) + K(\rho)$$

with $K(\rho) \overset{\text{def}}{=} \nabla_\perp (\Delta \psi(\rho))$.

• This equation is directly related to a locally formulated grating equation

$$\kappa^{\text{out}}(\rho) = \kappa^{\text{in}}(\rho) + m \left( \frac{2\pi}{d_x(\rho)}, \frac{2\pi}{d_y(\rho)} \right)$$

with the local grating period $d(\rho) = (d_x(\rho), d_y(\rho))$.

• That leads to the basic principle of a diffractive layer via:

$$d(\rho) = 2\pi \left( \left( \frac{\partial \psi(\rho)}{\partial x} \right)^{-1}, \left( \frac{\partial \psi(\rho)}{\partial y} \right)^{-1} \right)$$
Wavefront Surface Response of Focusing Lens

• In order to transform a plane incident field into a spherical con-
  vernegaone the wavefront surface response should be:

\[
\Delta \psi(\rho) = k_0 n \left( f - \sqrt{\|\rho\|^2 + f^2} \right)
\]
Structure Design

- Wrap the WSR: $(\Delta \psi(\rho))^{\text{DOE}} = \mod_{p2\pi} \left\{ k_0 n \left( f - \sqrt{\|\rho\|^2 + f^2} \right) \right\}$ with $p \in \mathbb{N}$.
- For $p = 1$ local radial period follows with $d(\rho) = 2\pi / \Delta \psi'(\rho)$.
- Structure design by inverse Thin Element Approximation (TEA): The height profile $h^{\text{DOE}}$ is given by:

$$h^{\text{DOE}}(\rho) = \frac{\lambda}{2\pi \Delta n} \Delta \psi(\rho)^{\text{DOE}}$$
Focusing Diffractive Lens with NA=0.2: Ray and Field Tracing

5 mm

Data for Wavelength of 532 nm [1E6 (V/m)^2]
Focusing Diffractive Lens with NA=0.2: Ray and Field Tracing

Amplitude

Phase

Ex
Effects due to Rayleigh matrix:
\[ B(\rho; \psi^\text{in}) U^\text{in}_\perp(\rho) \]
Focusing Diffractive Lens with NA=0.2: Ray and Field Tracing

- Amplitudes in Focus (Same scaling!)
Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing
Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing

Amplitude

Phase
Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing

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Effects due to Rayleigh matrix:
\[ B(\rho; \psi^{in}) U^{in}(\rho) \]
Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing

- Amplitudes in Focus (Same scaling!)
Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing

- Amplitudes in Focus (Same scaling!)

Vectorial grating effects reduce spot quality for high NA
Combination of OpticStudio® and VirtualLab Fusion

Complementary workflows
Color Aberration Correction by DOE

- **plane wave**
  - wavelength (486, 587, 656) nm
  - field of view along y-axis (0; 20)°
  - linearly polarized along x-axis
  - aperture 5 mm × 5 mm

- **detectors**
  - point spread function
  - modulation transfer function (MTF)

- **lens solution**
  - wavefront response function
  - diffractive lens
  - meta lens

Optimization of Binary Surface = Wavefront Surface Response in OpticStudio®
Inclusion of Higher Orders: On-Axis

- Electric Energy Density $[1 \times 10^6 (\text{V/m})^2]$
- Electric Energy Density $[1 \times 10^3 (\text{V/m})^2]$
- Electric Energy Density $[(\text{V/m})^2]$

simulation time per order $\sim$ seconds

+1$^{\text{st}}$ diffraction order
0$^{\text{th}}$ diffraction order
-1$^{\text{st}}$ diffraction order
Results: MTF for Various Diffractive Lens Structures

MTF w/o DOE

MTF w/ DOE: Continuous, 8, and 4 levels

MTF w/ DOE: Binary
Metasurfaces

Realization of wavefront surface responses by nanostructured layers
Physical Effects for Realizing Metasurfaces

- Propagation phase delay
  - Centrosymmetric (polarization insensitive)
  - Rotationally asymmetric (form birefringence)

- Resonance phase delay
  - Y. F. Yu et al., Laser Photonics Rev. 9, 412-418 (2015).
Physical Optics Modeling: Metasurface Layer

• In general a real surface layer structure does not just realize the desired wavefront response but additional effects and fields:

\[
V_{\perp}^{\text{out}}(\rho) = \{B(\rho; \psi^{\text{in}})U_{\perp}^{\text{in}}(\rho)\} \exp(i\psi^{\text{in}}(\rho) + \Delta\psi(\rho)) + V_{\perp}^{\text{res}}(\rho)
\]

• For nanofin-based metalayers the typical result can be written as:

\[
V_{\perp}^{\text{out}}(\rho) = \{B^{+}(\rho; \psi^{\text{in}})U_{\perp}^{\text{in}}(\rho)\} \exp(i\psi^{\text{in}}(\rho) + \Delta\psi(\rho)) \\
+ \{B^{-}(\rho; \psi^{\text{in}})U_{\perp}^{\text{in}}(\rho)\} \exp(i\psi^{\text{in}}(\rho) - \Delta\psi(\rho))
\]

Physical Optics Modeling: Metasurface Layer

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\[ + \{ B^-(\rho; \psi_{\text{in}}) U_{\text{in}}^{\perp}(\rho) \} \exp(i \psi_{\text{in}}(\rho) - \Delta \psi(\rho)) \]

Calculation by Fourier modal method (FMM), a.k.a. RCWA

Rigorous Analysis of Nanopillar Metasurface Building Block
Modeling Task

By varying the nanopillar diameter, the metasurface building block is supposed to have phase modulation covering $2\pi$. How to evaluate such nanopillar structure rigorously?

