

Photonics West 2019: VirtualLab Fusion Workshop in cooperation with Zemax

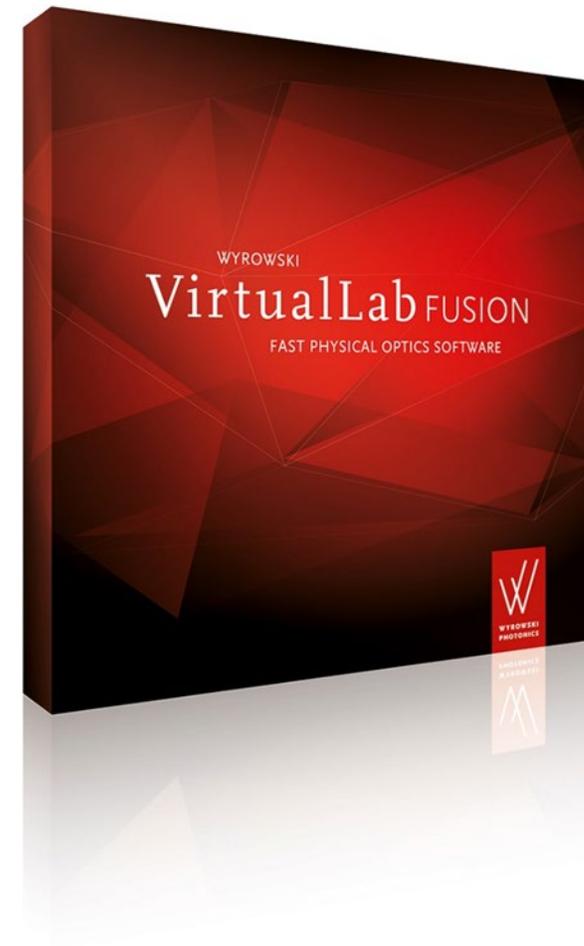
Modeling and Design of Diffractive- and Meta-lenses with VirtualLab Fusion

Frank Wyrowski, Akil Bhagat, Roberto Knoth, Site Zhang

VirtualLab Meta- and Diffractive Surface Solutions

- We prepare a new VirtualLab product for design and modeling of meta- and diffractive surfaces to be released in 2019.
- It will be based on the theory presented in this talk. The examples shown in this talk will be included in the product as special Use Cases together with suitable workflows.

This workshop gives a product preview by presenting its theoretical concepts, the working principles, and application examples.



Structure of Workshop

- Introduction of theory
 - Frank Wyrowski
- Design of binary surfaces in OpticStudio®
 - Akil Bhagat
- Physical-optics analysis of imported systems from OpticStudio: Diffractive lenses
 - Roberto Knoth
- Metalenses theory and modeling
 - Site Zhang
- Fabrication export
 - Roberto Knoth

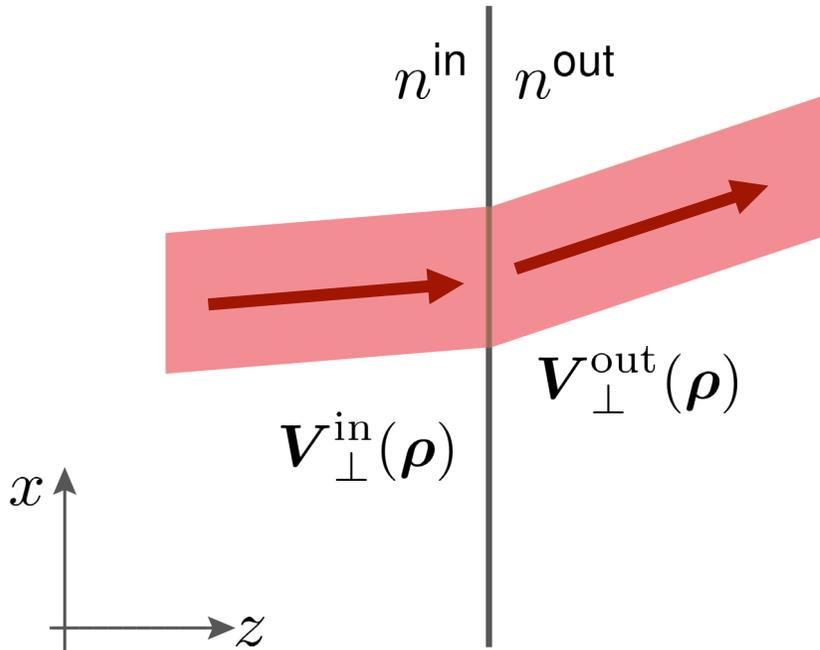
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Wavefront Surface Response Function

Unifying approach for dealing with different structure concepts in optical modeling and design

Plane Wave Interaction with Plane Surface



- For a field in a plane we use the notation $\boldsymbol{\rho} = (x, y)$ and $\mathbf{V}_{\perp}(\boldsymbol{\rho}) = (E_x(\boldsymbol{\rho}), E_y(\boldsymbol{\rho}))$.

- A plane input field is given by

$$\mathbf{V}_{\perp}^{\text{in}}(\boldsymbol{\rho}) = \mathbf{U}_{\perp}^{\text{in}} \exp(i\boldsymbol{\kappa}^{\text{in}} \boldsymbol{\rho})$$

with $\boldsymbol{\kappa} = (k_x, k_y)$.

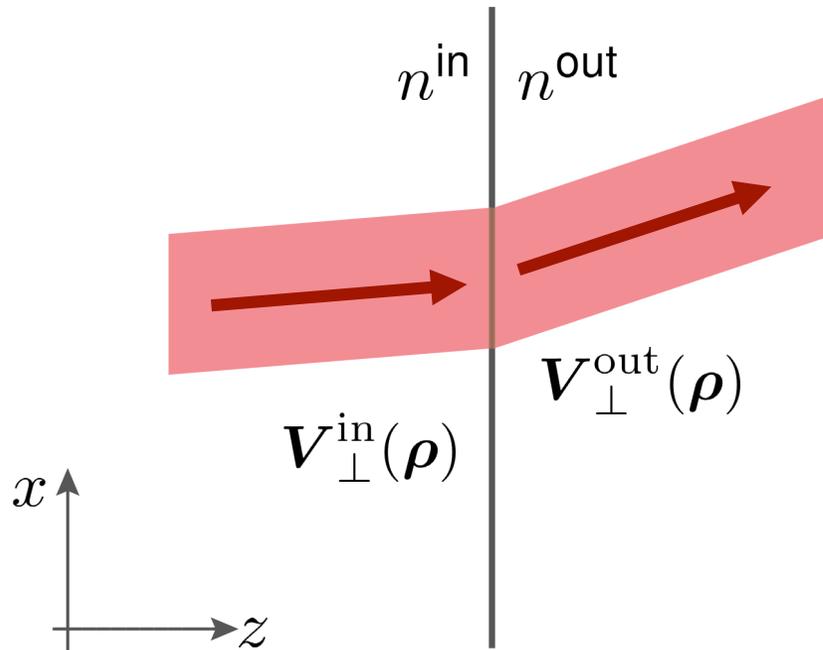
- The transmitted plane output field is given by

$$\mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = (\mathbf{B}(\boldsymbol{\kappa}^{\text{in}}) \mathbf{U}_{\perp}^{\text{in}}) \exp(i\boldsymbol{\kappa}^{\text{out}} \boldsymbol{\rho})$$

with the Fresnel effect at the surface is expressed by the 2×2 matrix $\mathbf{B}(\boldsymbol{\kappa}^{\text{in}})$.

- From the boundary conditions follows: $\boldsymbol{\kappa}^{\text{out}} \stackrel{!}{=} \boldsymbol{\kappa}^{\text{in}}$

Plane Wave Interaction with Plane Surface

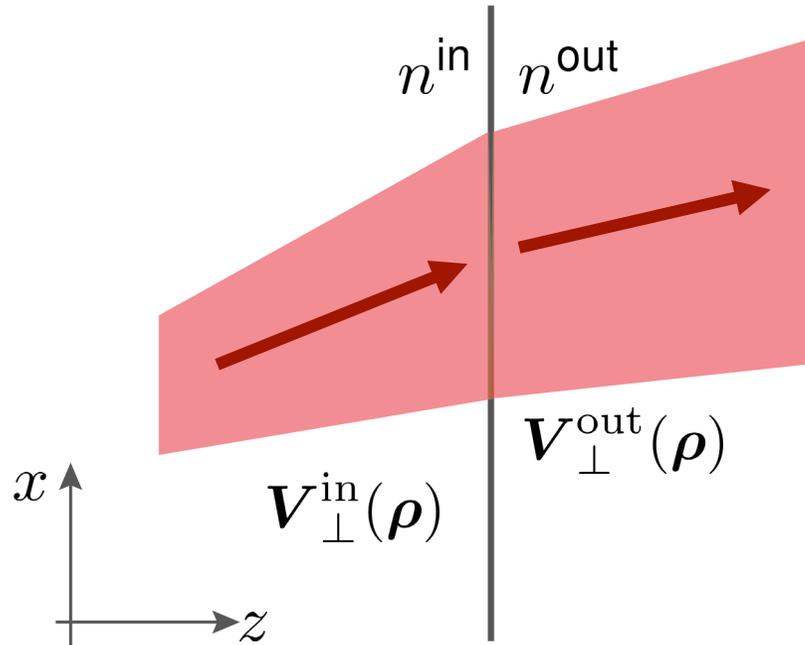


- From the boundary conditions follows: $\boldsymbol{\kappa}^{\text{out}} \stackrel{!}{=} \boldsymbol{\kappa}^{\text{in}}$
- With $\boldsymbol{\kappa} = k_0 n \hat{\boldsymbol{s}}_{\perp} = k_0 n (\sin \theta \cos \phi, \sin \theta \cos \phi)$ and $\phi = 0$ the law of refraction follows: $n^{\text{in}} \sin \theta^{\text{in}} = n^{\text{out}} \sin \theta^{\text{out}}$.
- For plane wave fields the wavefront phase is given by $\psi(\boldsymbol{\rho}) = \boldsymbol{\kappa} \cdot \boldsymbol{\rho}$.
- With $\boldsymbol{\kappa}^{\text{out}} \stackrel{!}{=} \boldsymbol{\kappa}^{\text{in}}$ we conclude $\psi^{\text{out}}(\boldsymbol{\rho}) = \psi^{\text{in}}(\boldsymbol{\rho})$.
- With $\nabla_{\perp} = \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right\}$ we find for plane wave fields:

$$\boldsymbol{\kappa} = \nabla_{\perp} \psi(\boldsymbol{\rho})$$

- Then $\boldsymbol{\kappa}^{\text{out}} \stackrel{!}{=} \boldsymbol{\kappa}^{\text{in}}$, that is the law of refraction, can be expressed by $\nabla_{\perp} \psi^{\text{in}}(\boldsymbol{\rho}) = \nabla_{\perp} \psi^{\text{out}}(\boldsymbol{\rho})$.

General Field Interaction with Plane Surface



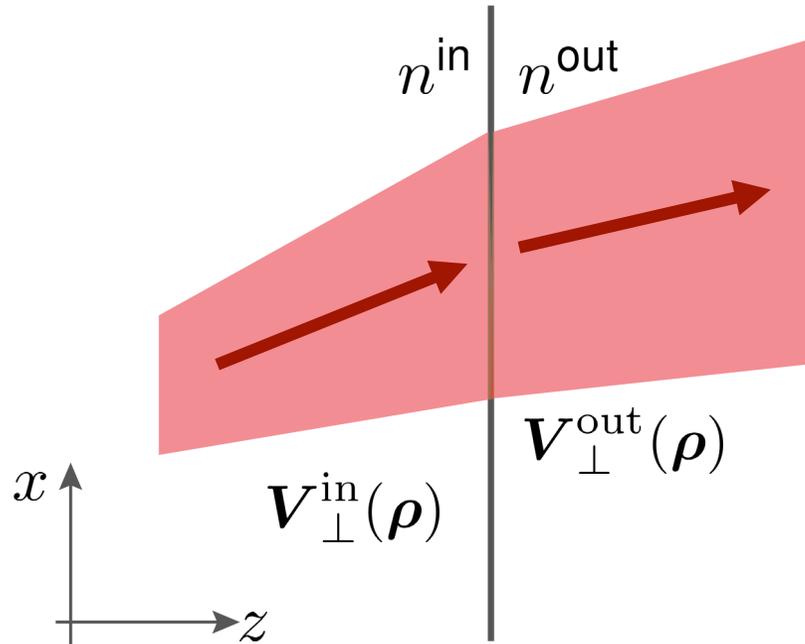
- Consider a general input field $\mathbf{V}_{\perp}^{\text{in}}(\boldsymbol{\rho}) = \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \exp(i\psi^{\text{in}}(\boldsymbol{\rho}))$.
- In general the effect of the plane surface on the input field can be expressed by the operator equation $\mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \mathbf{B}\mathbf{V}_{\perp}^{\text{in}}(\boldsymbol{\rho})$.
- Next we assume that we are allowed to apply the plane wave results locally. In physical optics that means the fields are in its homeomorphic field zone (HFZ).
- In the homeomorphic field zone we can express the effect of the plane surface on the input field locally:

$$\mathbf{U}_{\perp}^{\text{out}}(\boldsymbol{\rho}) \exp(i\psi^{\text{out}}(\boldsymbol{\rho})) = \mathbf{B}(\boldsymbol{\rho}) \{ \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \exp(i\psi^{\text{in}}(\boldsymbol{\rho})) \}$$

- The local use of the law of refraction can be expressed by:

$$\nabla_{\perp} \psi^{\text{in}}(\boldsymbol{\rho}) = \nabla_{\perp} \psi^{\text{out}}(\boldsymbol{\rho})$$

General Field Interaction with Plane Surface



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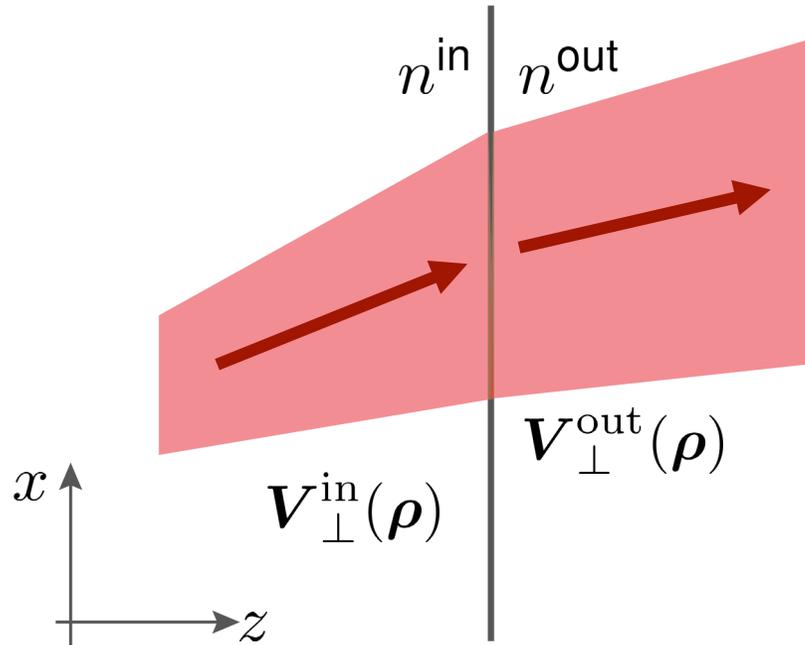
- The local use of the law of refraction is expressed by:

$$\nabla_{\perp} \psi^{\text{in}}(\boldsymbol{\rho}) = \nabla_{\perp} \psi^{\text{out}}(\boldsymbol{\rho})$$

- In conclusion the effect on the phase is described by the local Fresnel effects:

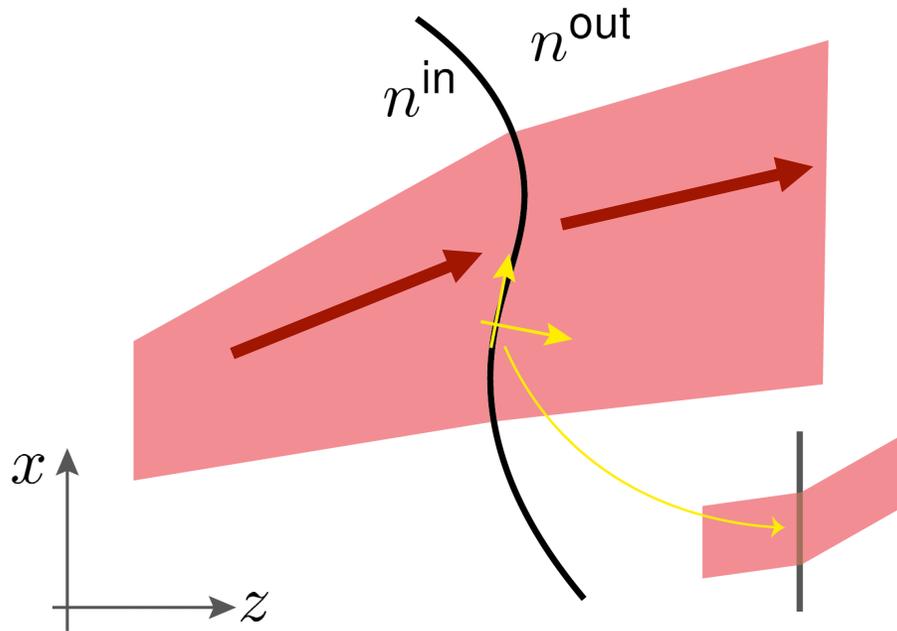
$$U_{\perp}^{\text{out}}(\boldsymbol{\rho}) \exp(i\psi^{\text{in}}(\boldsymbol{\rho})) = \{\mathbf{B}(\boldsymbol{\rho}; \psi^{\text{in}}) U_{\perp}^{\text{in}}(\boldsymbol{\rho})\} \exp(i\psi^{\text{in}}(\boldsymbol{\rho}))$$

General Field Interaction with Plane Surface



- Intermediate result: Plane surfaces do not change the wavefront phase of the incident field.

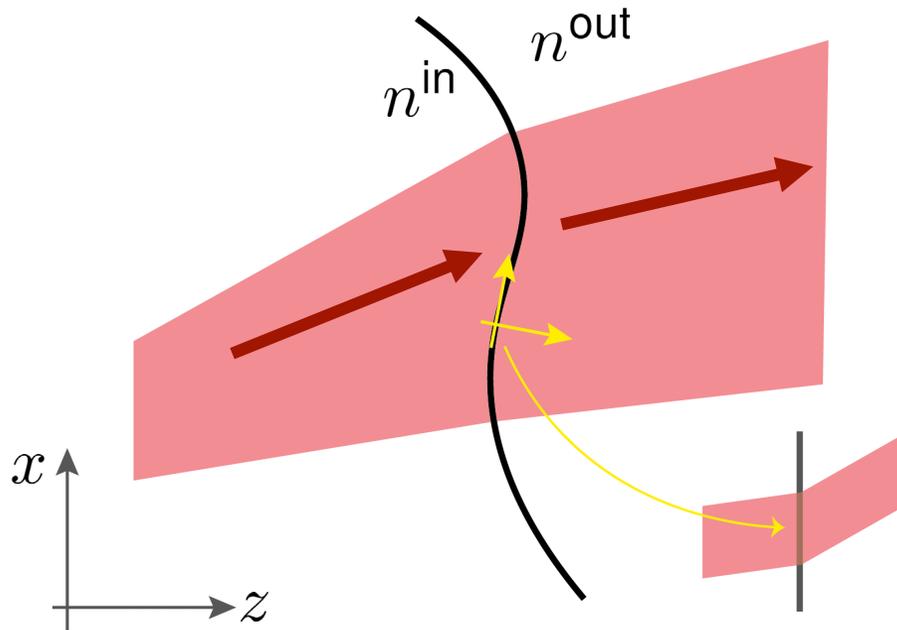
General Field Interaction with Curved Surface



- Intermediate result: Plane surfaces do not change the wavefront phase of the incident field.
- For curved surfaces the modeling can be still applied locally. Then the wavefront phase can be shaped by the shape of the surface:

$$\psi^{\text{out}}(\boldsymbol{\rho}) \neq \psi^{\text{in}}(\boldsymbol{\rho})$$

General Field Interaction with Curved Surface



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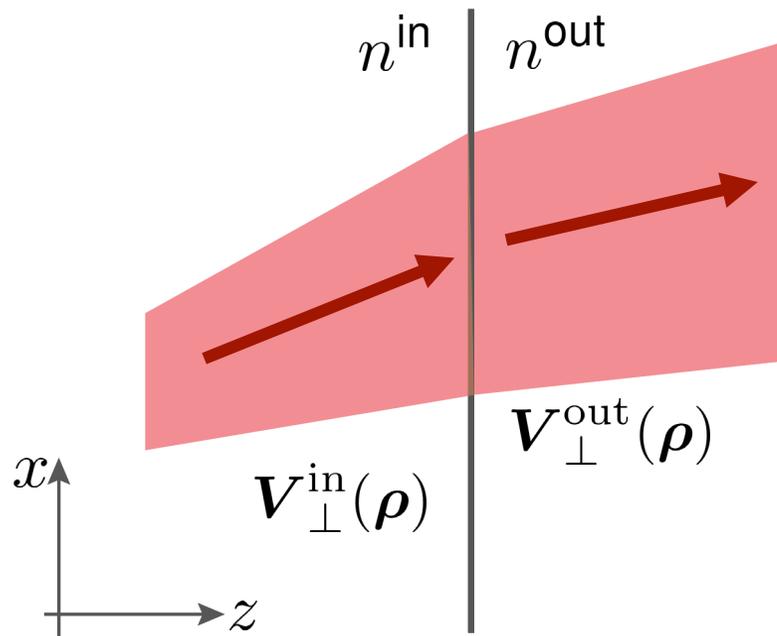
$$\psi^{\text{out}}(\boldsymbol{\rho}) \neq \psi^{\text{in}}(\boldsymbol{\rho})$$

- In what follows a change of the wavefront phase should be enforced by assuming a Wavefront Response Function $\Delta\psi(\boldsymbol{\rho})$ instead of (in addition to) a curved surface.

Wavefront Surface Response (WSR)

- For plane surfaces we found $\nabla_{\perp}\psi^{\text{in}}(\boldsymbol{\rho}) = \nabla_{\perp}\psi^{\text{out}}(\boldsymbol{\rho})$ and in conclusion

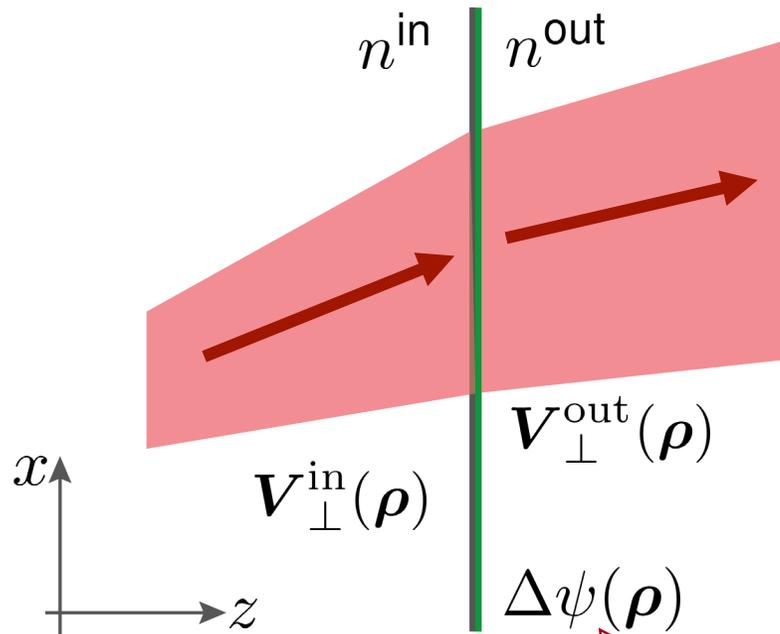
$$U_{\perp}^{\text{out}}(\boldsymbol{\rho}) \exp(i\psi^{\text{in}}(\boldsymbol{\rho})) = \{\mathbf{B}(\boldsymbol{\rho}; \psi^{\text{in}})U_{\perp}^{\text{in}}(\boldsymbol{\rho})\} \exp(i\psi^{\text{in}}(\boldsymbol{\rho})).$$



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Wavefront Surface
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- By introducing the wavefront surface response we assume an effect at the surface of the form

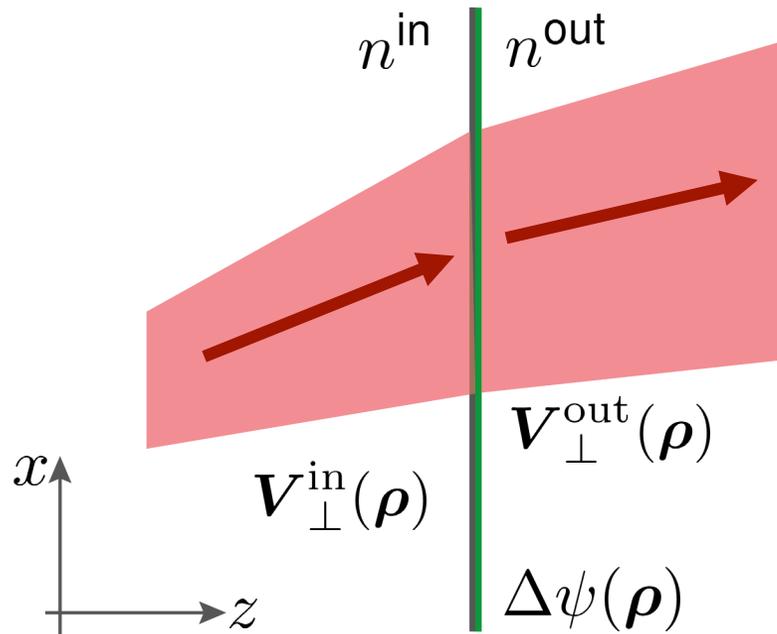
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with

$$\psi^{\text{out}}(\boldsymbol{\rho}) = \psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})$$

and thus

$$\nabla_{\perp}\psi^{\text{out}}(\boldsymbol{\rho}) = \nabla_{\perp}\psi^{\text{in}}(\boldsymbol{\rho}) + \nabla_{\perp}(\Delta\psi(\boldsymbol{\rho})).$$



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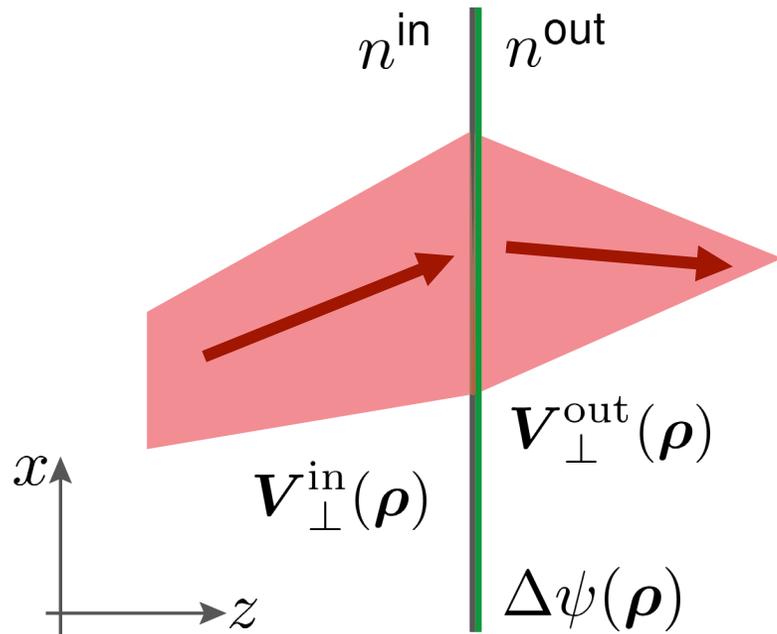
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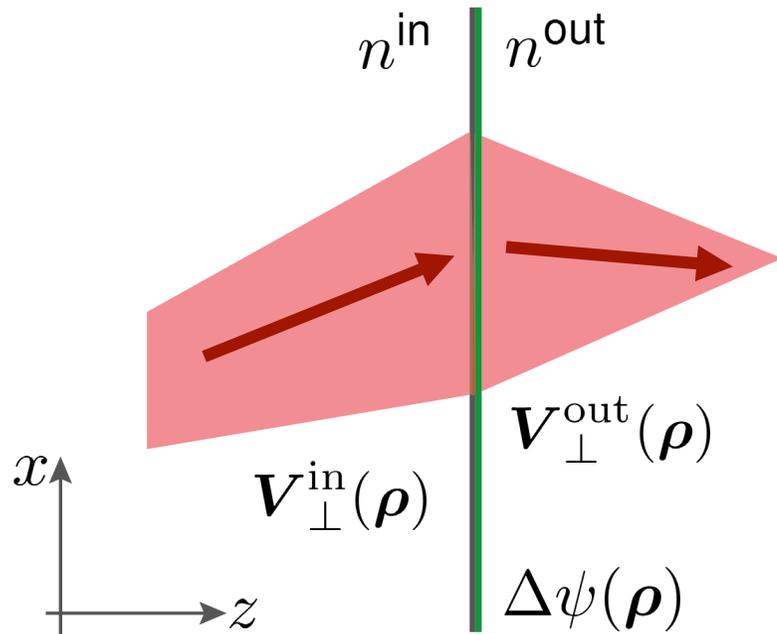
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Wavefront Surface Response (WSR)



- The equation

$$\nabla_{\perp}\psi^{\text{out}}(\boldsymbol{\rho}) = \nabla_{\perp}\psi^{\text{in}}(\boldsymbol{\rho}) + \nabla_{\perp}(\Delta\psi(\boldsymbol{\rho}))$$

results because of the local plane wave assumption (homeomorphic zone) into

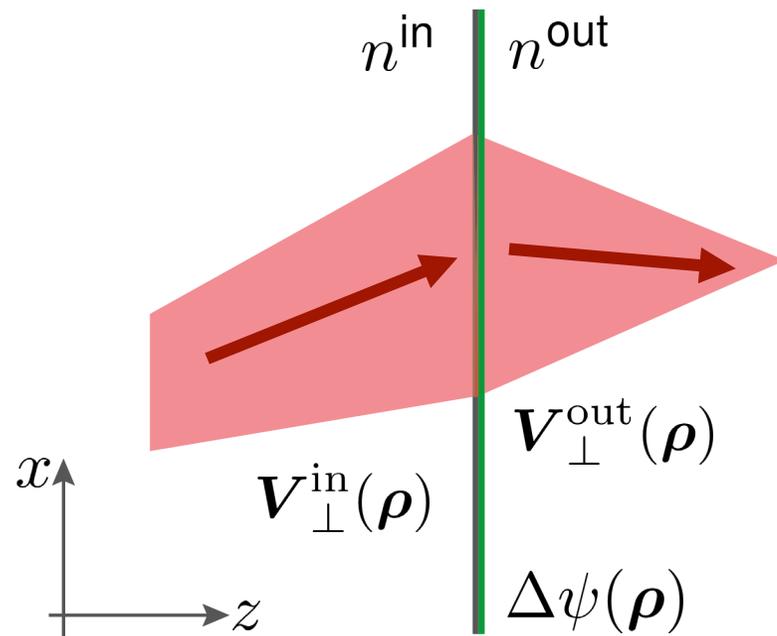
$$\boldsymbol{\kappa}^{\text{out}}(\boldsymbol{\rho}) = \boldsymbol{\kappa}^{\text{in}}(\boldsymbol{\rho}) + \mathbf{K}(\boldsymbol{\rho})$$

with $\mathbf{K}(\boldsymbol{\rho}) \stackrel{\text{def}}{=} \nabla_{\perp}(\Delta\psi(\boldsymbol{\rho}))$.

- That gives a direct access to ray tracing with $\boldsymbol{\kappa}(\boldsymbol{\rho}) = k_0 n \hat{\mathbf{s}}_{\perp}(\boldsymbol{\rho})$ via

$$n^{\text{out}} \hat{\mathbf{s}}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = n^{\text{in}} \hat{\mathbf{s}}_{\perp}^{\text{in}}(\boldsymbol{\rho}) + \mathbf{K}(\boldsymbol{\rho})/k_0.$$

How to Realize a Desired Wavefront Surface Response (WSR)?

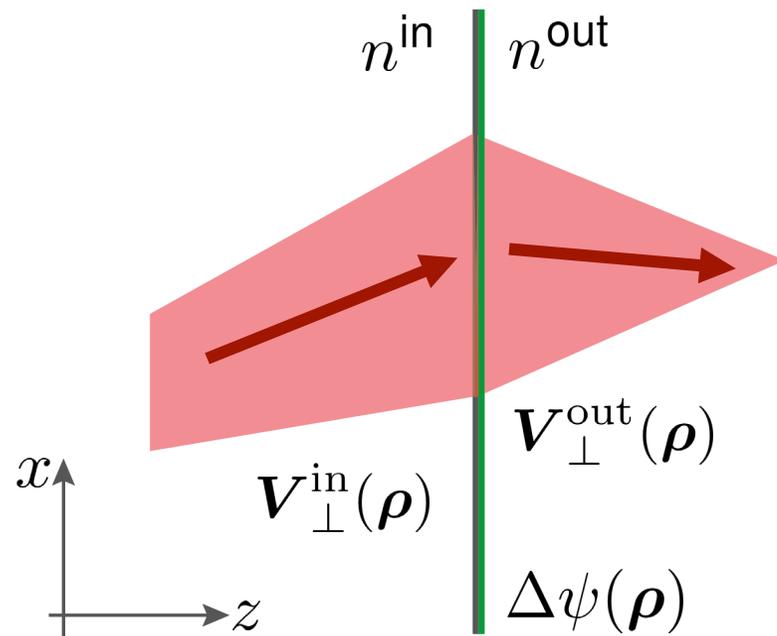


- From a physical-optics point of view the question arises, if there exists any manipulation of the structure of the surface, which provides an effect of the form:

$$U_{\perp}^{\text{out}}(\boldsymbol{\rho}) \exp(i\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})) = \{\mathbf{B}(\boldsymbol{\rho}; \psi^{\text{in}})U_{\perp}^{\text{in}}(\boldsymbol{\rho})\} \exp(i\psi^{\text{in}}(\boldsymbol{\rho}))$$

- A detailed answer can only be given for a specific surface structure.
- By introducing microstructured layers onto the surface a wavefront surface response can be implemented:
 - Graded-index layer
 - Volume hologram layer
 - Diffractive layer
 - Metamaterial layer

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Physical Optics Modeling: Metasurface Layer

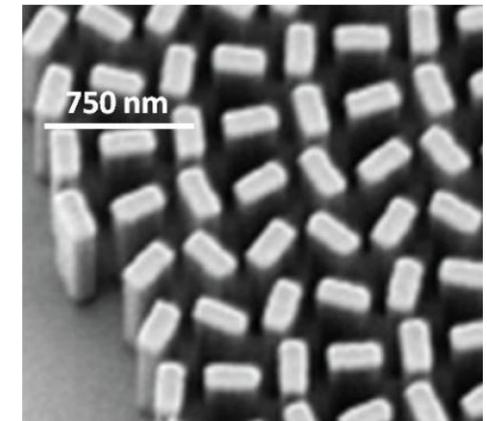
- In general a real surface layer structure does not just realize the desired wavefront response but additional effects and fields:

$$\mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \{ \mathbf{B}(\boldsymbol{\rho}; \psi^{\text{in}}) \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \} \exp(i\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})) + \mathbf{V}_{\perp}^{\text{res}}(\boldsymbol{\rho})$$

- For nanofin-based metalayers the typical result can be written as:

$$\begin{aligned} \mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = & \{ \mathbf{B}^{+}(\boldsymbol{\rho}; \psi^{\text{in}}) \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \} \exp(i\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})) \\ & + \{ \mathbf{B}^{-}(\boldsymbol{\rho}; \psi^{\text{in}}) \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \} \exp(i\psi^{\text{in}}(\boldsymbol{\rho}) - \Delta\psi(\boldsymbol{\rho})) \end{aligned}$$

- In depth investigation reveals, that $+\Delta\psi$ occurs for R-circularly polarized input fields and the conjugate phase for L-circularly polarized input.

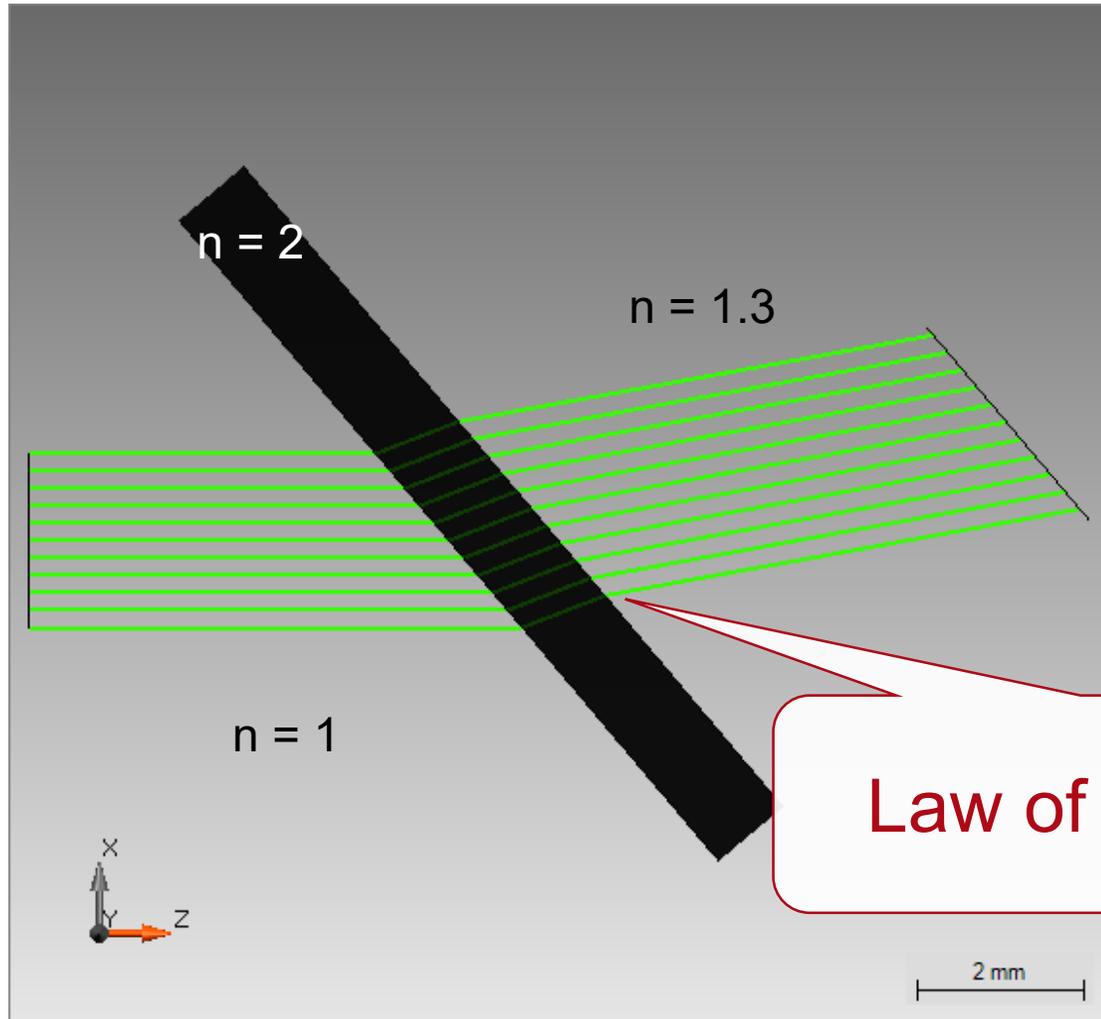


M. Khorasaninejad *et al.*,
Science **352**, 1190-1194 (2016).

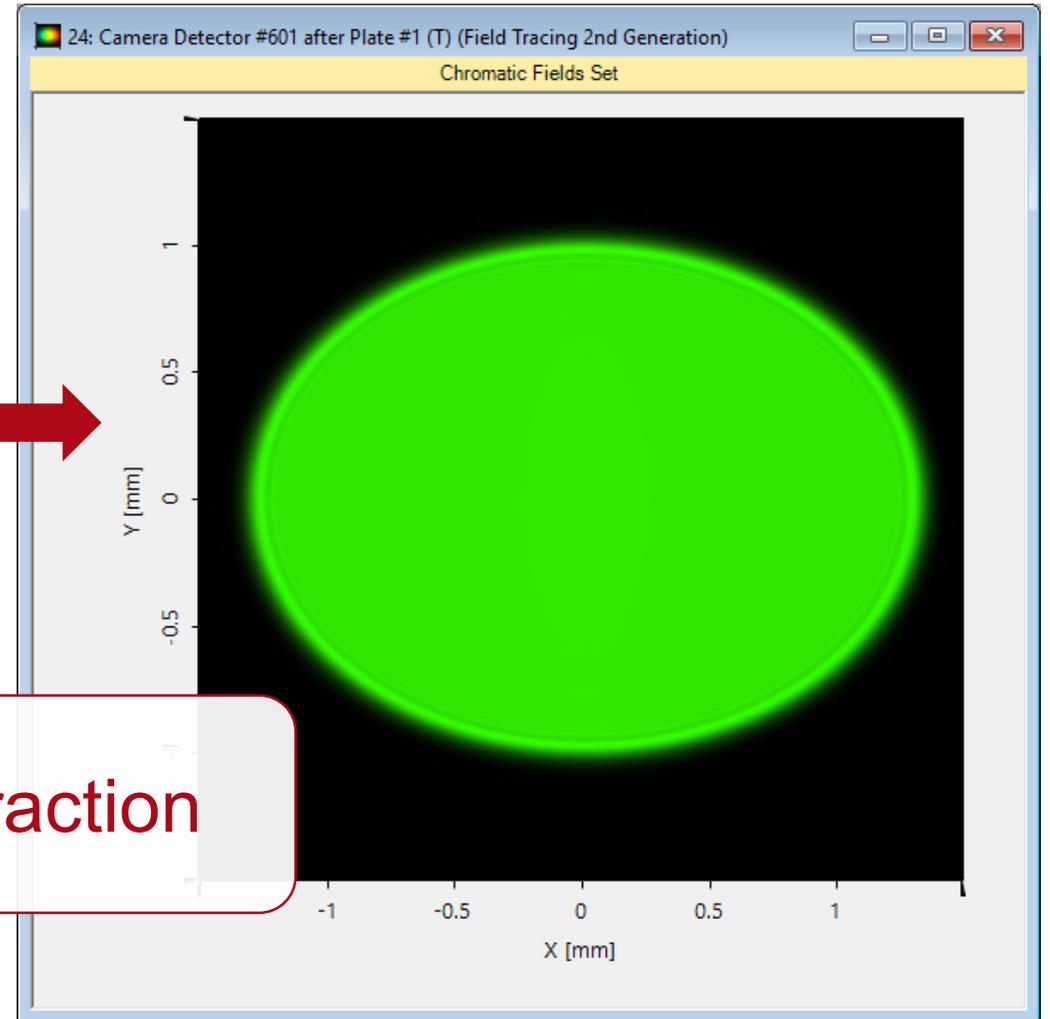
Metasurface layer

Linear Wavefront Response Function

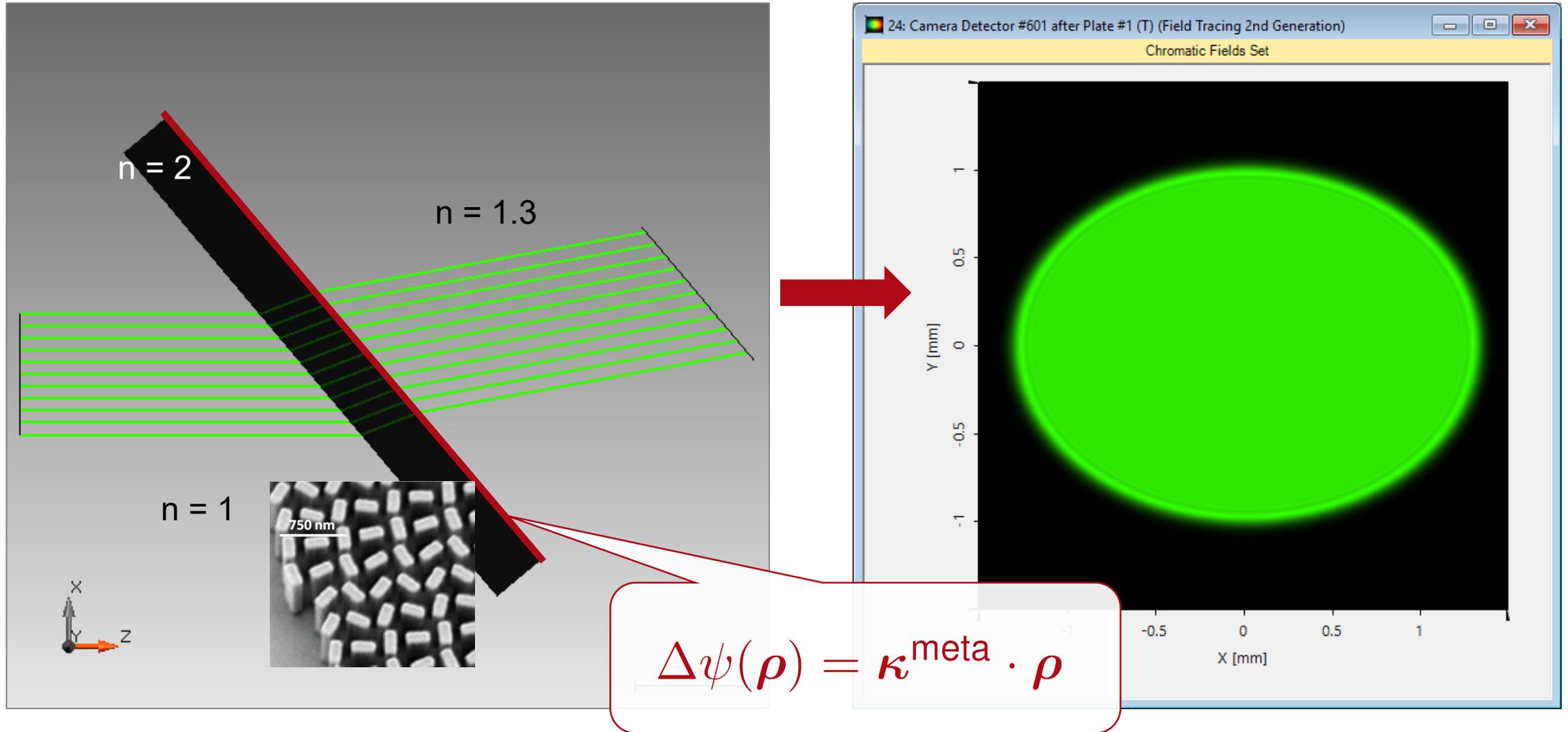
Plane Wave through Plate: Ray and Field Tracing



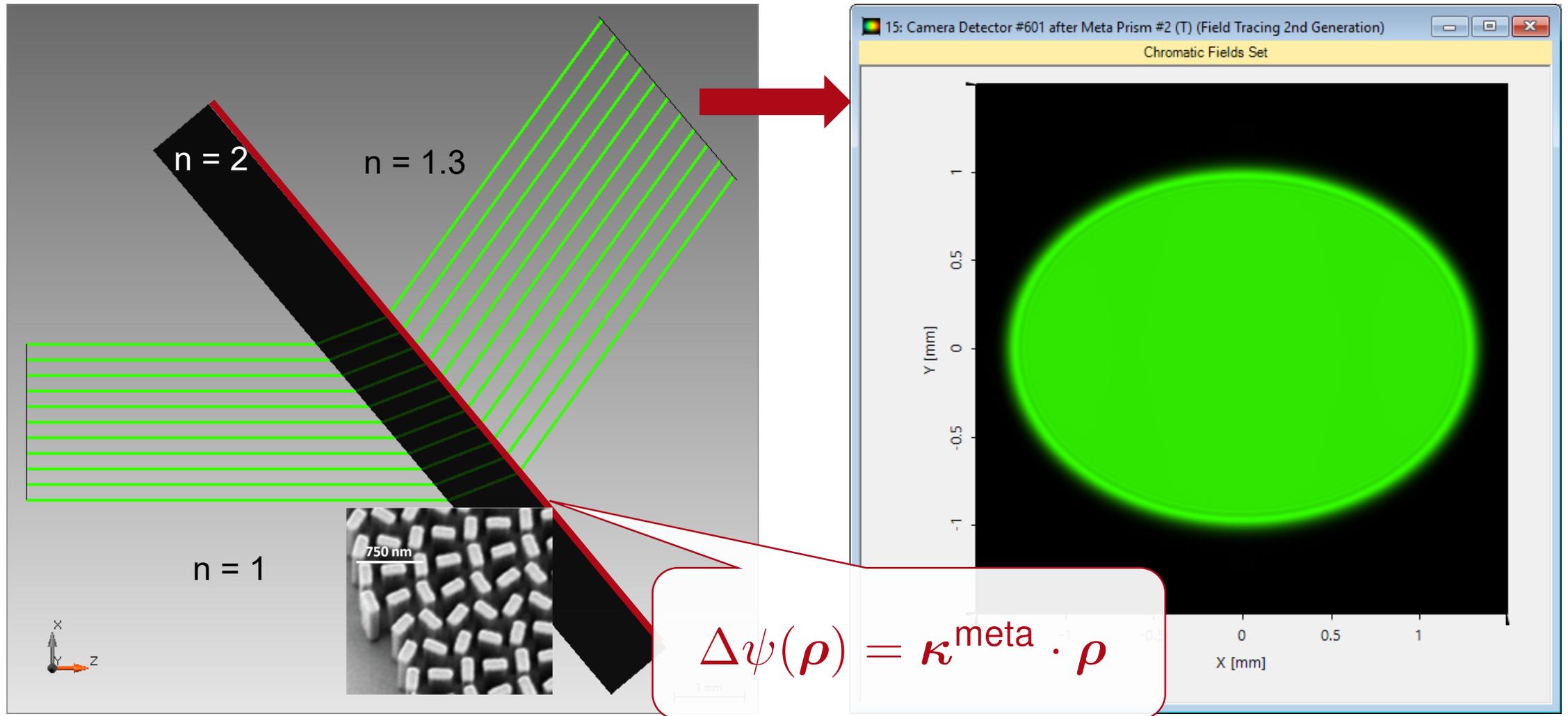
Law of refraction



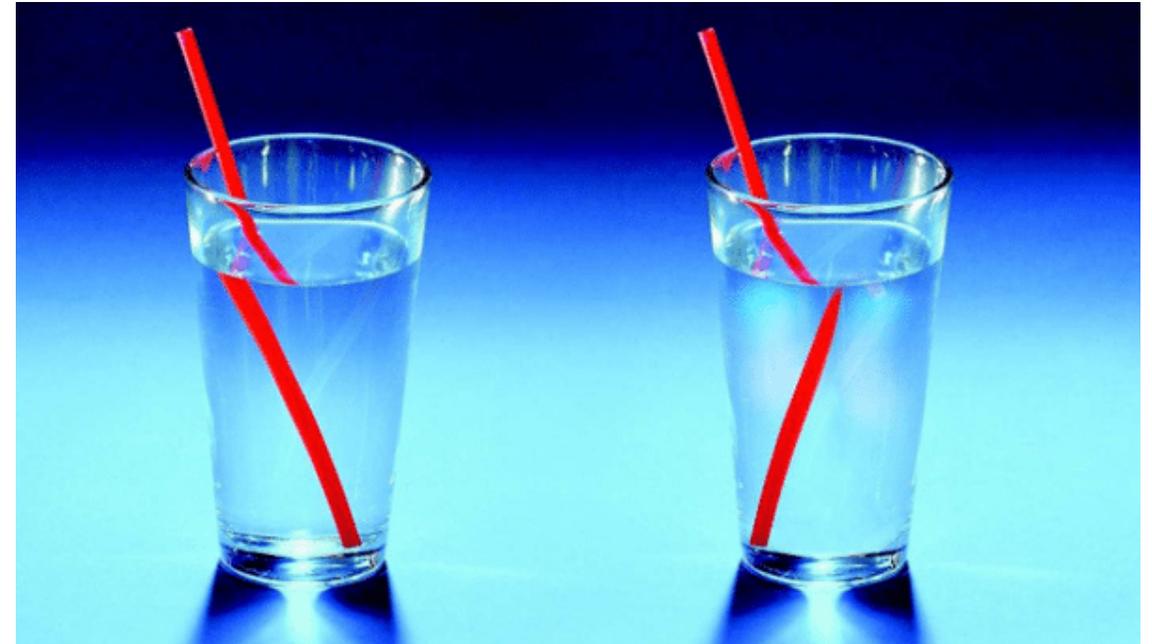
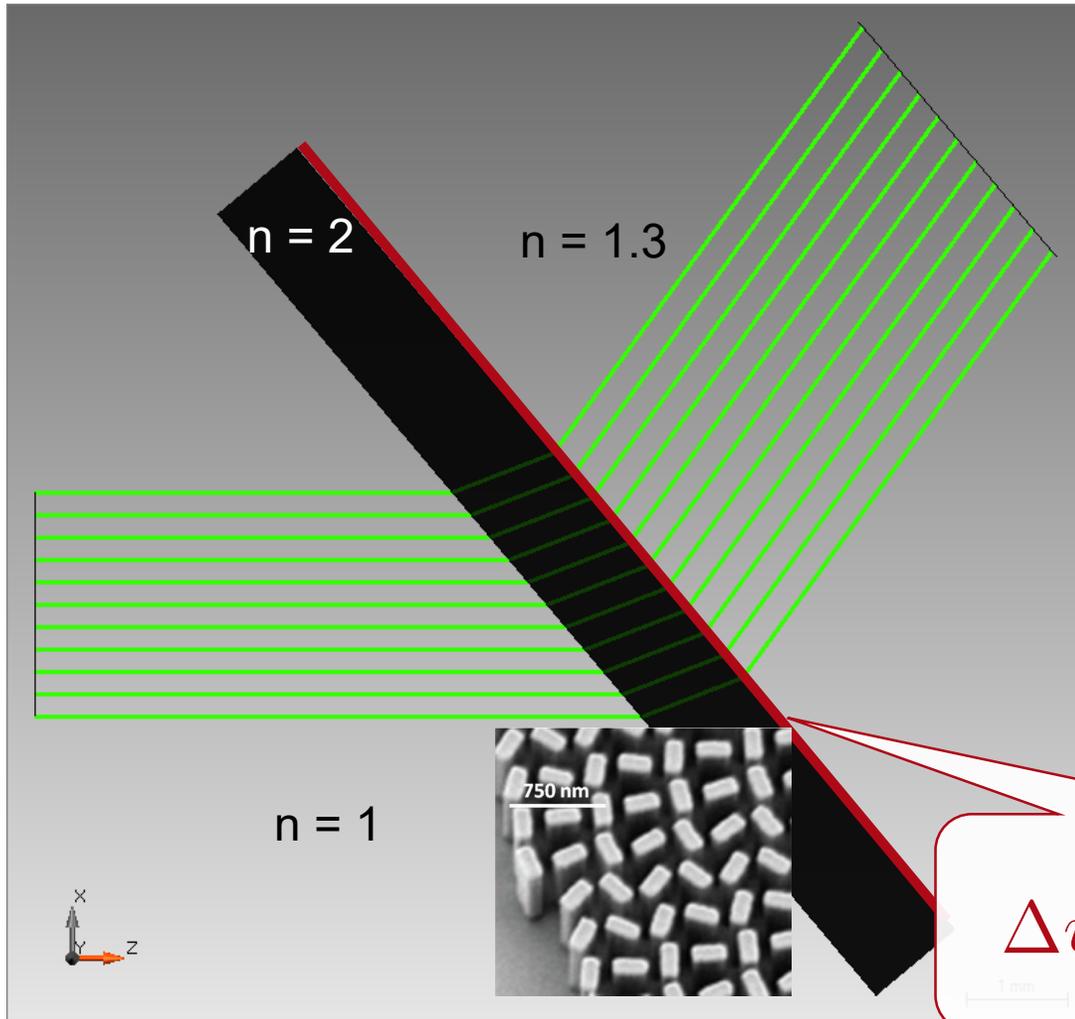
Plane Wave through Plate: Introducing Metasurface



Propagation Plate with Metasurface: Ray and Field Tracing



Propagation Plate with Metasurface: Ray and Field Tracing



Plane Wave through Plate: Ray and Field Tracing

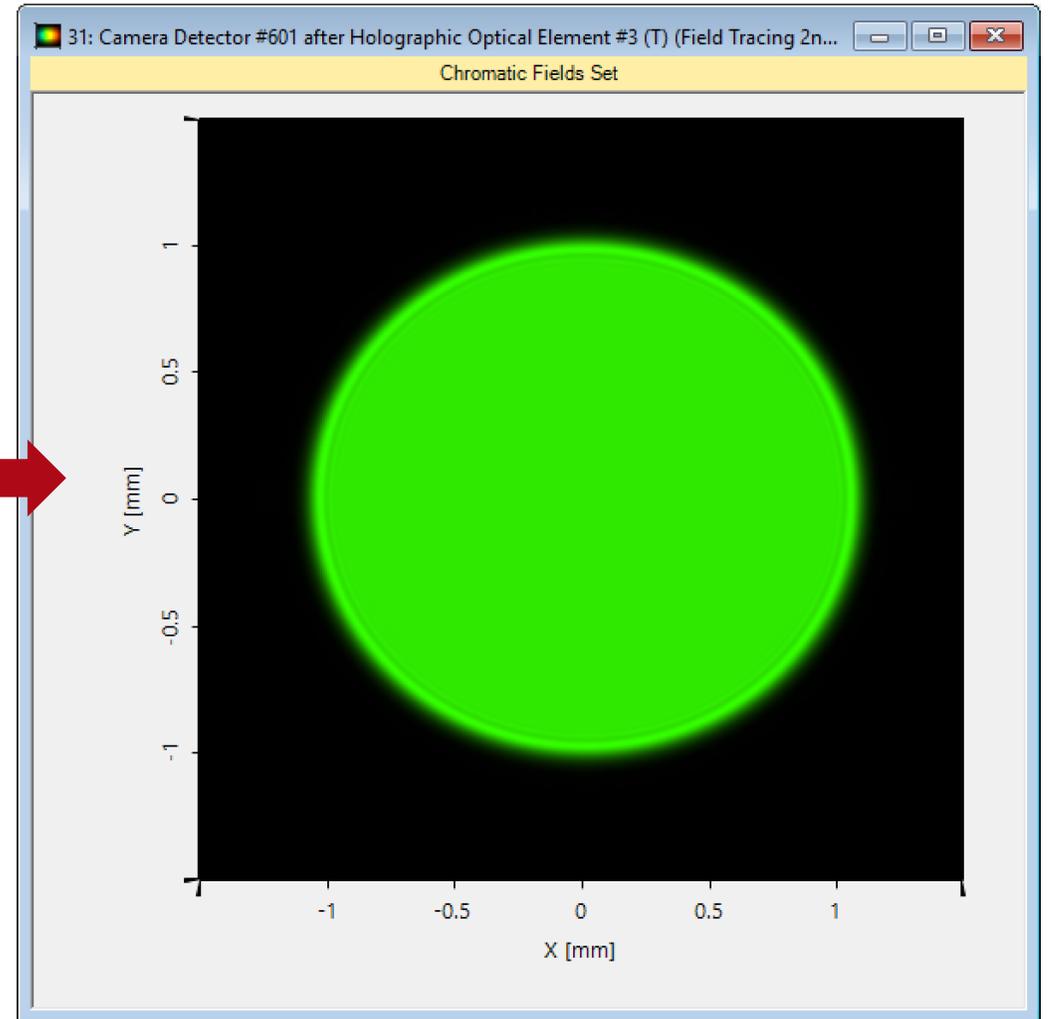
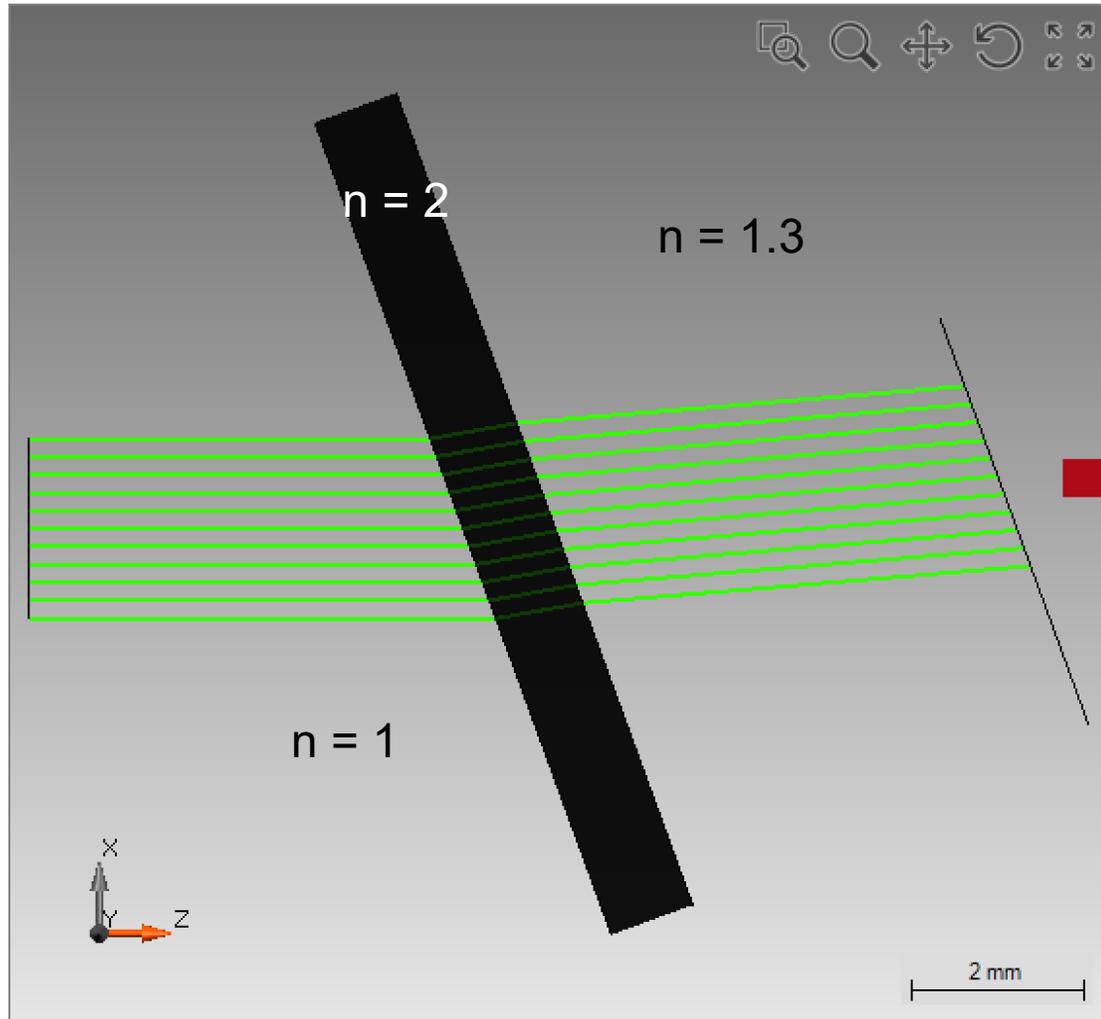


Plate with Metasurface: Ray and Field Tracing

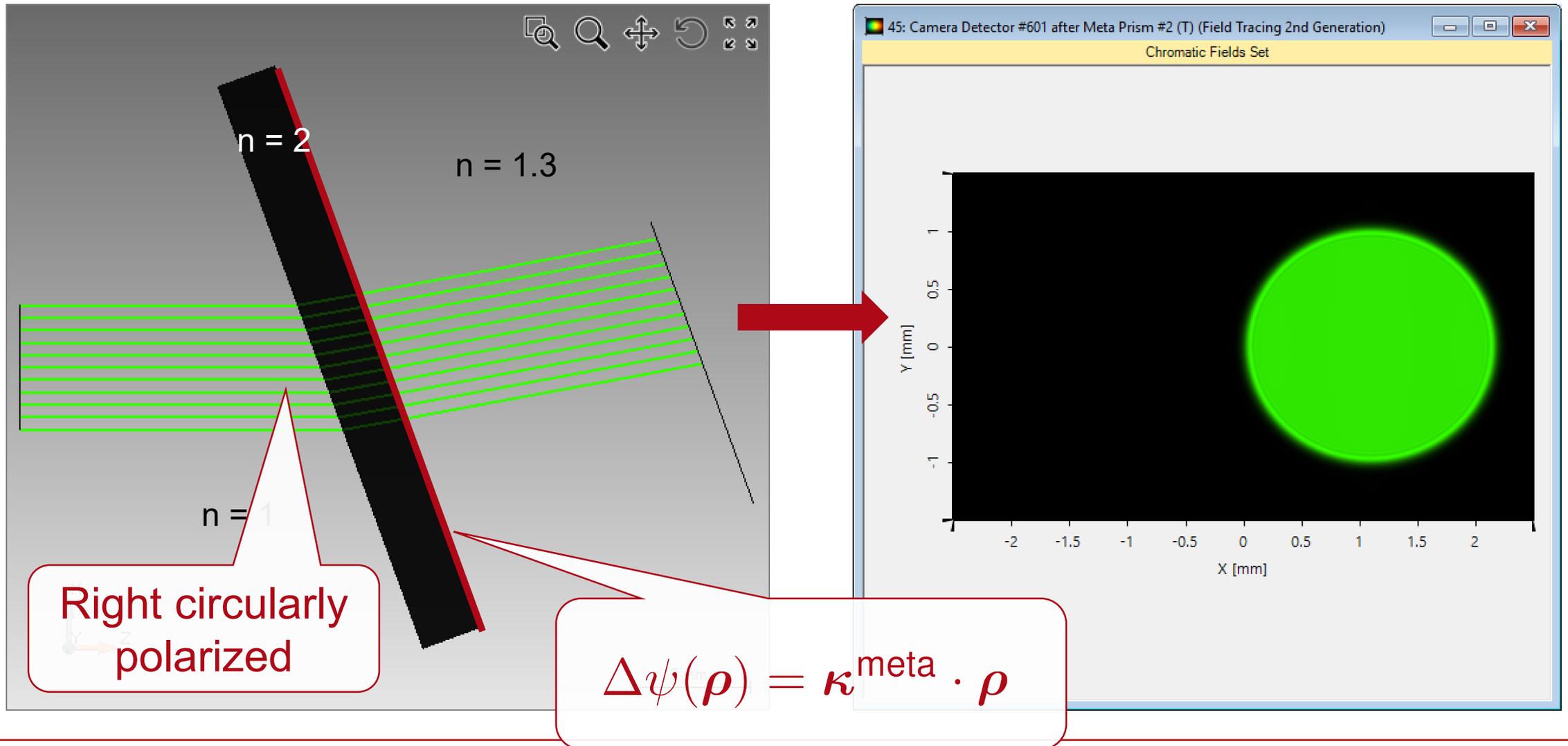


Plate with Metasurface: Ray and Field Tracing

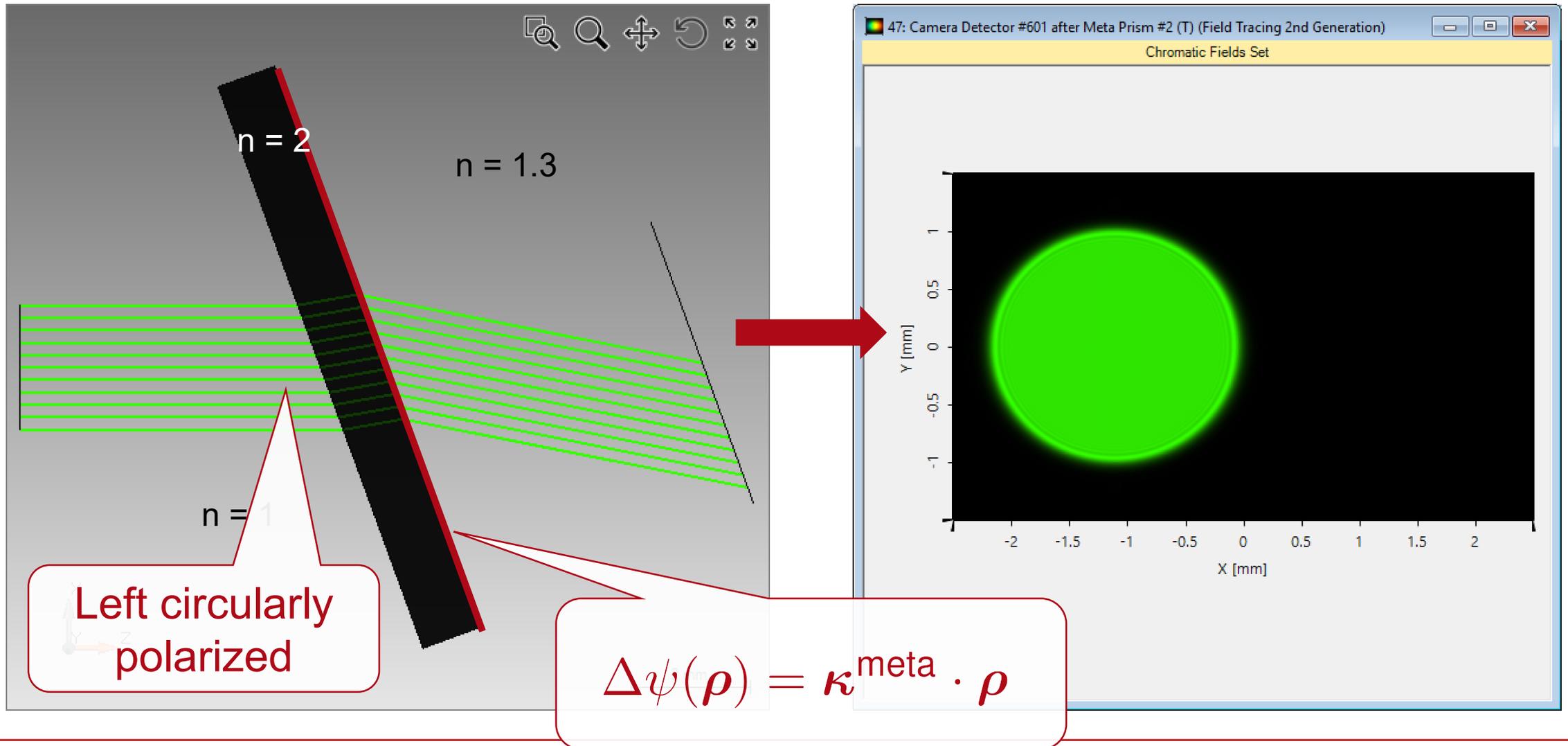
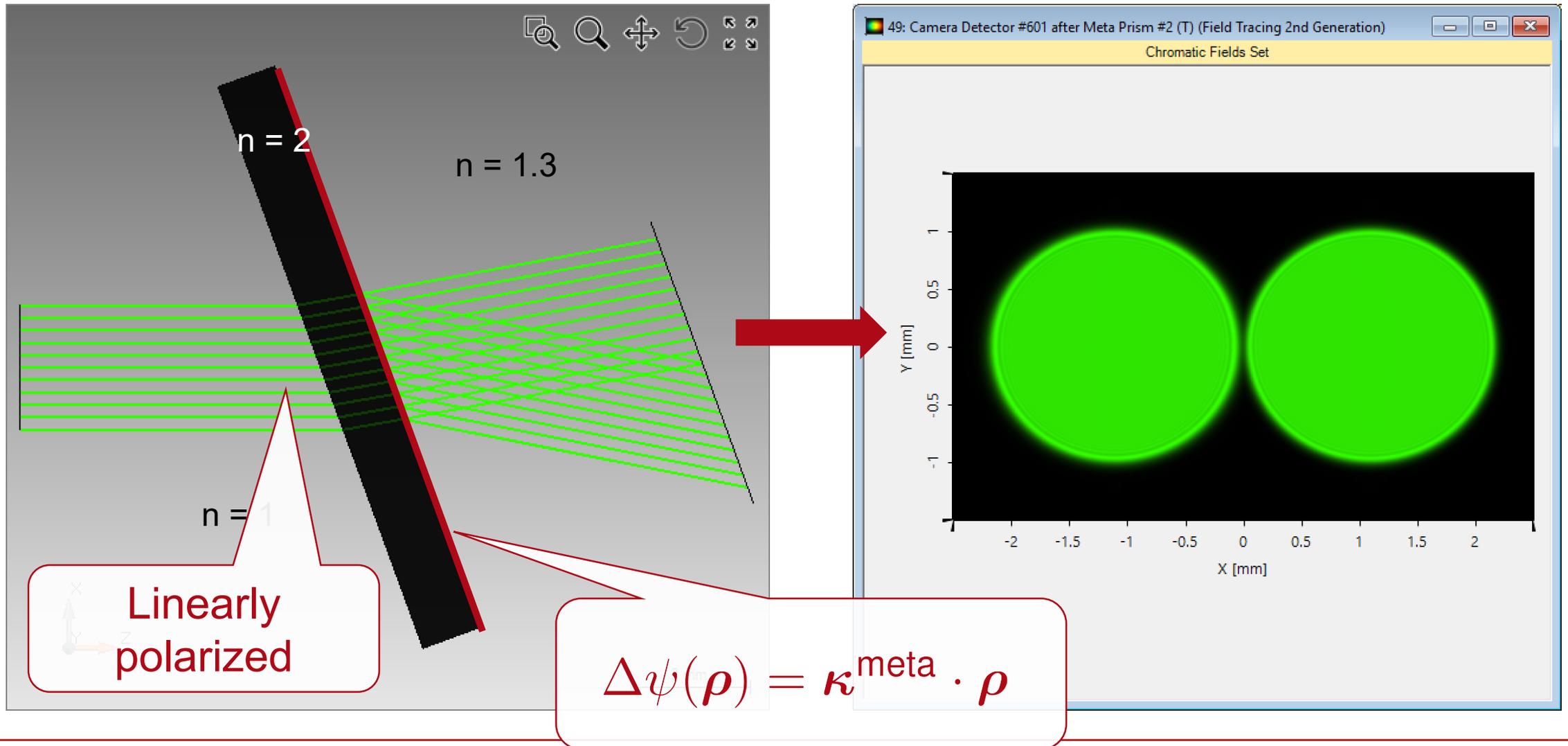
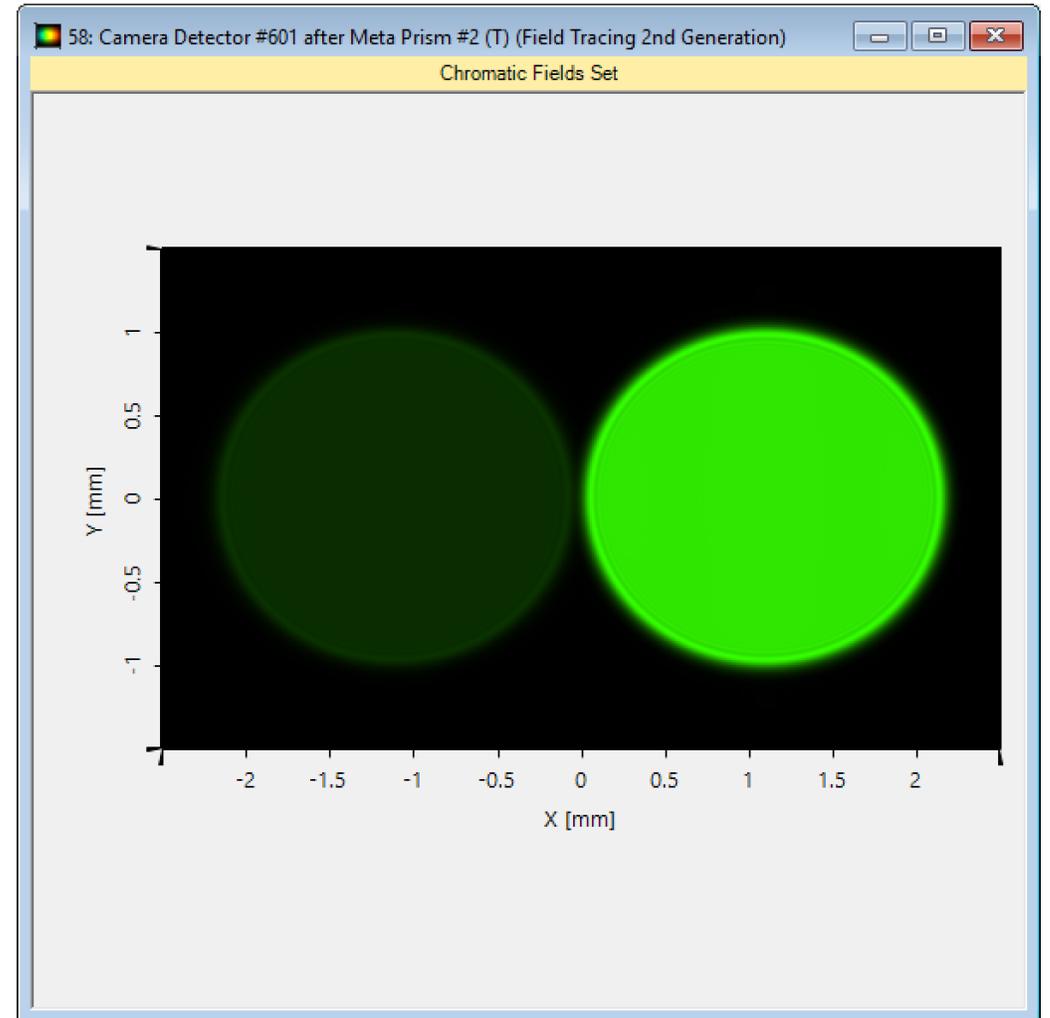
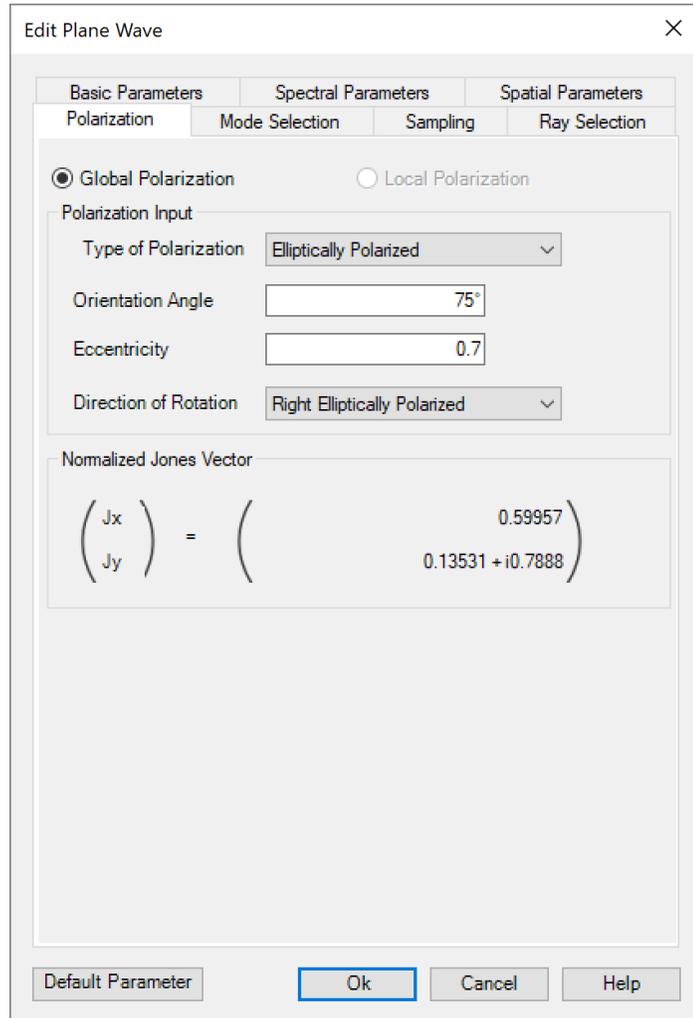


Plate with Metasurface: Ray and Field Tracing



Polarization Dependent Function of Nanofin Metalayer

Elliptical Polarization:



Diffraction layer

... with application to lenses

Physical Optics Modeling: Diffractive Layer

- In general a wavefront surface response $\Delta\psi(\boldsymbol{\rho})$ leads to the equation $\nabla_{\perp}\psi^{\text{out}}(\boldsymbol{\rho}) = \nabla_{\perp}\psi^{\text{in}}(\boldsymbol{\rho}) + \nabla_{\perp}(\Delta\psi(\boldsymbol{\rho}))$ and because of the local plane wave assumption (homeomorphic zone) into

$$\boldsymbol{\kappa}^{\text{out}}(\boldsymbol{\rho}) = \boldsymbol{\kappa}^{\text{in}}(\boldsymbol{\rho}) + \mathbf{K}(\boldsymbol{\rho})$$

with $\mathbf{K}(\boldsymbol{\rho}) \stackrel{\text{def}}{=} \nabla_{\perp}(\Delta\psi(\boldsymbol{\rho}))$.

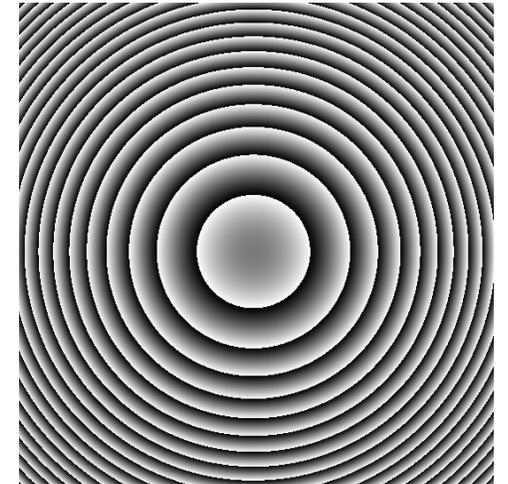
- This equation is directly related to a locally formulated grating equation

$$\boldsymbol{\kappa}^{\text{out}}(\boldsymbol{\rho}) = \boldsymbol{\kappa}^{\text{in}}(\boldsymbol{\rho}) + m(2\pi/d_x(\boldsymbol{\rho}), 2\pi/d_y(\boldsymbol{\rho}))$$

with the local grating period $\mathbf{d}(\boldsymbol{\rho}) = (d_x(\boldsymbol{\rho}), d_y(\boldsymbol{\rho}))$.

- That leads to the basic principle of a diffractive layer via:

$$\mathbf{d}(\boldsymbol{\rho}) = 2\pi \left((\partial\psi(\boldsymbol{\rho})/\partial x)^{-1}, (\partial\psi(\boldsymbol{\rho})/\partial y)^{-1} \right)$$

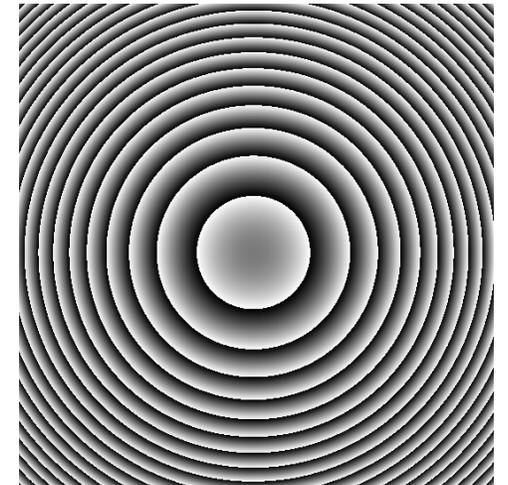


Physical Optics Modeling: Diffractive Layer

- This design and modeling understanding results in the decomposition of the output field into a series of local grating orders:

$$\mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \left\{ \mathbf{B}^{(1)}(\boldsymbol{\rho}; \psi^{\text{in}}) \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \right\} \exp(i\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})) \\ + \sum_{m=-\infty, m \neq 1}^{\infty} \left\{ \mathbf{B}^{(m)}(\boldsymbol{\rho}; \psi^{\text{in}}) \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \right\} \exp(i\psi^{\text{in}}(\boldsymbol{\rho}) + m\Delta\psi(\boldsymbol{\rho}))$$

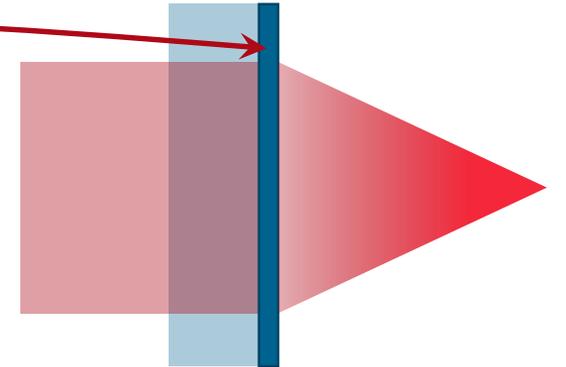
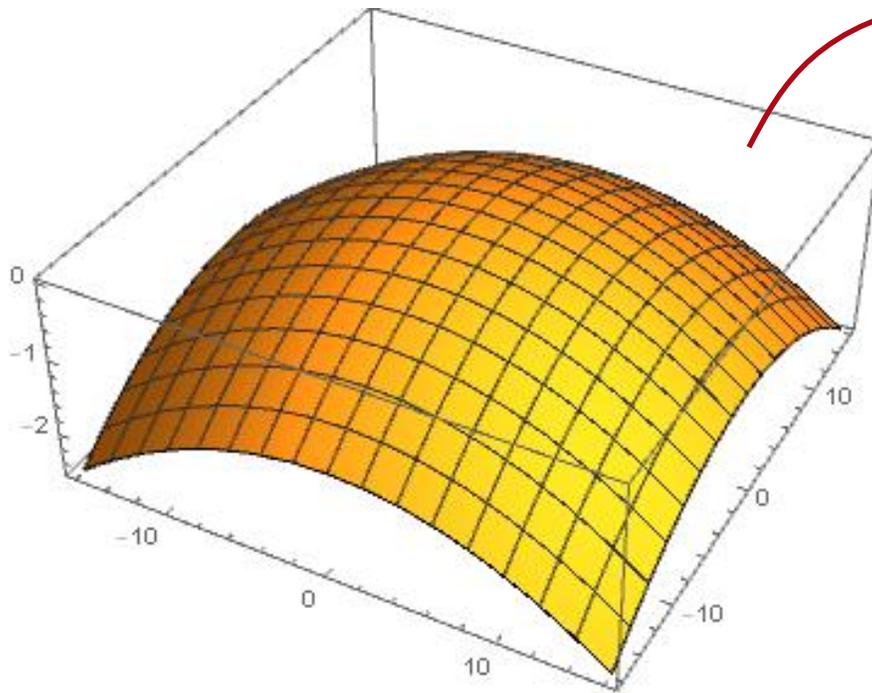
- The 2×2 matrix $\mathbf{B}^{(m)}(\boldsymbol{\rho}; \psi^{\text{in}})$ expresses the Rayleigh-matrix of grating theory and is rigorously calculated per direction and period by the Fourier Modal Method (FMM). Design should minimize Rayleigh coefficients for all undesired orders.
- Complications: Period drastically varies over surface, which results in a laterally varying number of propagating subfields. Full treatment available in **VirtualLab Lens Solutions**.



Wavefront Surface Response of Focusing Lens

- In order to transform a plane incident field into a spherical convergent one the wavefront surface response should be:

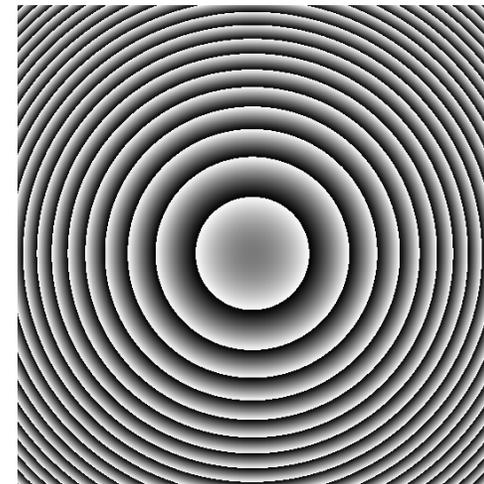
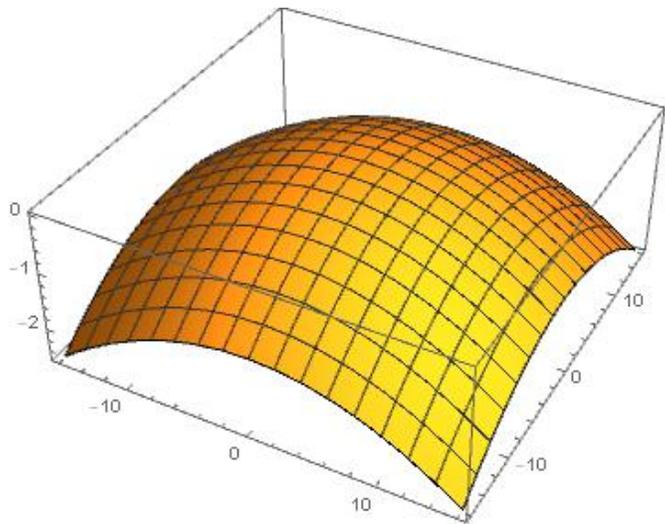
$$\Delta\psi(\boldsymbol{\rho}) = k_0 n \left(f - \sqrt{\|\boldsymbol{\rho}\|^2 + f^2} \right)$$



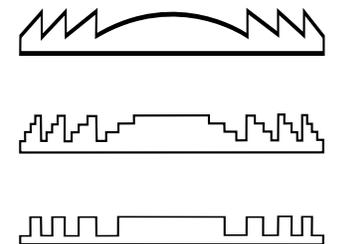
Structure Design

- Wrap the WSR: $(\Delta\psi(\boldsymbol{\rho}))^{\text{DOE}} = \text{mod}_{p2\pi} \left\{ k_0 n \left(f - \sqrt{\|\boldsymbol{\rho}\|^2 + f^2} \right) \right\}$ with $p \in \mathbb{N}$.
- For $p = 1$ local radial period follows with $d(\rho) = 2\pi / \Delta\psi'(\rho)$.
- Structure design by inverse Thin Element Approximation (TEA): The height profile h^{DOE} is given by:

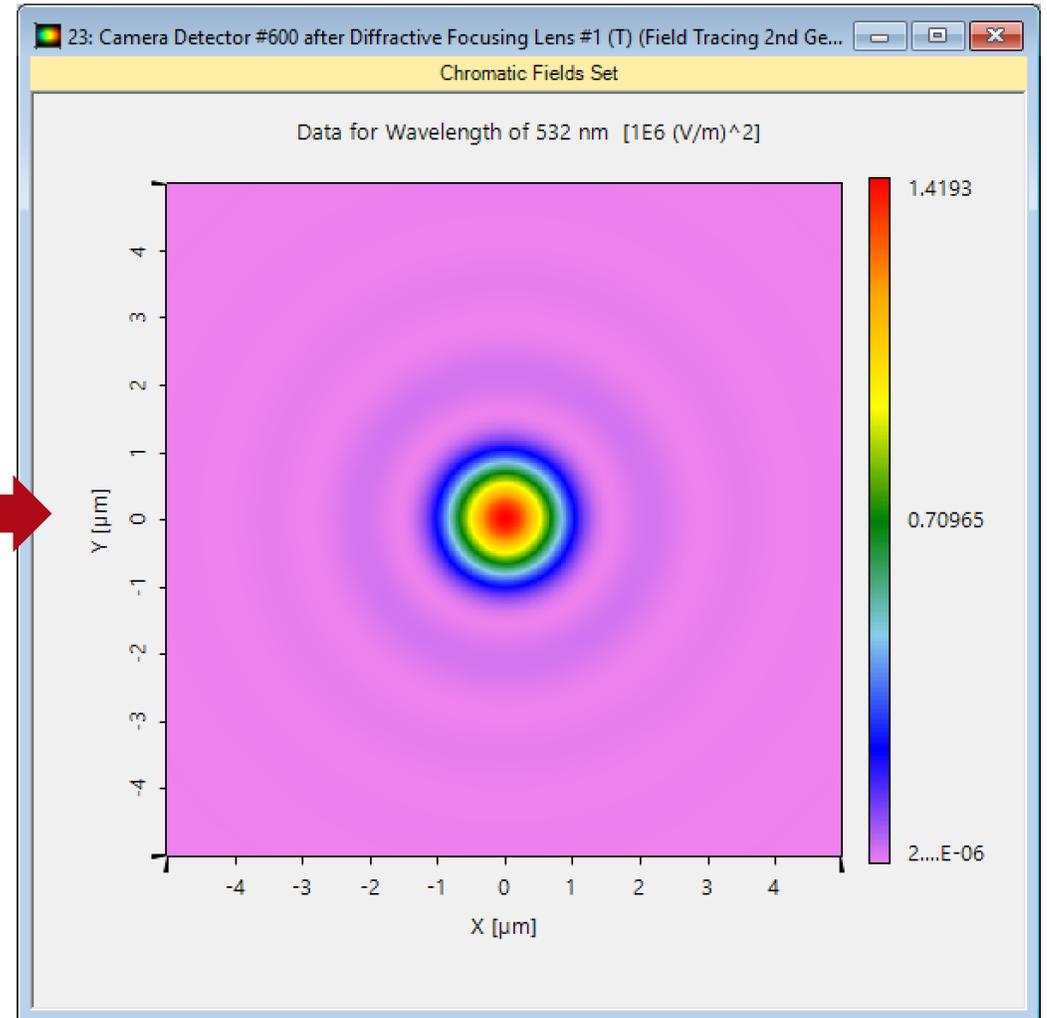
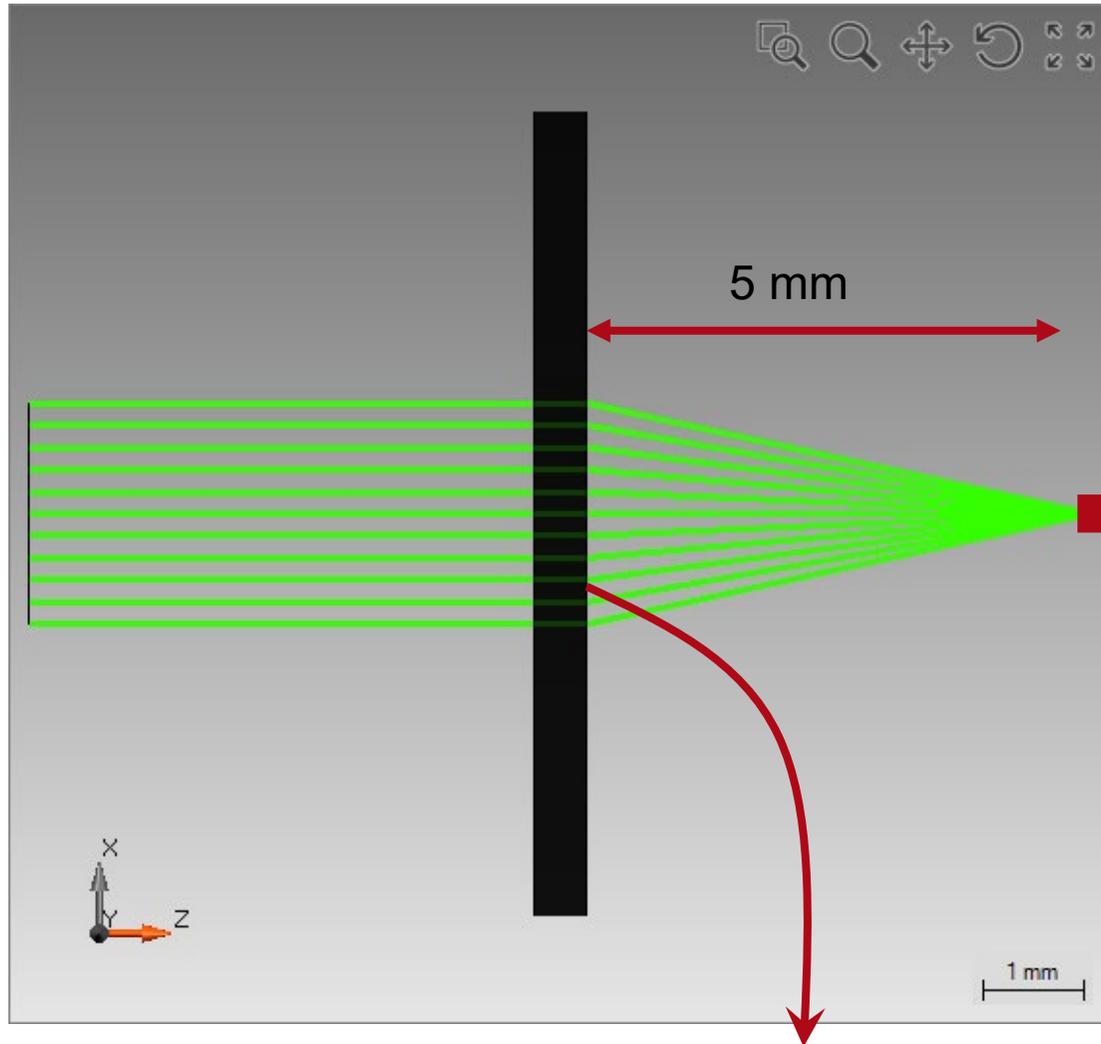
$$h^{\text{DOE}}(\rho) = \frac{\lambda}{2\pi\Delta n} \Delta\psi(\rho)^{\text{DOE}}$$



Profile quantization

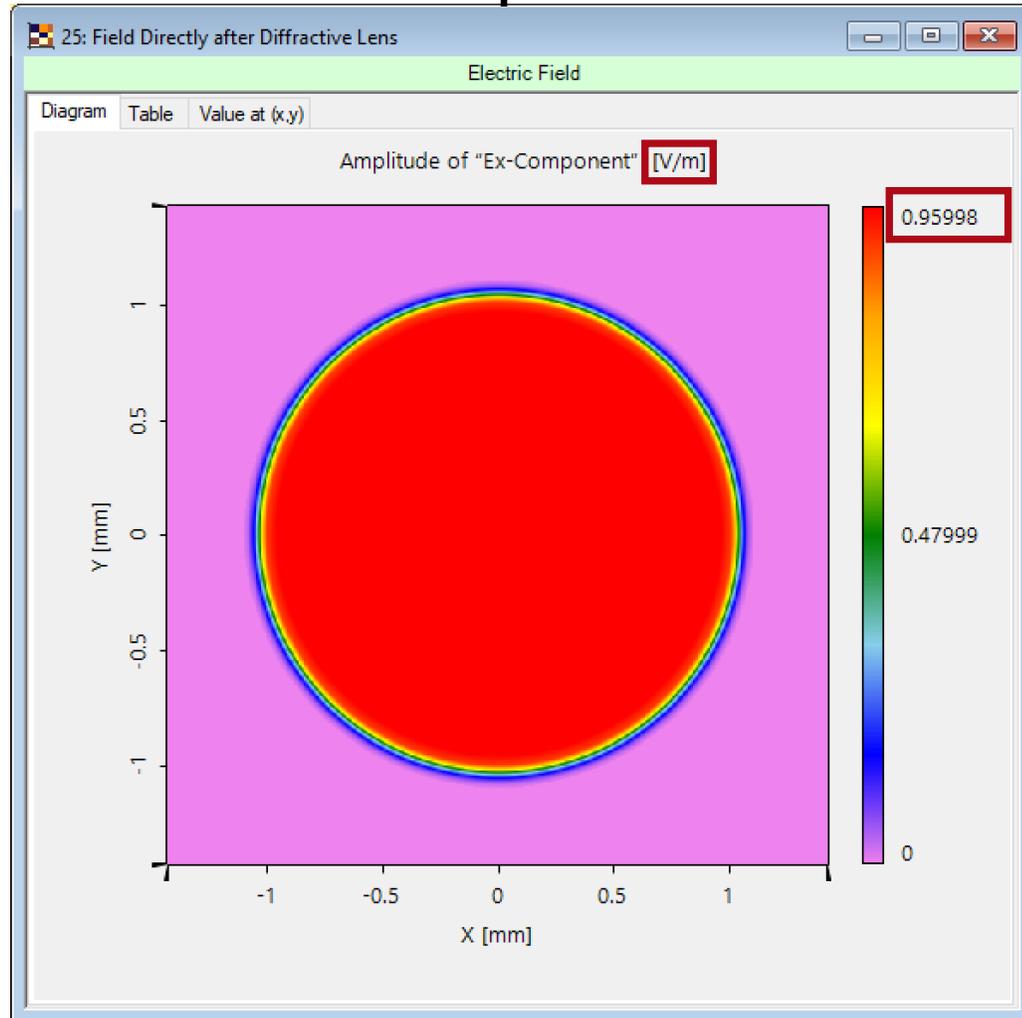


Focusing Diffractive Lens with NA=0.2: Ray and Field Tracing



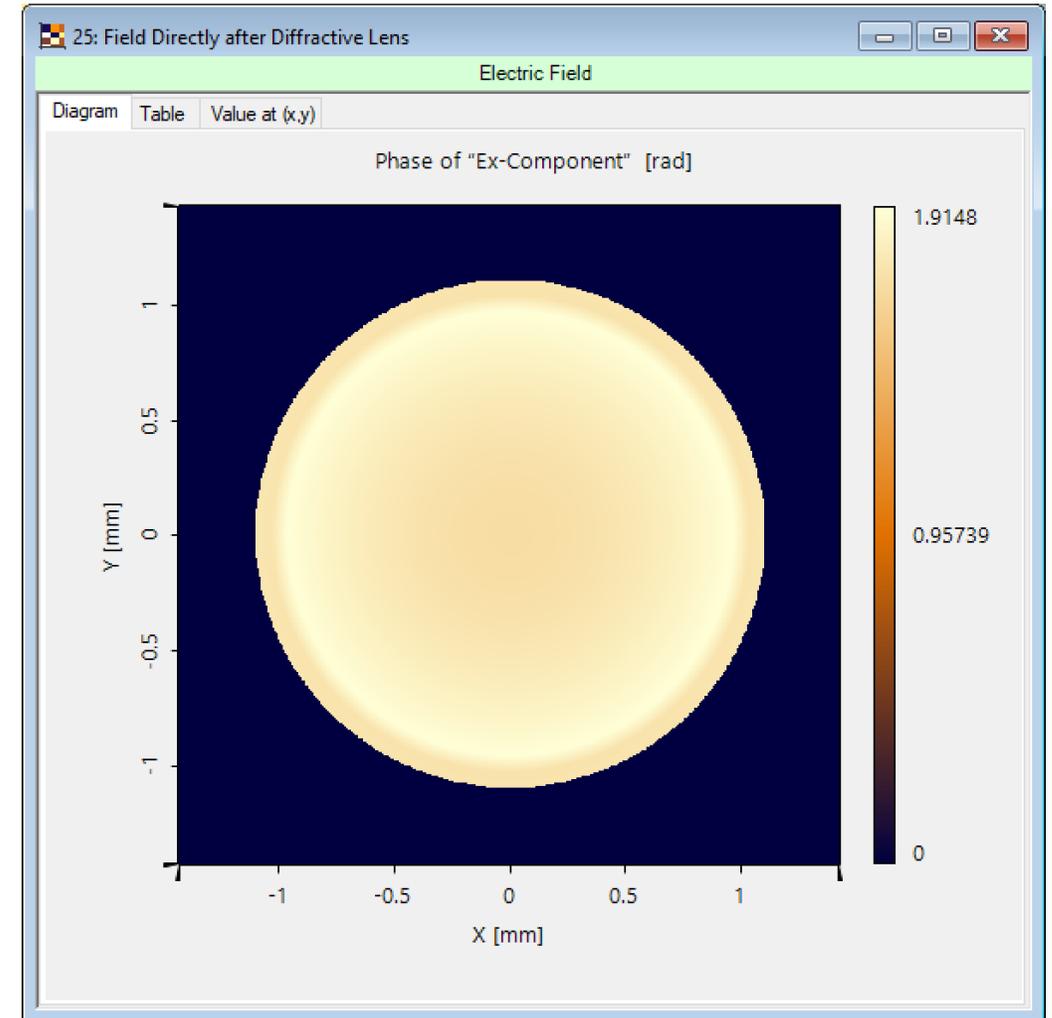
Focusing Diffractive Lens with NA=0.2: Ray and Field Tracing

Amplitude



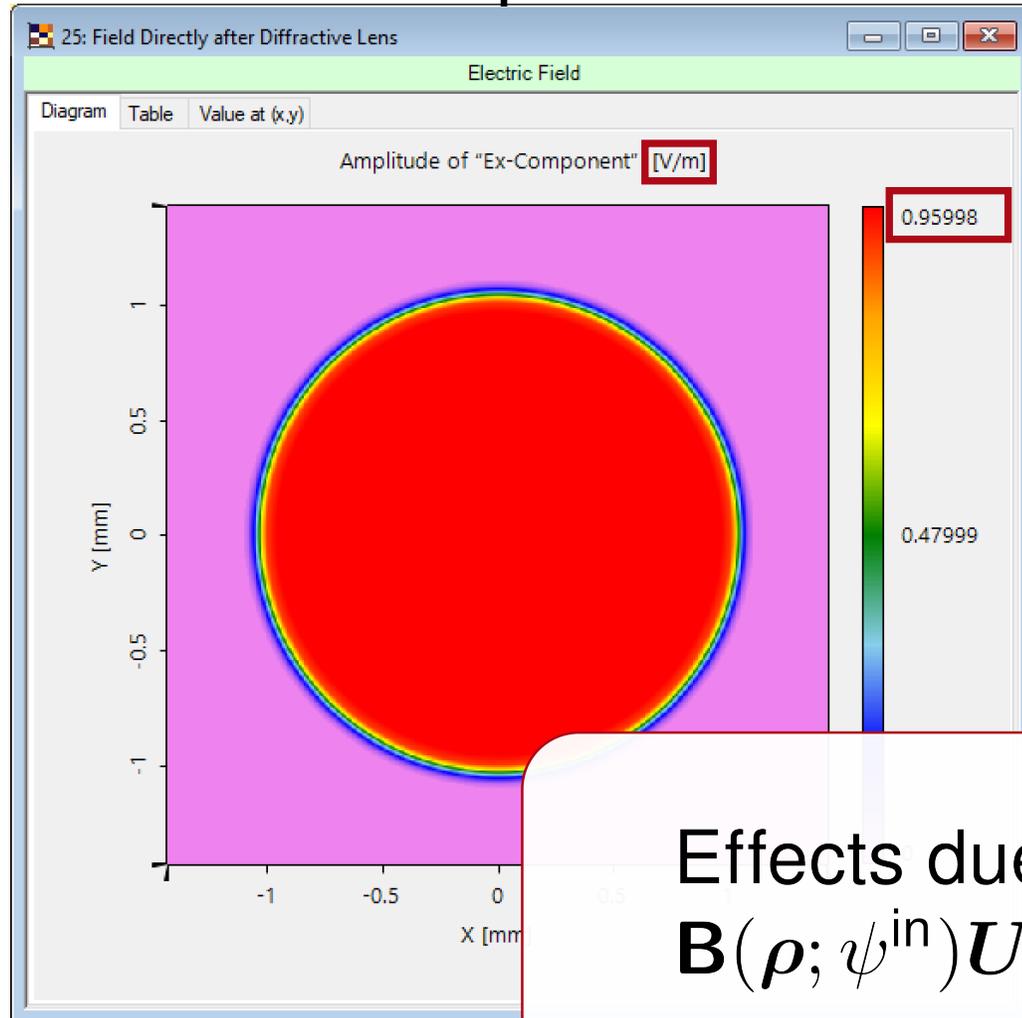
Ex

Phase



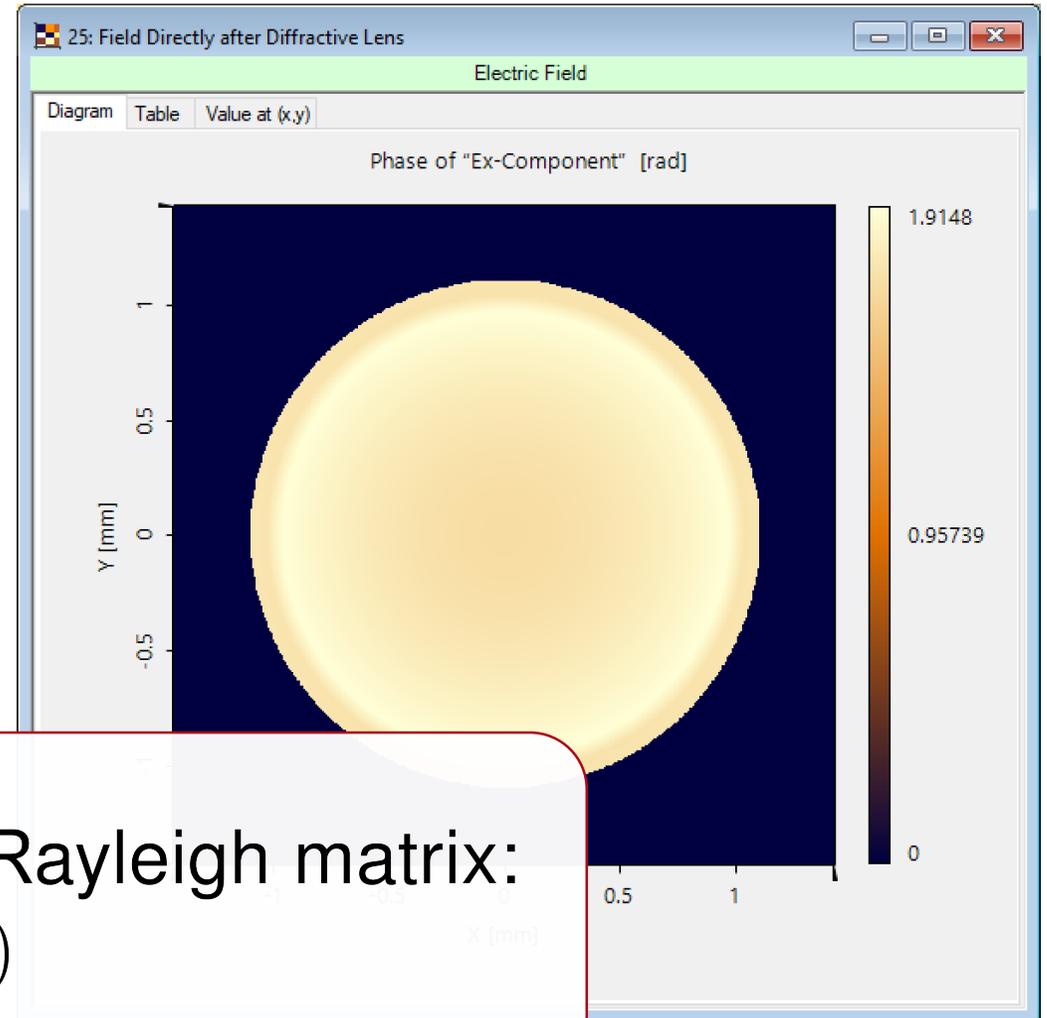
Focusing Diffractive Lens with NA=0.2: Ray and Field Tracing