Parameters from M. Khorasaninejad, *Nano Lett.* 2016, 16, 7229-7234

<table>
<thead>
<tr>
<th>Nanopillars No.</th>
<th>#1 (405 nm)</th>
<th>#2 (532 nm)</th>
<th>#3 (660 nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td>180 nm</td>
<td>250 nm</td>
<td>350 nm</td>
</tr>
<tr>
<td>$H$</td>
<td>400 nm</td>
<td>600 nm</td>
<td>600 nm</td>
</tr>
<tr>
<td>$D$ (variable)</td>
<td>80-155 nm</td>
<td>100-220 nm</td>
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Nanopillar Analysis vs. Pillar Diameter

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Nanopillar Analysis vs. Pillar Diameter

- The phase modulation covers $2\pi$ range, and it changes almost linearly with pillar diameter, which enables convenient phase control.
- The transmission efficiency remains above 90% for varying pillar diameter over the design range.

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Form Birefringence Analysis

plane wave
- normal incidence
- wavelengths 405, 532, 660 nm

plane wave
- wavelength @405nm
- incidence angle [-25°, 25°]

How to analyze the form birefringence for the metasurface building block – nanofin structure?
Nanofin Structural Designs

<table>
<thead>
<tr>
<th>Case</th>
<th>660 nm Design</th>
<th>532 nm Design</th>
<th>405 nm Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>refractive index</td>
<td>$n=2.36$</td>
<td>$n=2.43$</td>
<td>$n=2.63$</td>
</tr>
<tr>
<td>$W$</td>
<td>85 nm</td>
<td>95 nm</td>
<td>40 nm</td>
</tr>
<tr>
<td>$L$</td>
<td>410 nm</td>
<td>250 nm</td>
<td>150 nm</td>
</tr>
<tr>
<td>$H$</td>
<td>600 nm</td>
<td>600 nm</td>
<td>600 nm</td>
</tr>
<tr>
<td>$S$</td>
<td>430 nm</td>
<td>325 nm</td>
<td>200 nm</td>
</tr>
</tbody>
</table>

We analyze this case as an example.

• B-matrix for metasurface building block

\[
\begin{pmatrix}
  b_{xx} & b_{xy} \\
  b_{yx} & b_{yy}
\end{pmatrix}
\]

• Ideally, it should function as an almost half-wave plate effect, with:

\[
\begin{align*}
  b_{yx} & = 0, \\
  b_{xx} & \approx -b_{yy}.
\end{align*}
\]
Angular Analysis for Nanofin – 405nm Design

• B-matrix for metasurface building block

\[
\begin{pmatrix}
b_{xx} & b_{xy} \\
b_{yx} & b_{yy}
\end{pmatrix}
\]

• Ideally, it should function as a half-wave plate, i.e.

\[
\begin{align*}
b_{xy} & \approx 0, \\
b_{yx} & \approx 0, \\
b_{xx} & \approx -b_{yy}.
\end{align*}
\]

calculation by Fourier modal method (FMM), a.k.a. RCWA
Modeling of A Polarization-Dependent Metalens
High-NA Metalens Simulation

Desired to function as a focusing lens

How to calculate the point spread function (PSF) at the focal plane of a metalens, with polarization effects considered?

\[ \Delta \psi(\rho) = k_0 n \left( f - \sqrt{|\rho|^2 + f^2} \right) \]

- input plane wave
  - normal incidence
  - wavelength @532nm
  - beam diameter 2mm
- polarization state
  a) R-circular
  b) Linear
  c) L-circular


f=725µm
High-NA Metalens Simulation

\[ V_{\perp}^{\text{out}}(\rho) = \{ B^{+}(\rho; \psi^{\text{in}}) U_{\perp}^{\text{in}}(\rho) \} \exp(i\psi^{\text{in}}(\rho) + \Delta\psi(\rho)) + \{ B^{-}(\rho; \psi^{\text{in}}) U_{\perp}^{\text{in}}(\rho) \} \exp(i\psi^{\text{in}}(\rho) - \Delta\psi(\rho)) \]

- **Only desired mode**
- **Both desired and conjugate modes**
- **Only conjugate mode**

R-circular polarization input

linear polarization input

L-circular polarization input
High-NA Metalens Simulation

\[ V_{\text{out}}^{\perp}(\rho) = \{ B^+(\rho; \psi^\text{in}) U_{\perp}^{\text{in}}(\rho) \} \exp(i\psi^\text{in}(\rho) + \Delta\psi(\rho)) + \{ B^-(\rho; \psi^\text{in}) U_{\perp}^{\text{in}}(\rho) \} \exp(i\psi^\text{in}(\rho) - \Delta\psi(\rho)) \]

FWHM=355µm

R-circular polarization input

linear polarization input

L-circular polarization input
VirtualLab Fusion – Connecting Field Solvers

Booth #110

- connecting field solvers
- free space prisms, plates, cubes, ...
- lenses & freeforms
- apertures & boundaries
- gratings
- diffractive, Fresnel, meta lenses
- HOE, CGH, DOE
- micro lens & freeform arrays
- SLM & adaptive components
- diffractive beam splitters
- diffusers
- scatterer
- waveguides & fibers
- nonlinear components
- crystals & anisotropic components
- free space