Amplitude



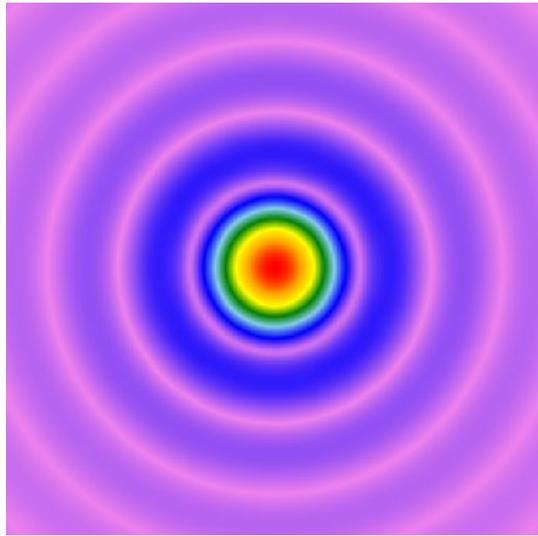
Ex

Phase



Effects due to Rayleigh matrix:
 $\mathbf{B}(\boldsymbol{\rho}; \psi^{\text{in}}) \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})$

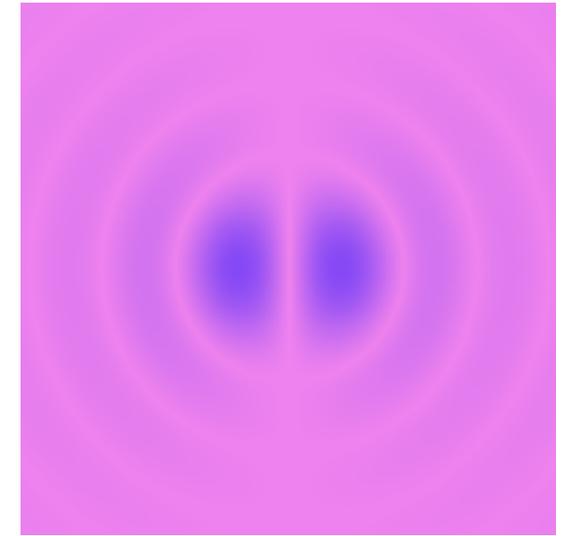
Focusing Diffractive Lens with NA=0.2: Ray and Field Tracing



Amplitude Ex



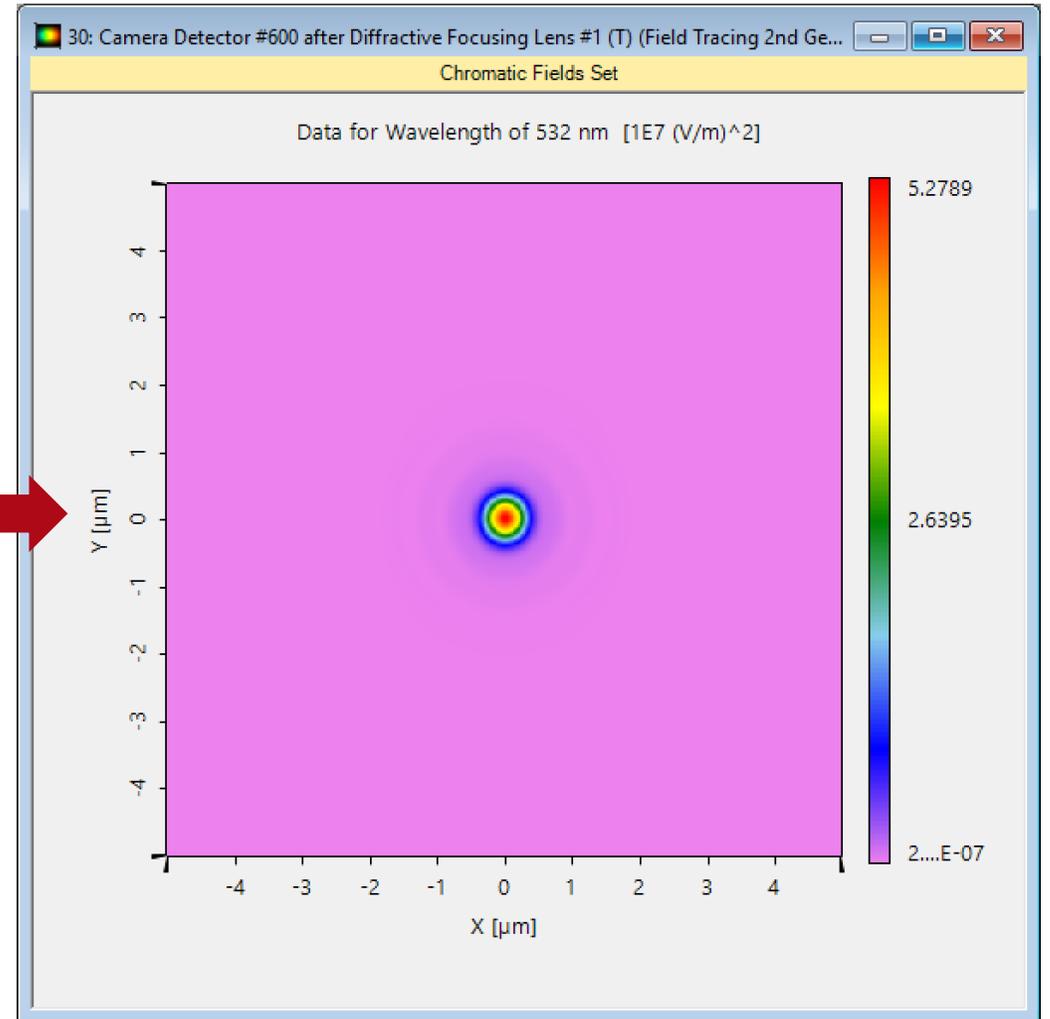
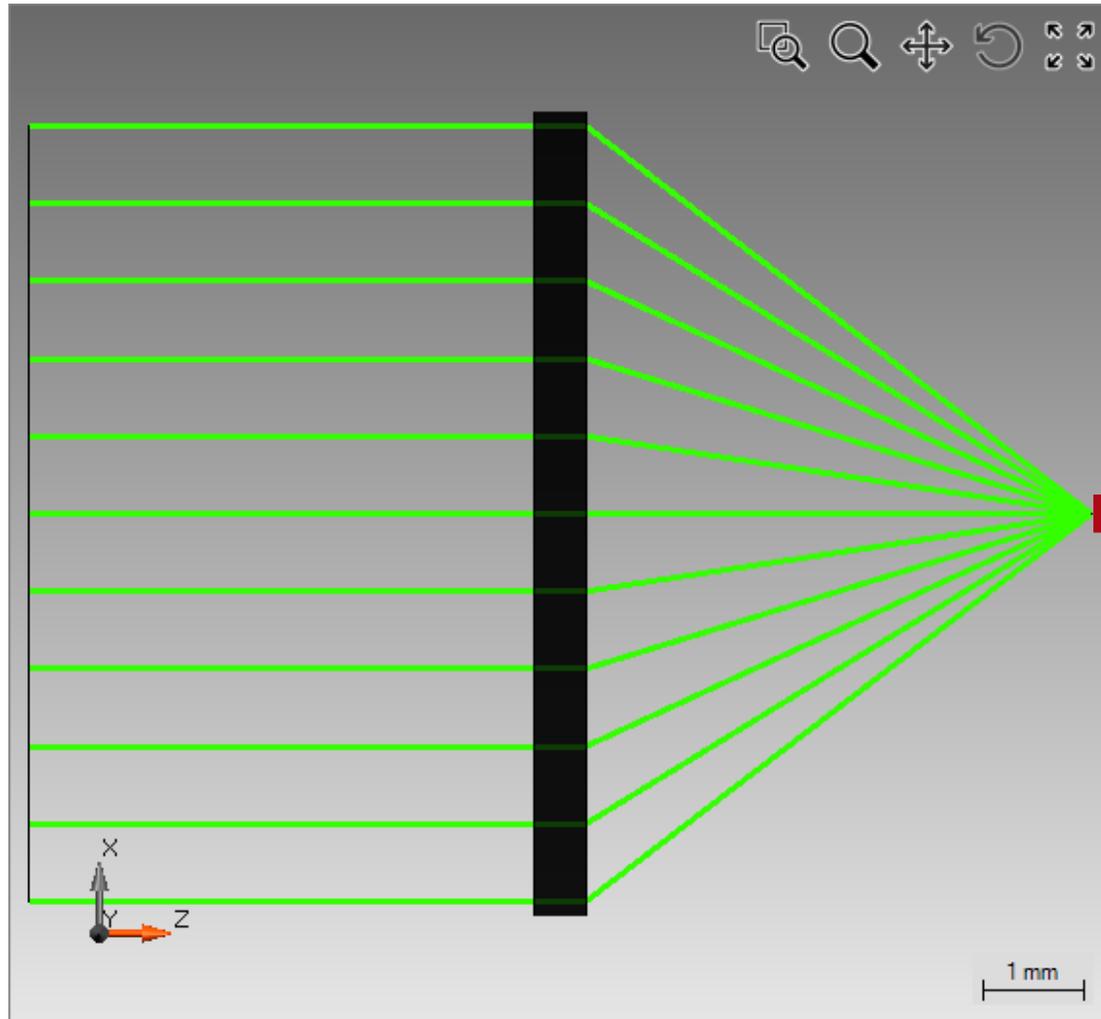
Amplitude Ey



Amplitude Ez

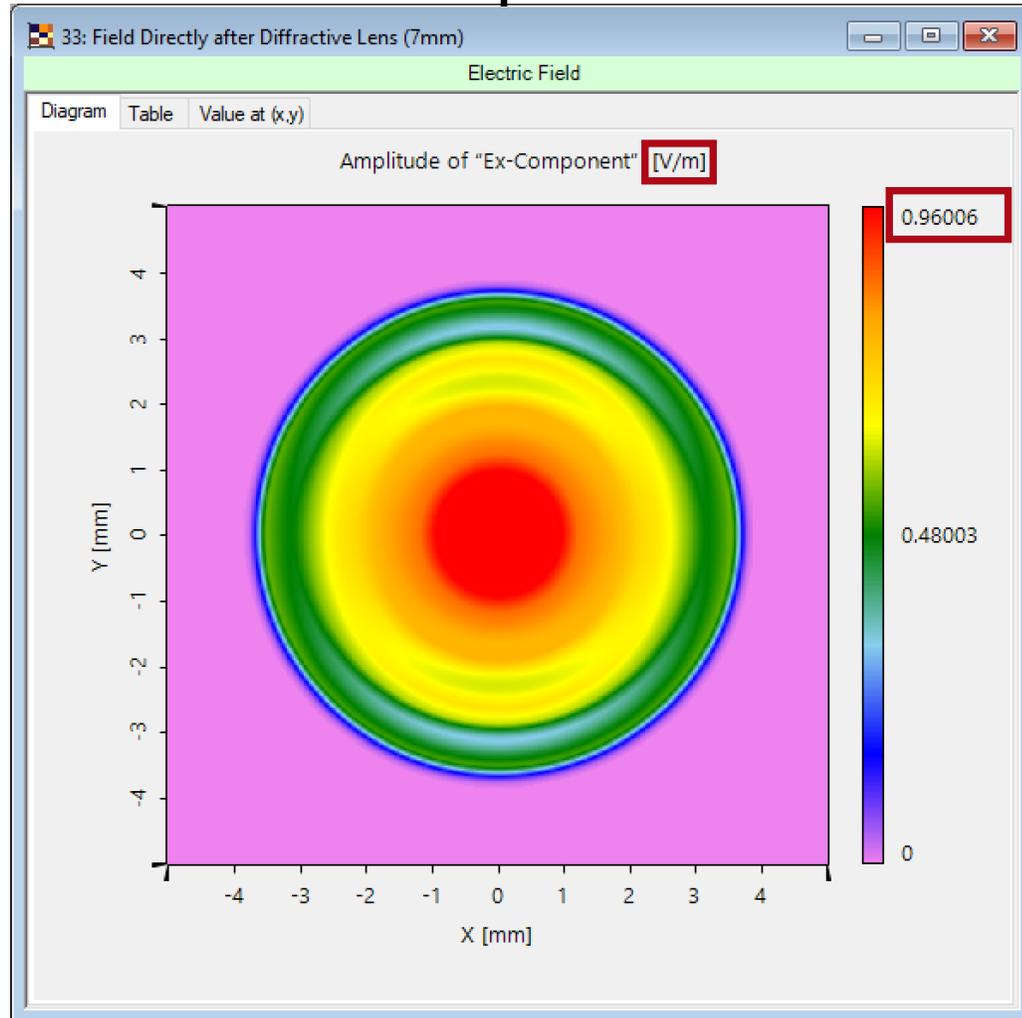
- Amplitudes in Focus (Same scaling!)

Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing



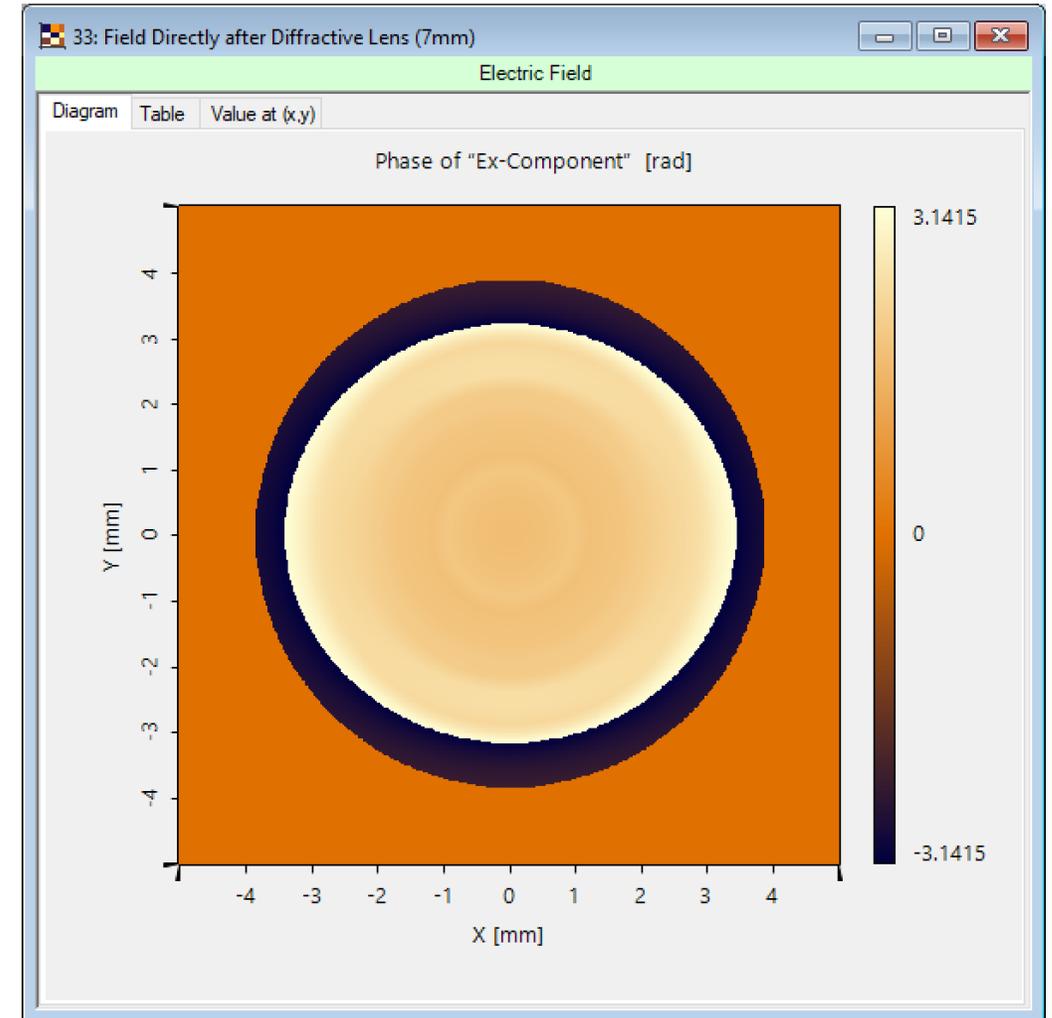
Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing

Amplitude



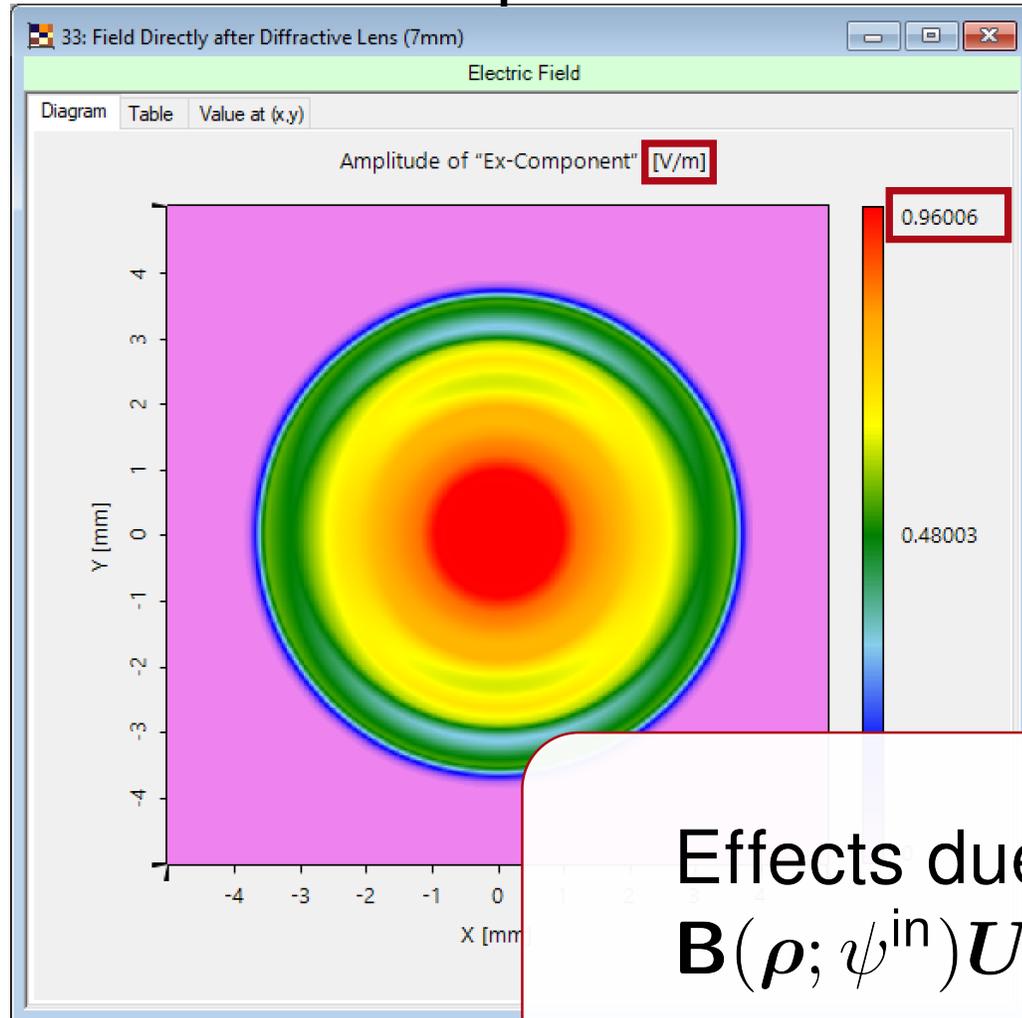
Ex

Phase



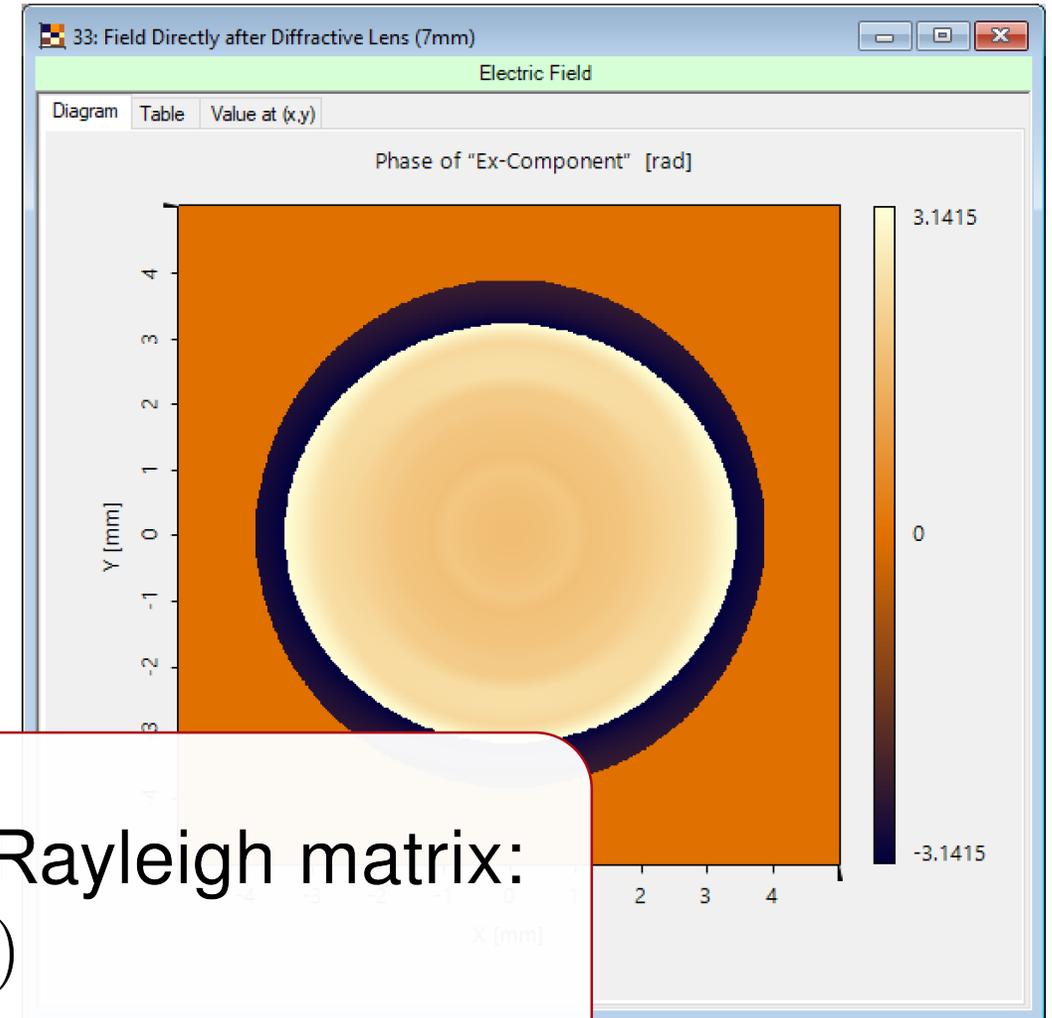
Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing

Amplitude



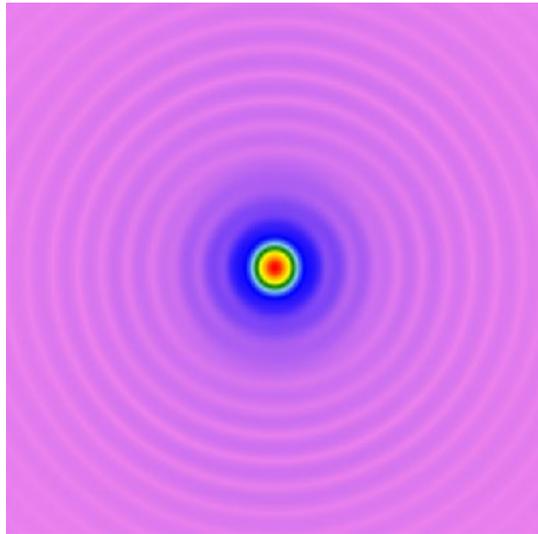
Ex

Phase

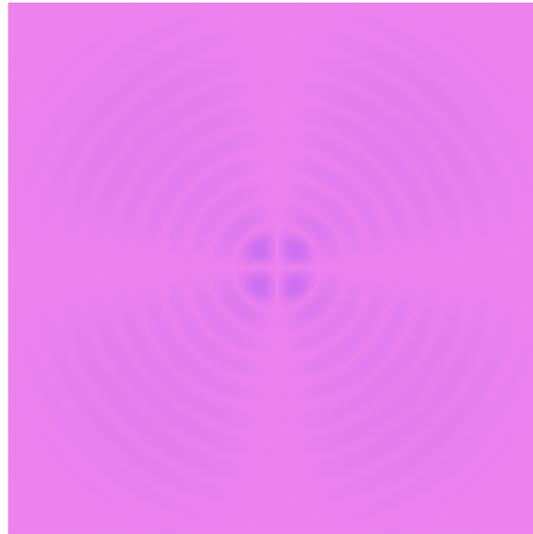


Effects due to Rayleigh matrix:
 $\mathbf{B}(\boldsymbol{\rho}; \psi^{\text{in}}) \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho})$

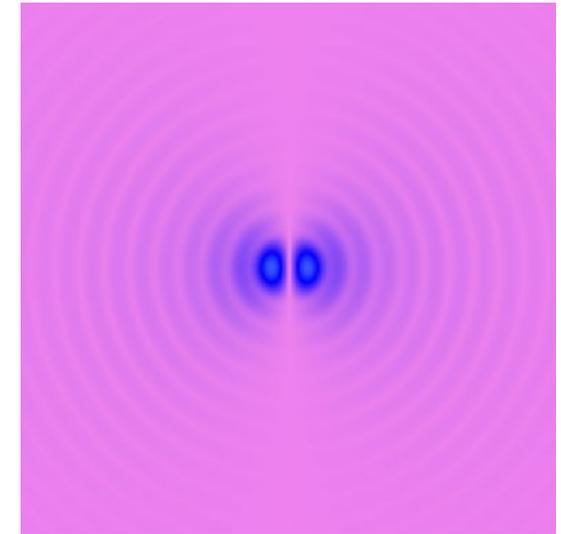
Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing



Amplitude Ex



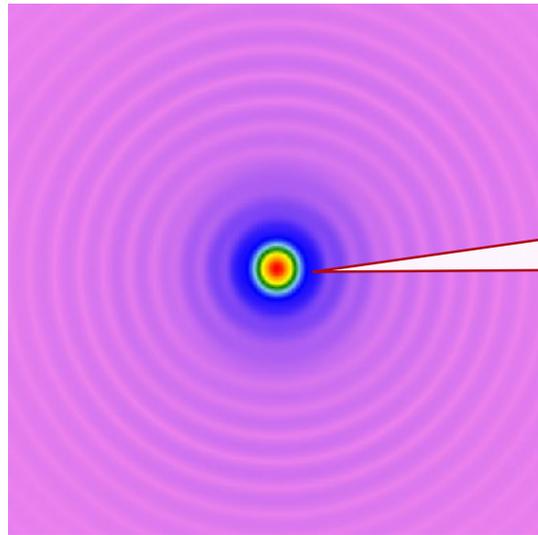
Amplitude Ey



Amplitude Ez

- Amplitudes in Focus (Same scaling!)

Focusing Diffractive Lens with NA=0.57: Ray and Field Tracing

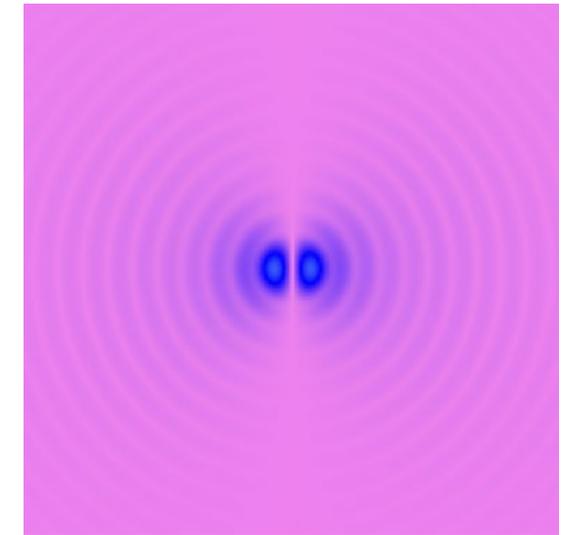


Amplitude Ex



Vectorial grating effects reduce spot quality

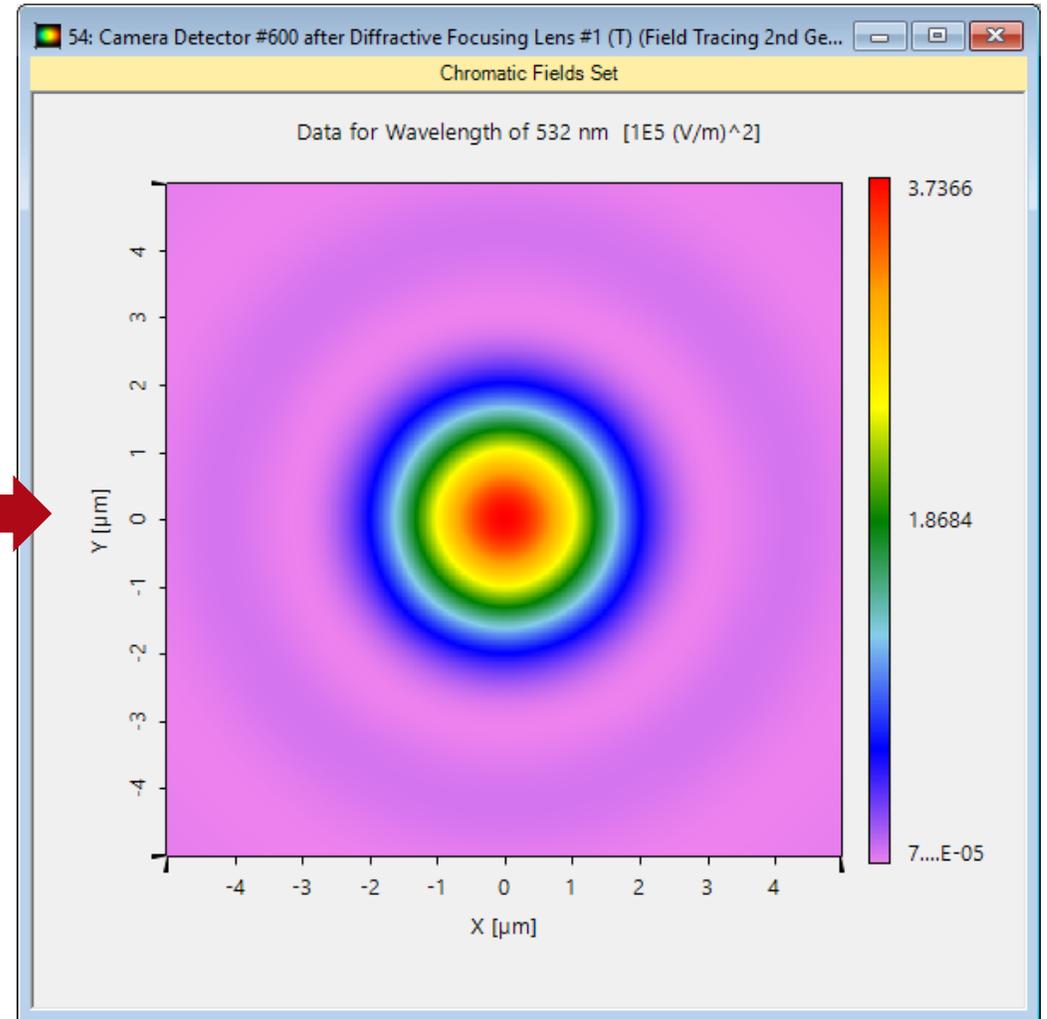
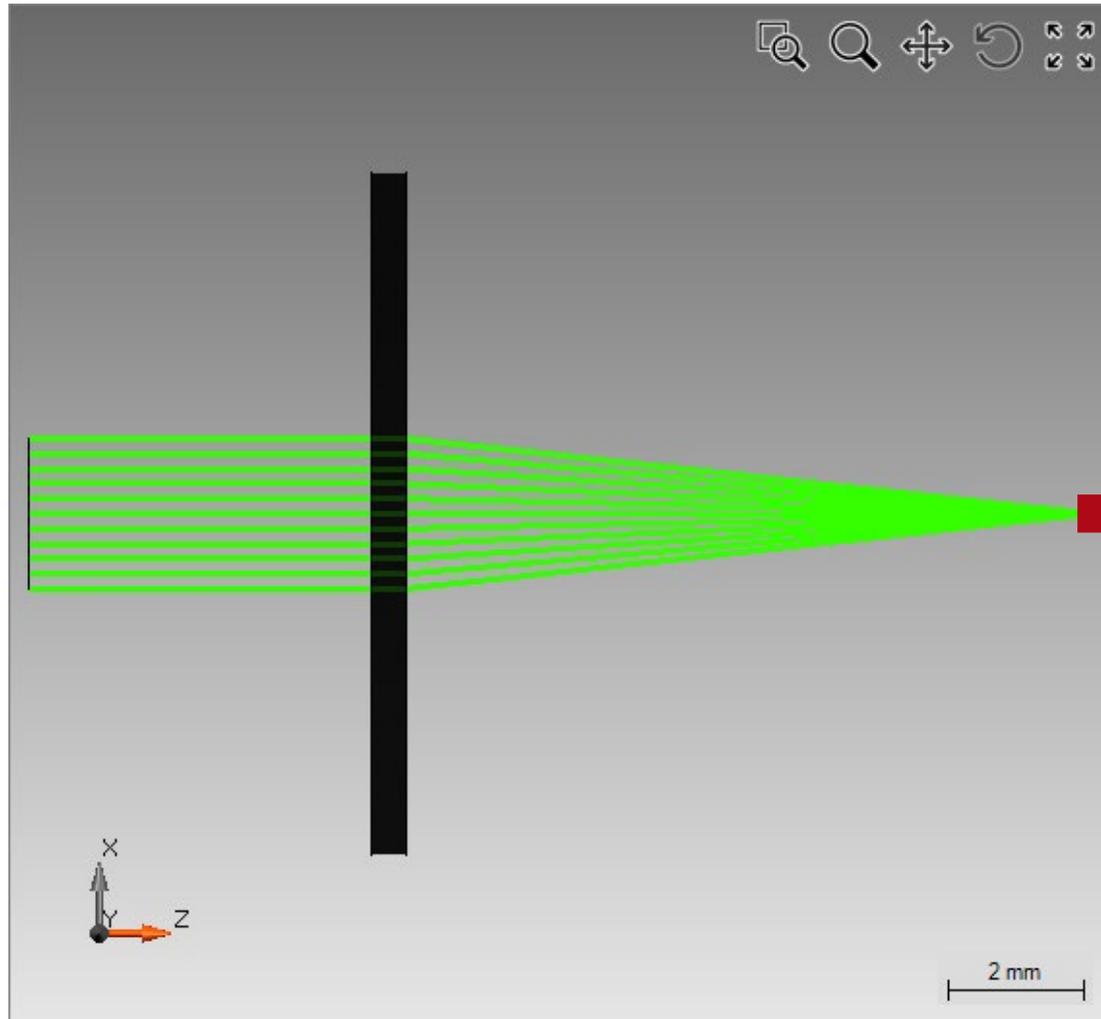
Amplitude Ey



Amplitude Ez

- Amplitudes in Focus (Same scaling!)

Lens with NA=0.1 and 8 Height Values: Ray and Field Tracing



Inclusion of Higher Orders

Design Wavelength

Height Scaling Factor

Use Profile Quantization

No. of Height Levels

Order for Simulation

Order
-2
-1
0
+1
+2

70: Ray Distribution 3D

3D View 2D View

2 mm

$$\mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \left\{ \mathbf{B}^{(1)}(\boldsymbol{\rho}; \psi^{\text{in}}) \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \right\} \exp(i\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho}))$$
$$+ \sum_{m=-\infty, m \neq 1}^{\infty} \left\{ \mathbf{B}^{(m)}(\boldsymbol{\rho}; \psi^{\text{in}}) \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \right\} \exp(i\psi^{\text{in}}(\boldsymbol{\rho}) + m\Delta\psi(\boldsymbol{\rho}))$$

Inclusion of Higher Orders

Design Wavelength

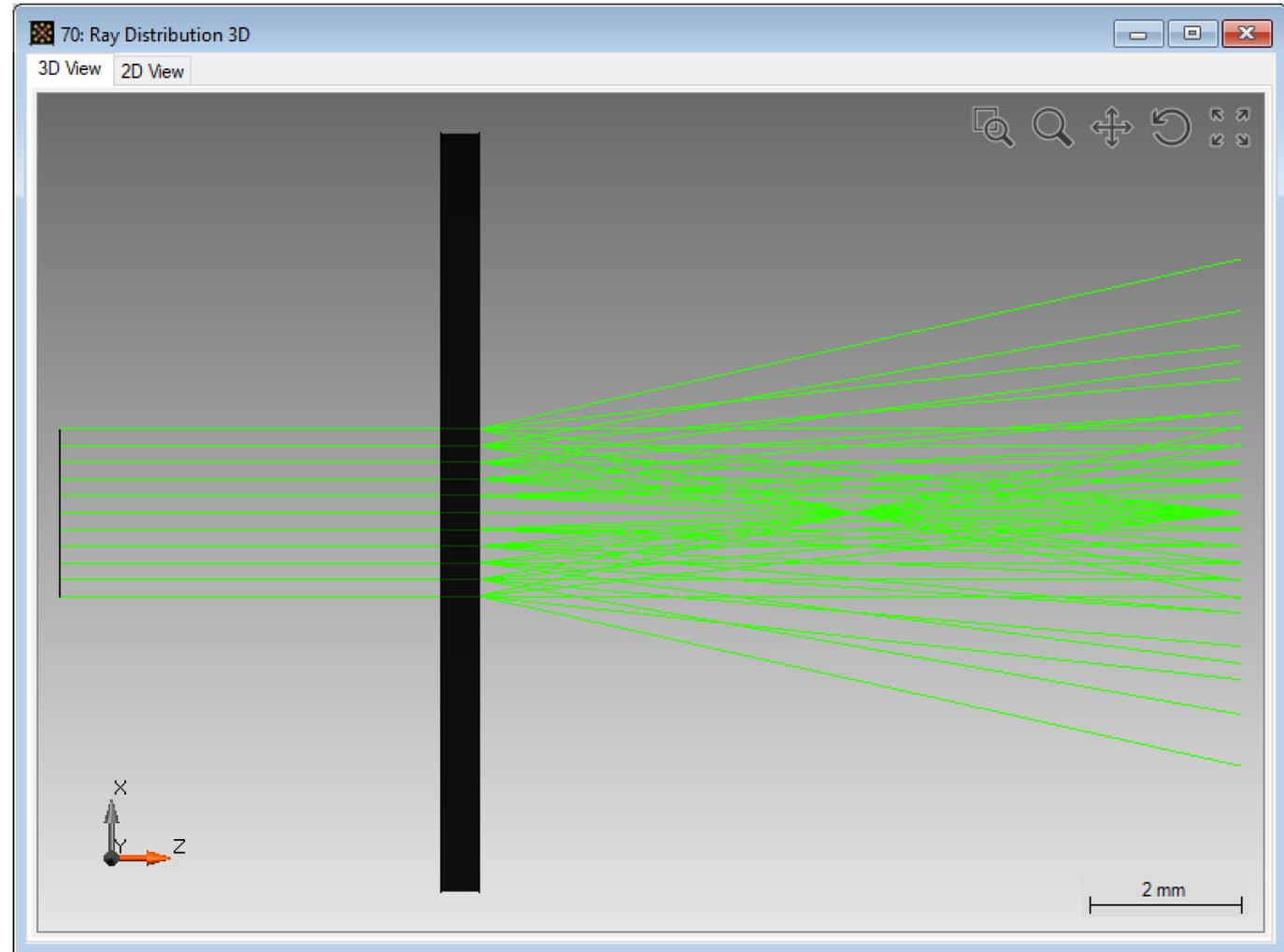
Height Scaling Factor

Use Profile Quantization

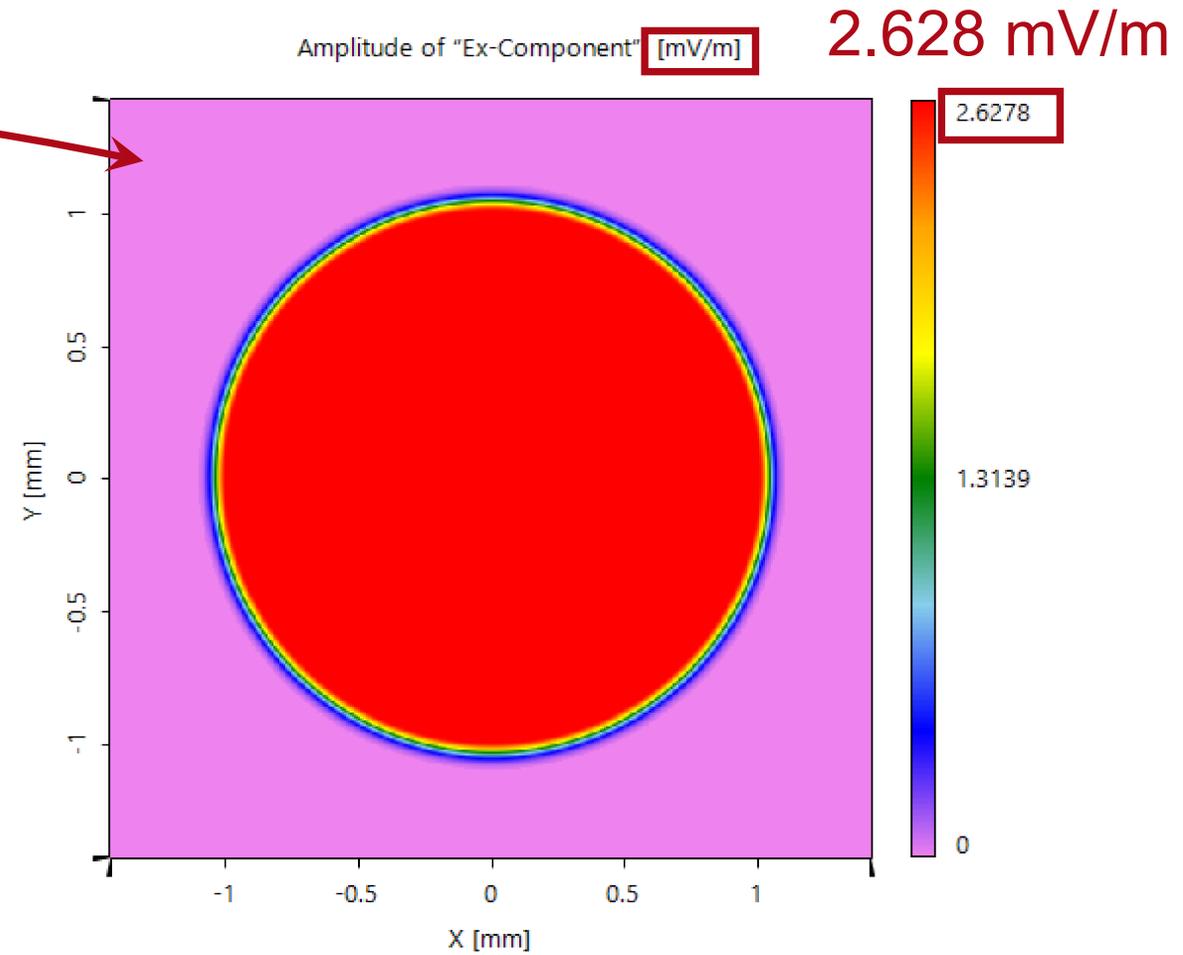
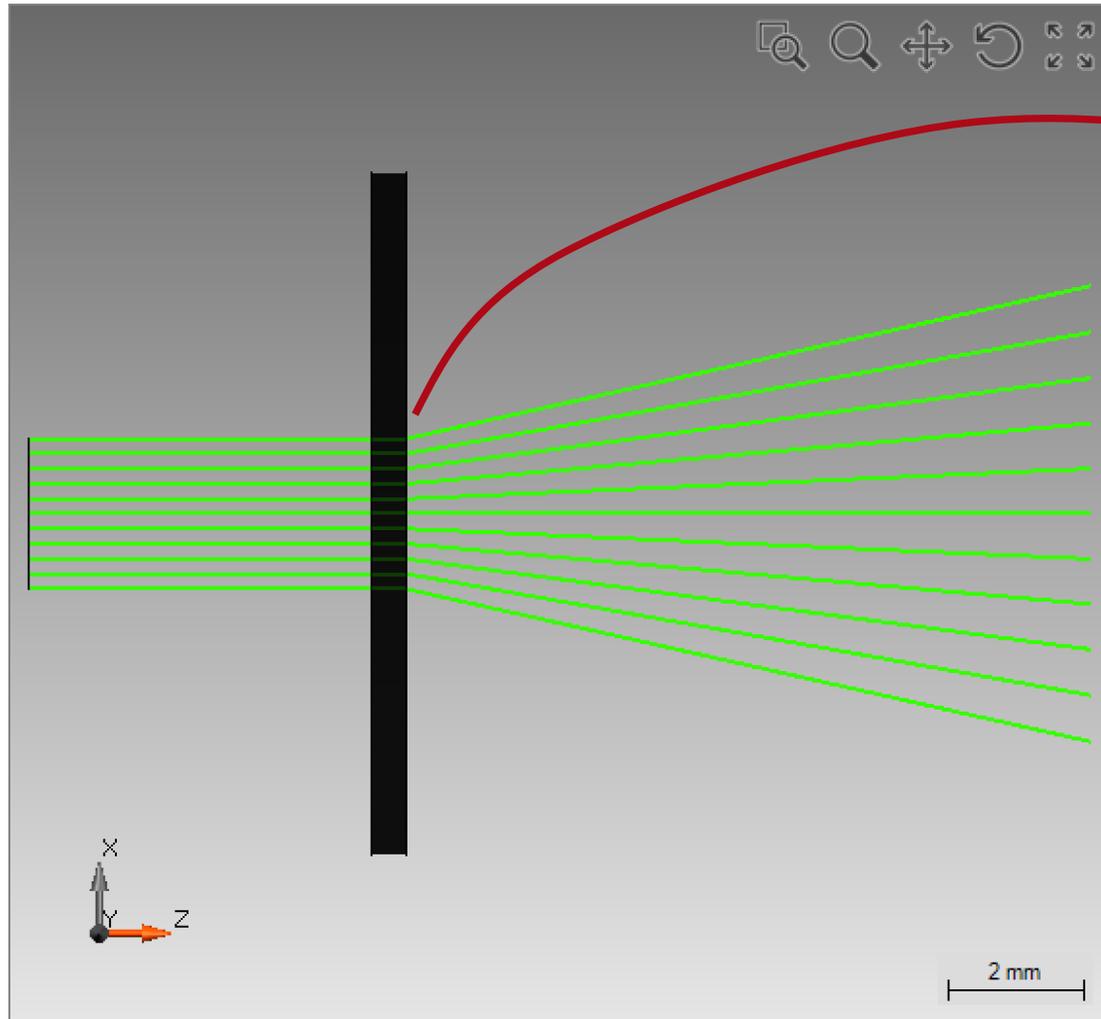
No. of Height Levels

Order for Simulation

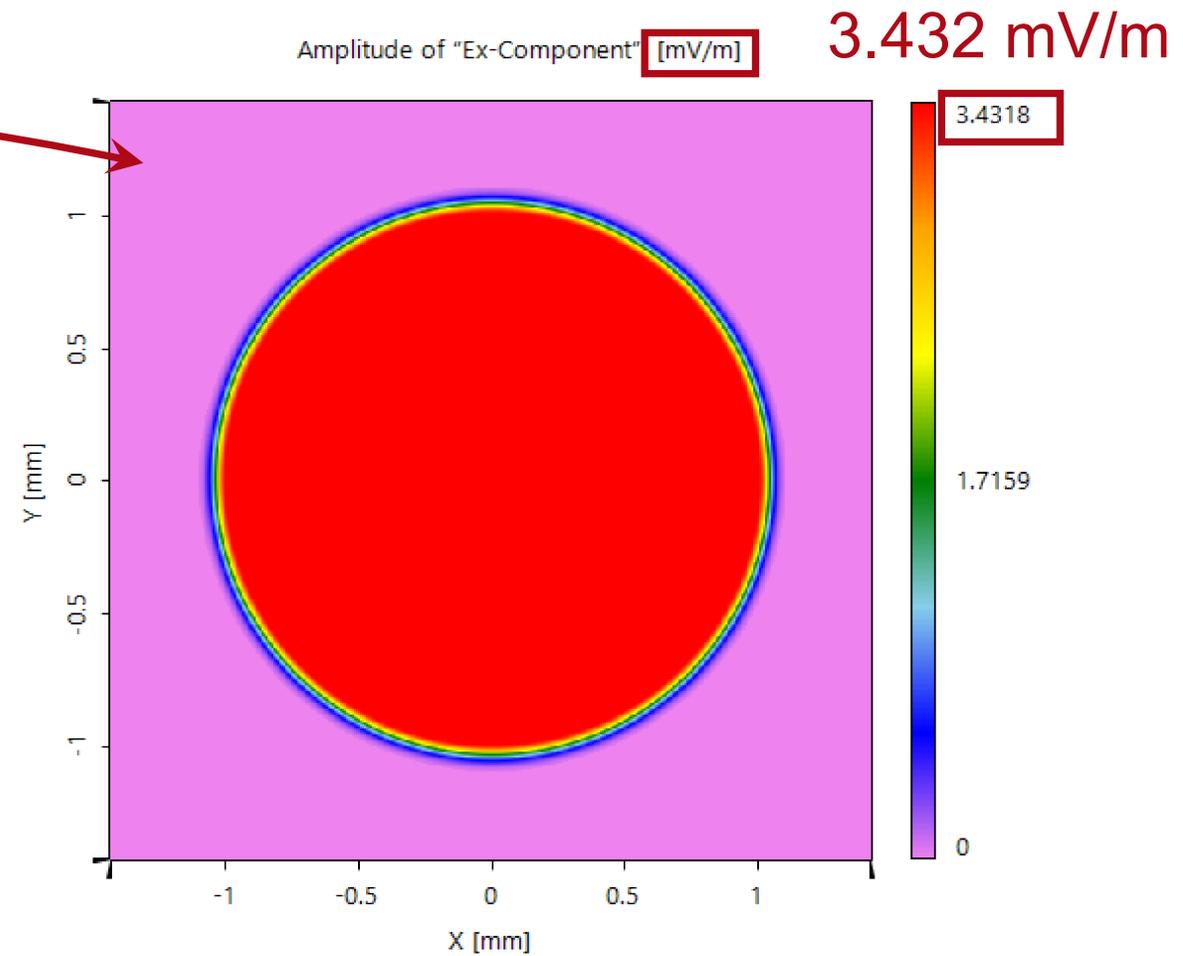
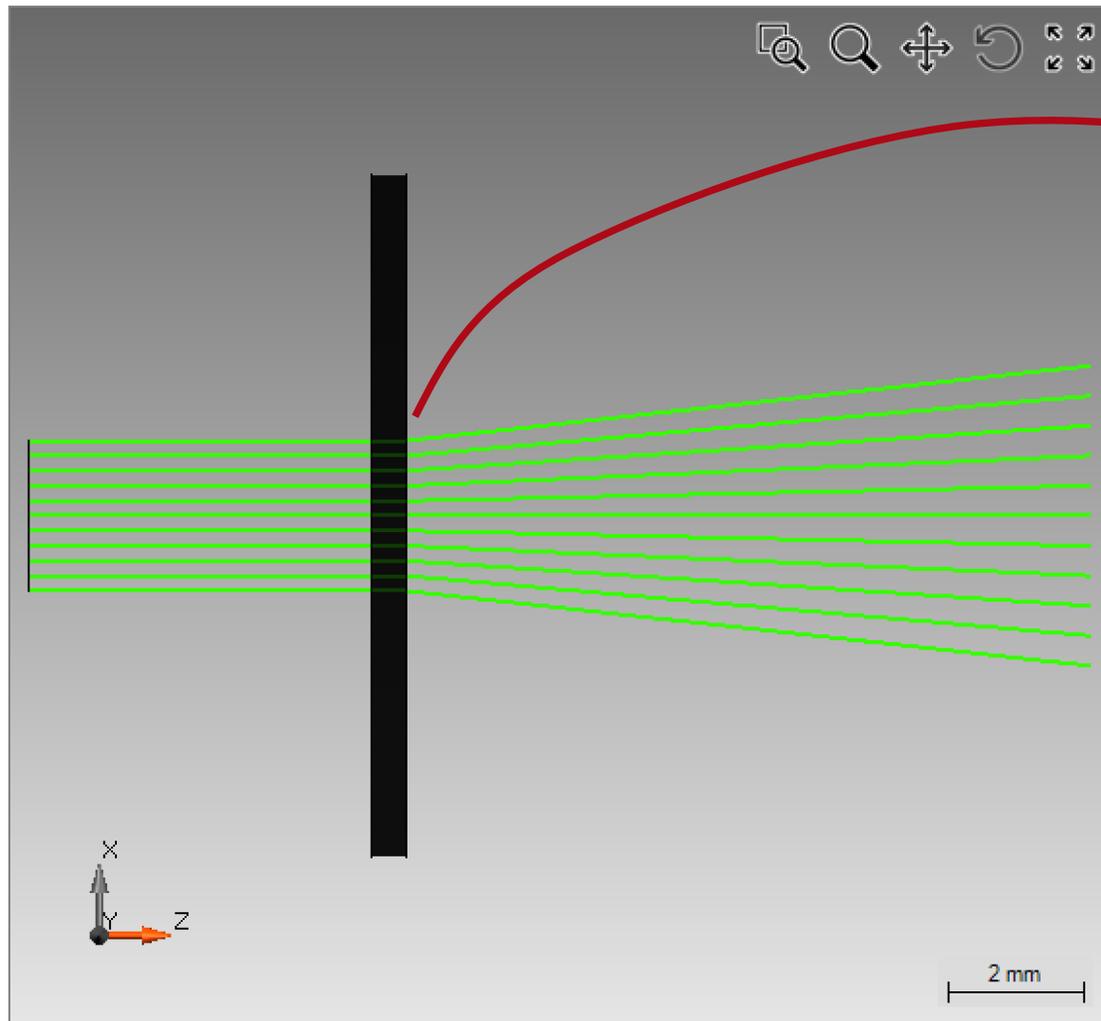
Order
-2
-1
0
+1
+2



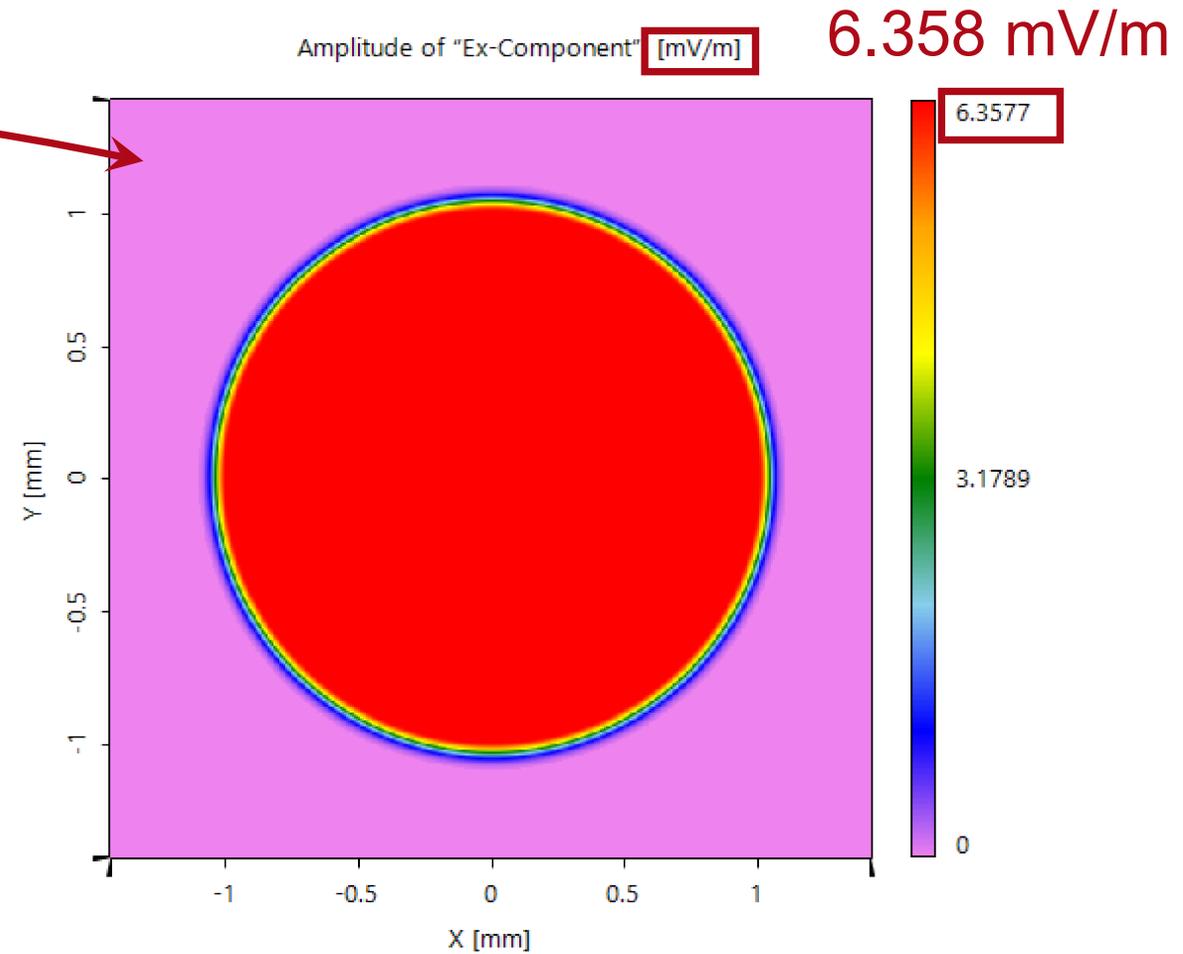
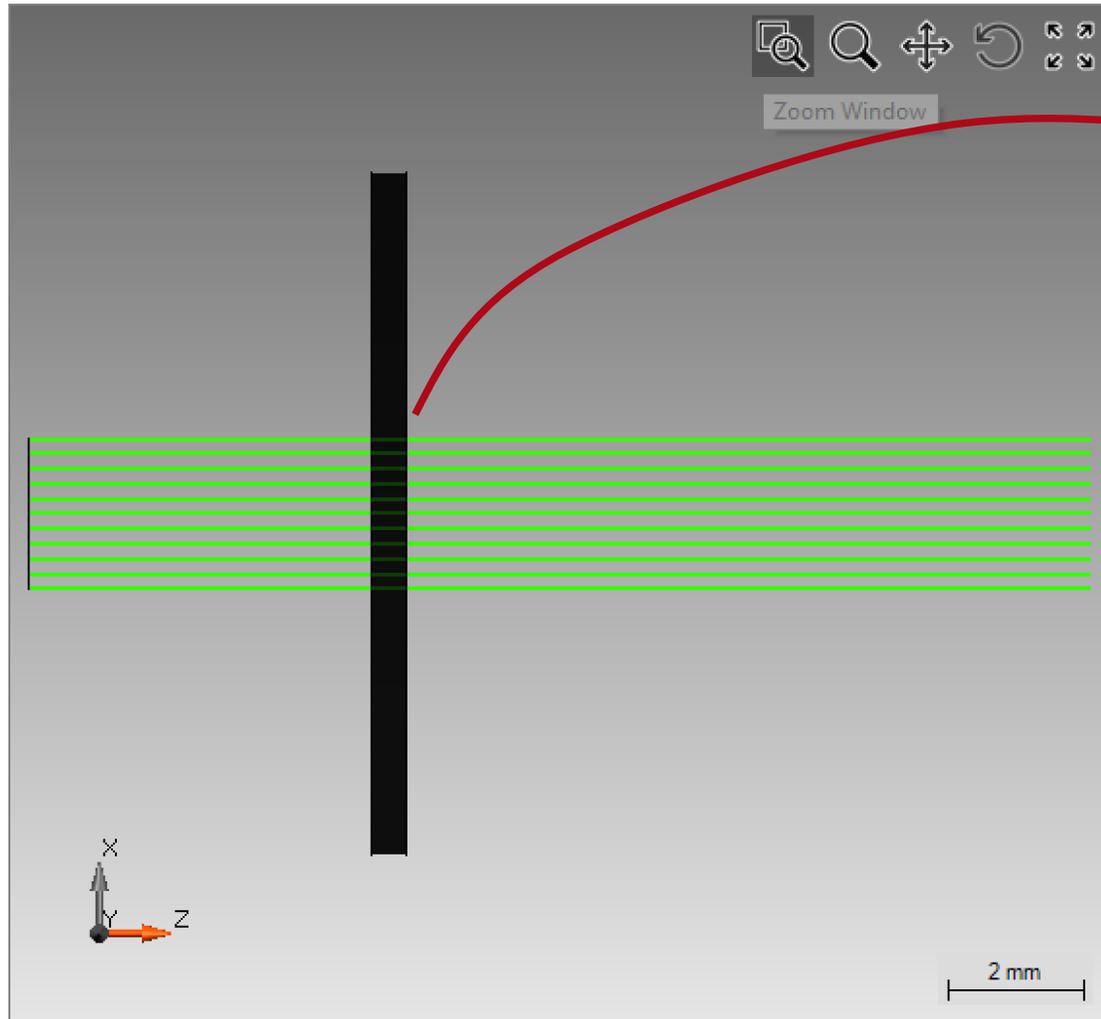
Higher Orders (8 Level): -2nd Order



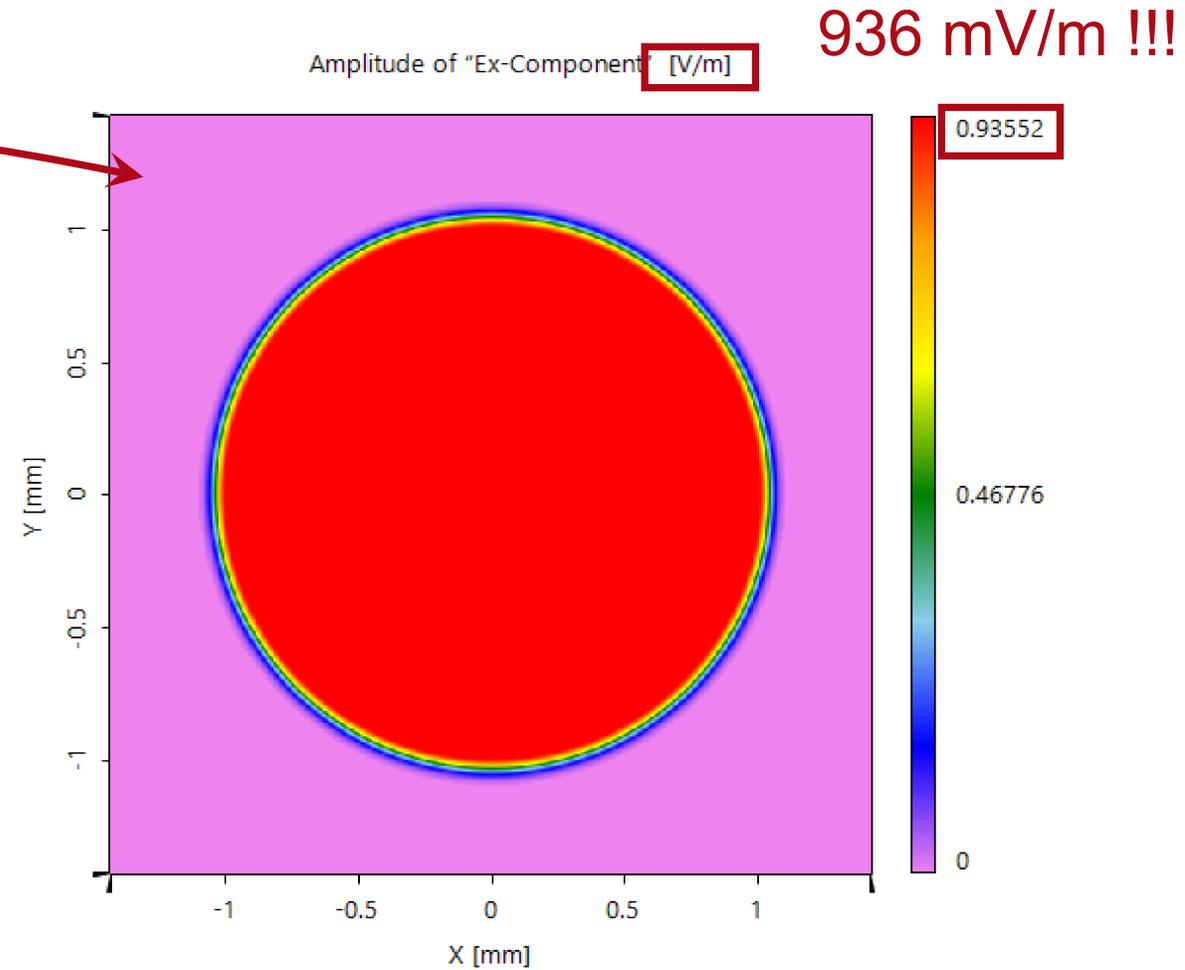
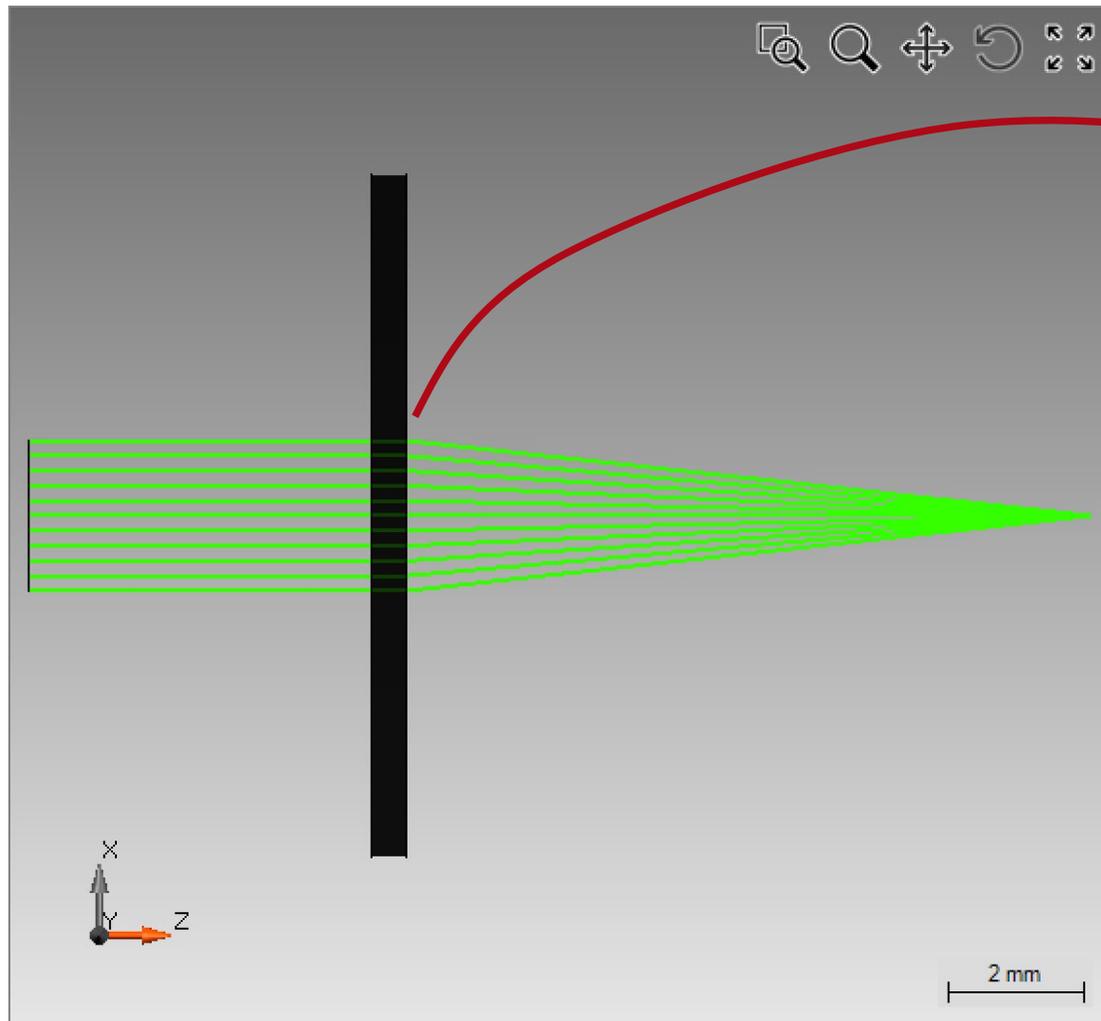
Higher Orders (8 Level): -1st Order



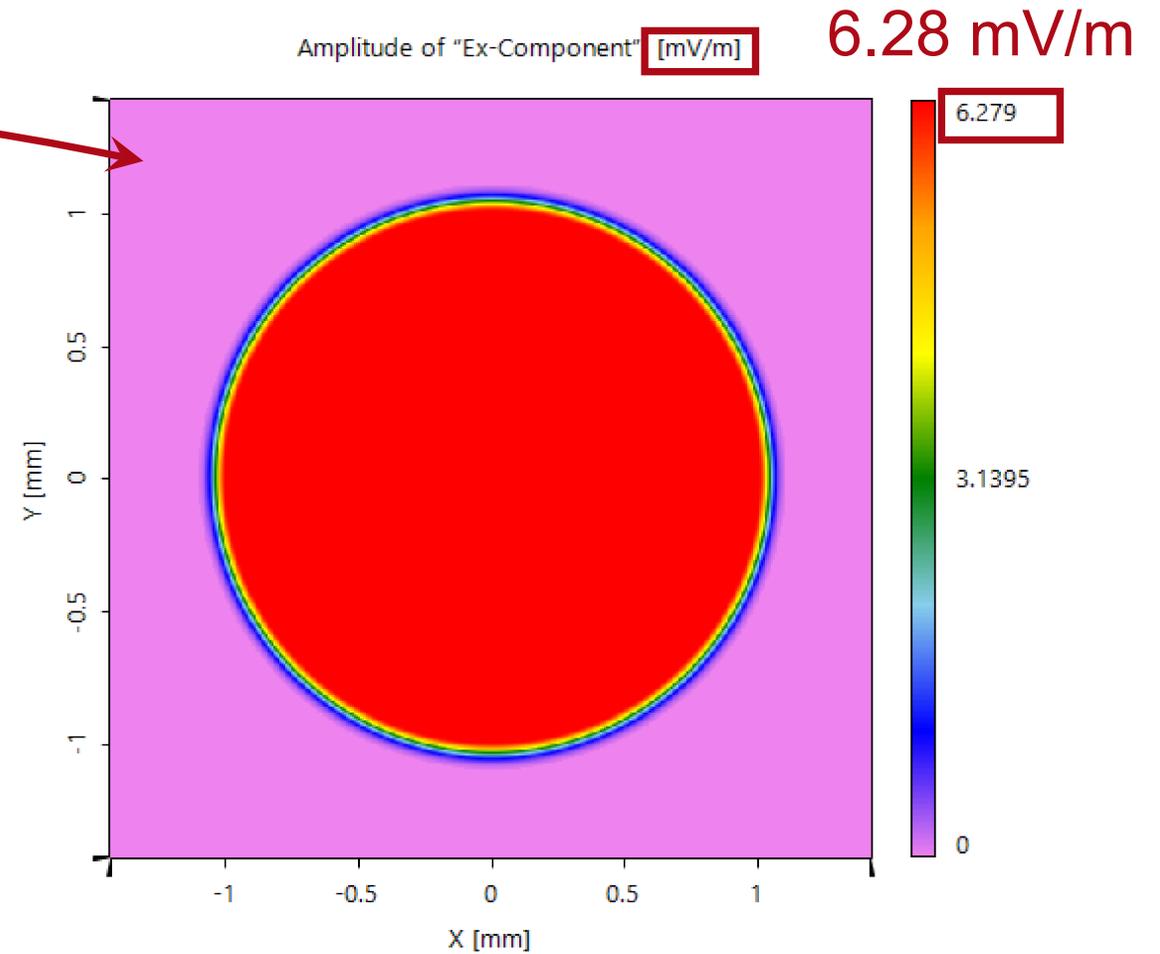
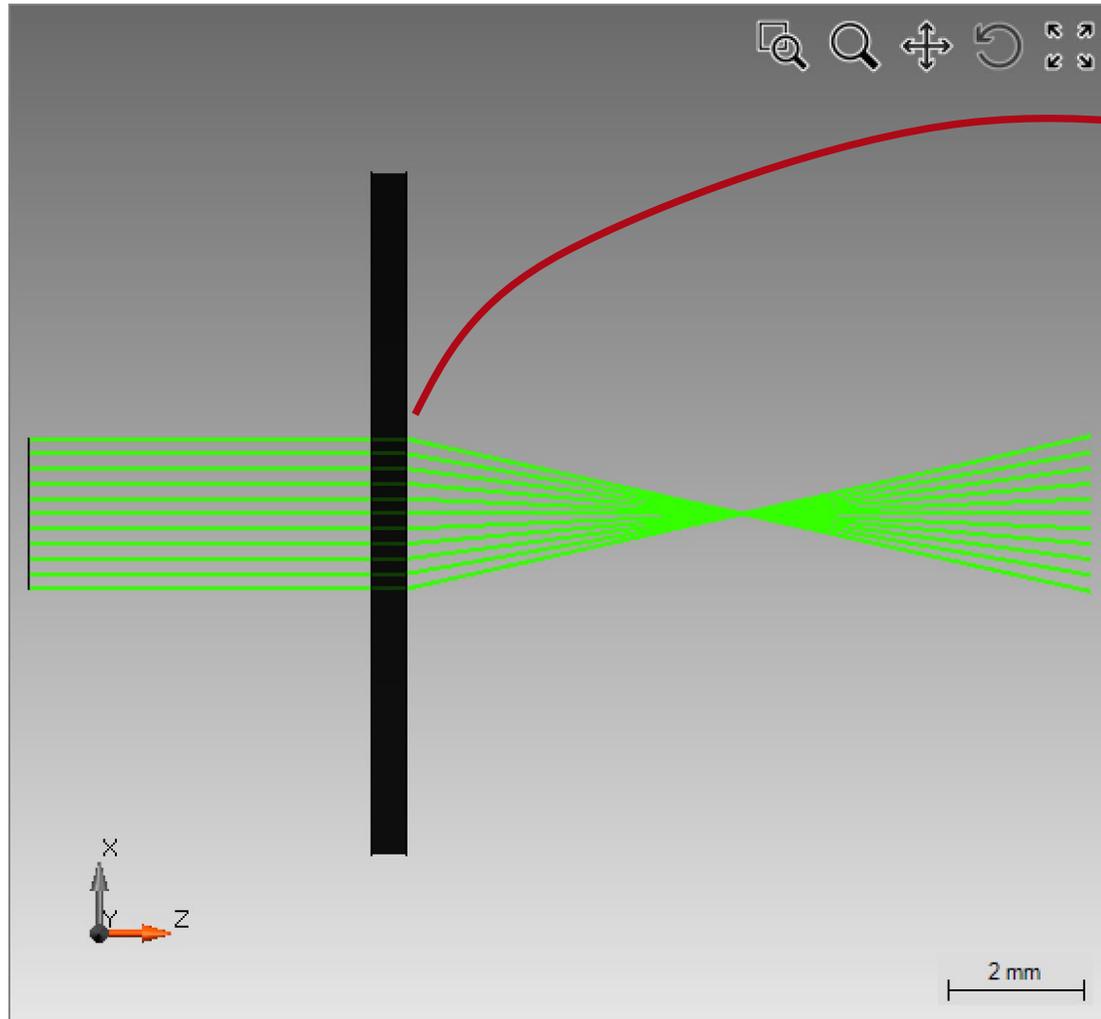
Higher Orders (8 Level): 0th Order



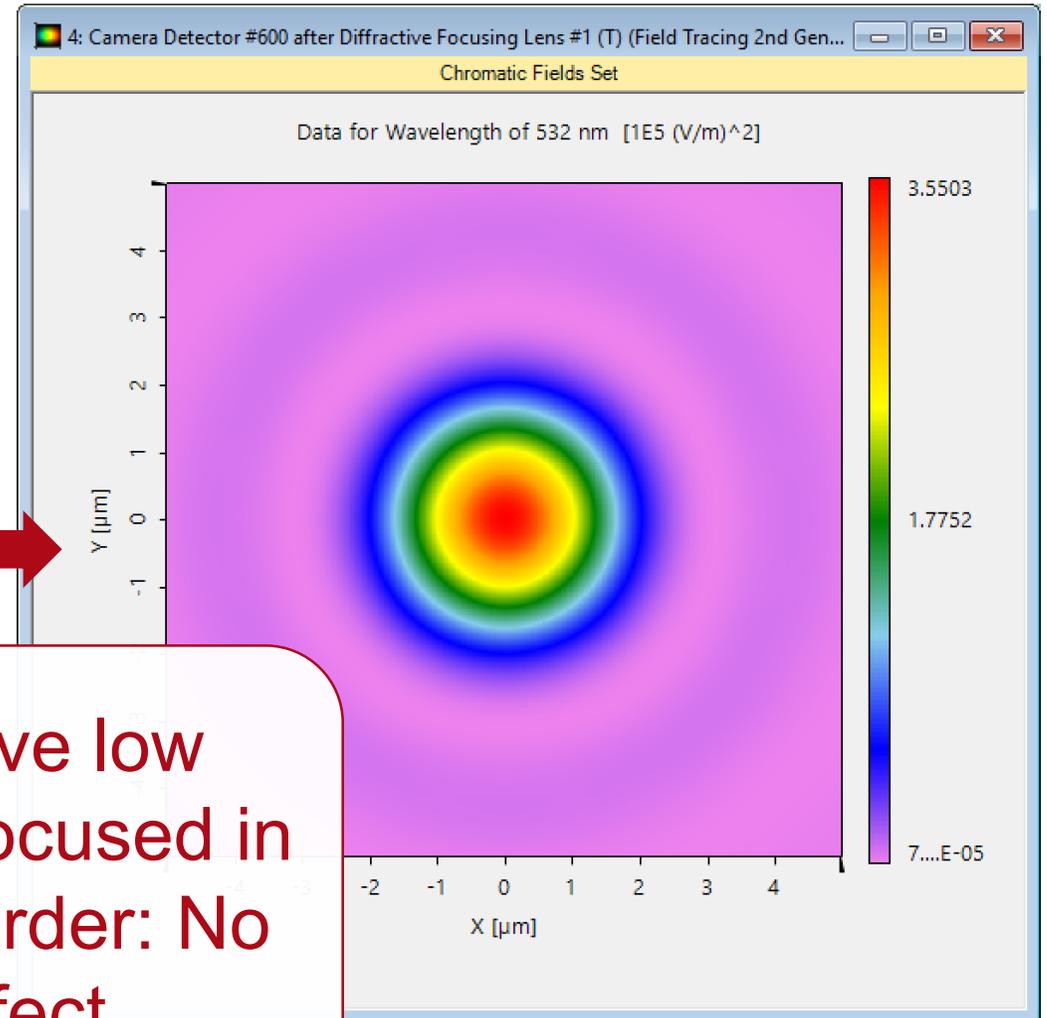
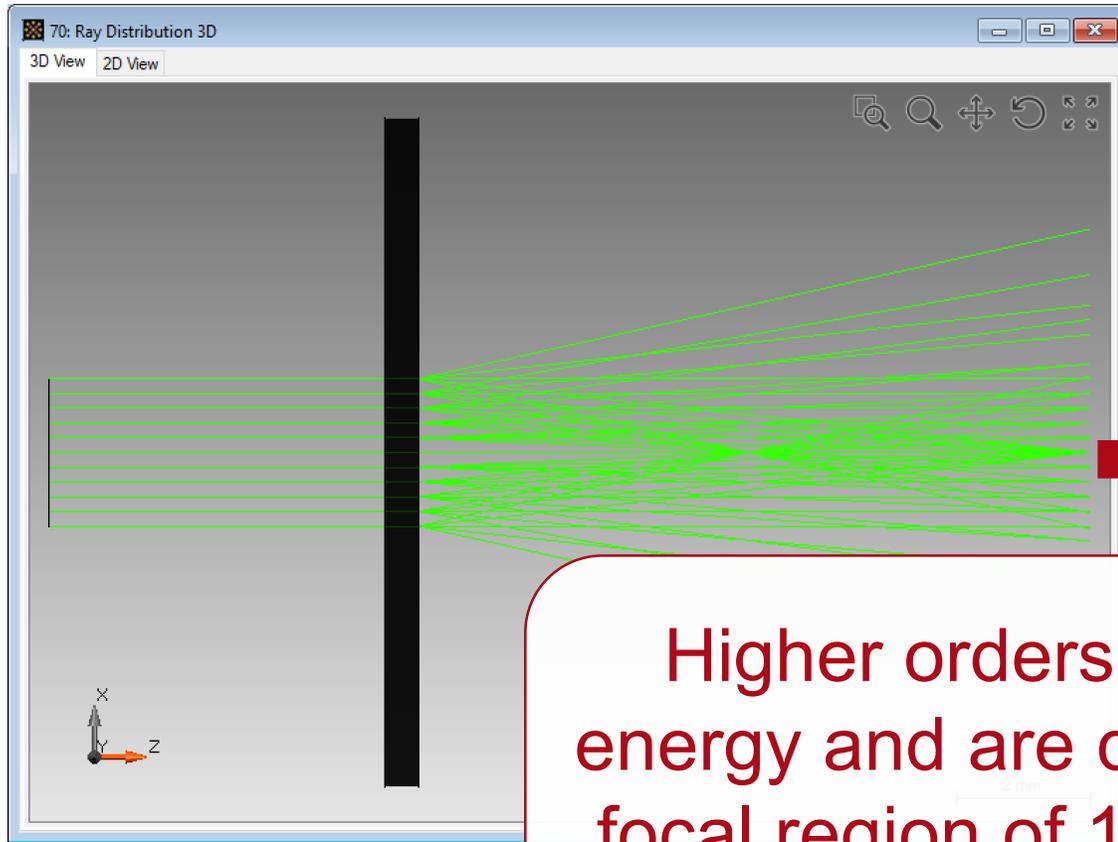
Higher Orders (8 Level): 1st Order (Working Order)



Higher Orders (8 Level): +2nd Order



Lens with NA=0.1 and 8 Height Values: Ray and Field Tracing

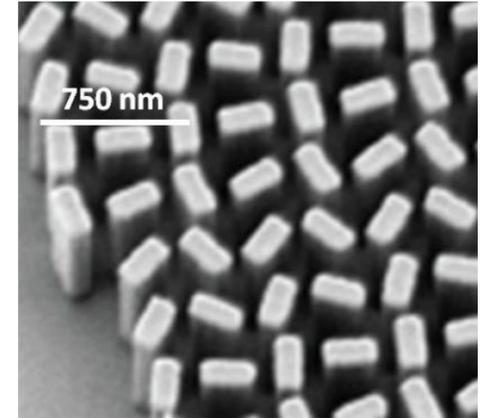


Higher orders have low energy and are defocused in focal region of 1st order: No detrimental effect

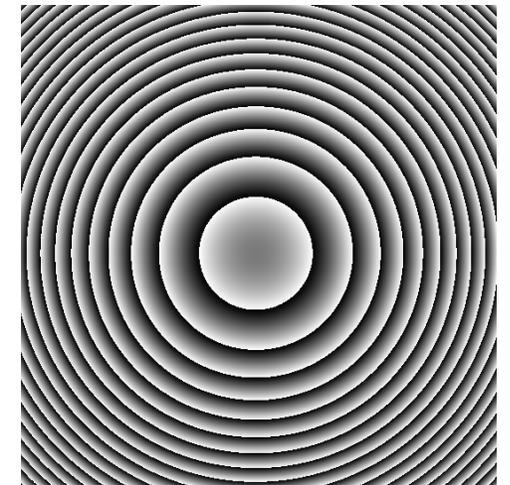
Standard Workflow

- Design of system with optional inclusion of ideal Wavefront Surface Response (WSR) function.
- Decision about most suitable flat optics layer to realize WSR.
- Design of layer structure.
- Analysis of the performance and detrimental additional subfields.

Example VR/AR
projection approach

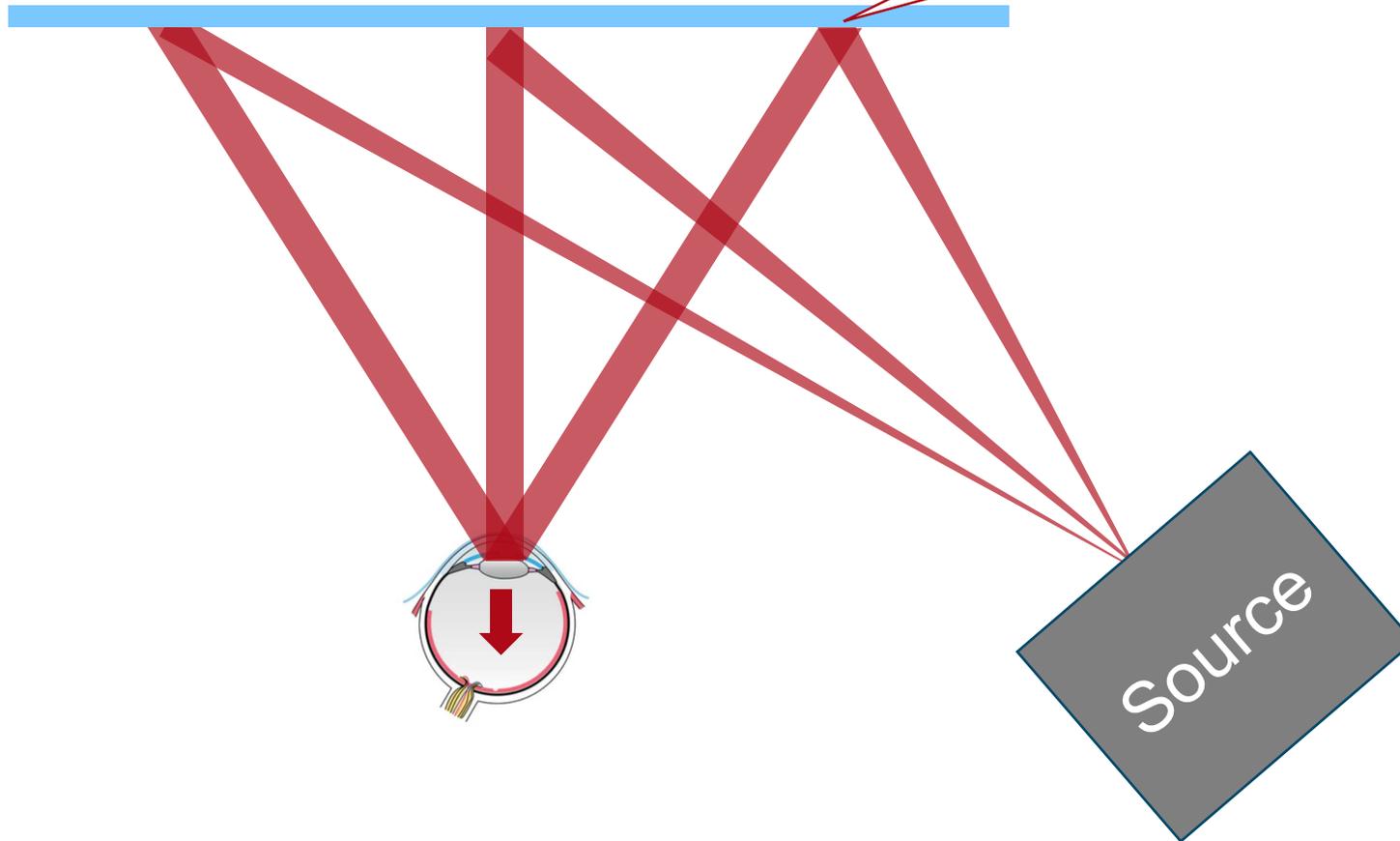


M. Khorasaninejad *et al.*,
Science **352**, 1190-1194 (2016).

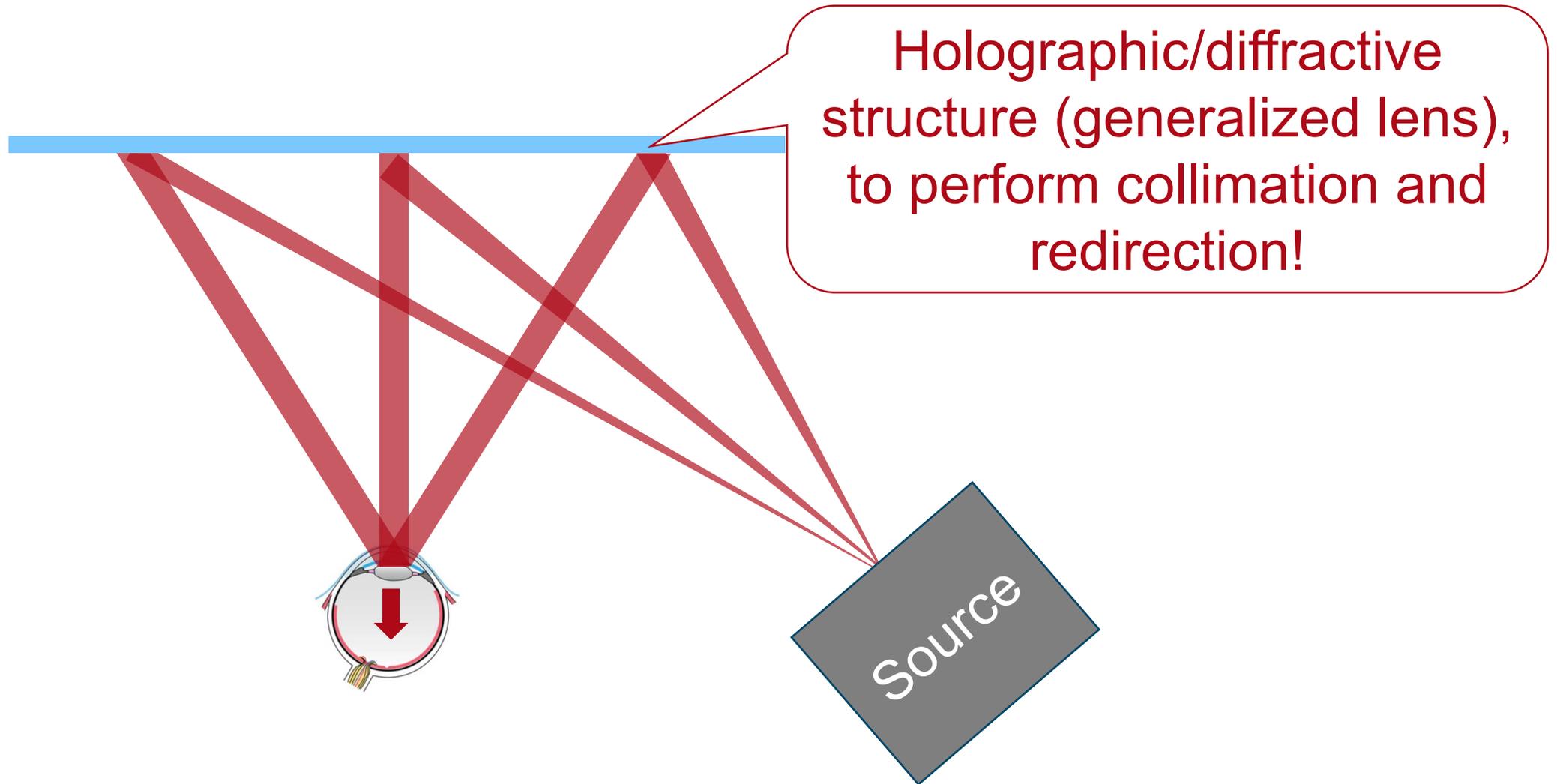


Projection Approach in AR/VR

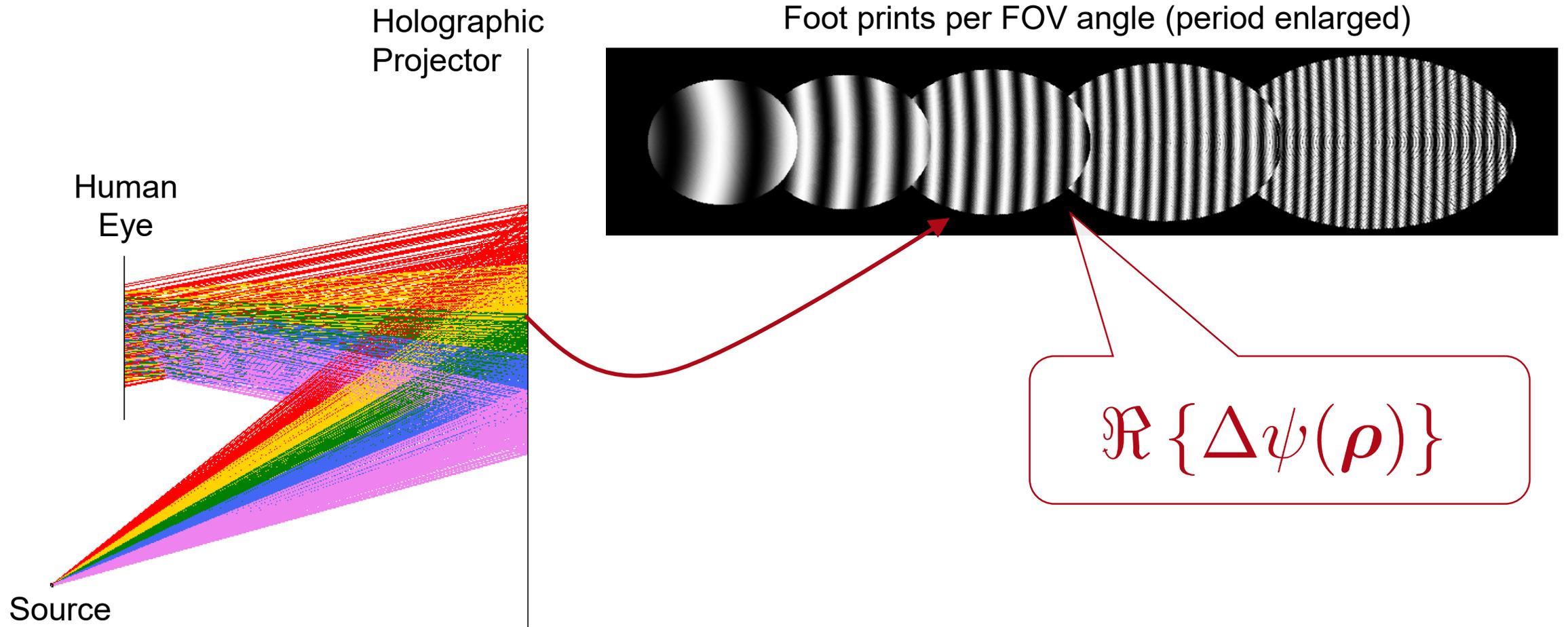
Local optical function:
Collimation and
redirection of input beam.



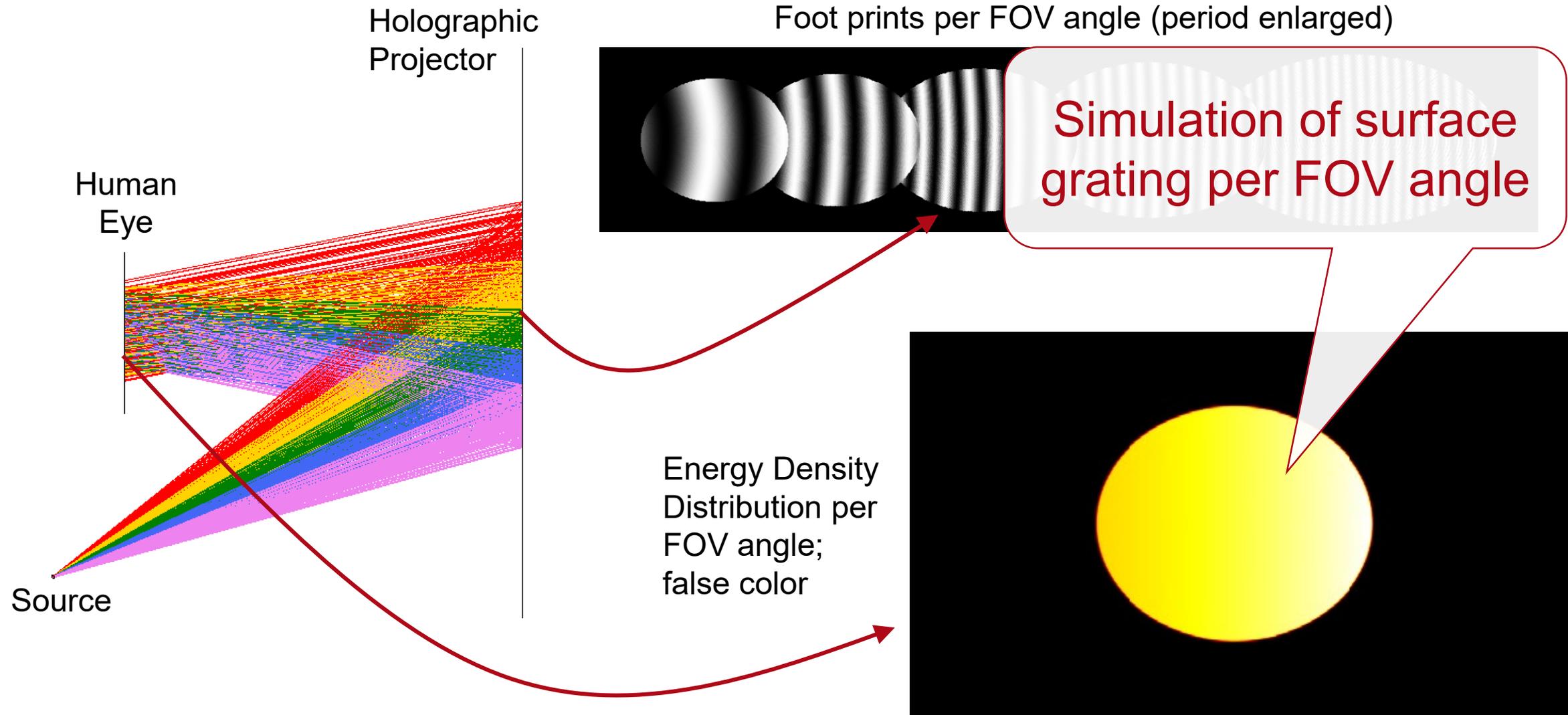
Projection: Holographic/Diffractive Lens Approach



Design and Simulation for Different FOV Angles

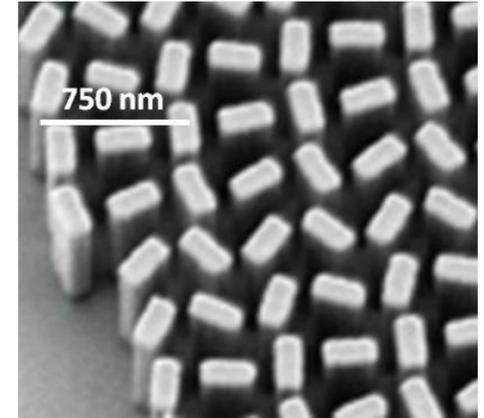


Design and Simulation for Different FOV Angles

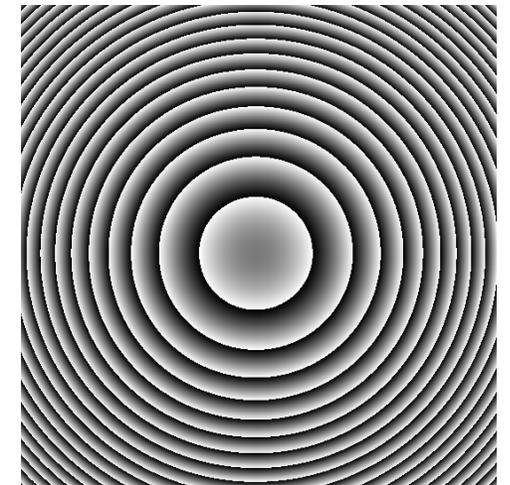


Standard Workflow

- Design of system with optional inclusion of ideal Wavefront Surface Response (WSR) function.
- Decision about most suitable flat optics layer to realize WSR.
- Design of layer structure.
- Analysis of the performance and detrimental additional subfields.
- Tolerancing.
- Export of fabrication data.



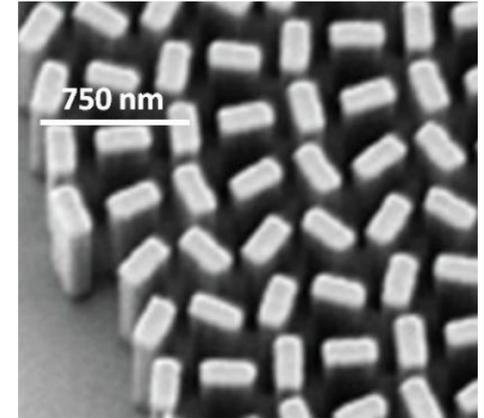
M. Khorasaninejad *et al.*,
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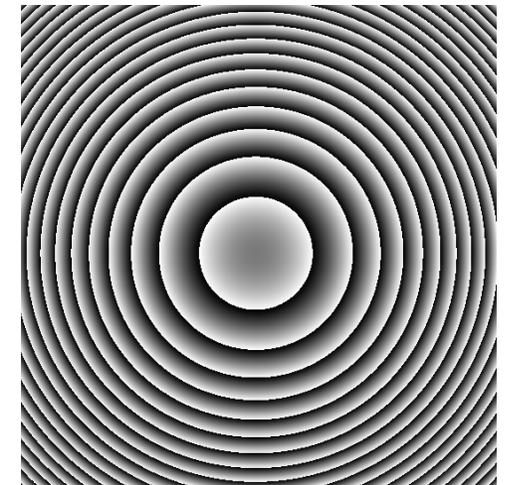
Standard Workflow

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Combination with OpticStudio® via Binary Surfaces, which represent a special form of a WSR.



M. Khorasaninejad *et al.*,
Science **352**, 1190-1194 (2016).



Structure of Workshop

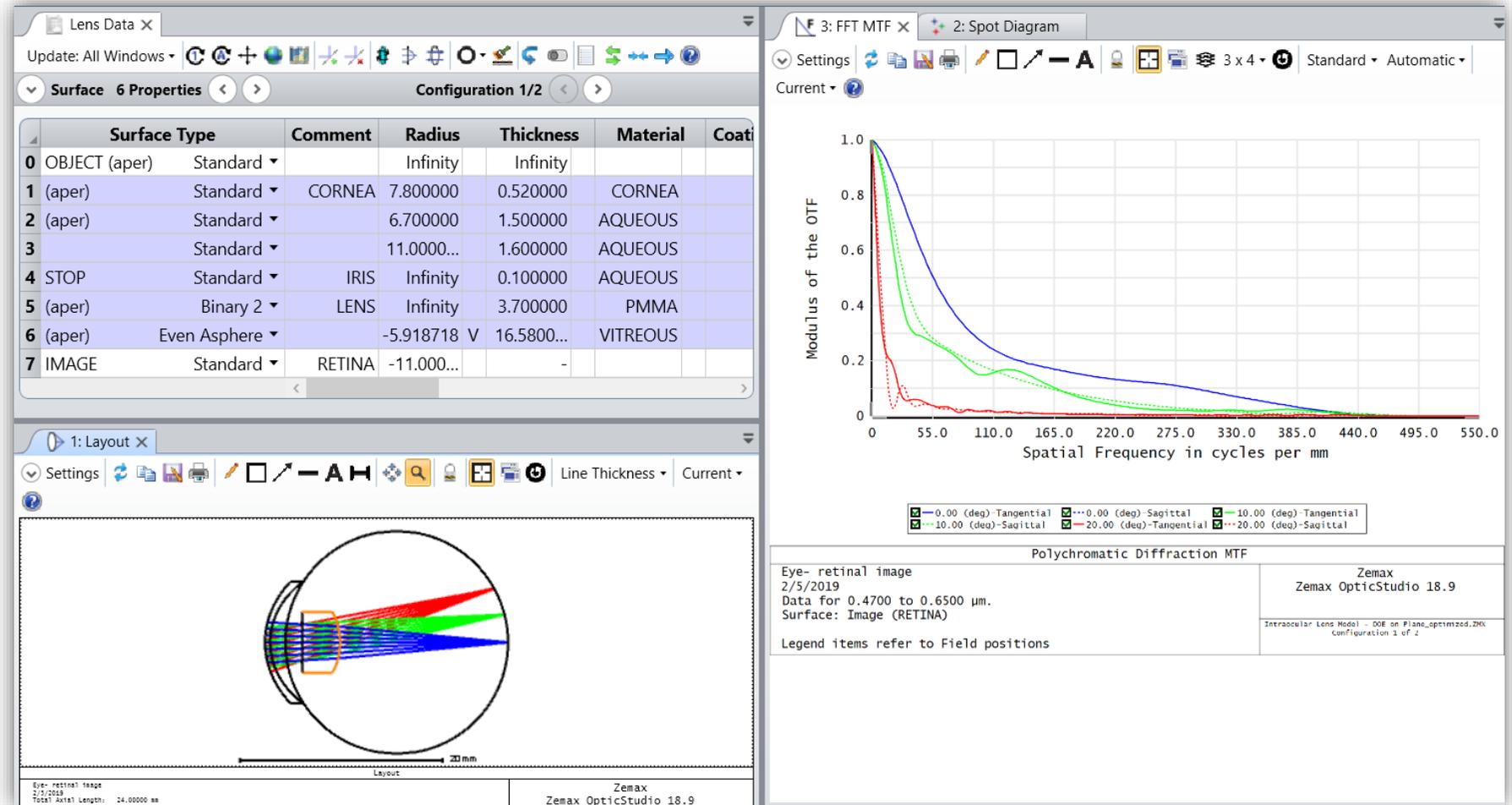
- Introduction of theory
 - Frank Wyrowski
- **Design of binary surfaces in OpticStudio®**
 - **Akil Bhagat**
- Physical-optics analysis of imported systems from OpticStudio: Diffractive lenses
 - Roberto Knoth
- Metalenses theory and modeling
 - Site Zhang
- Fabrication export
 - Roberto Knoth

Binary Surfaces in OpticStudio®

Wavefront Surface Response in OpticStudio®

Intraocular Lens Model using OpticStudio

- Using the Binary 2 surface to model the lens
- Setup and design
- Analysis
- Next Steps



Binary 2 Surface

- In this system, the diffractive intraocular lens is modelled using a Binary 2 surface
- The Binary 2 surface has an even asphere base shape, with a grating period defined by a rotationally symmetric polynomial
 - Even Asphere sag:

$$z = \frac{cr^2}{1 + \sqrt{1 - (1+k)c^2r^2}} + \alpha_1r^2 + \alpha_2r^4 + \alpha_3r^6 + \alpha_4r^8 + \alpha_5r^{10} + \alpha_6r^{12} + \alpha_7r^{14} + \alpha_8r^{16}$$

- Additional Phase added by the grating:

$$\Phi = M \sum_{i=1}^N A_i \rho^{2i}$$

- For a complete definition of these terms, please see the Binary 2 OpticStudio help file or the article below:
 - <https://customers.zemax.com/os/resources/learn/knowledgebase/how-diffractive-surfaces-are-modeled-in-zemax>

Setup and Design

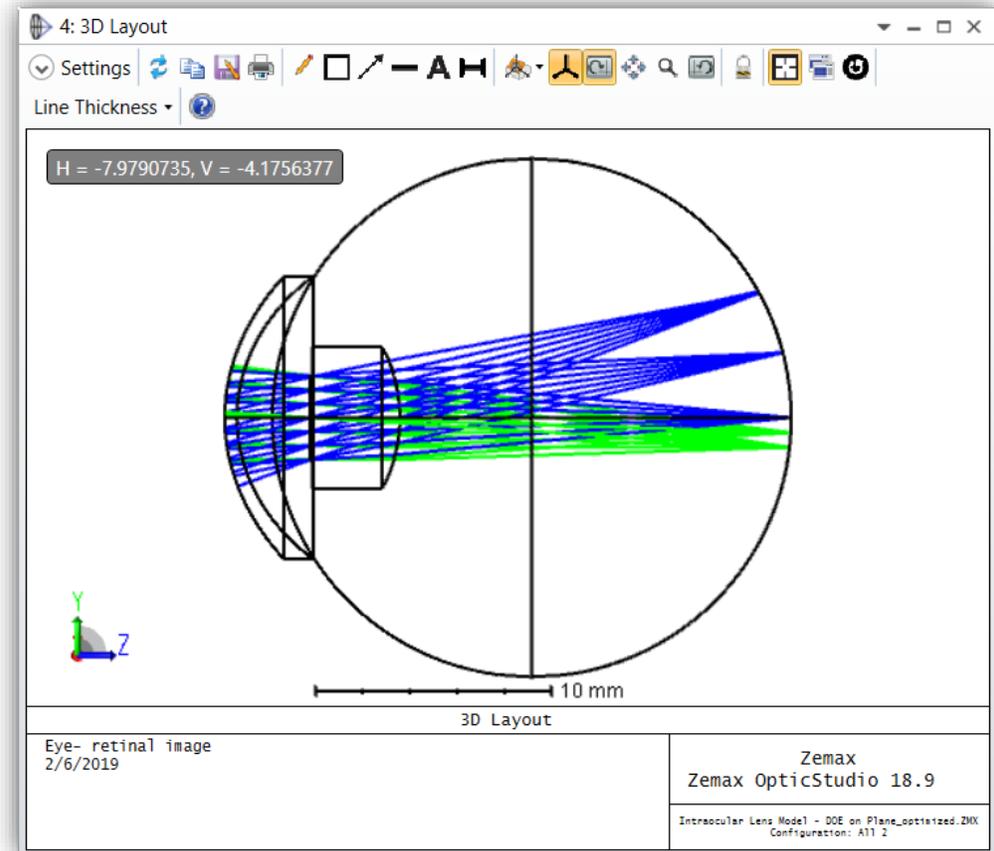
- OpticStudio models only 1 diffractive order at a time, which means we will utilize multiple configurations in this file to model 2 orders

Multi-Configuration Editor

Update: All Windows

Operand 1 Properties Configuration 1/2

	Active : 1/2	Config 1*	Config 2
1	PRAM ▾ 5/0	0.000000	1.000000
2	FLTP ▾ -	0	1
3	THIC ▾ 0	1.000000E+10	250.000000
4	YFIE ▾ 2	10.000000	10.000000
5	YFIE ▾ 3	20.000000	20.000000



Setup and Design

- Once the initial system is setup, OpticStudio can be used to optimize the system to achieve the best performance for both configurations
- This system was optimized for minimum RMS spot size along both configurations using the built in Optimization Wizard

Wizards and Operands Merit Function: Click Update to Calculate

Optimization Wizard
Current Operand (1)

Optimization Function
Criterion: **Spot**
Spatial Frequency: 30
X Weight: 1
Y Weight: 1
Type: RMS
Reference: Centroid

Pupil Integration
 Gaussian Quadrature
 Rectangular Array
Rings: 3
Arms: 6
Obscuration: 0

Boundary Values
 Glass Min: 0 Max: 1e+03 Edge Thickness: 0
 Air Min: 0 Max: 1e+03 Edge Thickness: 0

Start At: 6
Overall Weight: 1
Configuration: **All**
Field: **All**

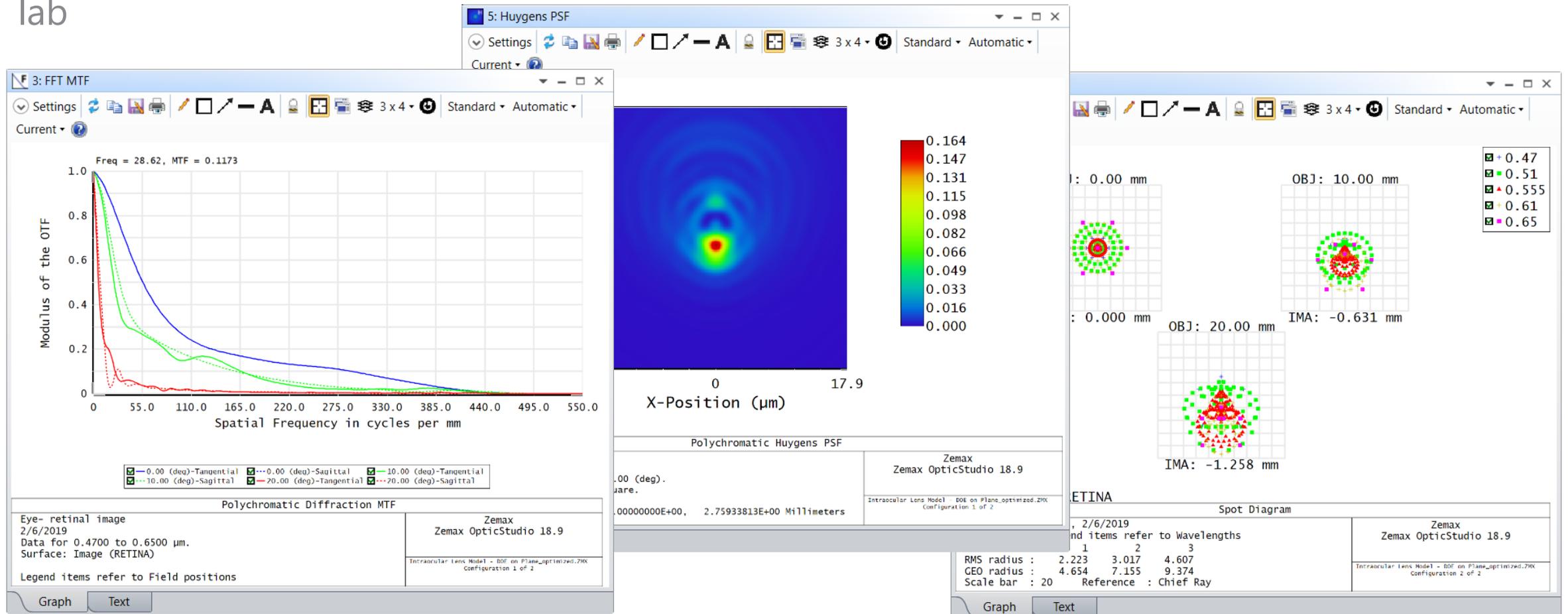
Assume Axial Symmetry:
Ignore Lateral Color:
Add Favorite Operands:

OK Apply Close Save Settings Load Settings Reset Settings

Operand	Weight	Value	% Contrib
000	1.000000	-0.000152	0.032412
000	1.000000	2.452991E-07	8.398604E-08
000	0.005294	0.008927	0.588899
000	0.008471	0.011388	1.533277
000	0.005294	0.013701	1.387183
000	0.029263	0.006116	1.527951
14	TRAC	2 0.000000 0.000000 0.707107 0.000000	0.000000 0.046821 0.005438 1.932467
15	TRAC	2 0.000000 0.000000 0.941965 0.000000	0.000000 0.029263 0.005767 1.358308
16	TRAC	2 0.000000 0.000000 0.335711 0.000000	0.000000 0.058178 0.003589 1.046020

Analysis

- There are many analysis tools in OpticStudio
- In general, we recommend using a tool that mimics the measurements you will make in the lab



Next Steps

- Our system is designed, and the analyses indicate it meets specifications, however our Binary 2 surface is still just an equation and has no real geometry.
- At this point our friends at VirtualLab can take the Binary 2 surface designed in OpticStudio, and create an actual optic out of it which will have the same properties we modelled.

Structure of Workshop

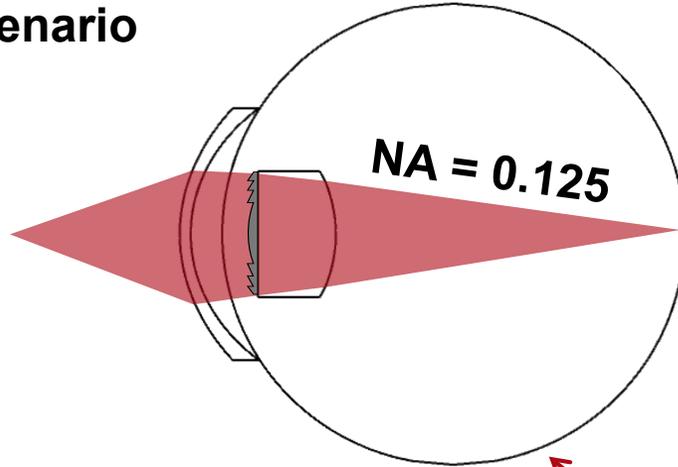
- Introduction of theory
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- **Physical-optics analysis of imported systems from OpticStudio: Diffractive lenses**
 - **Roberto Knoth**
- Metalenses theory and modeling
 - Site Zhang
- Fabrication export
 - Roberto Knoth

An Example of Diffractive Layer Design

Design and Analysis of Intraocular Diffractive Lens

Design Task for a Diffractive Layer

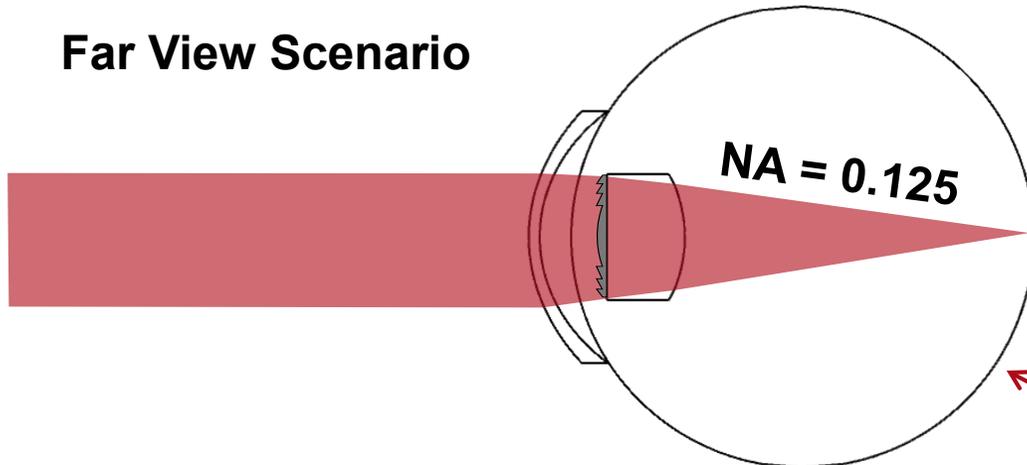
Near View Scenario



Both configuration of the setups require two wavefront surface response functions.

From this reason a diffractive layer design is a good choice to achieve different wavefront effects for the two configurations.

Far View Scenario



$m = 1$

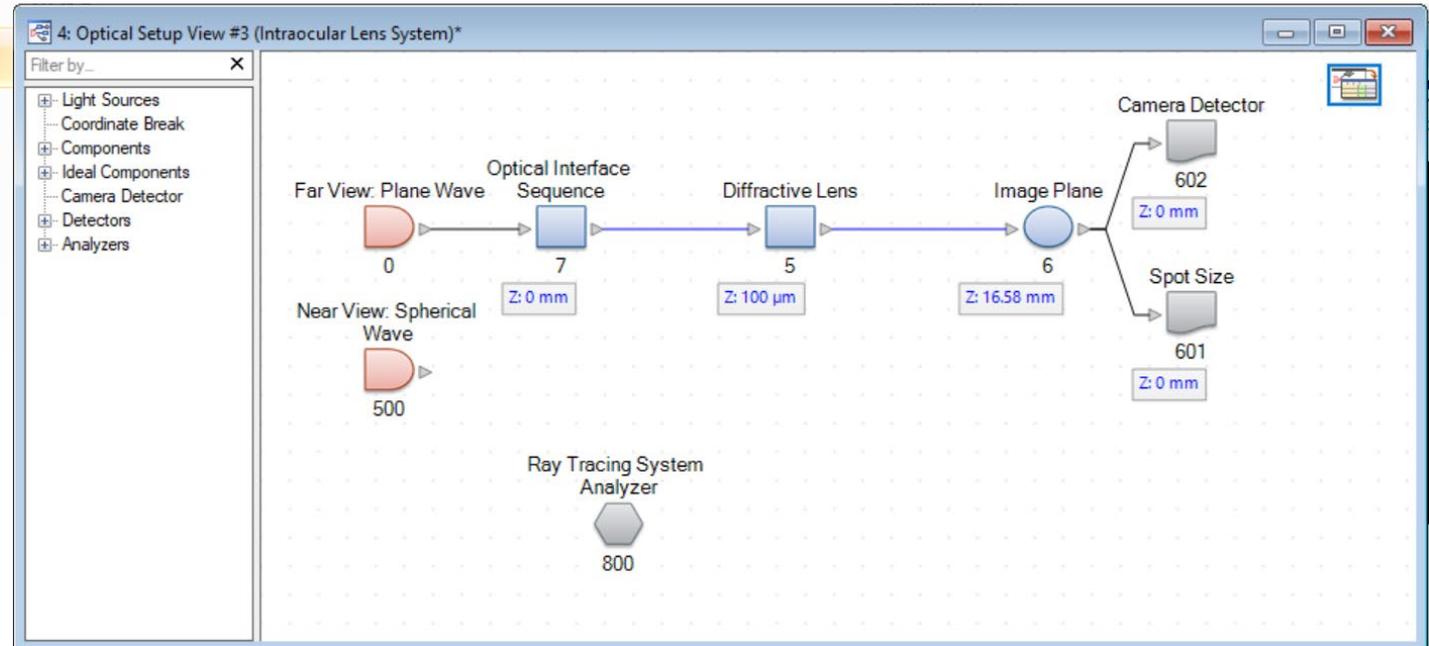
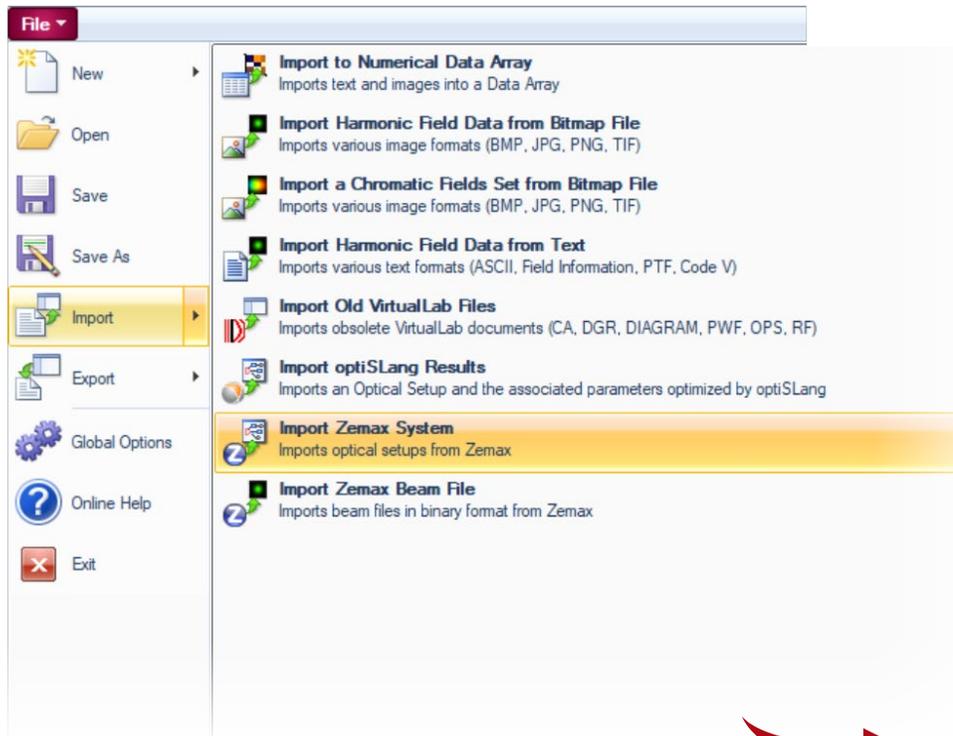
$$V_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \left\{ \mathbf{B}^{(1)}(\boldsymbol{\rho}; \psi^{\text{in}}) U_{\perp}^{\text{in}}(\boldsymbol{\rho}) \right\} \exp(i\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho})) \\ + \sum_{m=-\infty, m \neq 1}^{\infty} \left\{ \mathbf{B}^{(m)}(\boldsymbol{\rho}; \psi^{\text{in}}) U_{\perp}^{\text{in}}(\boldsymbol{\rho}) \right\} \exp(i\psi^{\text{in}}(\boldsymbol{\rho}) + m\Delta\psi(\boldsymbol{\rho}))$$

$m = 0$

Import of Optical System from OpticStudio

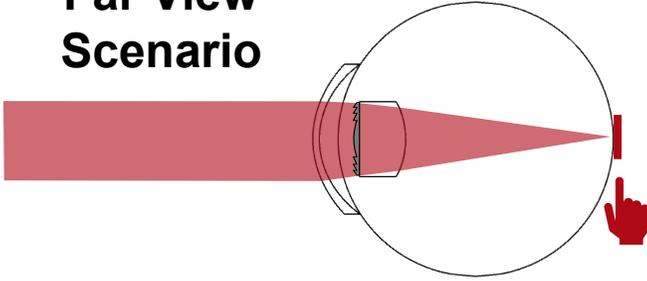
The configuration of the optical setup as well as the design of the wavefront surface response by a Binary-2 surface was generated in OpticStudio.

VirtualLab Fusion provides the capability to import the optical setups and to merge them in a single optical setup configuration.



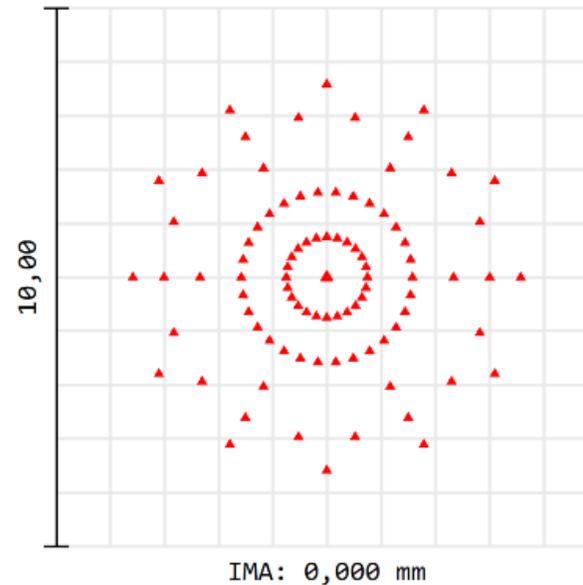
Far View: Conformity of OpticStudio Import

Far View
Scenario



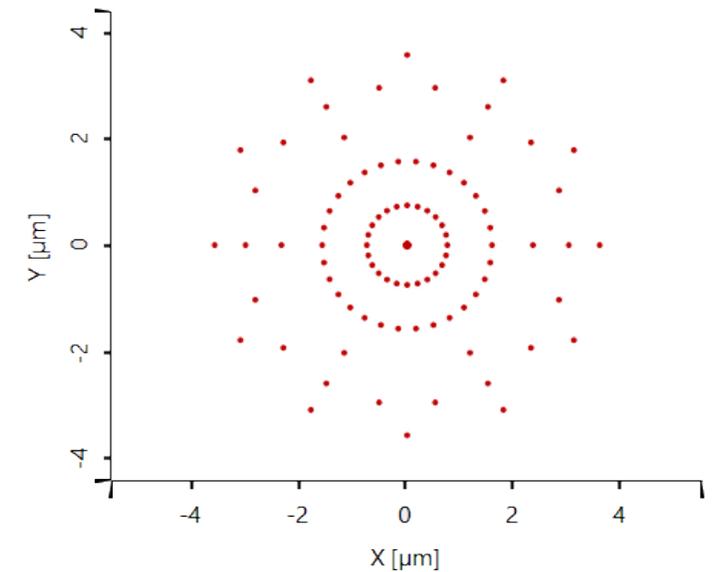
spot diagram calculated by
OpticStudio

- central wavelength of 555 nm
- on-axis mode



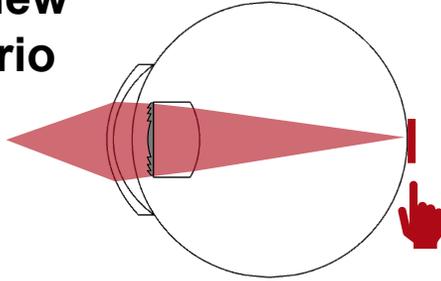
spot diagram calculated by
VirtualLab Fusion

- central wavelength of 555 nm
- on-axis mode



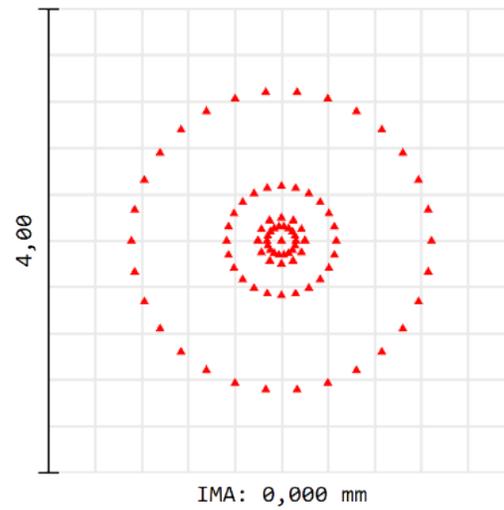
Near View: Conformity of OpticStudio Import

Near View
Scenario



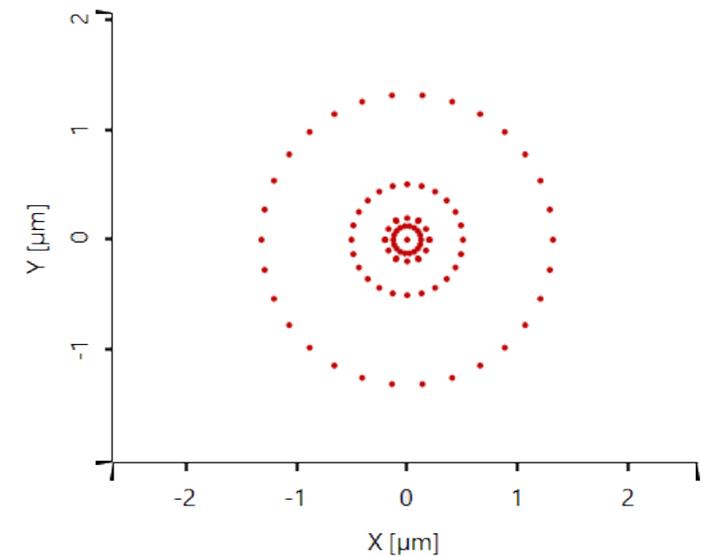
spot diagram calculated by
OpticStudio

- central wavelength of 555 nm
- on-axis mode



spot diagram calculated by
VirtualLab Fusion

- central wavelength of 555 nm
- on-axis mode



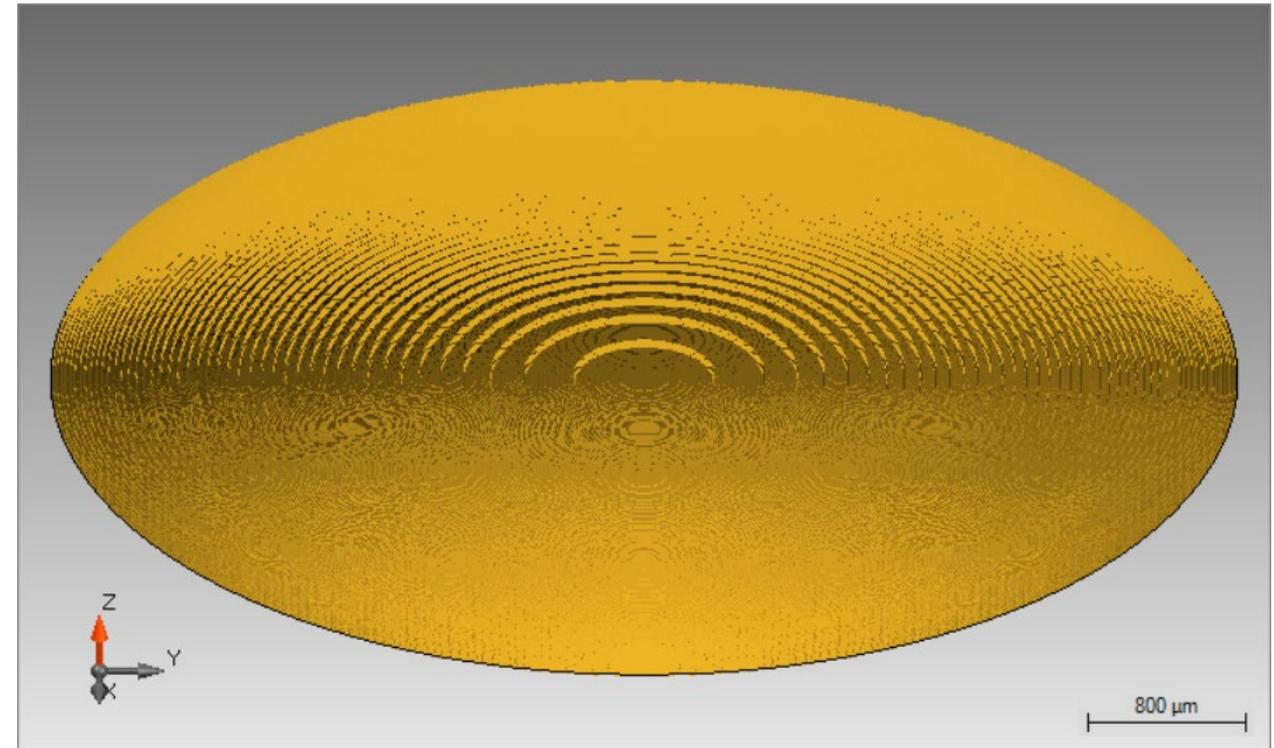
Structure Design: Diffractive Layer Profile Height

- The structure profile of the diffractive layer is calculated by TEA according to the wavefront surface response:

$$h^{DOE}(\rho) = \frac{\lambda}{2\pi\Delta n} \Delta\psi(\rho)^{DOE}$$

TEA provides directly a very high efficiency for the 1st order

$$V_{\perp}^{\text{out}}(\rho) = \left\{ \mathbf{B}^{(1)}(\rho; \psi^{\text{in}}) U_{\perp}^{\text{in}}(\rho) \right\} \exp(i\psi^{\text{in}}(\rho) + \Delta\psi(\rho)) + \sum_{m=-\infty, m \neq 1}^{\infty} \left\{ \mathbf{B}^{(m)}(\rho; \psi^{\text{in}}) U_{\perp}^{\text{in}}(\rho) \right\} \exp(i\psi^{\text{in}}(\rho) + m\Delta\psi(\rho))$$

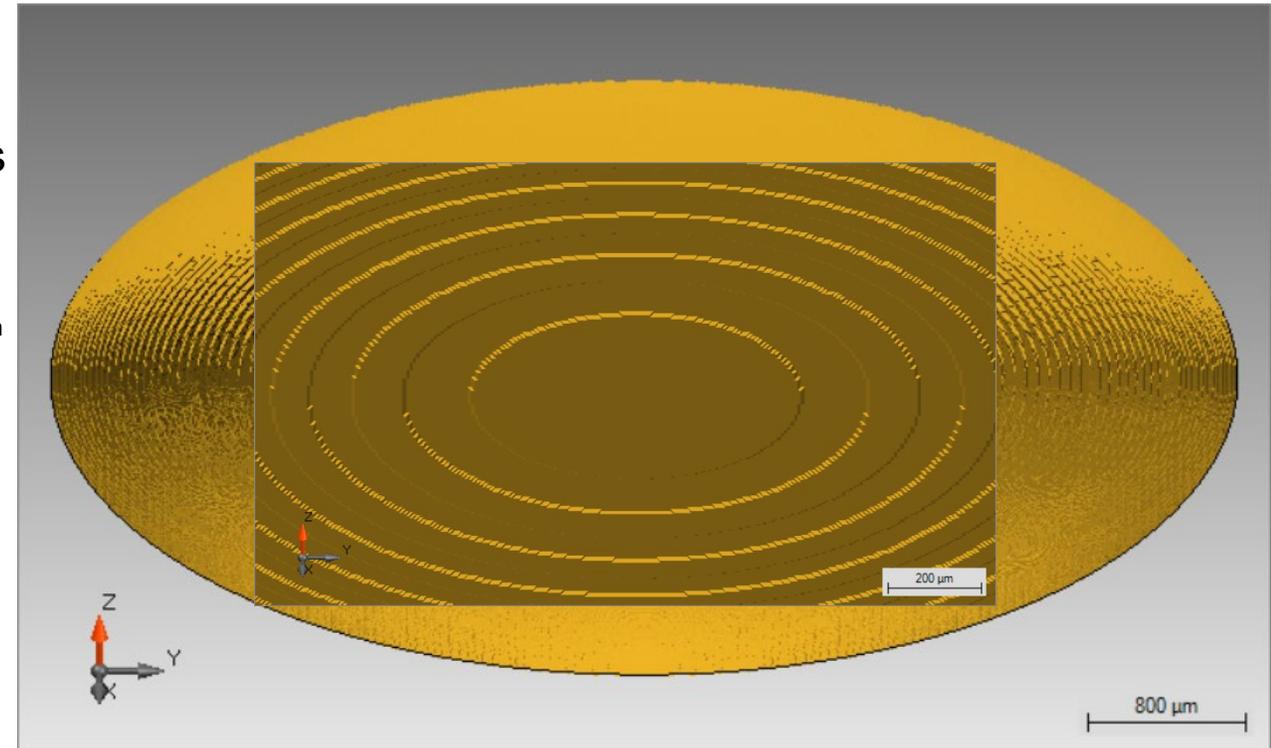


Structure Design: Diffractive Layer Profile Height

- With the introduction of a scaling factor in the TEA formula, the resulting structure height is modulated to control the efficiencies of the diffraction orders:

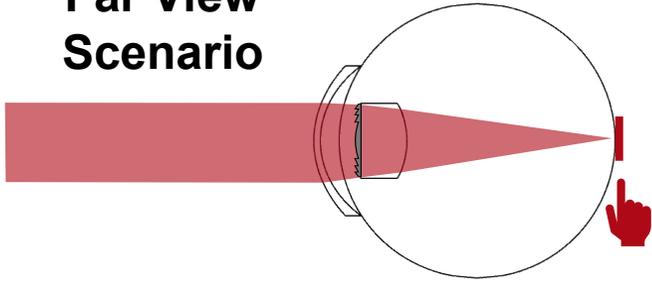
$$h^{DOE}(\rho) = \beta \frac{\lambda}{2\pi\Delta n} \Delta\psi(\rho)^{DOE}$$

- A quantization of the structure with 2 height levels is chosen because binary diffractive lenses
 - are beneficial for manufacturing (cost, easier to fabricate)
 - give a better control of the efficiencies especially for the 0th and 1st order using the height modulation approach

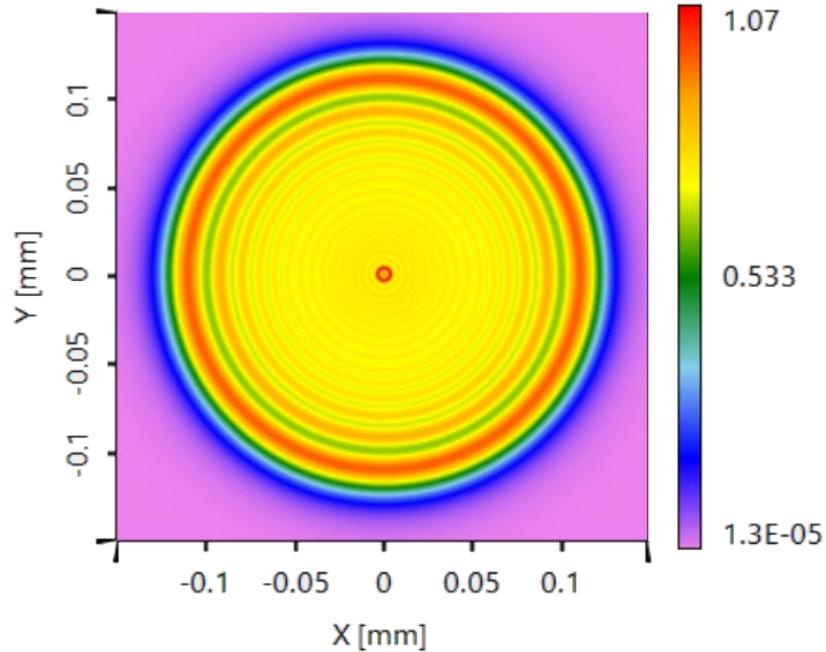


Structure Design: Height Modulation of 1.00

Far View Scenario

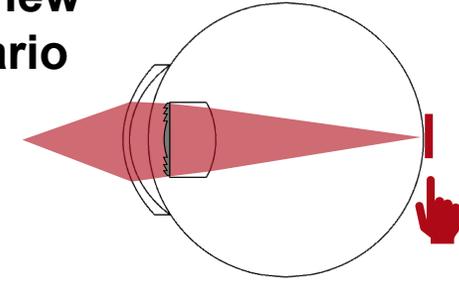


\propto Electric Energy Density $[1E2](V/m)^2$

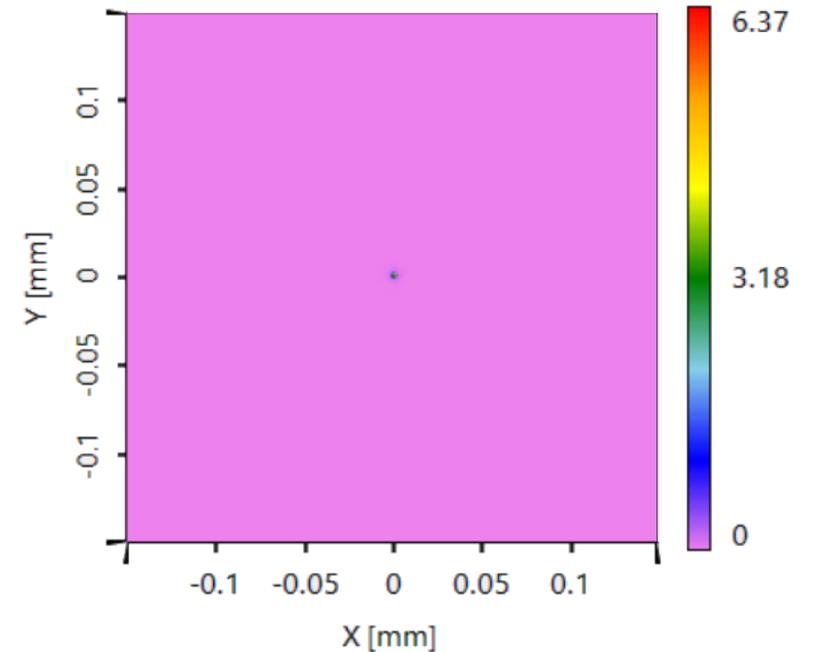


scaling of modulation height by $\beta = 1.00$

Near View Scenario

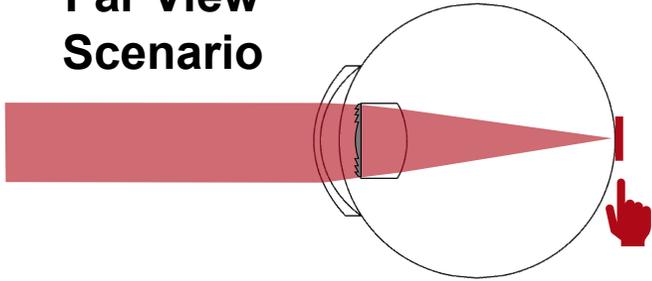


\propto Electric Energy Density $[1E5](V/m)^2$

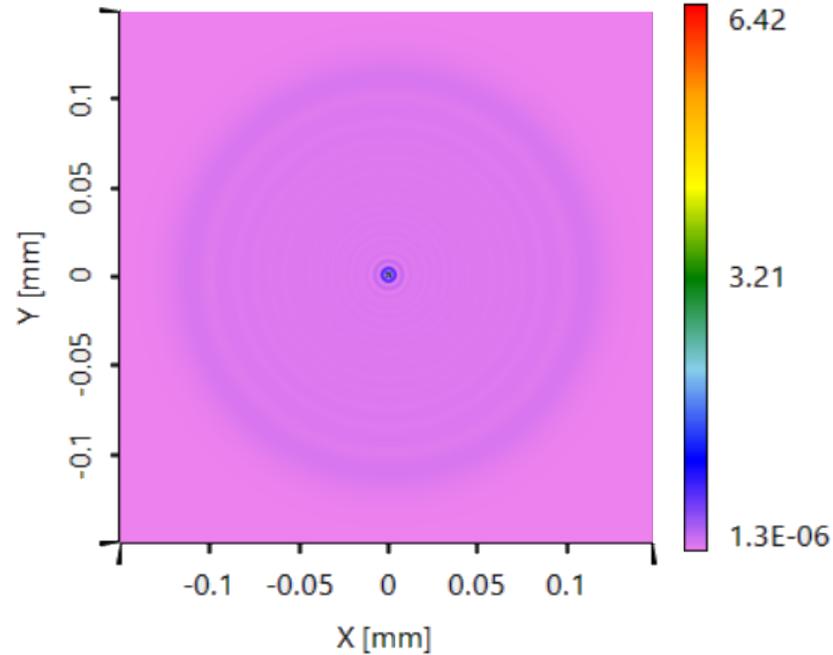


Structure Design: Height Modulation of 0.95

Far View Scenario

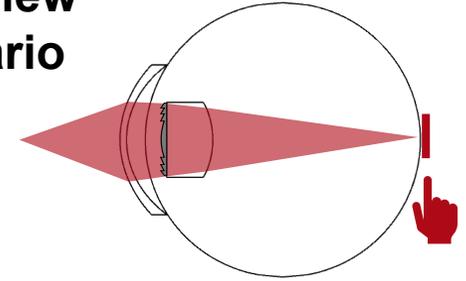


\propto Electric Energy Density $[1E3](V/m)^2]$

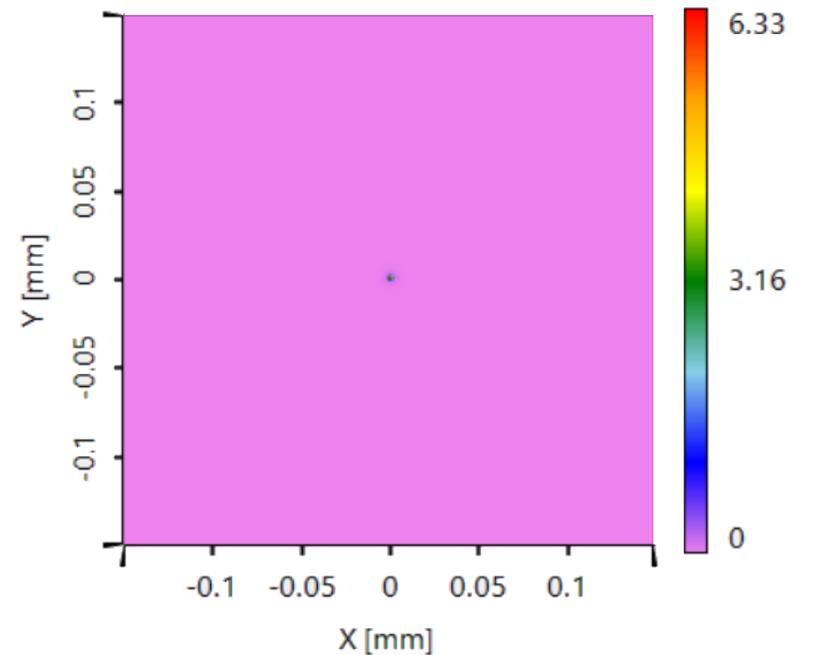


scaling of modulation height by $\beta = 0.95$

Near View Scenario

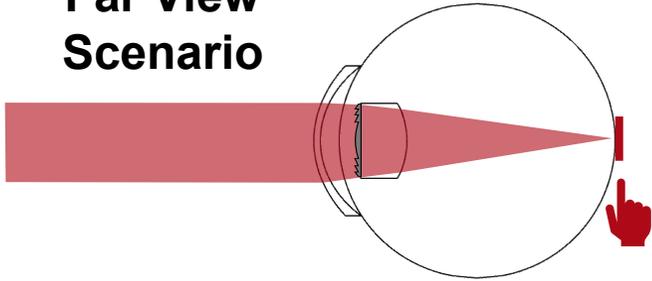


\propto Electric Energy Density $[1E5](V/m)^2]$

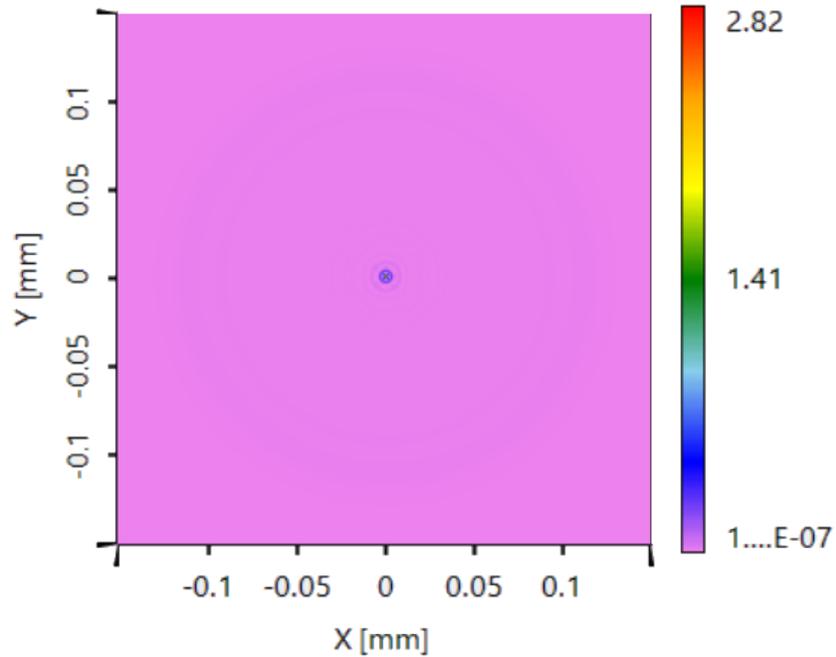


Structure Design: Height Modulation of 0.90

Far View Scenario

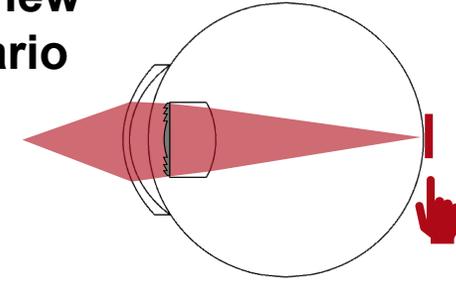


\propto Electric Energy Density $[1E4 (V/m)^2]$

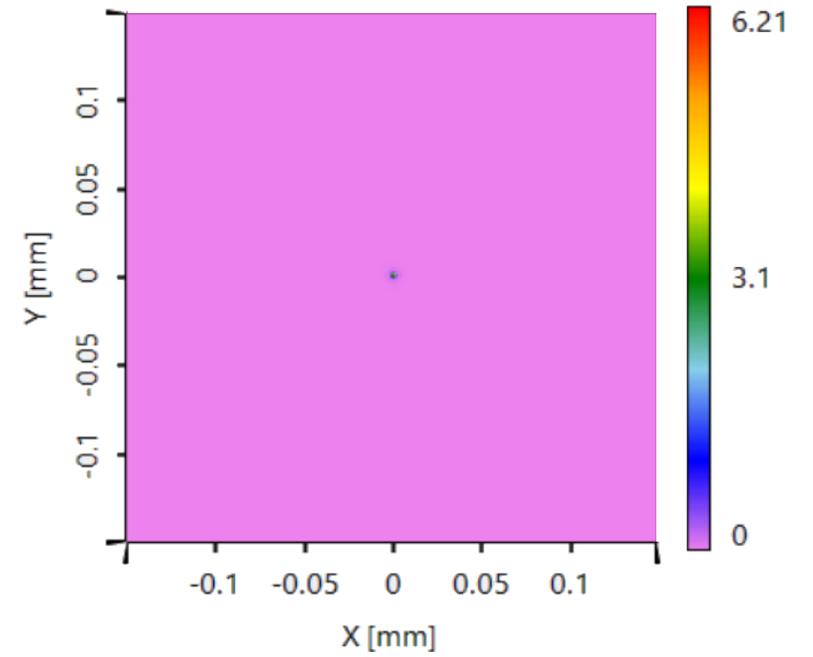


scaling of modulation height by $\beta = 0.90$

Near View Scenario

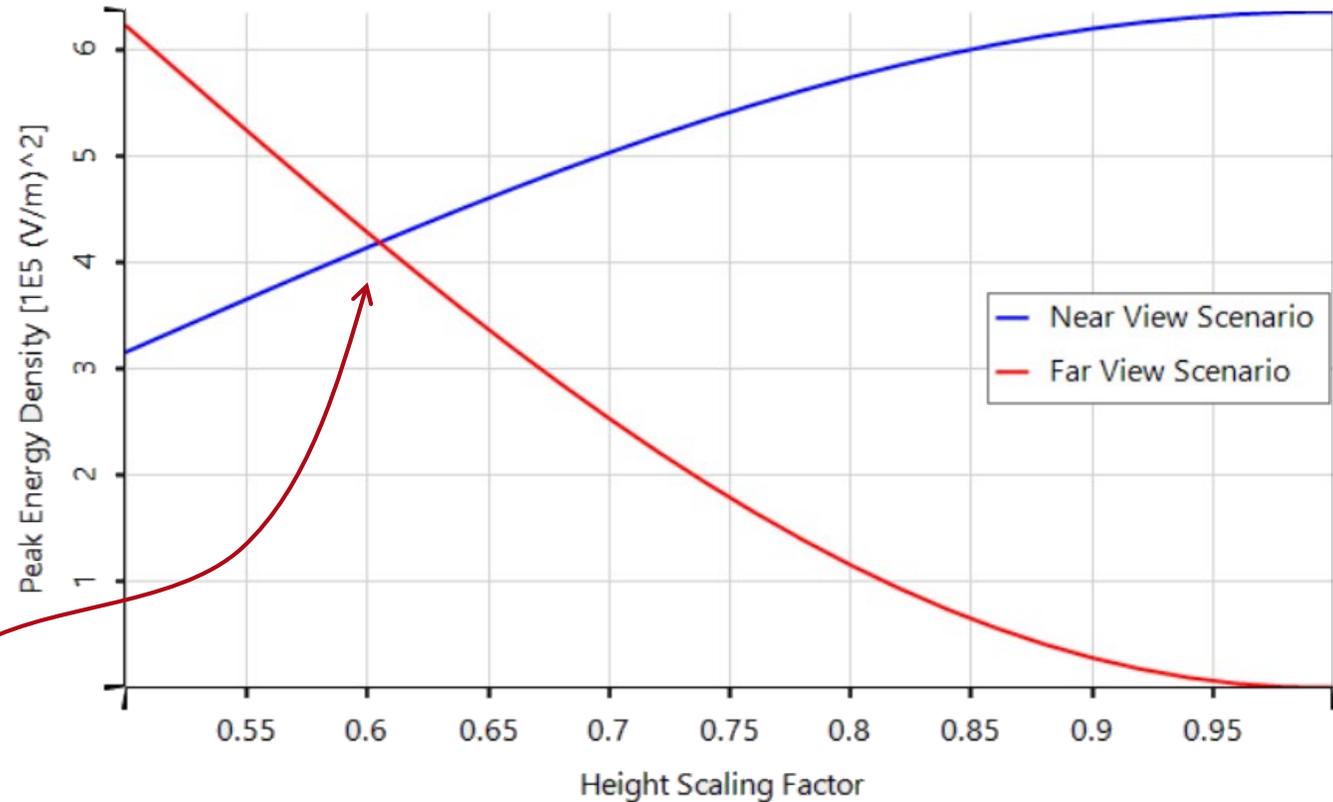


\propto Electric Energy Density $[1E5 (V/m)^2]$



Structure Design: Find the Optimum Scaling Factor

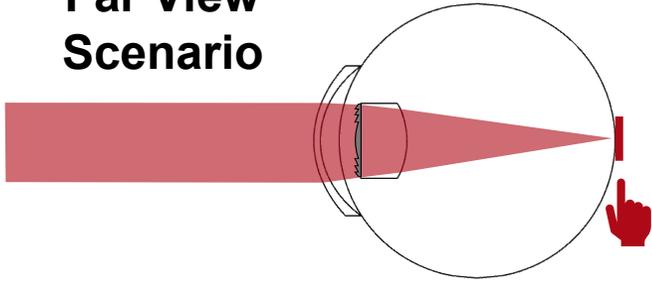
- As a goal the peak energy density of the both foci shall be the same.
- Therefore, the peak energy density is calculated according to the scaling factor for both scenarios.



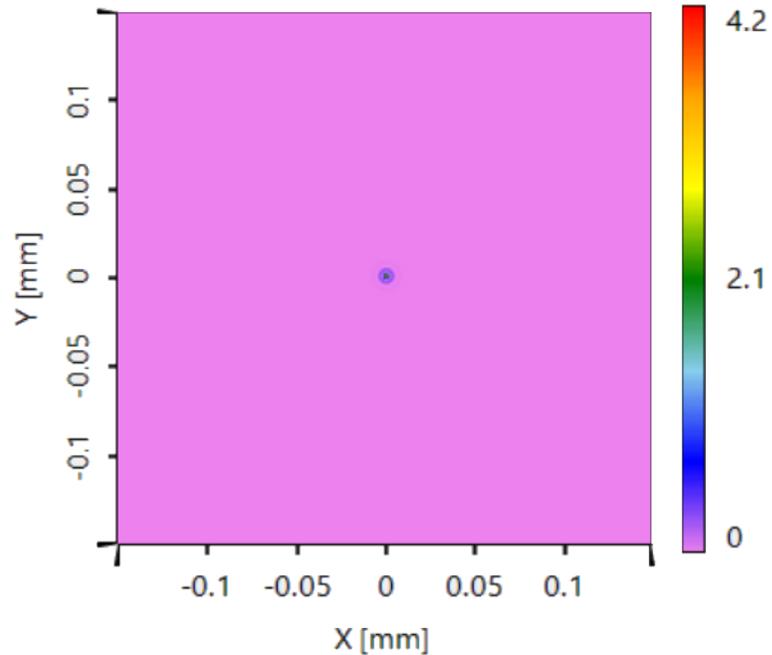
Optimum of the scaling factor for equivalent peak energy density for both foci (near and far view)

Structure Design: Optimum Height Modulation of 0.605

Far View Scenario



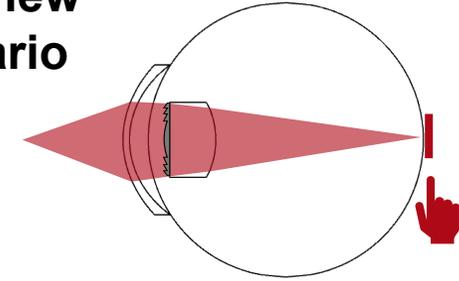
\propto Electric Energy Density $[1E5](V/m)^2]$



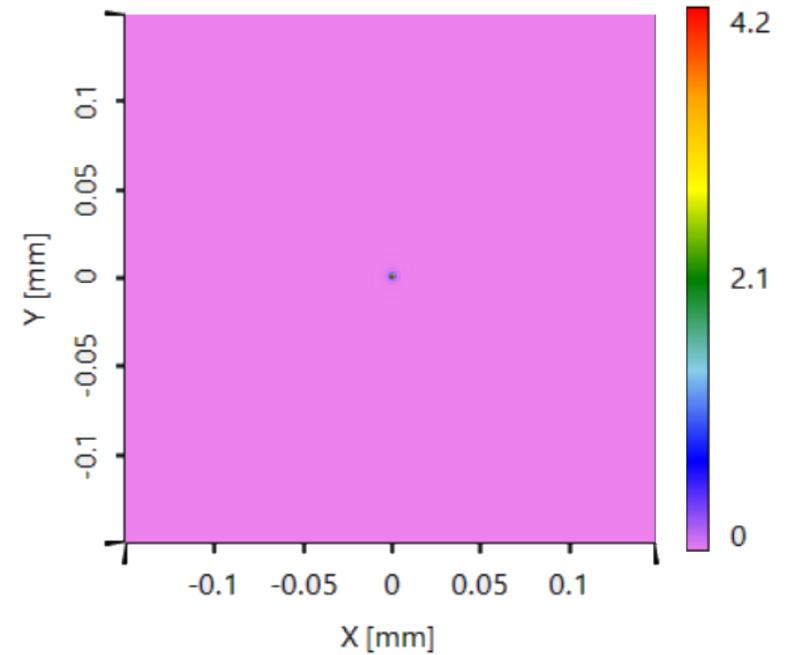
scaling of modulation height by
 $\beta = 0.605$

Goal of equivalent maximum energy density for both foci achieved!

Near View Scenario

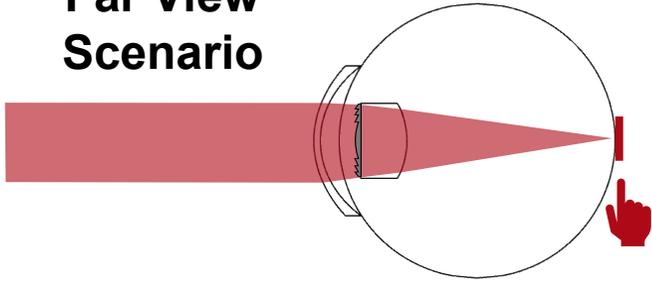


\propto Electric Energy Density $[1E5](V/m)^2]$

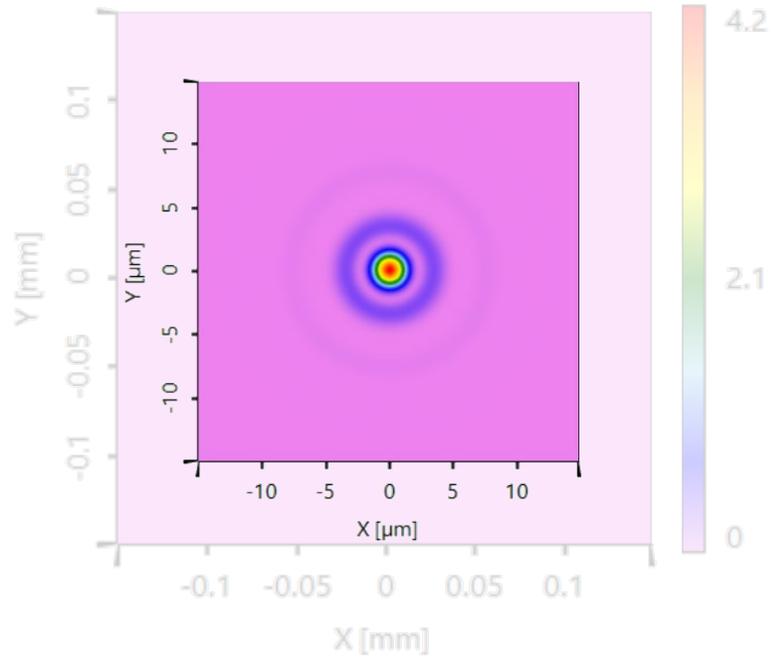


Analysis: PSF of Foci for Optimum Structure Design

Far View Scenario

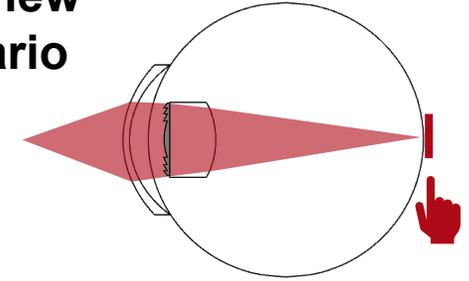


\propto Electric Energy Density [$1E5 (V/m)^2$]



scaling of modulation height by
 $\beta = 0.605$

Near View Scenario



\propto Electric Energy Density [$1E5 (V/m)^2$]

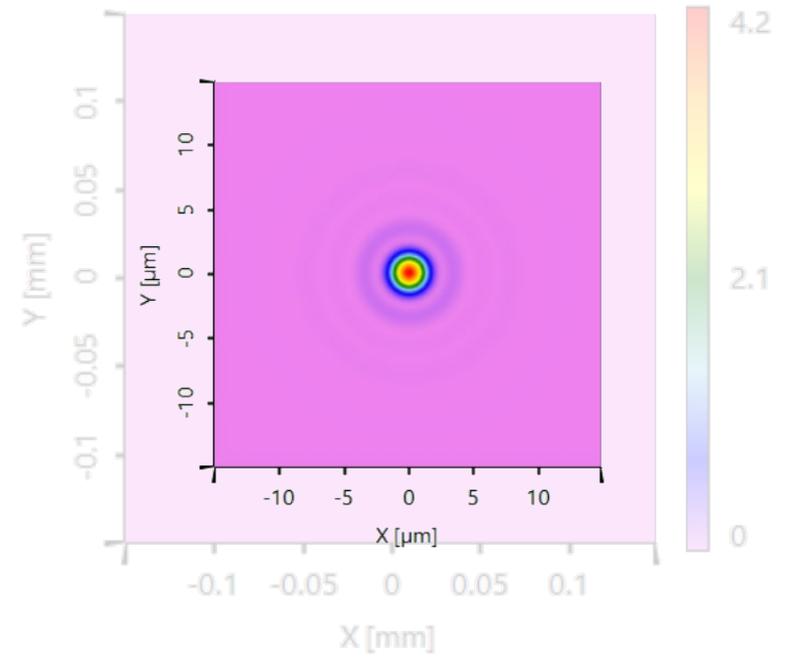
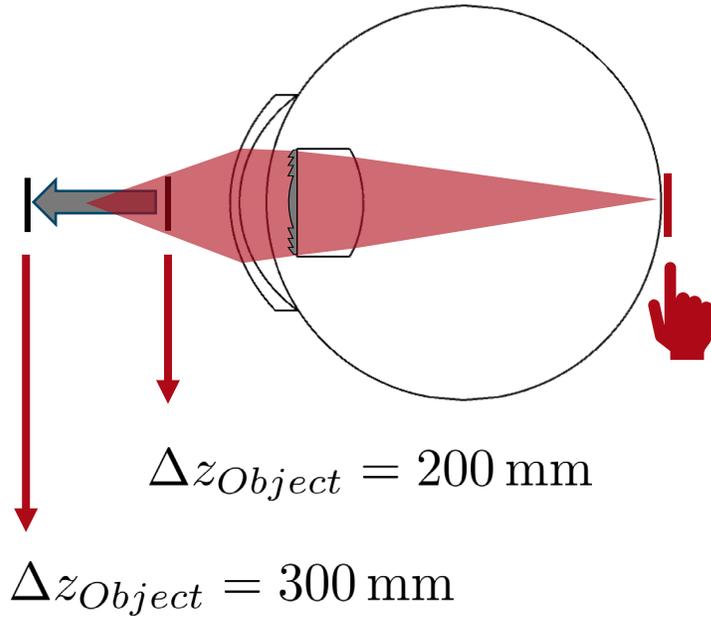


Illustration of Focus Development in Near View Region

Near View Scenario

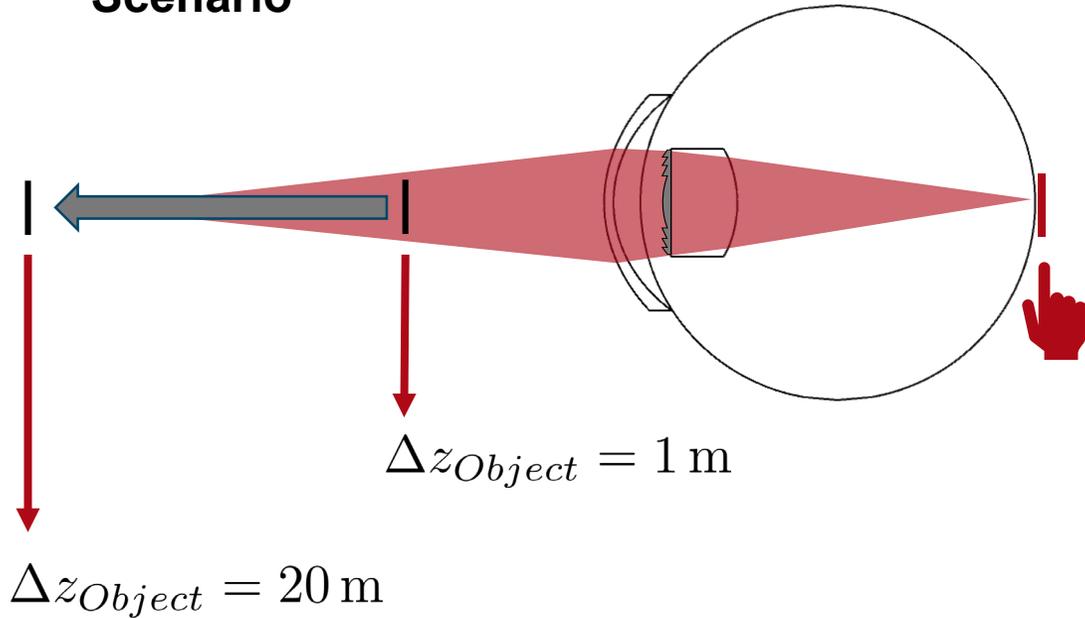


step size 10mm
→ 101 simulations
simulation time ~13min



Illustration of Focus Development in Far View Region

Far View
Scenario

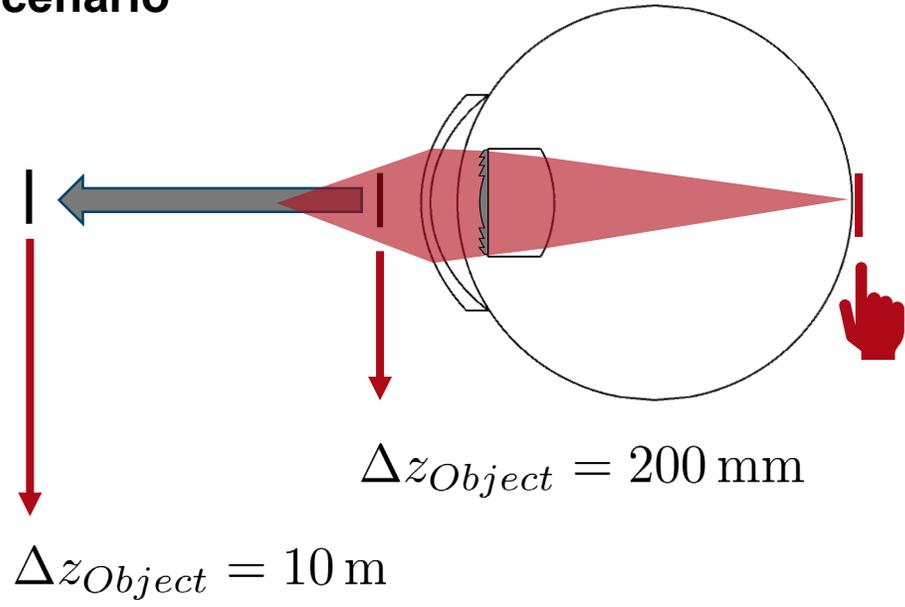


step size 250mm
→ 77 simulations
simulation time ~10min



Illustration of Focus Development from Near to Far Region

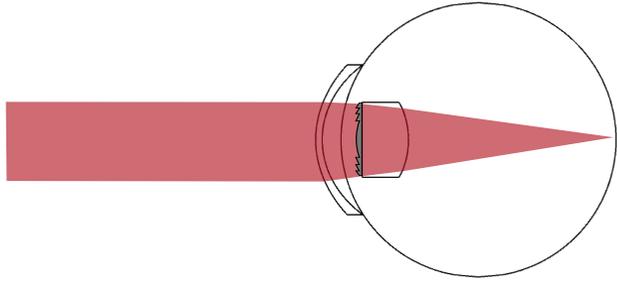
Near to Far
View Scenario



step size 50mm
→ 168 simulations
simulation time ~20min

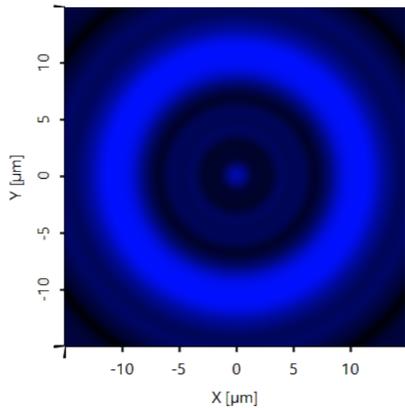


Far View Scenario: Analysis of the Focal Spots per Color

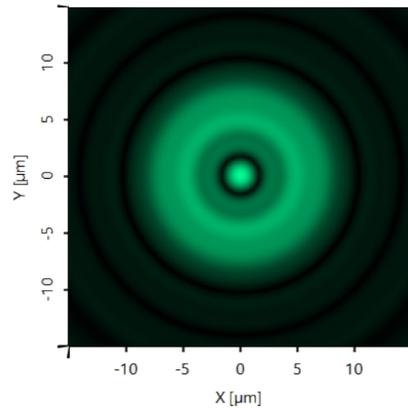


VirtualLab enables various analysis capabilities of the designed diffractive lens structure

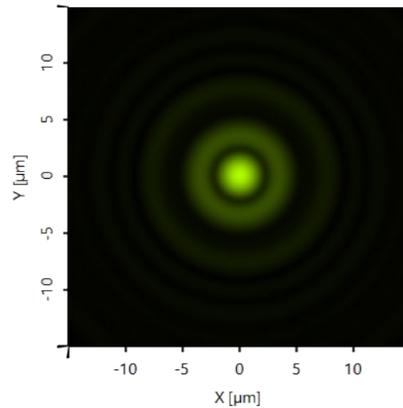
Wavelength 470 nm



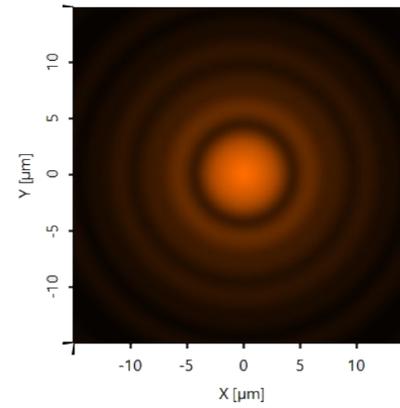
Wavelength 510 nm



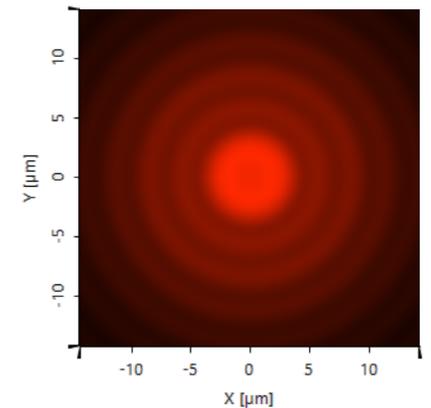
Wavelength 555 nm



Wavelength 610 nm



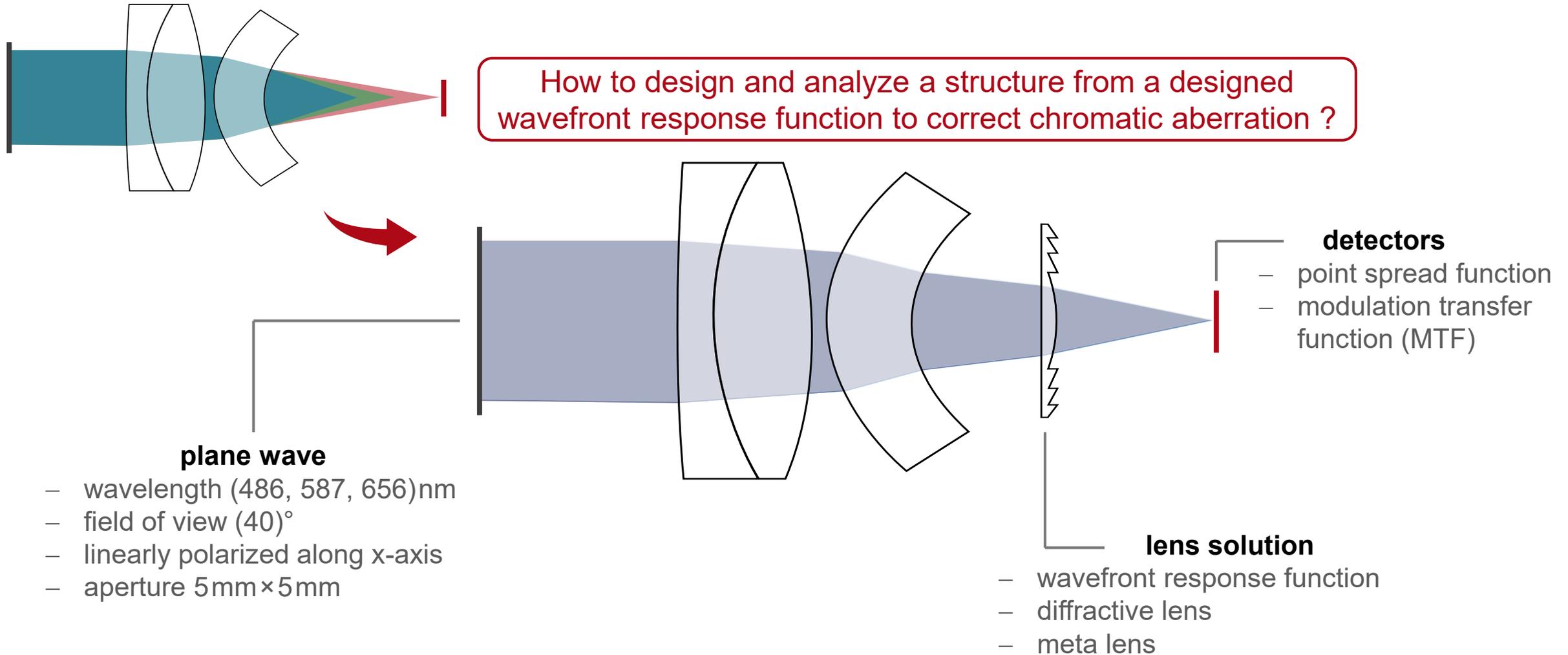
Wavelength 650 nm



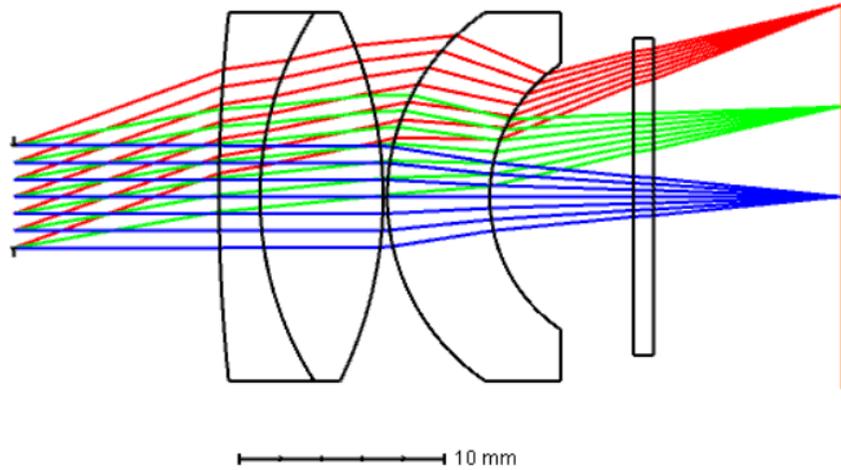
An Example of Diffractive Layer Design

Design and Analysis of a Hybrid Eyepiece for Correction of Chromatic Aberration

Modeling and Design Task

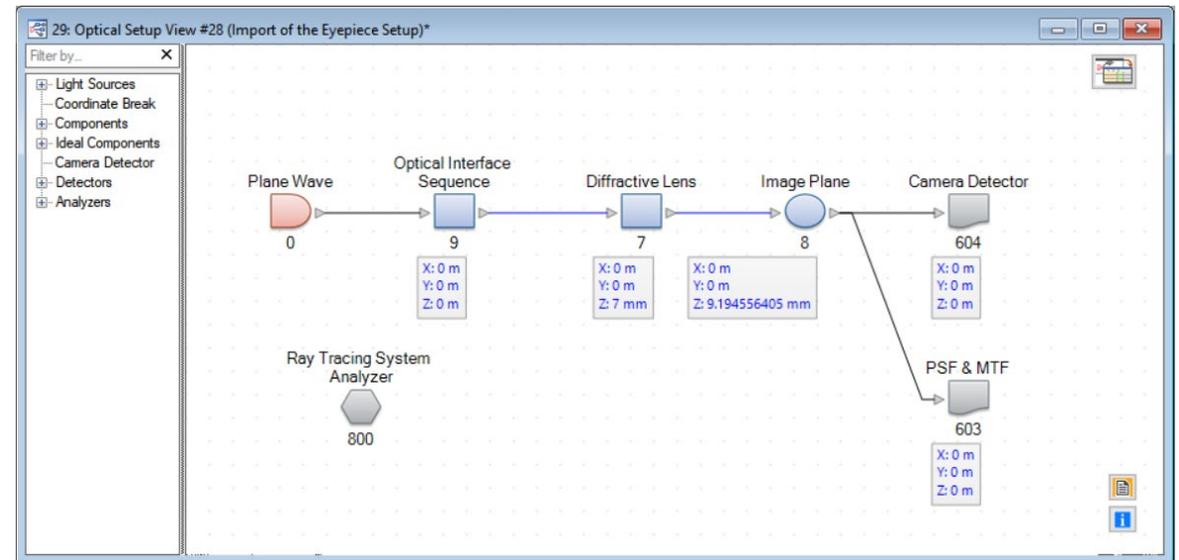


Design of Wavefront Surface Response in OpticStudio



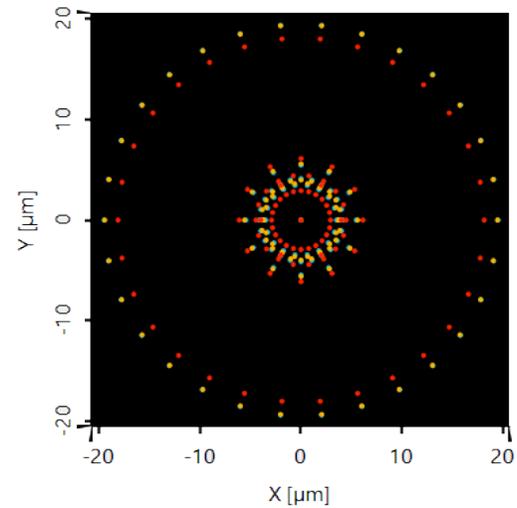
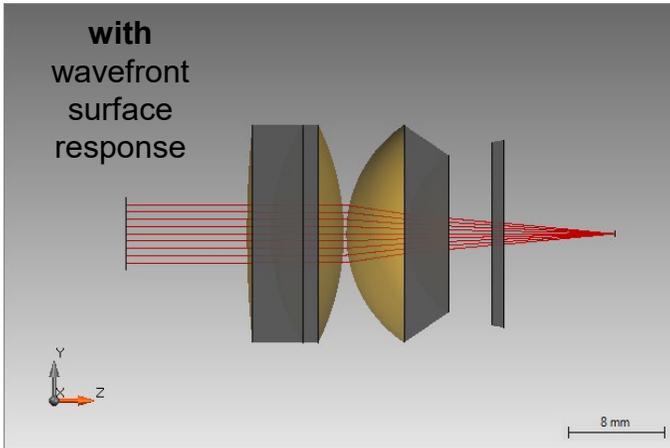
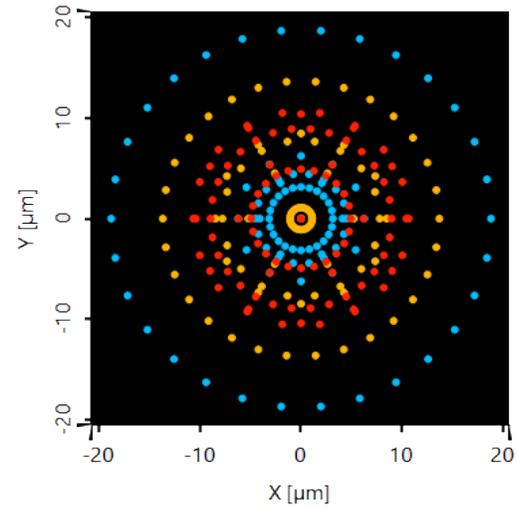
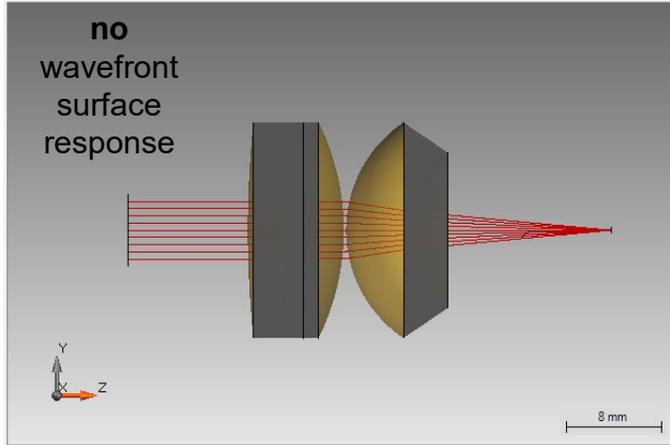
Optical setup including the wavefront surface response was originally designed in OpticStudio

Import of the OpticStudio file to VirtualLab Fusion

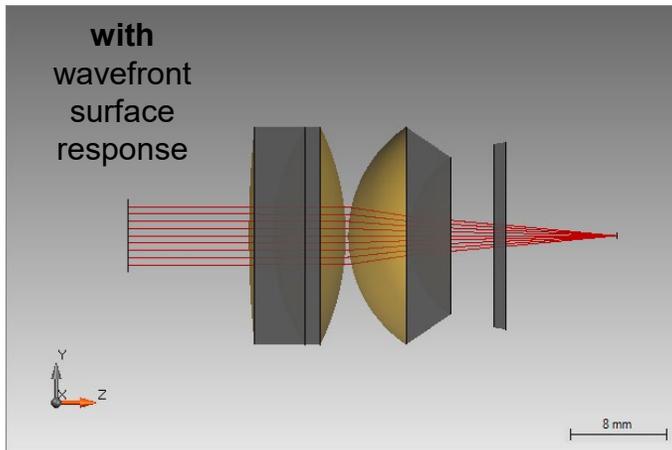
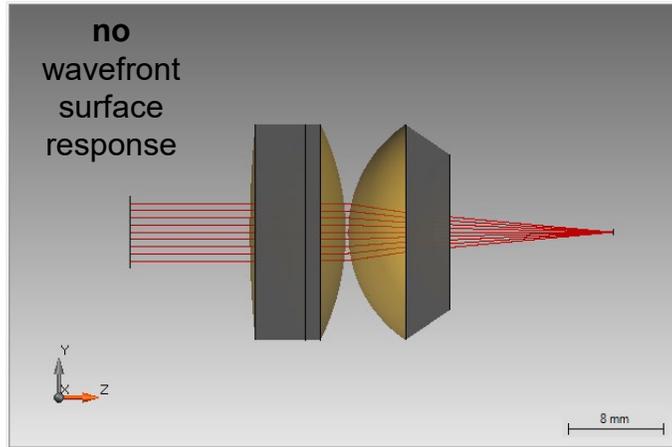


Wavefront Surface Response (WSR)

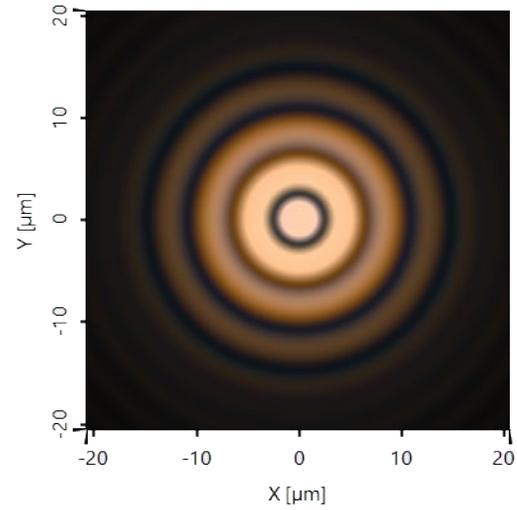
On-Axis Analysis: Comparison of Spot Diagram



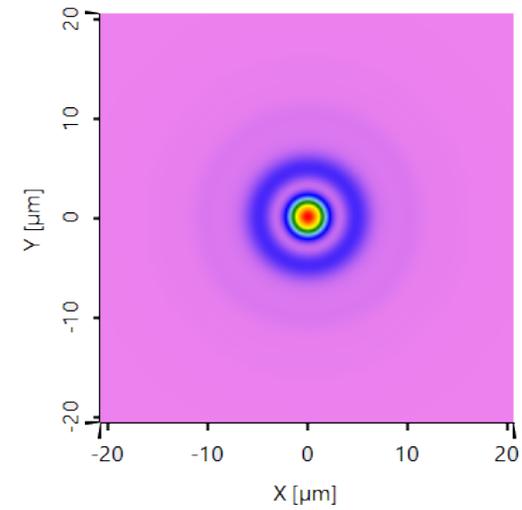
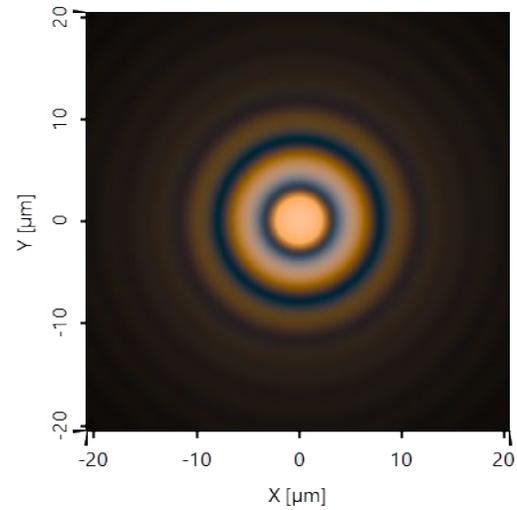
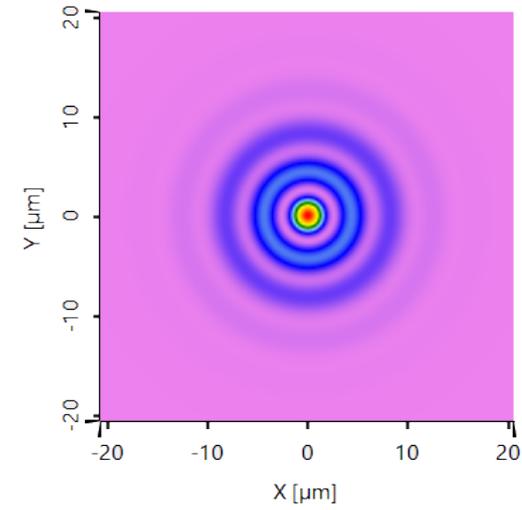
On-Axis Analysis: Comparison of PSF



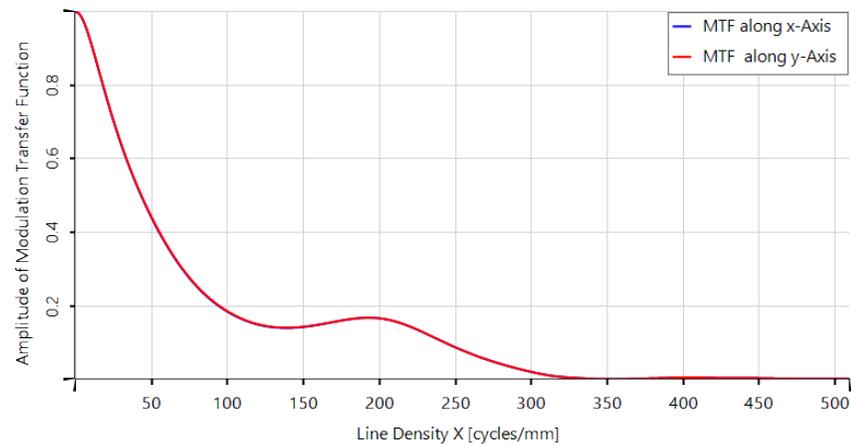
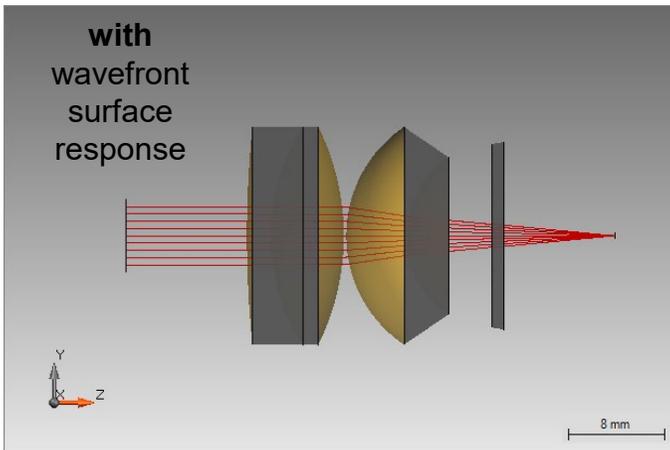
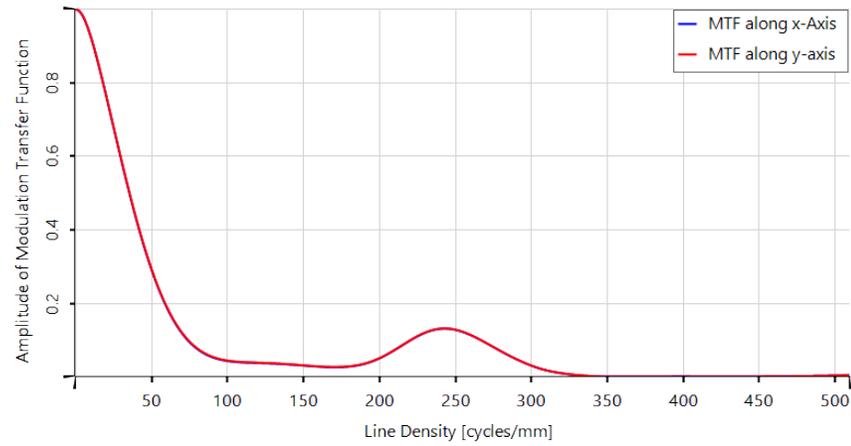
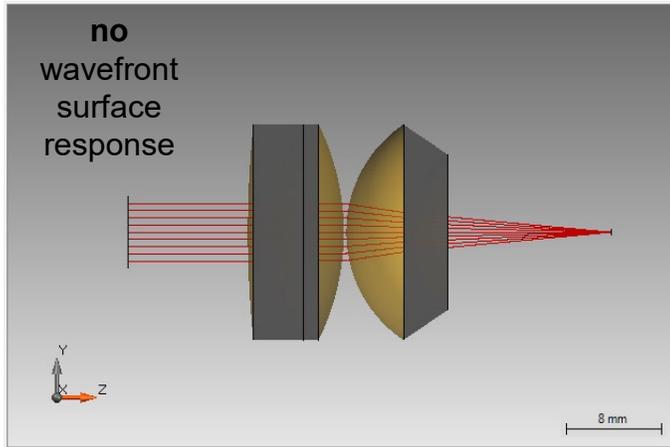
Real Color View



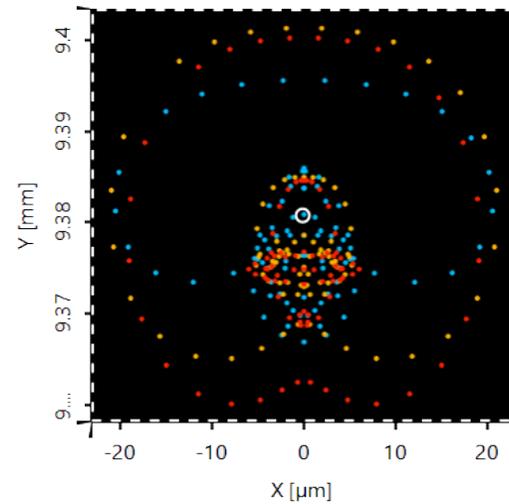
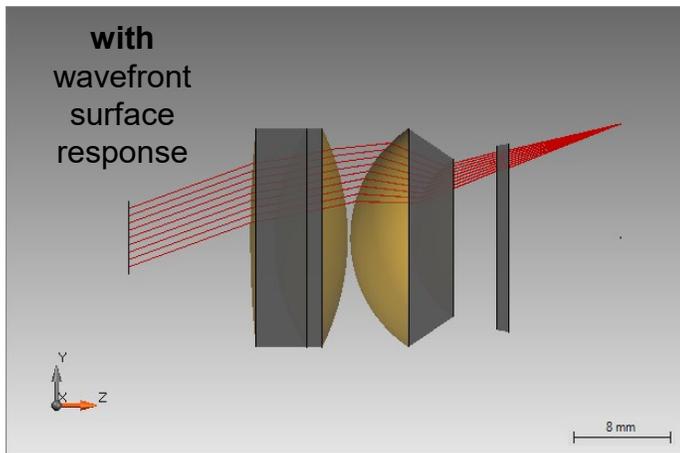
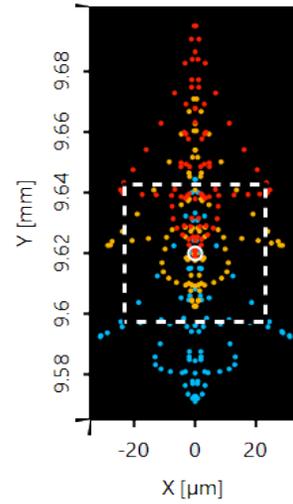
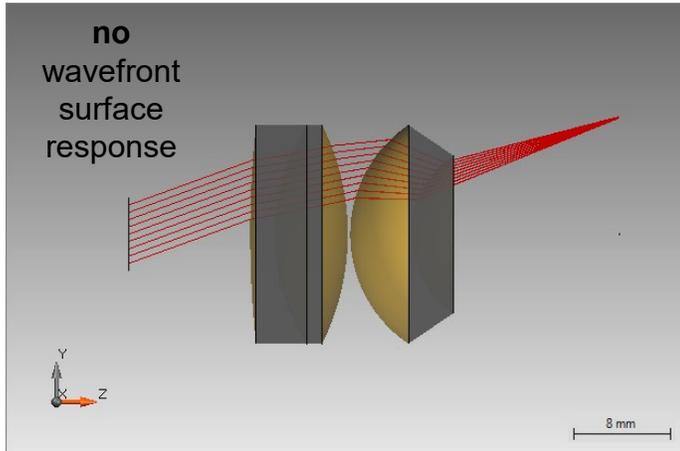
False Color View



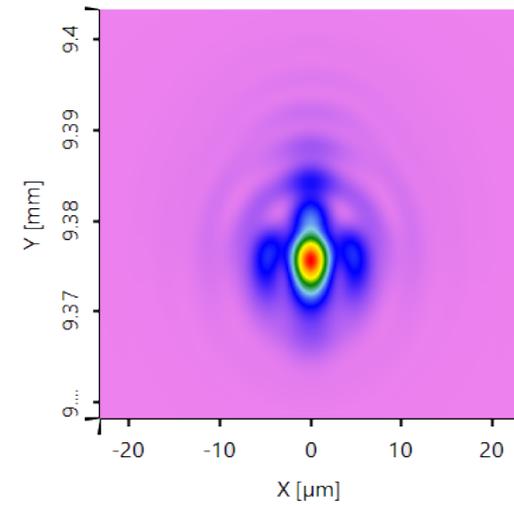
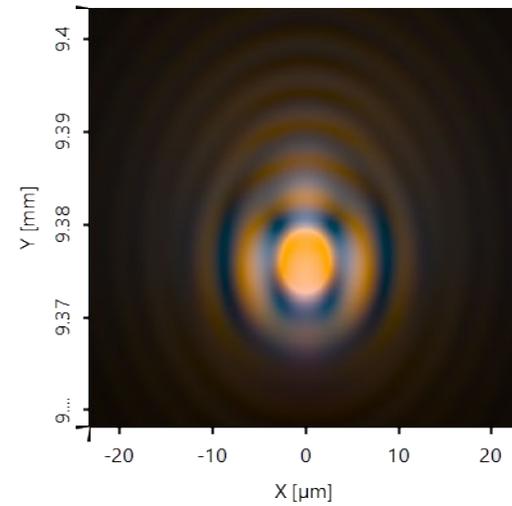
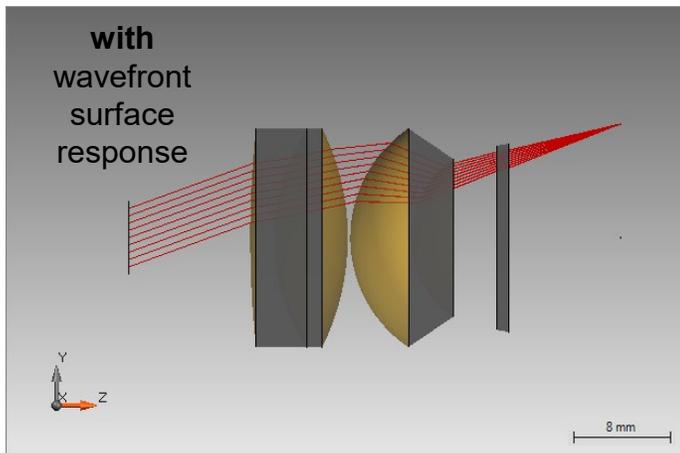
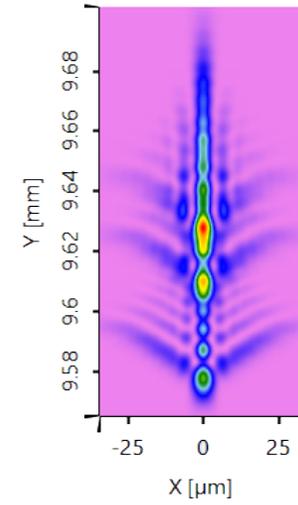
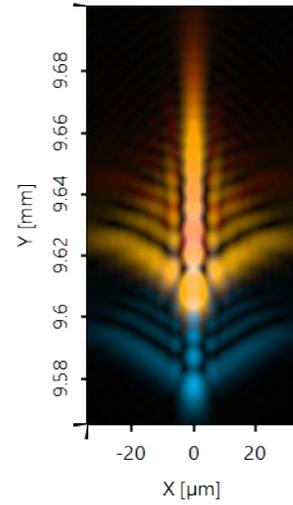
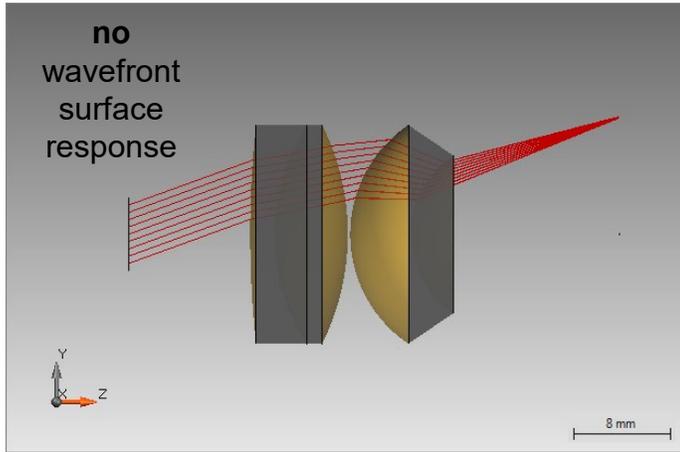
On-Axis Analysis: Comparison of MTF



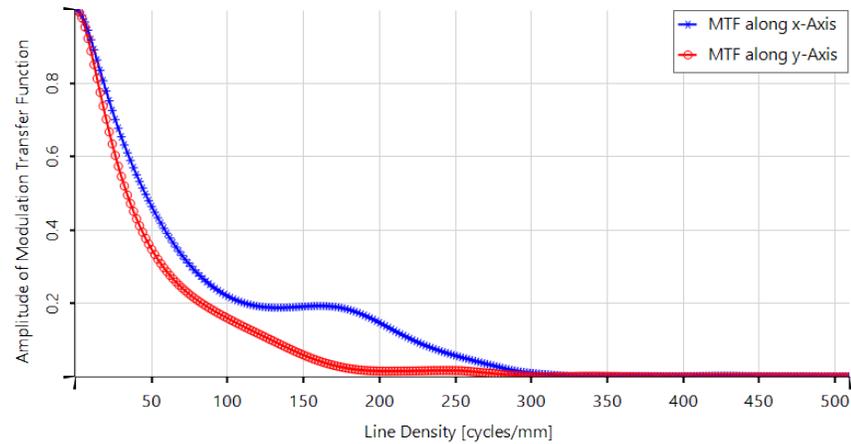
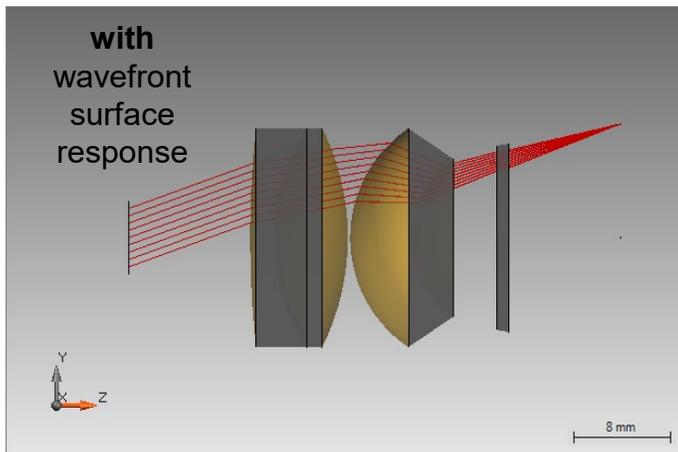
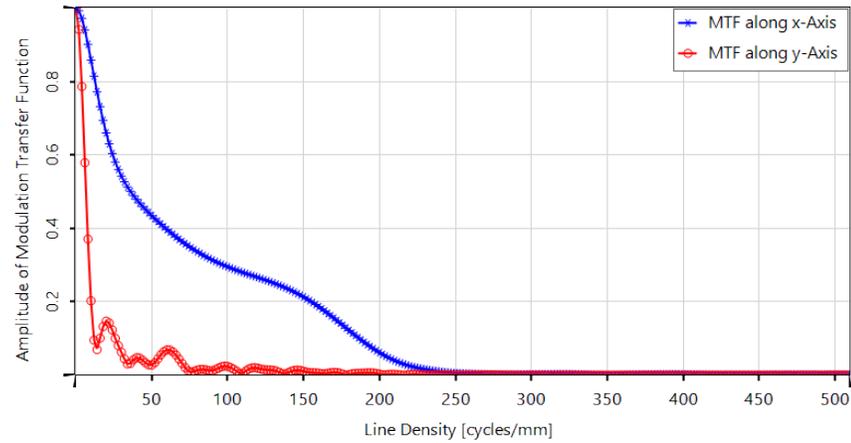
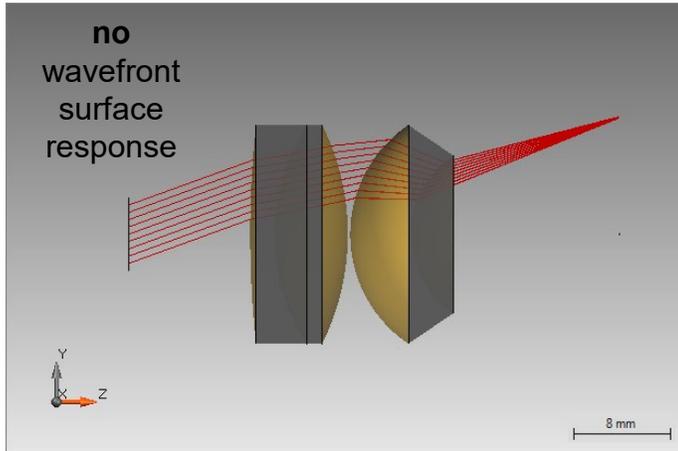
Off-Axis Analysis: Comparison of Spot Diagram



Off-Axis Analysis: Comparison of PSF



Off-Axis Analysis: Comparison of MTF

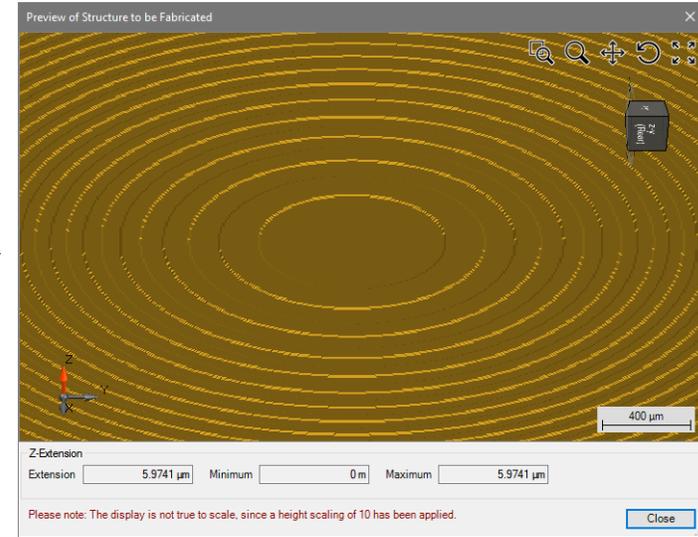
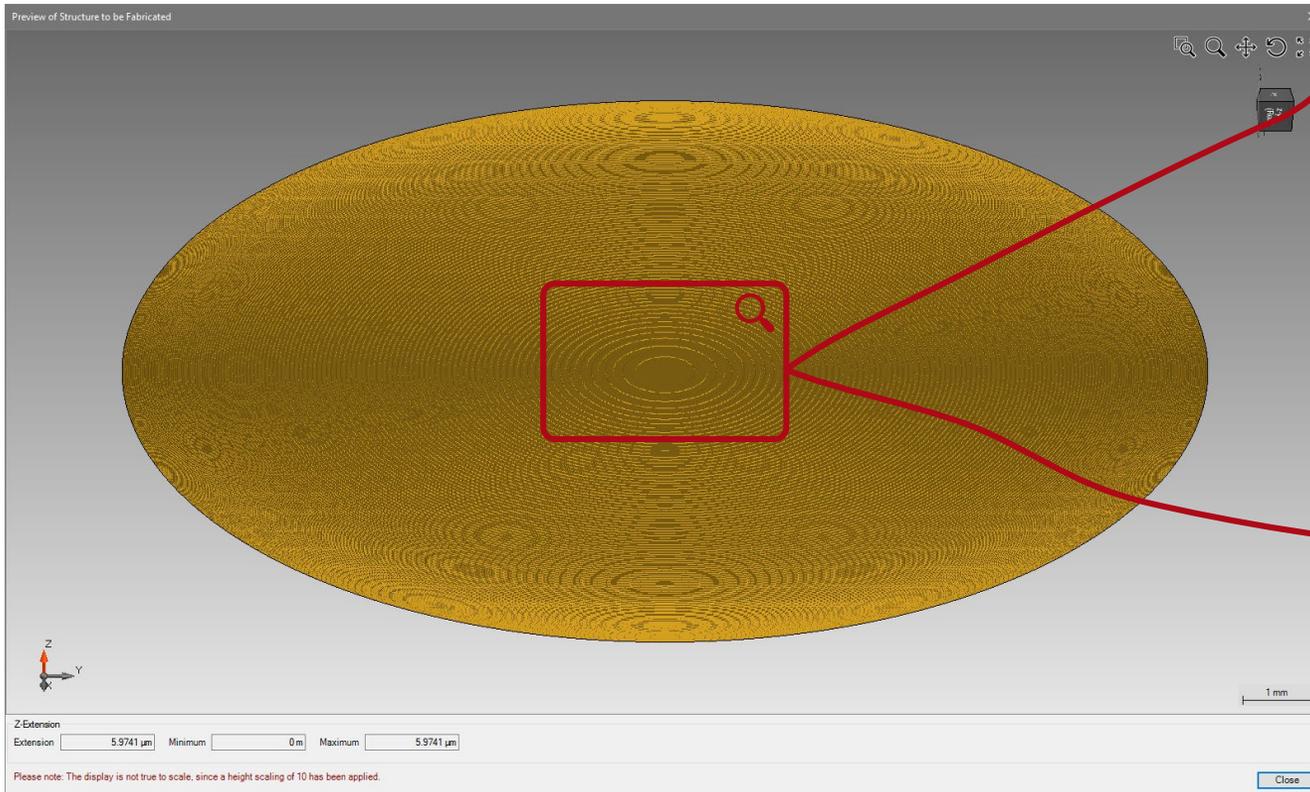


Structure Design – Diffractive Lens

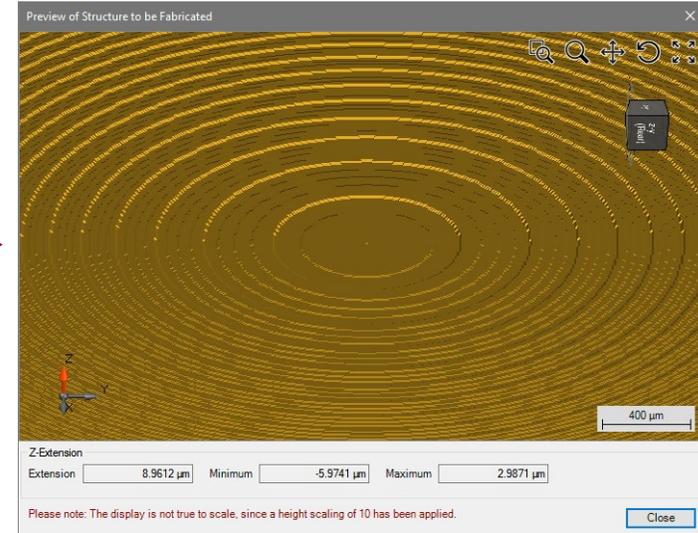
Visualization of Quantized Diffractive Lens Structure

The structure is designed by TEA:

$$h^{DOE}(\rho) = \frac{\lambda}{2\pi\Delta n} \Delta\psi(\rho)^{DOE}$$

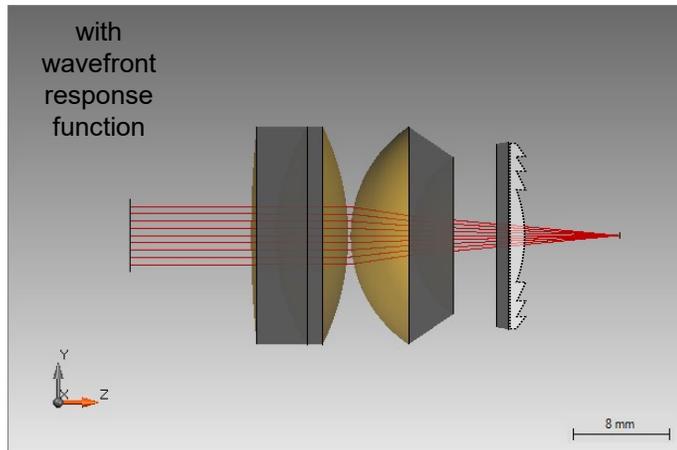


Preview for 2
quantization
levels

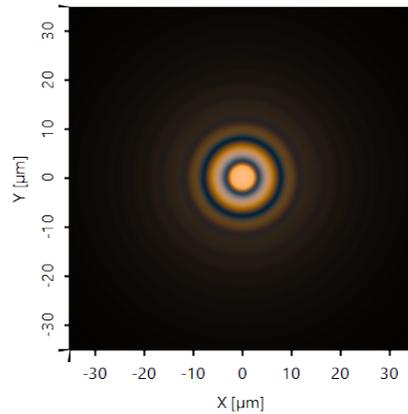
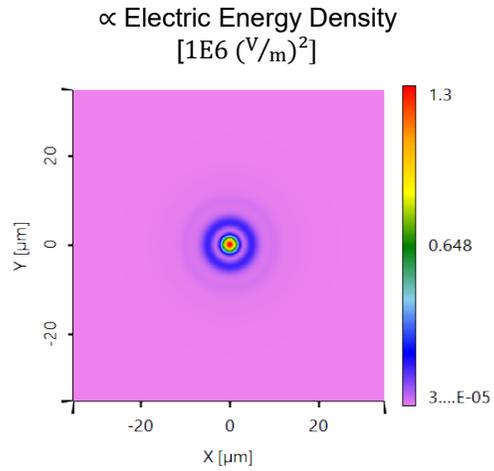


Preview for 4
quantization
levels

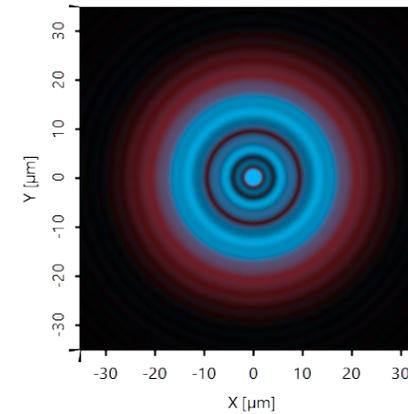
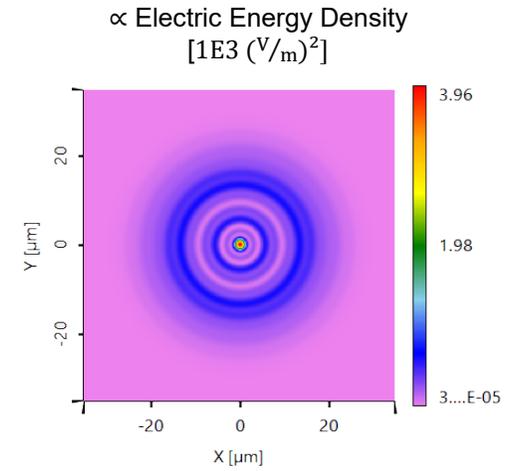
On-Axis Analysis: Inclusion of Higher Orders



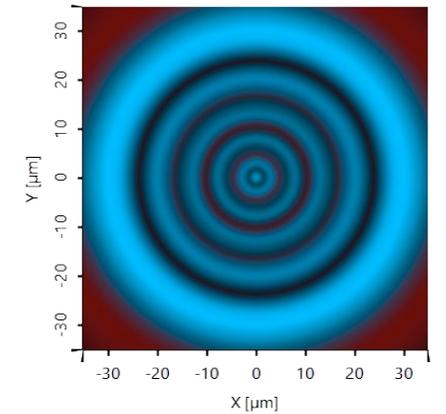
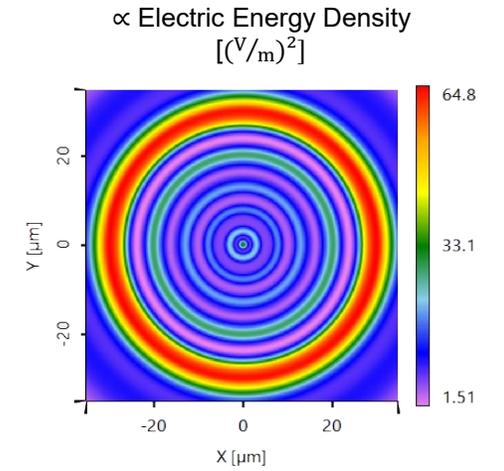
simulation time per order ~seconds



+1st diffraction order

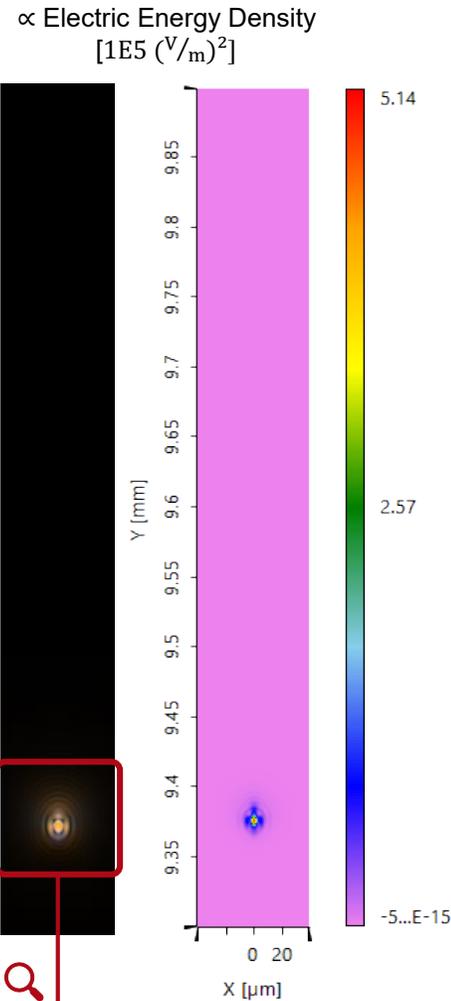
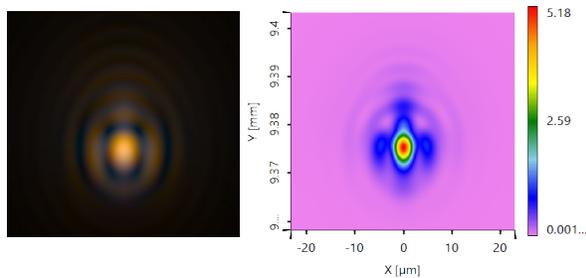
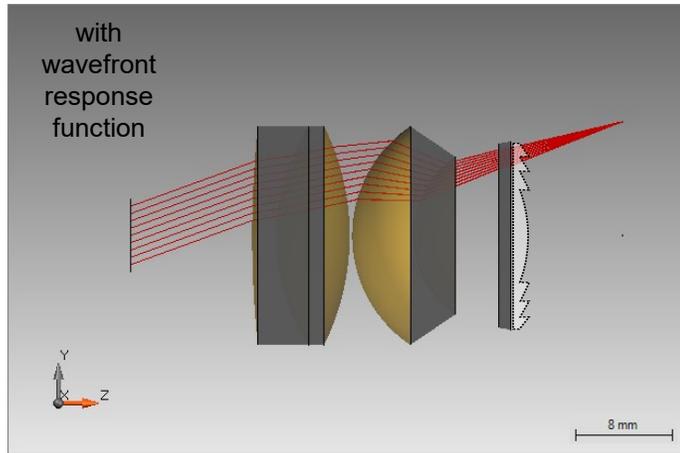


0th diffraction order

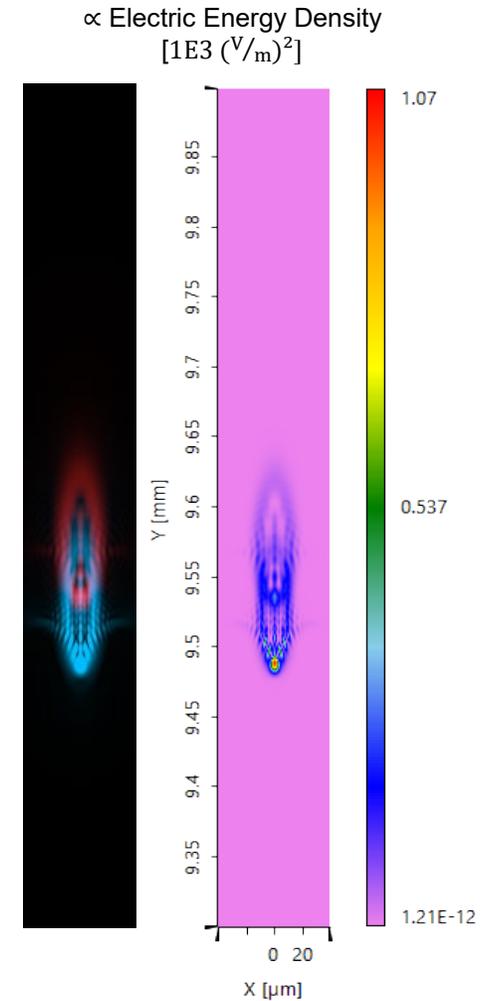


-1st diffraction order

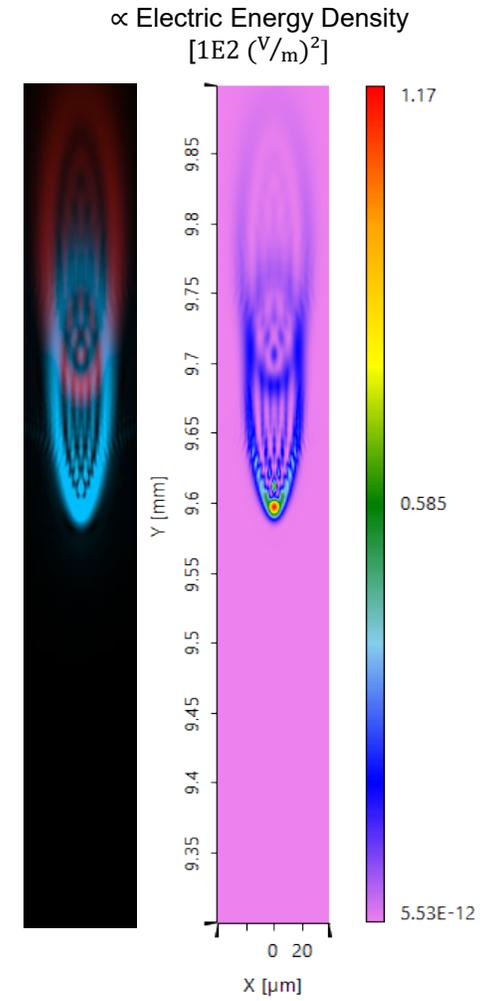
Off-Axis Analysis: Inclusion of Higher Orders



+1st diffraction order

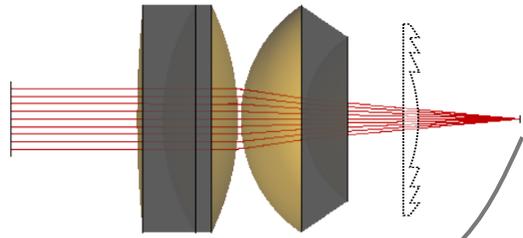


0th diffraction order

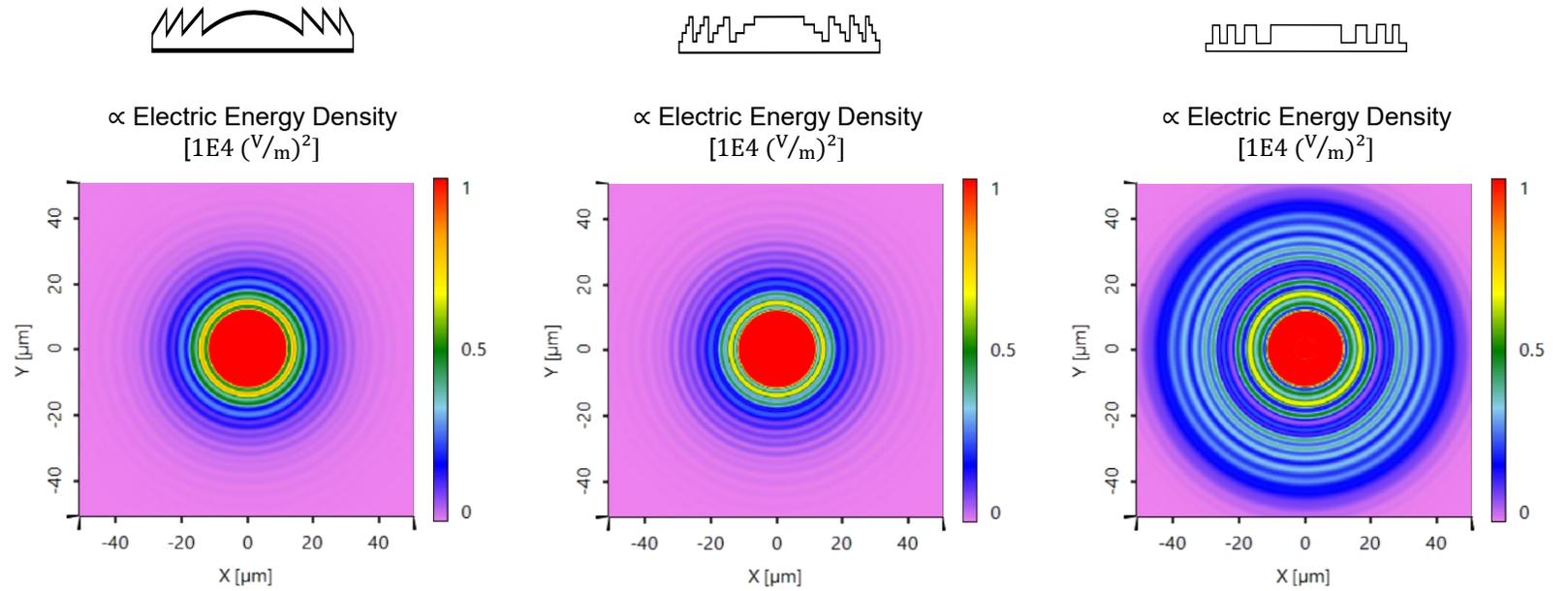
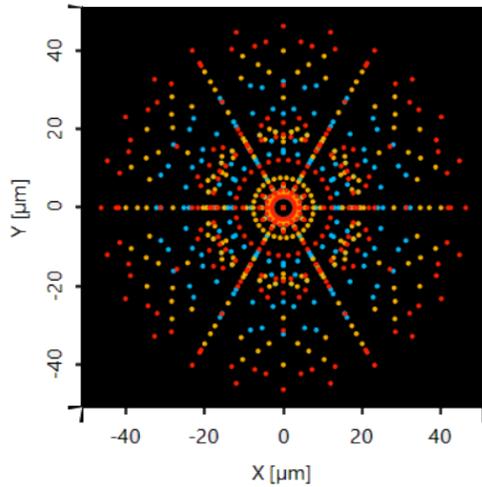


-1st diffraction order

On-Axis Analysis: Inclusion of Height Profile Quantization

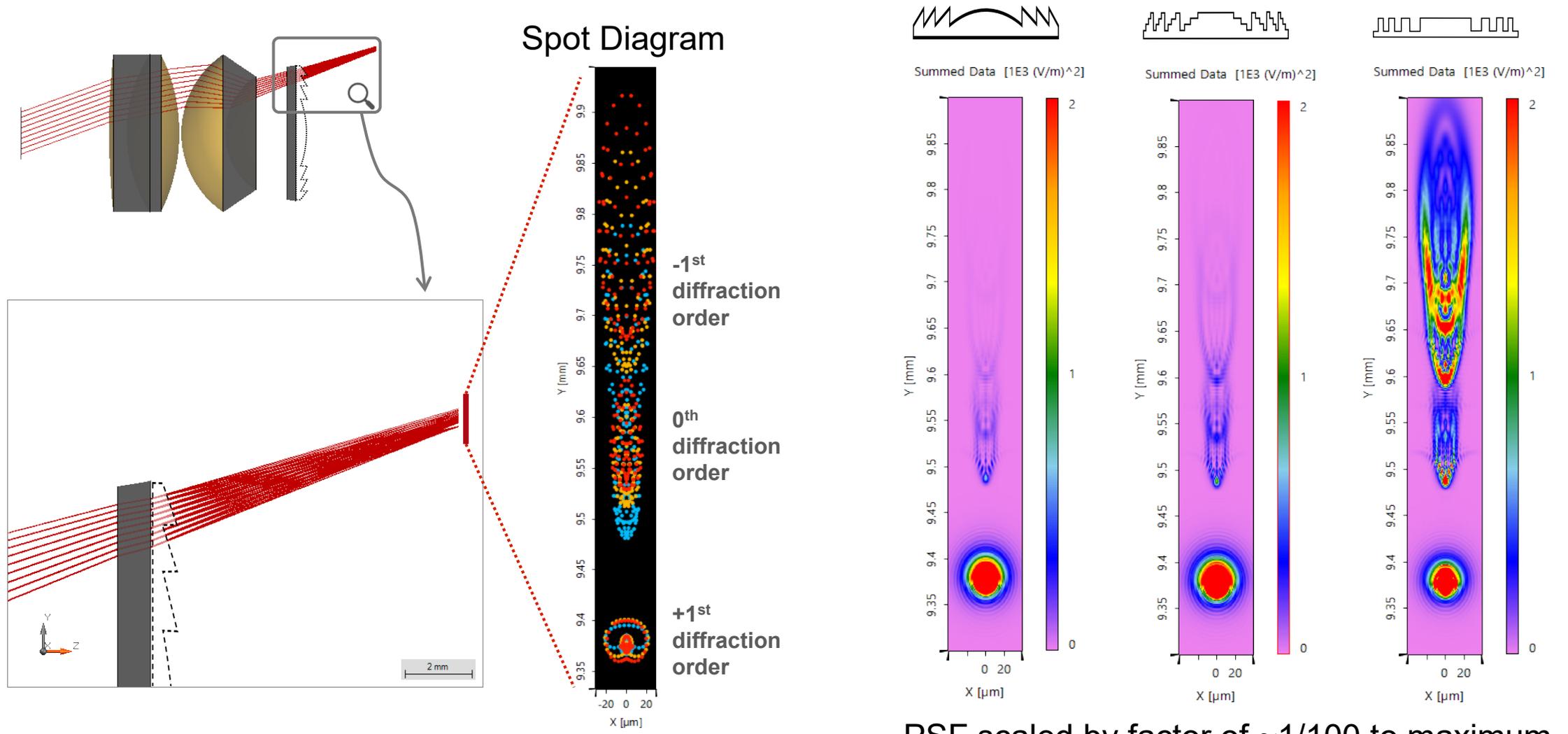


Spot Diagram



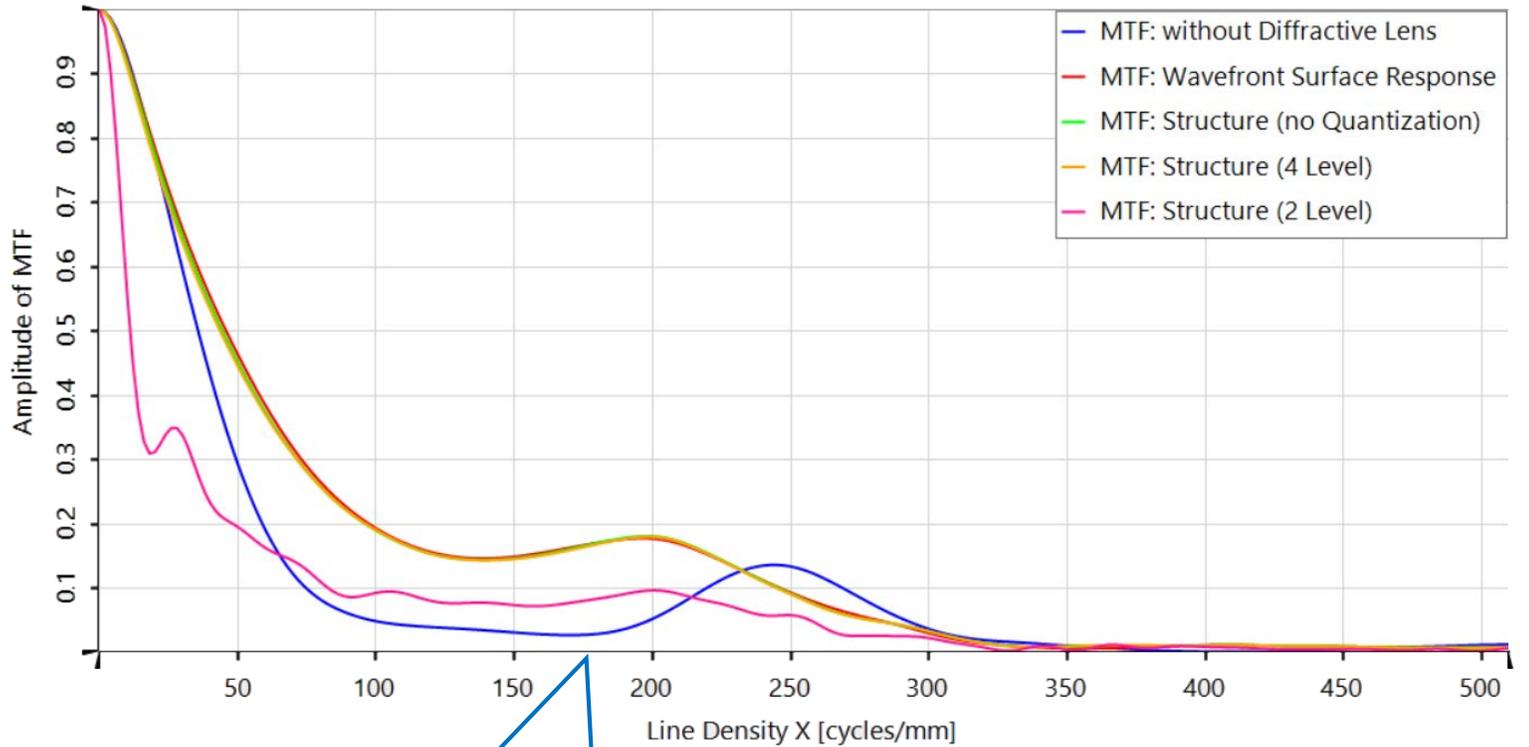
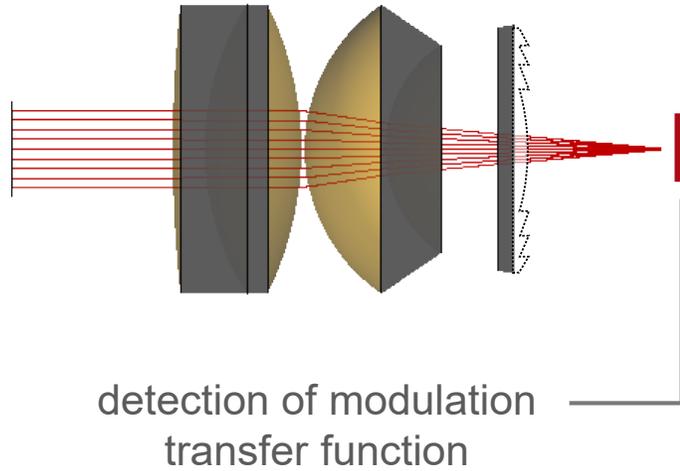
PSF scaled by factor of $\sim 1/100$ to maximum

Off-Axis Analysis: Inclusion of Height Profile Quantization



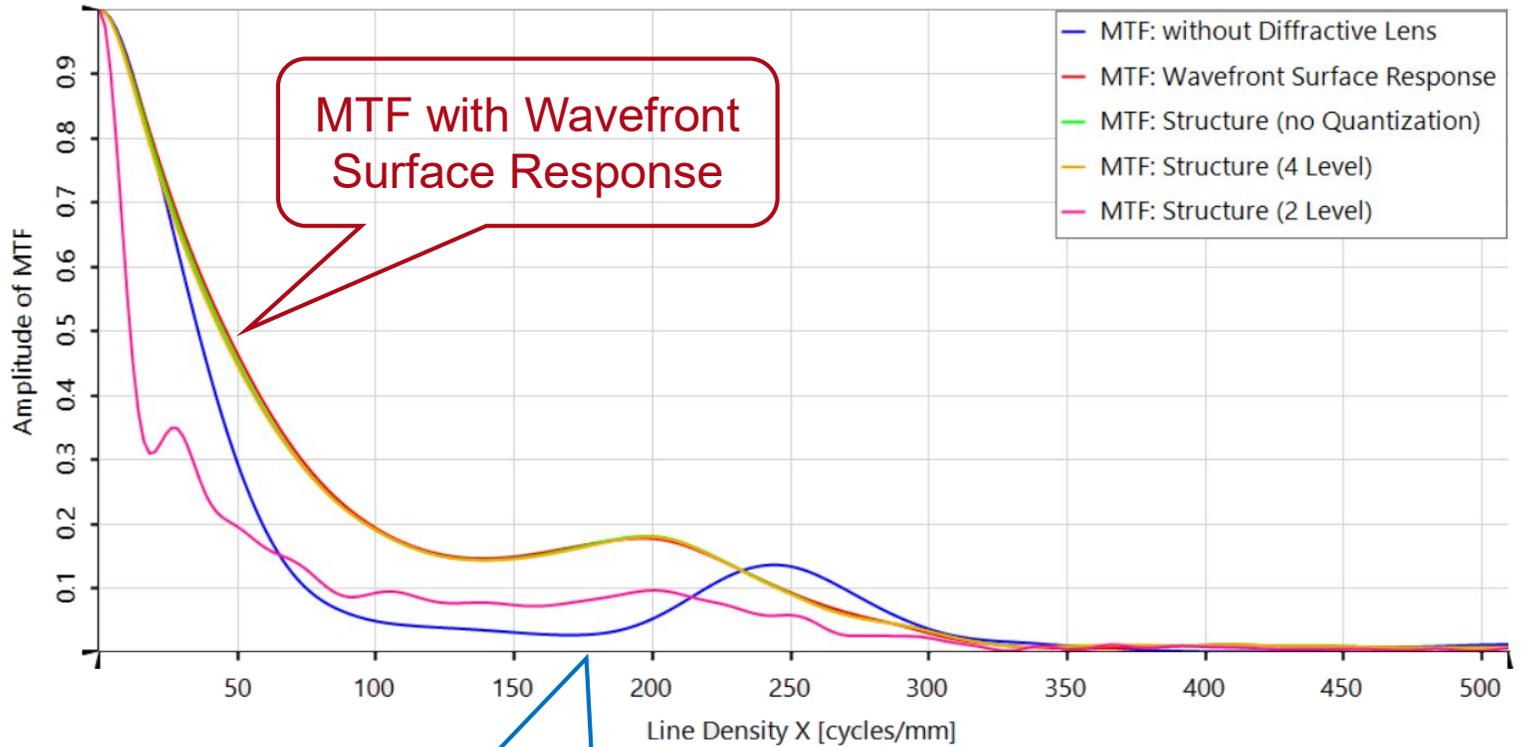
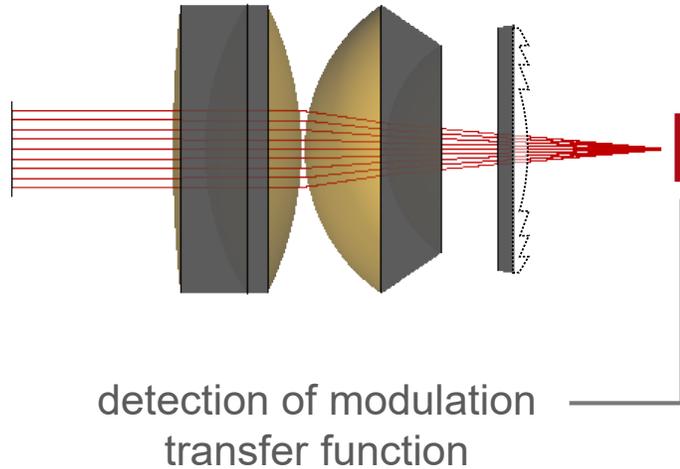
PSF scaled by factor of $\sim 1/100$ to maximum

MTF for Various Diffractive Lens Structures

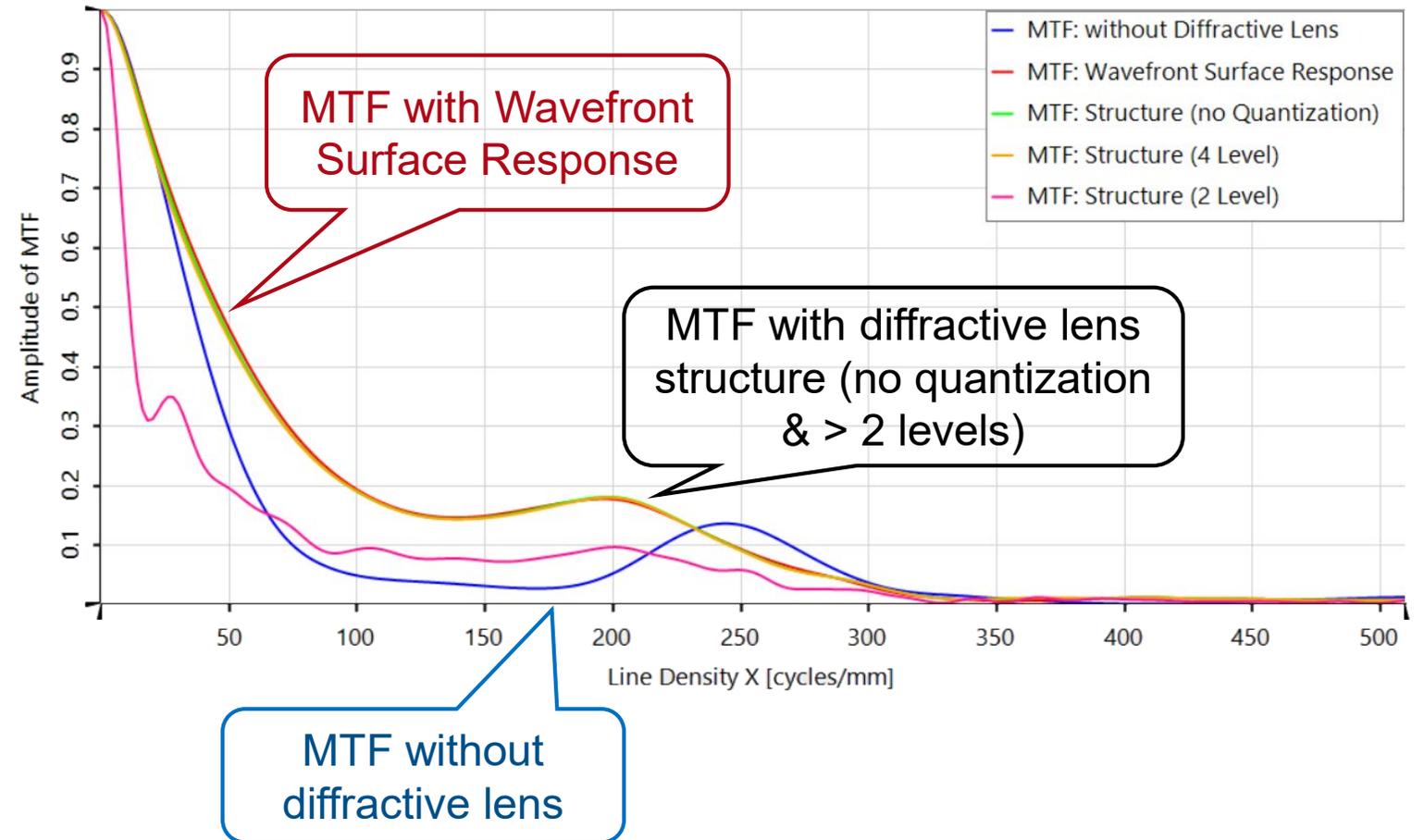
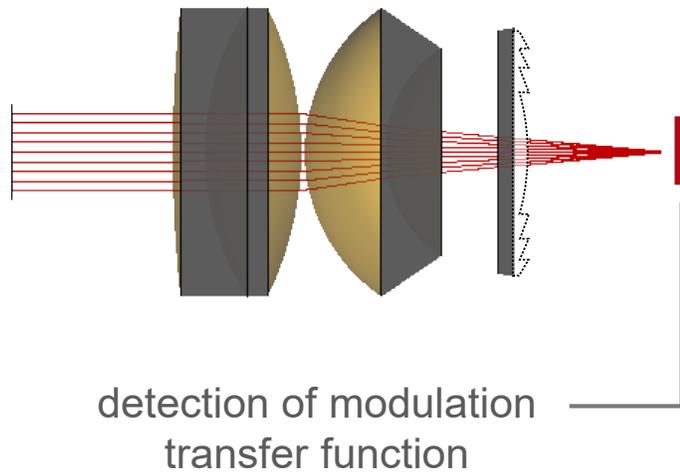


MTF without diffractive lens

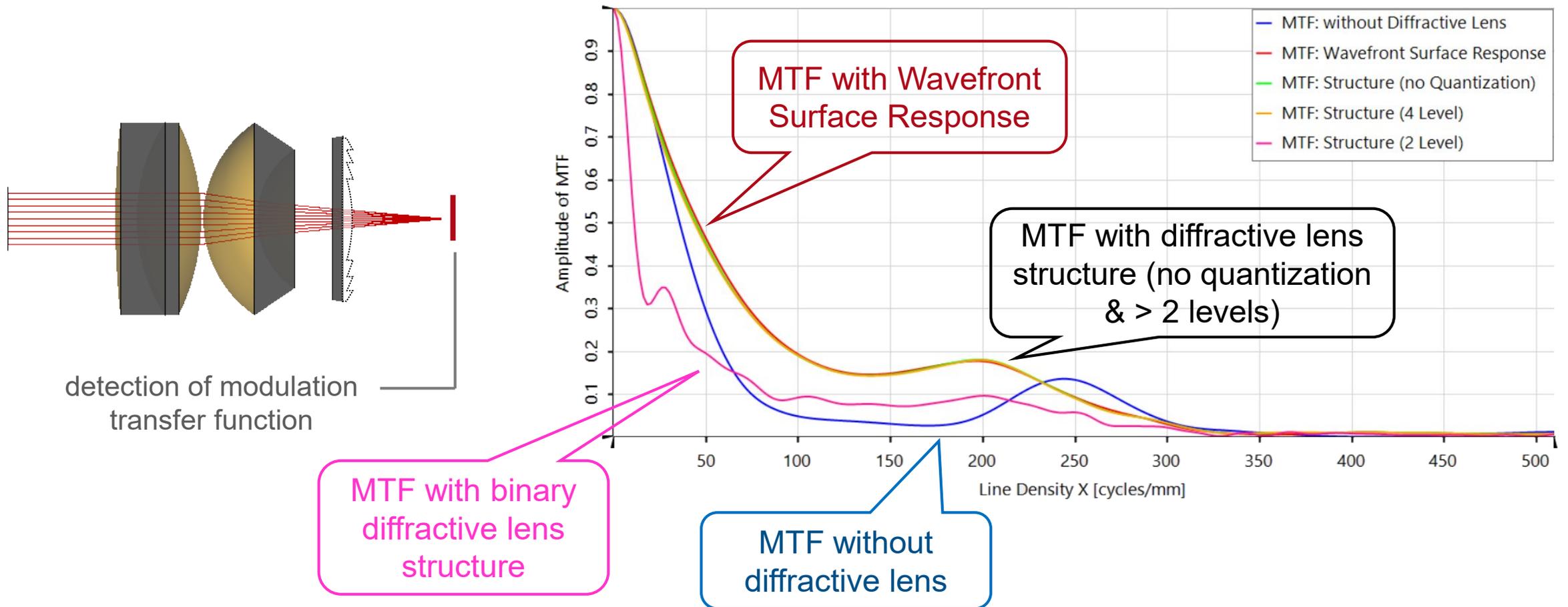
MTF for Various Diffractive Lens Structures



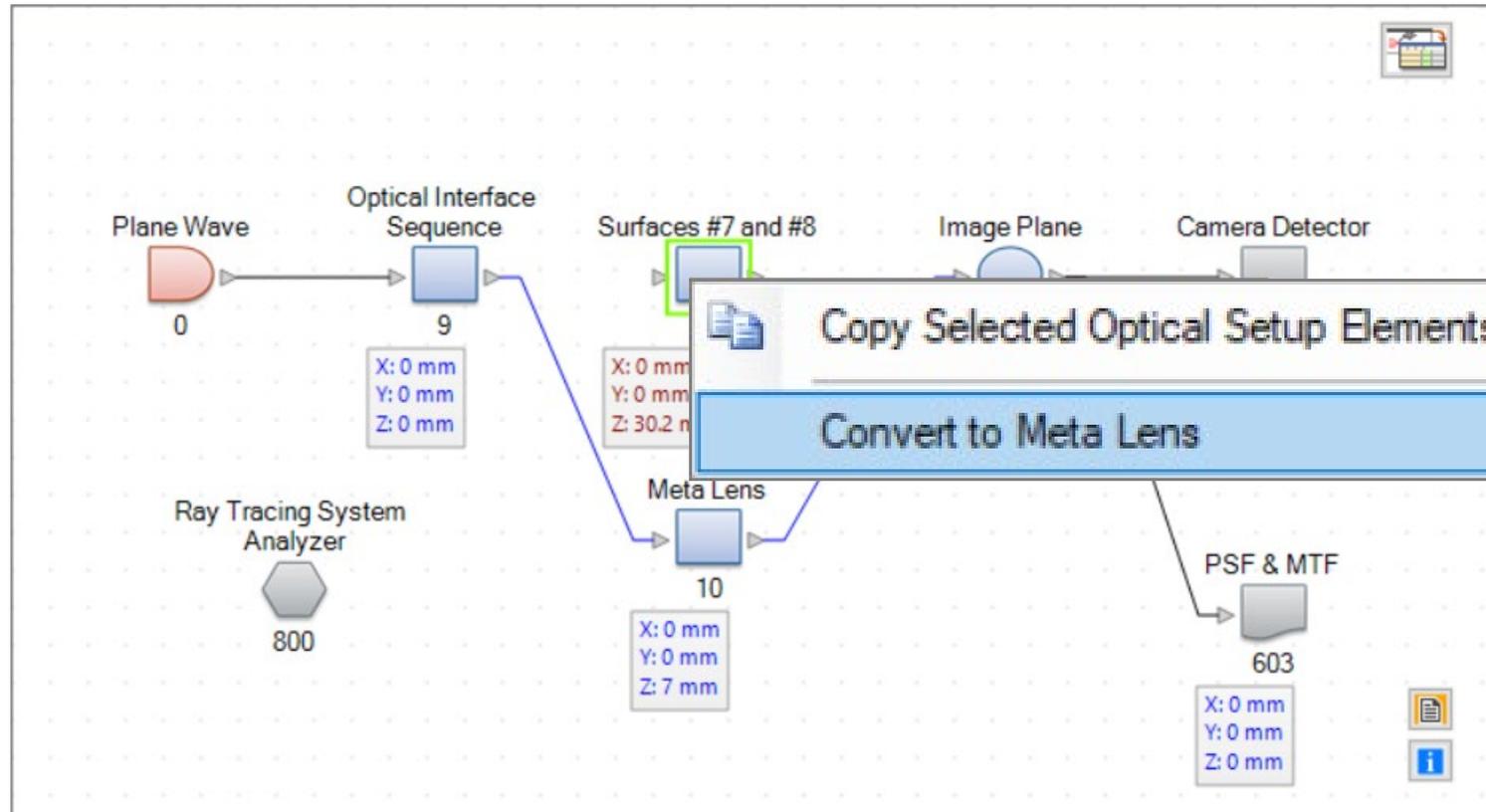
MTF for Various Diffractive Lens Structures



MTF for Various Diffractive Lens Structures



... convert to metalens



Structure of Workshop

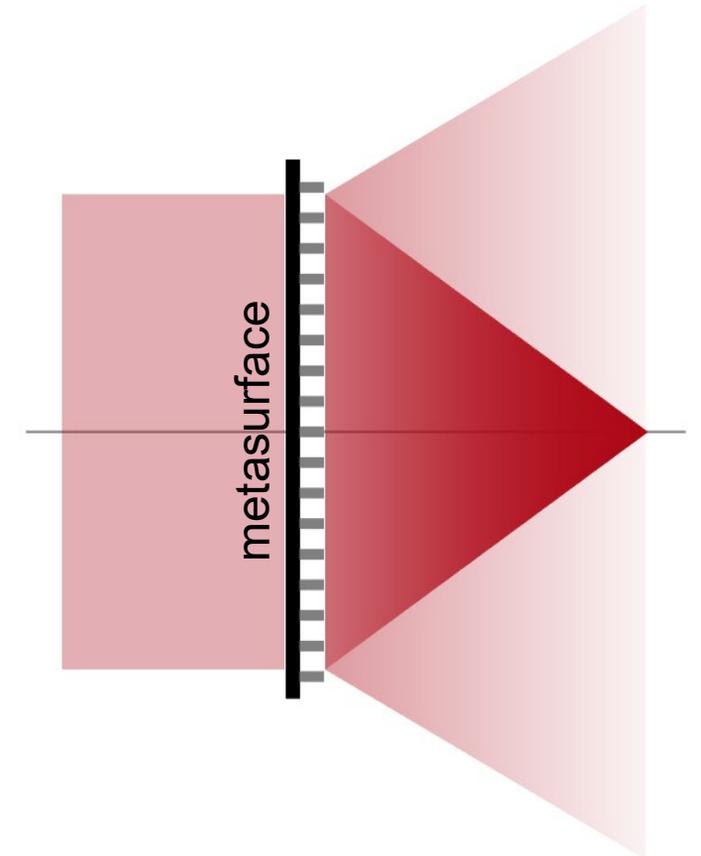
- Introduction of theory
 - Frank Wyrowski
- Design of binary surfaces in OpticStudio
 - Akil Bhagat
- Physical-optics analysis of imported systems from OpticStudio: Diffractive lenses
 - Roberto Knoth
- **Metalenses theory and modeling**
 - **Site Zhang**
- Fabrication export
 - Roberto Knoth

Metasurface Theory and Modeling

- In general a real surface layer structure does not just realize the desired wavefront response but additional effects and fields

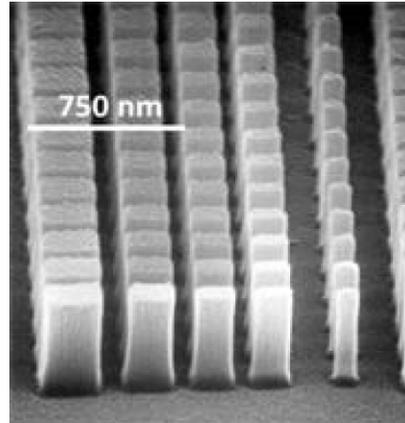
$$\mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \{ \mathbf{B}(\boldsymbol{\rho}; \psi^{\text{in}}) \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \} \exp(i(\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho}))) + \mathbf{V}_{\perp}^{\text{res}}(\boldsymbol{\rho}).$$

- There are different types of metasurfaces that can be used to realize the desired $\Delta\psi(\boldsymbol{\rho})$. The exact form of the B-operator and the residual terms depends on the employed type of metasurfaces.



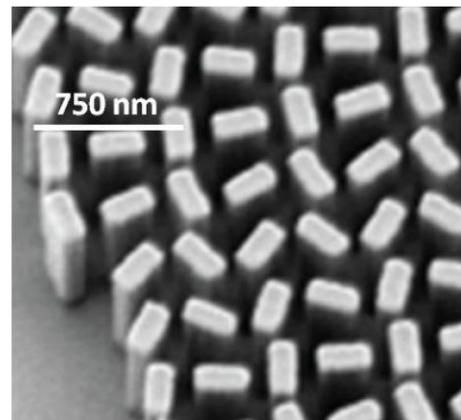
Physical Effects for Realizing Metasurfaces

- Propagation phase delay
 - Centrosymmetric (polarization insensitive)



P. Lalanne *et al.*, J. Opt. Soc. Am. A **16**, 1143-1156 (1999).

- Rotationally asymmetric (form birefringence)

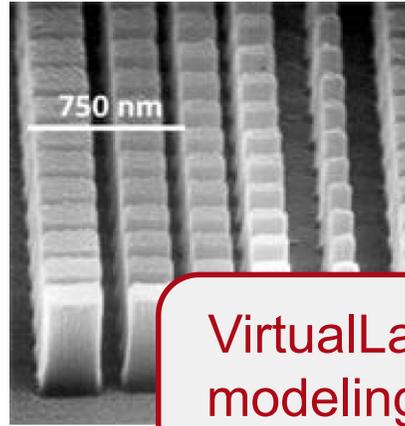


M. Khorasaninejad *et al.*, Science **352**, 1190-1194 (2016).

Physical Effects for Realizing Metasurfaces

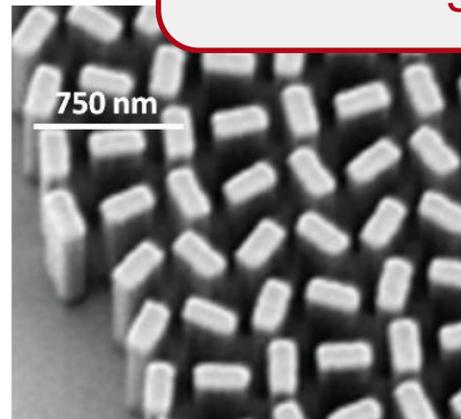
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- Rotationally asymmetric
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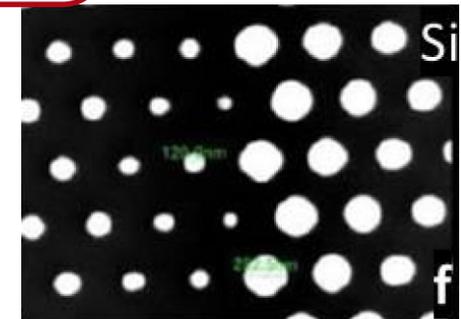


M. Khorasaninejad *et al.*, Science **352**, 1190-1194 (2016).

- Resonance phase delay



et al., Science **334**, 337 (2011).



Y. F. Yu *et al.*, Laser Photonics Rev. **9**, 412-418 (2015).

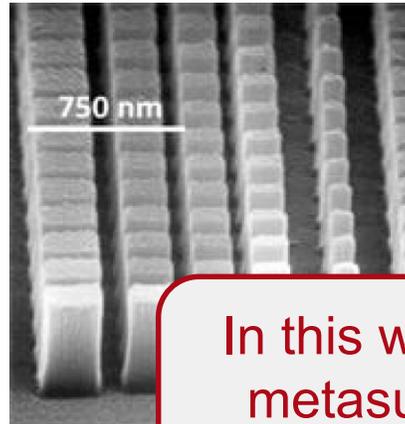
VirtualLab Fusion is capable of modeling different metasurface structures.

Physical Effects for Realizing Metasurfaces

- Propagation phase delay

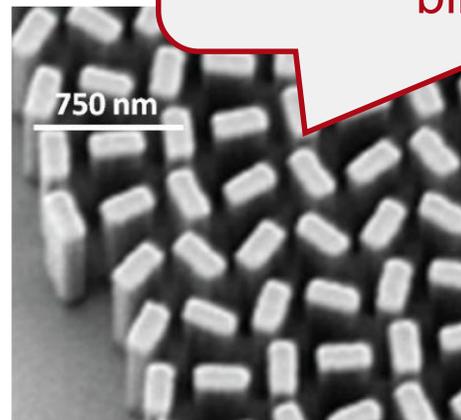
- Centrosymmetric
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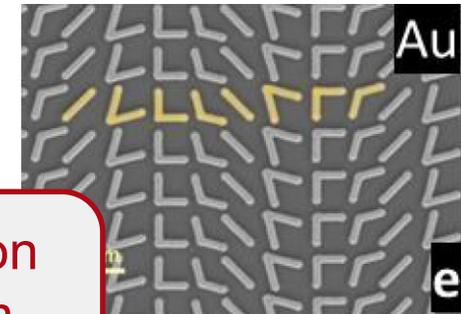
- Rotationally asymmetric
(form birefringence)

M. Khorasaninejad *et al.*, Science **352**, 1190-1194 (2016).

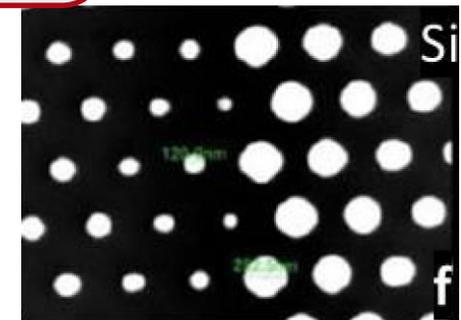


In this workshop, we focus on metasurface based on form birefringence

- Resonance phase delay



et al., Science **334**, 337 (2011).



Y. F. Yu *et al.*, Laser Photonics Rev. **9**, 412-418 (2015).

Metasurface Modeling

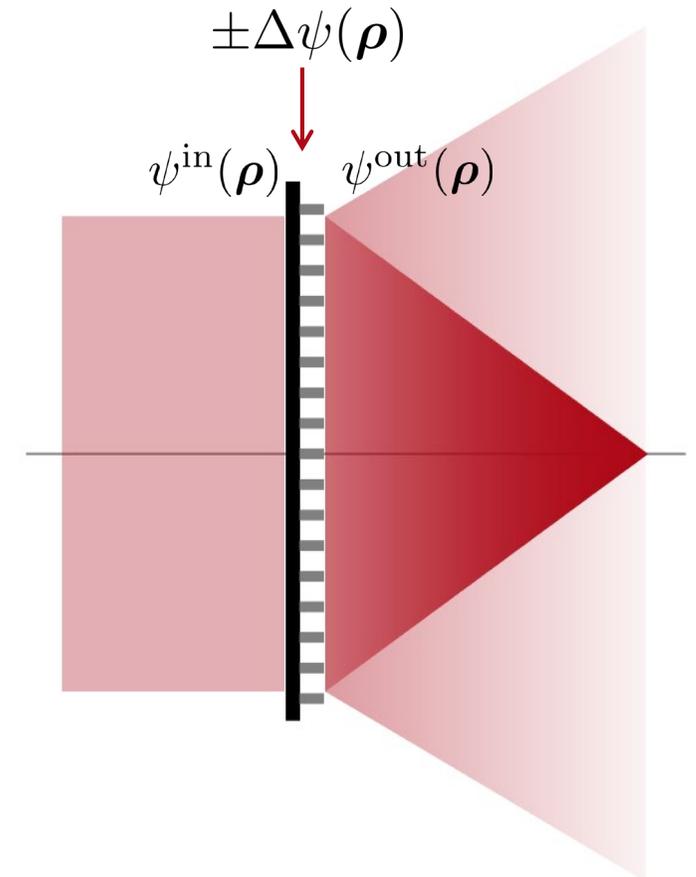
- In general a real surface layer structure does not just realize the desired wavefront response but additional effects and fields

$$\mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \{ \mathbf{B}(\boldsymbol{\rho}; \psi^{\text{in}}) \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \} \exp(i(\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho}))) + \mathbf{V}_{\perp}^{\text{res}}(\boldsymbol{\rho}).$$

- For metasurfaces made of rotated nanofins, the typical results can be written as

$$\begin{aligned} \mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = & \{ \mathbf{B}^{+}(\boldsymbol{\rho}; \psi^{\text{in}}) \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \} \exp(i(\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho}))) \\ & + \{ \mathbf{B}^{-}(\boldsymbol{\rho}; \psi^{\text{in}}) \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \} \exp(i(\psi^{\text{in}}(\boldsymbol{\rho}) - \Delta\psi(\boldsymbol{\rho}))) . \end{aligned}$$

- We will show that $+\Delta\psi$ occurs for R-circularly polarized input fields and the conjugate phase $-\Delta\psi$ for L-circularly polarized input.



Metasurface Building Block

- Locally at ρ , the B-operator for a single meta building block is

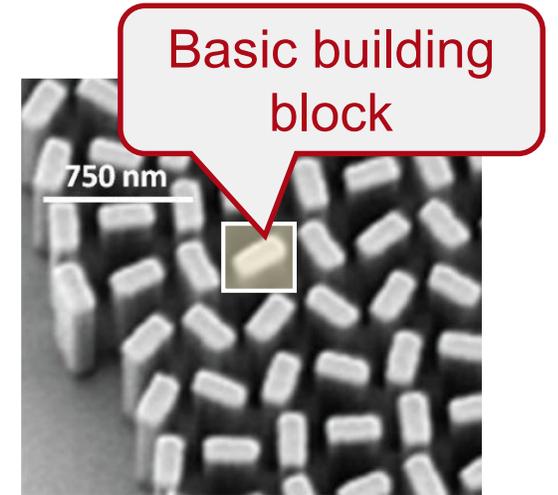
$$\mathbf{V}_{\perp}^{\text{out}}(\rho) = \mathcal{B}(\rho; \psi^{\text{in}}) \mathbf{V}_{\perp}^{\text{in}}(\rho),$$

and writing down the 2×2 -matrix B-operator explicitly, we have

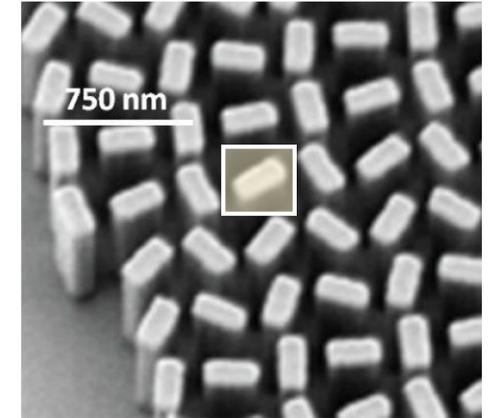
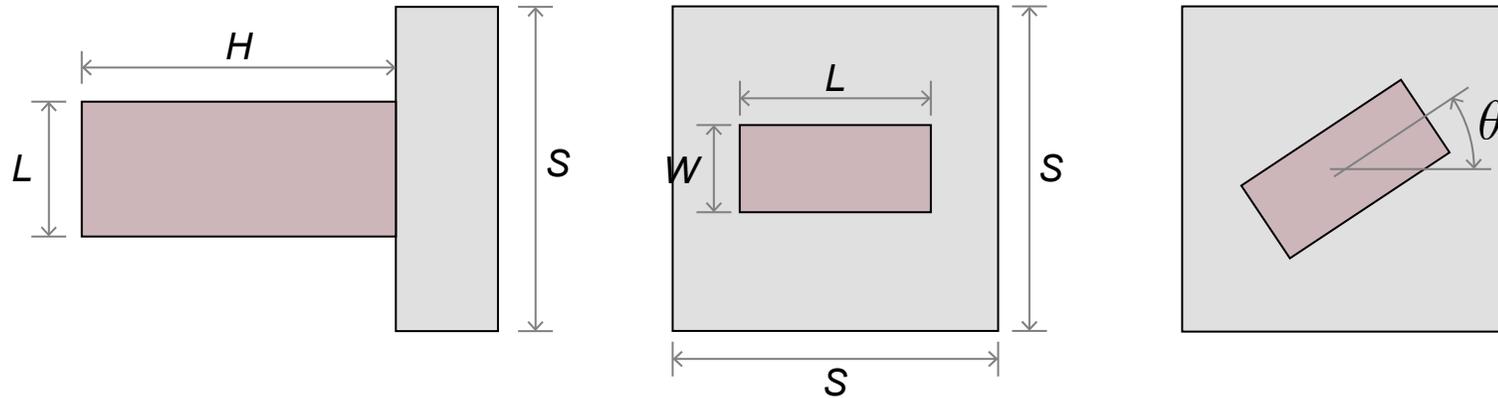
$$\mathbf{V}_{\perp}^{\text{out}}(\rho) = \begin{pmatrix} B_{xx}(\rho; \psi^{\text{in}}) & B_{xy}(\rho; \psi^{\text{in}}) \\ B_{yx}(\rho; \psi^{\text{in}}) & B_{yy}(\rho; \psi^{\text{in}}) \end{pmatrix} \mathbf{V}_{\perp}^{\text{in}}(\rho).$$

- Building blocks over the whole surface are same, but with different rotation angle $\theta(\rho)$. That can be expressed as

$$\mathbf{V}_{\perp}^{\text{out}}(\rho) = \begin{pmatrix} \cos \theta(\rho) & -\sin \theta(\rho) \\ \sin \theta(\rho) & \cos \theta(\rho) \end{pmatrix} \begin{pmatrix} b_{xx} & b_{xy} \\ b_{yx} & b_{yy} \end{pmatrix} \begin{pmatrix} \cos \theta(\rho) & \sin \theta(\rho) \\ -\sin \theta(\rho) & \cos \theta(\rho) \end{pmatrix} \mathbf{V}_{\perp}^{\text{in}}(\rho)$$



Metasurface Building Block



- Building blocks over the whole surface are same, different rotation angle $\theta(\boldsymbol{\rho})$. That can be expressed

... in the “eigen” coordinate system of the structure

$$\mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \begin{pmatrix} \cos \theta(\boldsymbol{\rho}) & -\sin \theta(\boldsymbol{\rho}) \\ \sin \theta(\boldsymbol{\rho}) & \cos \theta(\boldsymbol{\rho}) \end{pmatrix} \begin{pmatrix} b_{xx} & b_{xy} \\ b_{yx} & b_{yy} \end{pmatrix} \begin{pmatrix} \cos \theta(\boldsymbol{\rho}) & \sin \theta(\boldsymbol{\rho}) \\ -\sin \theta(\boldsymbol{\rho}) & \cos \theta(\boldsymbol{\rho}) \end{pmatrix} \mathbf{V}_{\perp}^{\text{in}}(\boldsymbol{\rho})$$

Metasurface Building Block

- Omitting the variables for conciseness in the expr

... to be analyzed
in detail later

$$\mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} b_{xx} & b_{xy} \\ b_{yx} & b_{yy} \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \mathbf{V}_{\perp}^{\text{in}}(\boldsymbol{\rho}),$$

and, without introducing any approximation, but rearrange the terms so that the rotation angle θ can be extracted, we obtain the following result

$$\begin{aligned} \mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = & \frac{1}{2} \left[(b_{xx} + b_{yy}) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + (b_{xy} - b_{yx}) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right] \mathbf{V}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \\ & + \frac{1}{4} \left[(b_{xx} - b_{yy}) \begin{pmatrix} 1 & -i \\ -i & -1 \end{pmatrix} + (b_{xy} + b_{yx}) \begin{pmatrix} i & 1 \\ 1 & -i \end{pmatrix} \right] \mathbf{V}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \exp(i2\theta) \\ & + \frac{1}{4} \left[(b_{xx} - b_{yy}) \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} + (b_{xy} + b_{yx}) \begin{pmatrix} -i & 1 \\ 1 & i \end{pmatrix} \right] \mathbf{V}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \exp(-i2\theta). \end{aligned}$$

Metasurface Modeling

- Omitting the variables for conciseness in the expression

$$\mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} b_{xx} & b_{xy} \\ b_{yx} & b_{yy} \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \mathbf{V}_{\perp}^{\text{in}}(\boldsymbol{\rho}),$$

and, without introducing any approximation, but rearrange the terms so that the rotation angle θ is the same in all terms, we obtain the following result

$$\begin{aligned} \mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = & \frac{1}{2} \left[(b_{xx} + b_{yy}) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + (b_{xx} - b_{yy}) \begin{pmatrix} 1 & -i \\ -i & -1 \end{pmatrix} \right] \mathbf{V}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \exp(i\Delta\psi(\boldsymbol{\rho})) \\ & + \frac{1}{4} \left[(b_{xx} - b_{yy}) \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} + (b_{xy} + b_{yx}) \begin{pmatrix} -i & 1 \\ 1 & i \end{pmatrix} \right] \mathbf{V}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \exp(-i\Delta\psi(\boldsymbol{\rho})), \end{aligned}$$

With spatially varying rotation angles, the desired wavefront surface response can be realized!

with $\Delta\psi(\boldsymbol{\rho}) := 2\theta(\boldsymbol{\rho})$, and $\theta(\boldsymbol{\rho})$ is the rotation angle.

Metasurface Modeling

- Omitting the variables for conciseness in the expression

$$\mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} b_{xx} & b_{xy} \\ b_{yx} & b_{yy} \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \mathbf{V}_{\perp}^{\text{in}}(\boldsymbol{\rho}),$$

and, without introducing any approximation, but rearrange the terms so that the rotation angle θ can be extracted, we obtain the following result

$$\begin{aligned} \mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = & \frac{1}{2} \left[(b_{xx} + b_{yy}) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + (b_{xy} - b_{yx}) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right] \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \exp(i\psi^{\text{in}}(\boldsymbol{\rho})) \\ & + \frac{1}{4} \left[(b_{xx} - b_{yy}) \begin{pmatrix} 1 & -i \\ -i & -1 \end{pmatrix} + (b_{xy} + b_{yx}) \begin{pmatrix} i & 1 \\ 1 & -i \end{pmatrix} \right] \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \exp\left(i(\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho}))\right) \\ & + \frac{1}{4} \left[(b_{xx} - b_{yy}) \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} + (b_{xy} + b_{yx}) \begin{pmatrix} -i & 1 \\ 1 & i \end{pmatrix} \right] \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \exp\left(i(\psi^{\text{in}}(\boldsymbol{\rho}) - \Delta\psi(\boldsymbol{\rho}))\right), \end{aligned}$$

with $\Delta\psi(\boldsymbol{\rho}) := 2\theta(\boldsymbol{\rho})$, and $\theta(\boldsymbol{\rho})$ is the rotation angle.

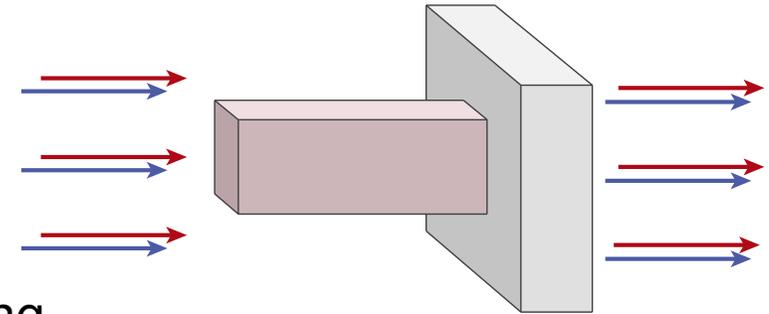
desired response $\Delta\psi(\boldsymbol{\rho})$

conjugate part $-\Delta\psi(\boldsymbol{\rho})$

Form Birefringence Analysis

- Next, we would like to examine the matrix

$$\begin{pmatrix} b_{xx} & b_{xy} \\ b_{yx} & b_{yy} \end{pmatrix}$$

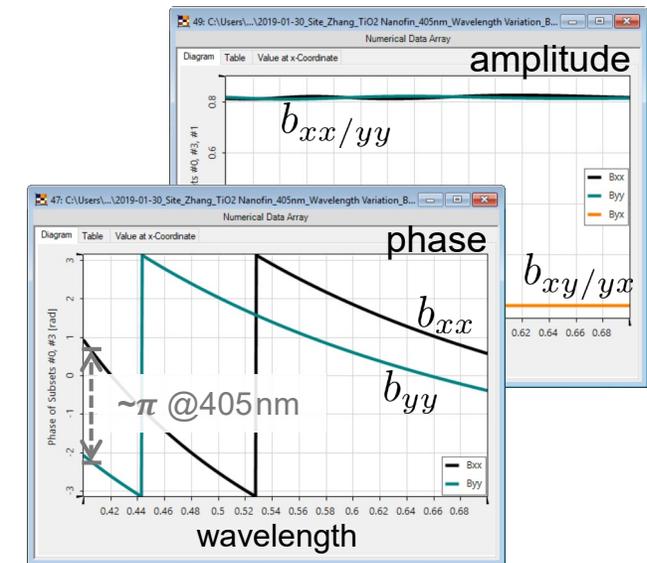


that describes the fundamental property of the meta building block, and we analyze it rigorously.

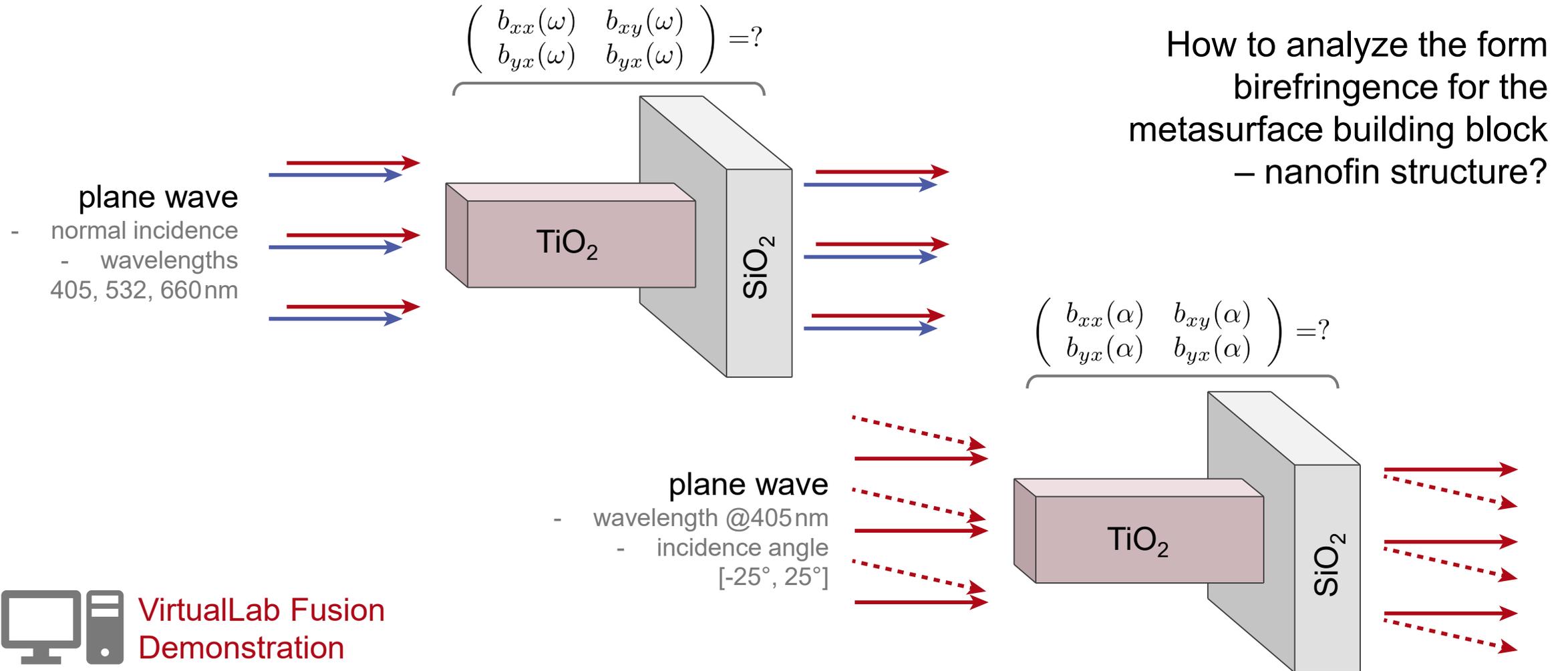
- Ideally, the building block is supposed to work as a half-wave plate, i.e.

$$\begin{aligned} b_{xy} &\approx 0, \\ b_{yx} &\approx 0, \\ b_{xx} &\approx -b_{yy}. \end{aligned}$$

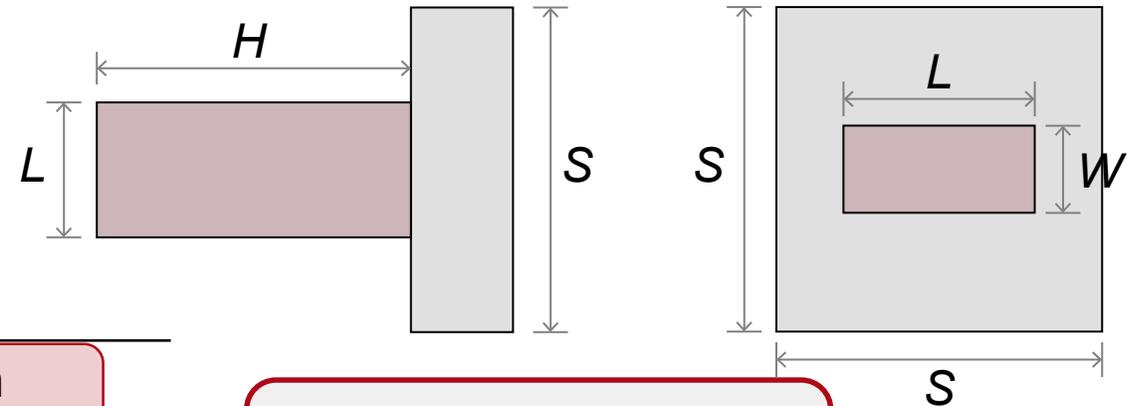
- We will substitute these approximations step by step in what follows.



Form Birefringence Analysis



Nanofin Structural Designs



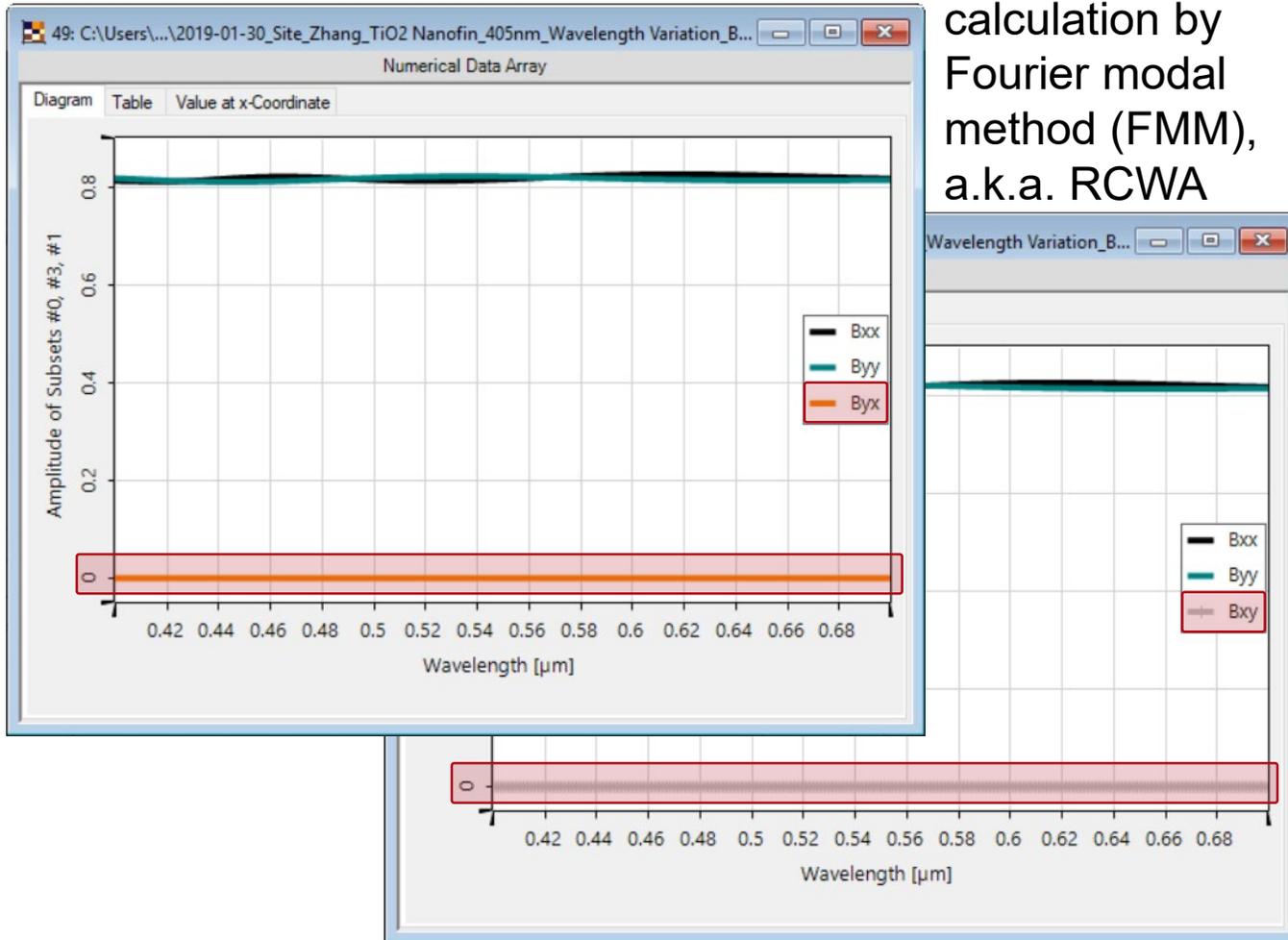
structural parameters

Case	660 nm Design	532 nm Design	405 nm Design
refractive index	$n=2.36$	$n=2.43$	$n=2.63$
W	85 nm	95 nm	40 nm
L	410 nm	250 nm	150 nm
H	600 nm	600 nm	600 nm
S	430 nm	325 nm	200 nm

We analyze this case as an example.

M. Khorasaninejad, W. T. Chen, R. C. Devlin, J. Oh, A. Y. Zhu, F. Capasso, *Science* **2016**, 352 (6290), 1190-1194

Spectral Analysis for Nanofin – 405nm Design

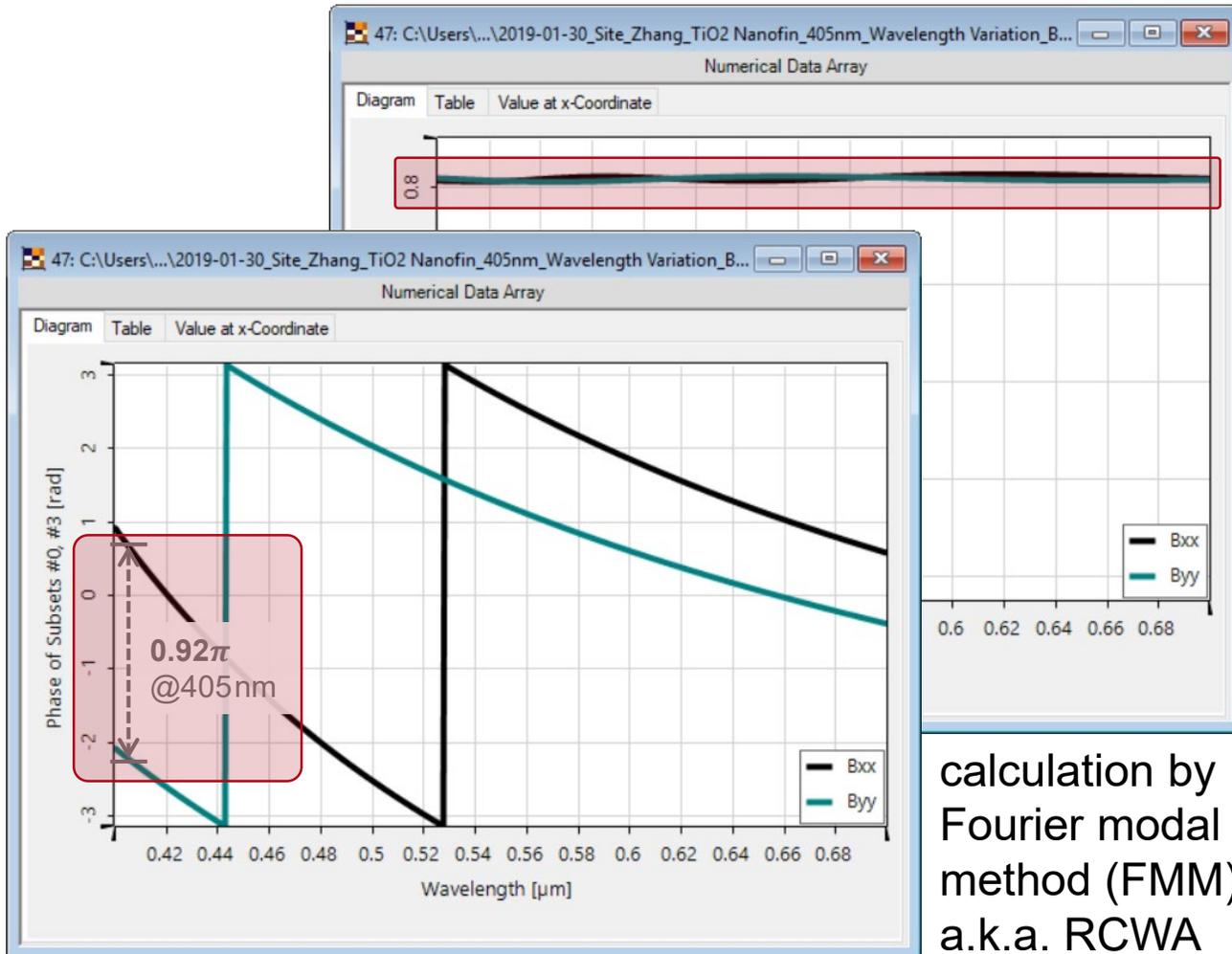


- B-matrix for metasurface building block

$$\begin{pmatrix} b_{xx} & b_{xy} \\ b_{yx} & b_{yy} \end{pmatrix}$$
 Almost no polarization crosstalk
 function as a half-wave plate, i.e.

$$\begin{aligned}
 b_{xy} &\approx 0, \\
 b_{yx} &\approx 0, \\
 b_{xx} &\approx -b_{yy}.
 \end{aligned}$$

Spectral Analysis for Nanofin – 405nm Design



- B-matrix for metasurface building block

$$\begin{pmatrix} b_{xx} & b_{xy} \\ b_{yx} & b_{yy} \end{pmatrix}.$$

- Ideally, it should function as .e.

Almost half-wave plate effect

$$b_{yx} = 0,$$

$$b_{xx} \approx -b_{yy}.$$

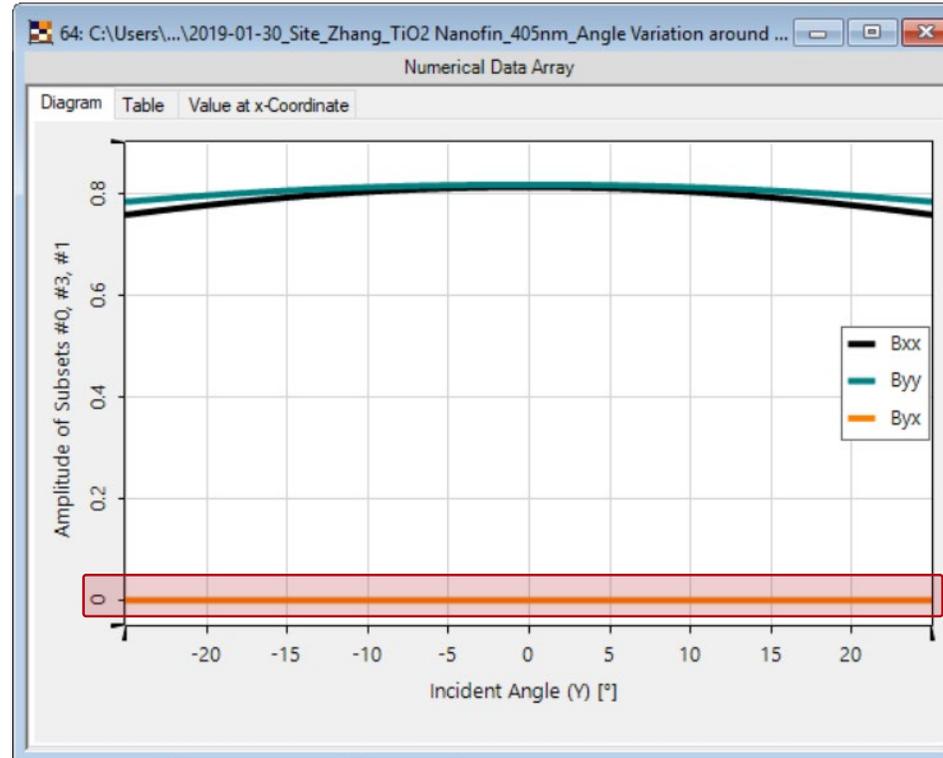
Angular Analysis for Nanofin – 405nm Design

- B-matrix for metasurface building block

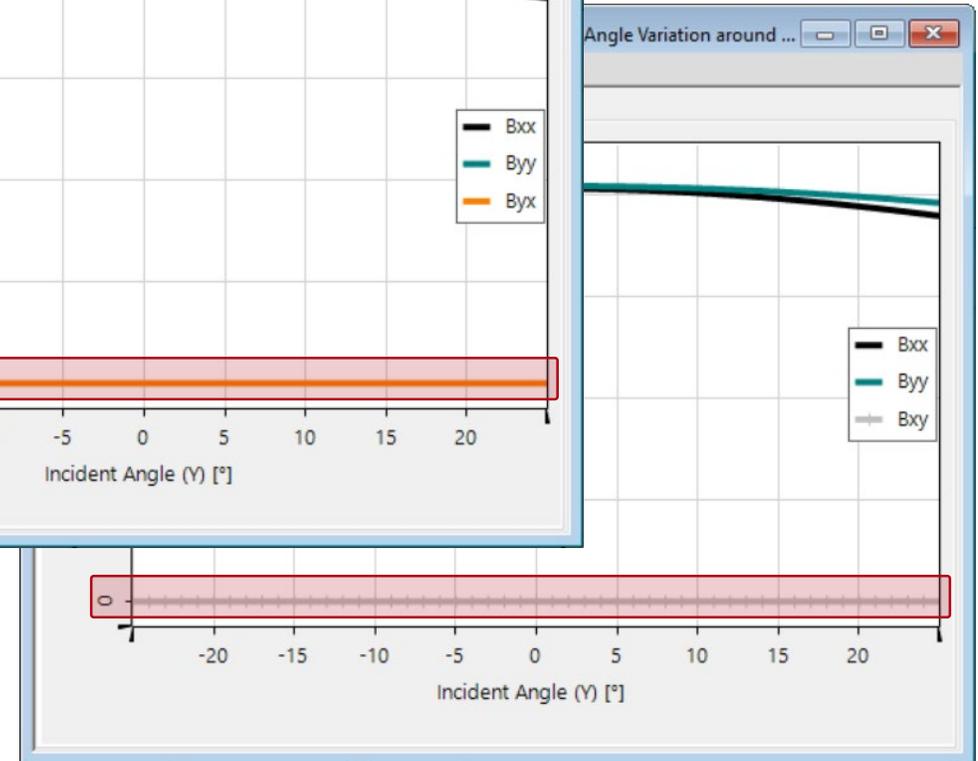
$$\begin{pmatrix} b_{xx} & b_{xy} \\ b_{yx} & b_{yy} \end{pmatrix}.$$

- Ideally, it should function as a half-wave plate, i.e.

$$\begin{aligned} b_{xy} &\approx 0, \\ b_{yx} &\approx 0, \\ b_{xx} &\approx -b_{yy}. \end{aligned}$$



calculation by Fourier modal method (FMM), a.k.a. RCWA



Angular Analysis for Nanofin – 405nm Design

- B-matrix for metasurface building block

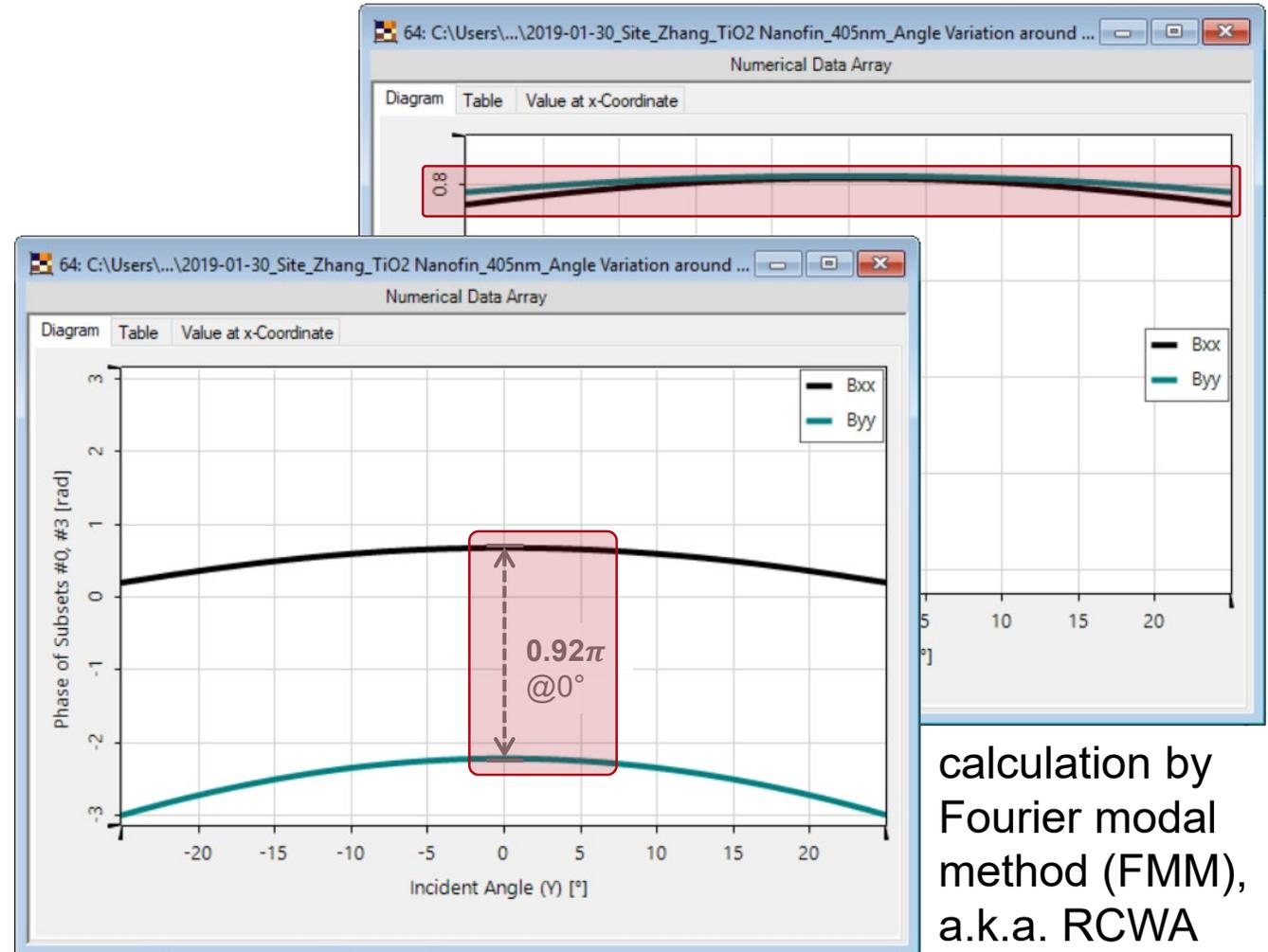
$$\begin{pmatrix} b_{xx} & b_{xy} \\ b_{yx} & b_{yy} \end{pmatrix}.$$

- Ideally, it should function as a half-wave plate, i.e.

$$b_{xy} \approx 0,$$

$$b_{yx} \approx 0,$$

$$b_{xx} \approx -b_{yy}.$$



calculation by Fourier modal method (FMM), a.k.a. RCWA

Metasurface Modeling

- Use the half-wave plate approximations

$$\left. \begin{aligned} b_{xy} &\approx 0, & b_{yx} &\approx 0, \\ b_{xx} &\approx -b_{yy}. \end{aligned} \right\}$$

As shown before, valid for certain wavelength and angle range

- Then, the metasurface response can be written as

$$\begin{aligned} \mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = & \frac{1}{2} \left[\cancel{(b_{xx} + b_{yy})} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \cancel{(b_{xy} - b_{yx})} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \right] \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \exp(i\psi^{\text{in}}(\boldsymbol{\rho})) \\ & + \frac{1}{4} \left[(b_{xx} - b_{yy}) \begin{pmatrix} 1 & -i \\ -i & -1 \end{pmatrix} + \cancel{(b_{xy} + b_{yx})} \begin{pmatrix} i & 1 \\ 1 & -i \end{pmatrix} \right] \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \exp\left(i(\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho}))\right) \\ & + \frac{1}{4} \left[(b_{xx} - b_{yy}) \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} + \cancel{(b_{xy} - b_{yx})} \begin{pmatrix} -i & 1 \\ 1 & i \end{pmatrix} \right] \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \exp\left(i(\psi^{\text{in}}(\boldsymbol{\rho}) - \Delta\psi(\boldsymbol{\rho}))\right). \end{aligned}$$

desired response $\Delta\psi(\boldsymbol{\rho})$

conjugate part $-\Delta\psi(\boldsymbol{\rho})$

Metasurface Modeling

- Use the half-wave plate approximations

$$b_{xy} \approx 0, \quad b_{yx} \approx 0, \\ b_{xx} \approx -b_{yy}.$$

- Then, the metasurface response can be written as

$$\begin{aligned} \mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) &= \frac{1}{4} (b_{xx} - b_{yy}) \begin{pmatrix} 1 & -i \\ -i & -1 \end{pmatrix} \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \exp\left(i(\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho}))\right) \longrightarrow \text{desired } \Delta\psi(\boldsymbol{\rho}) \\ &\quad + \frac{1}{4} (b_{xx} - b_{yy}) \begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \exp\left(i(\psi^{\text{in}}(\boldsymbol{\rho}) - \Delta\psi(\boldsymbol{\rho}))\right) \longrightarrow \text{conjugate part} \\ &= \left\{ \mathbf{B}^{+}(\boldsymbol{\rho}; \psi^{\text{in}}) \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \right\} \exp\left(i(\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho}))\right) \\ &\quad + \left\{ \mathbf{B}^{-}(\boldsymbol{\rho}; \psi^{\text{in}}) \mathbf{U}_{\perp}^{\text{in}}(\boldsymbol{\rho}) \right\} \exp\left(i(\psi^{\text{in}}(\boldsymbol{\rho}) - \Delta\psi(\boldsymbol{\rho}))\right) \end{aligned}$$

Metasurface Modeling – Polarization Effect

- If the input field is R-circularly polarized, i.e.

$$\mathbf{V}_{\perp}^{\text{in}}(\boldsymbol{\rho}) = \begin{pmatrix} 1 \\ i \end{pmatrix} U^{\text{in}}(\boldsymbol{\rho}) \exp(i\psi^{\text{in}}(\boldsymbol{\rho})),$$

Hint:

$$\begin{pmatrix} 1 & i \\ i & -1 \end{pmatrix} \begin{pmatrix} 1 \\ i \end{pmatrix} = 0$$

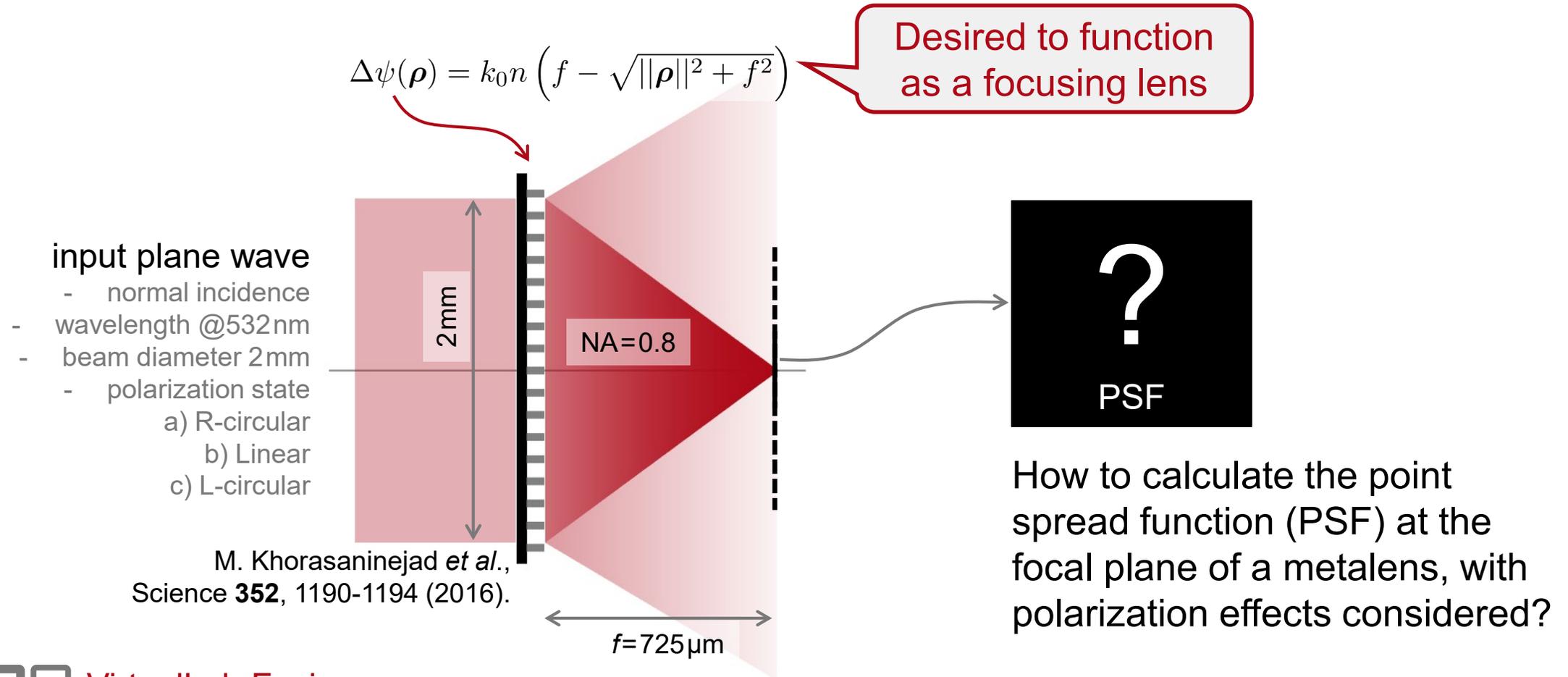
substituting it into the B-operator expression, yields the output field in the following form

$$\mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \frac{1}{2} (b_{xx} - b_{yy}) \begin{pmatrix} 1 \\ -i \end{pmatrix} U^{\text{in}}(\boldsymbol{\rho}) \exp(i(\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho}))) . \longrightarrow \text{desired } \Delta\psi(\boldsymbol{\rho}) \text{ only}$$

- In the idealized case, with $b_{xx} = 1$ and $b_{yy} = -1$, one gets

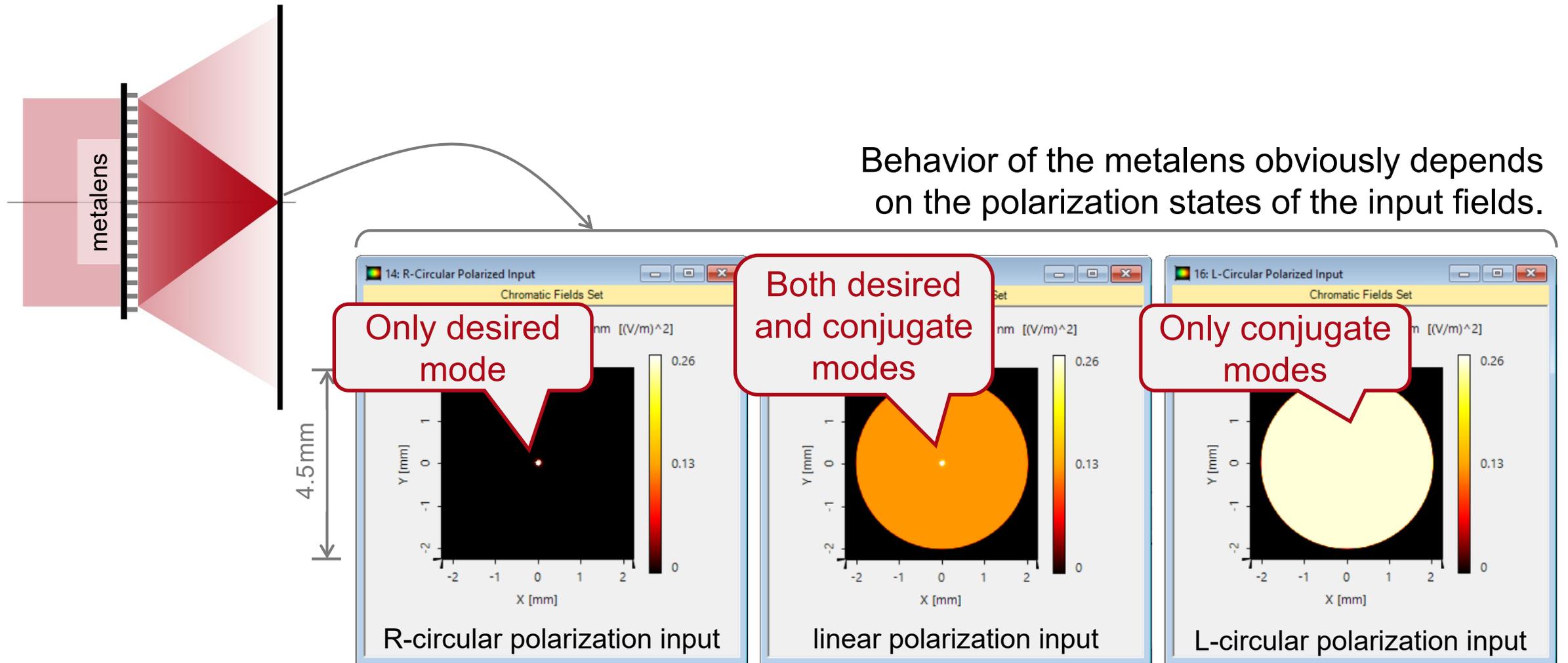
$$\mathbf{V}_{\perp}^{\text{out}}(\boldsymbol{\rho}) = \begin{pmatrix} 1 \\ -i \end{pmatrix} U^{\text{in}}(\boldsymbol{\rho}) \exp(i(\psi^{\text{in}}(\boldsymbol{\rho}) + \Delta\psi(\boldsymbol{\rho}))) .$$

High-NA Metalens Simulation

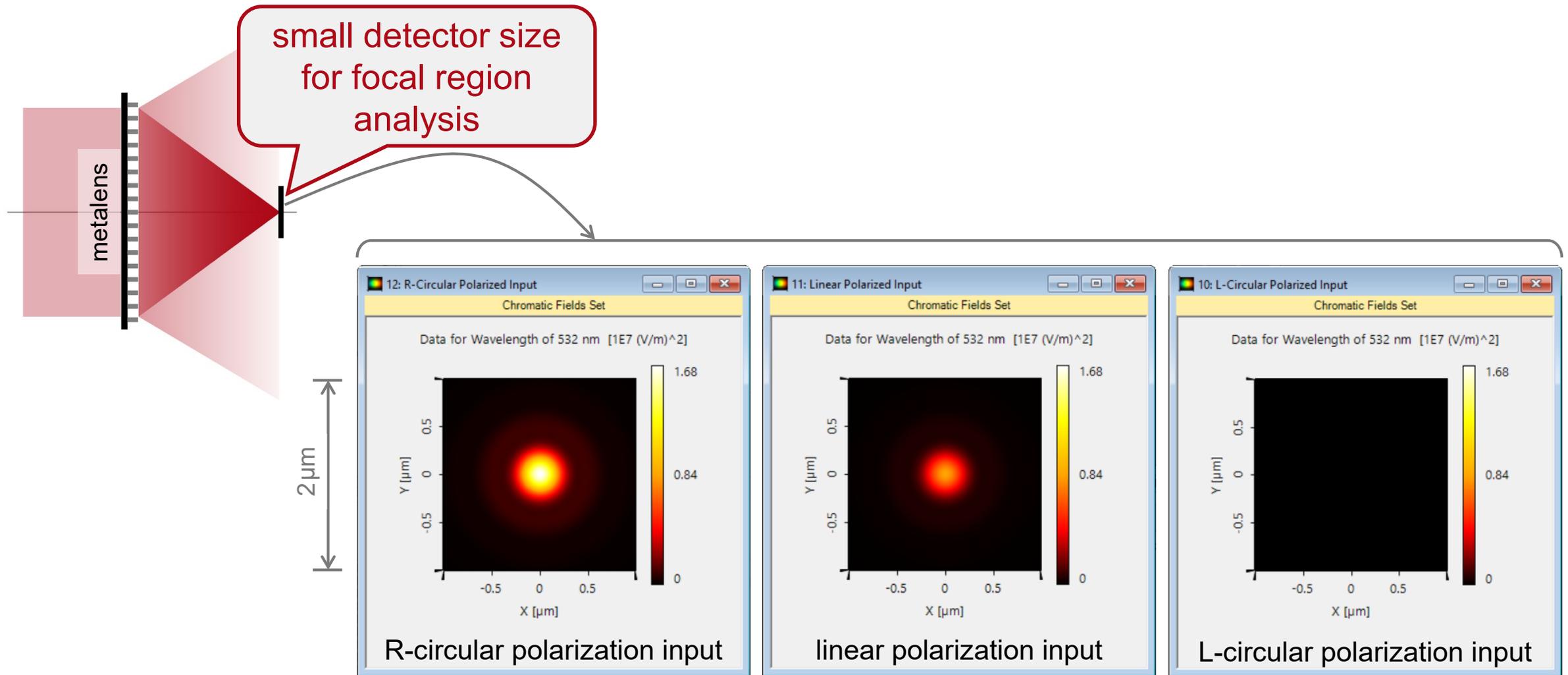


M. Khorasaninejad *et al.*,
Science **352**, 1190-1194 (2016).

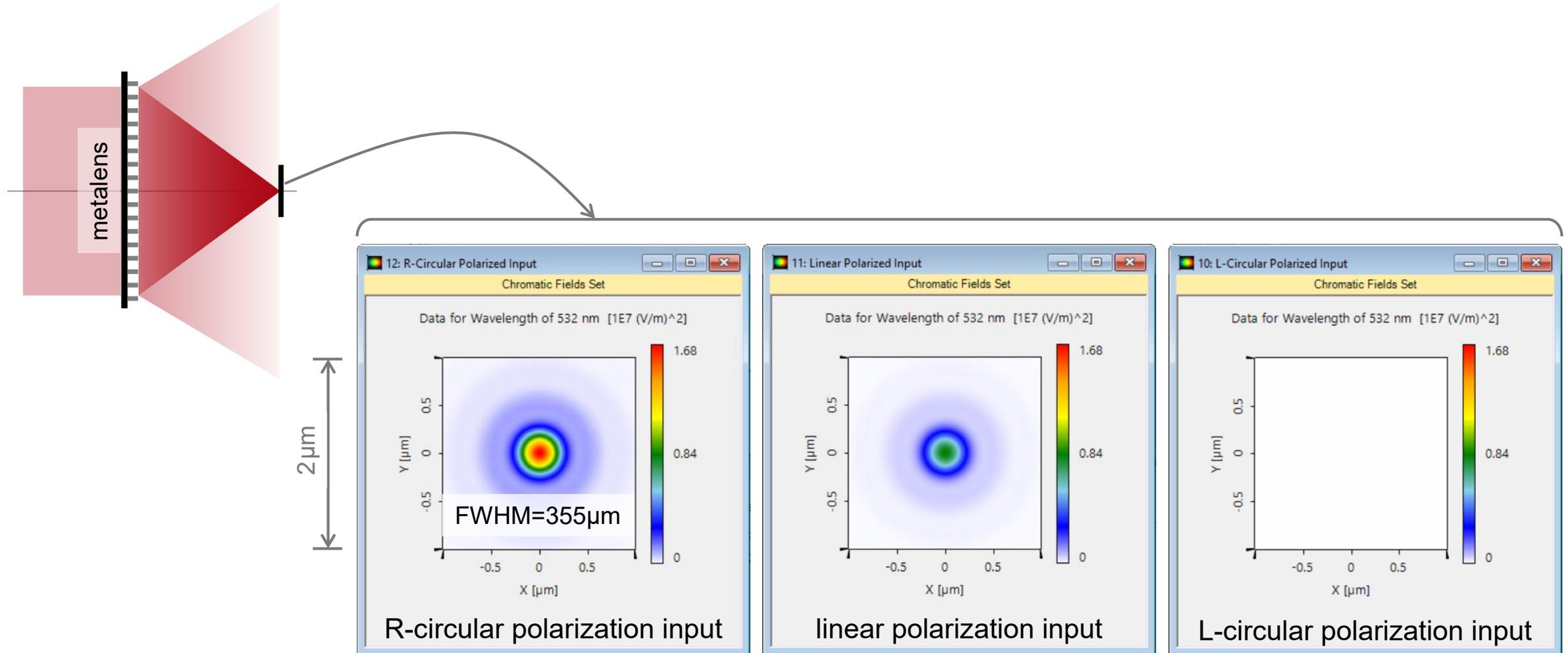
High-NA Metalens Simulation



High-NA Metalens Simulation



High-NA Metalens Simulation

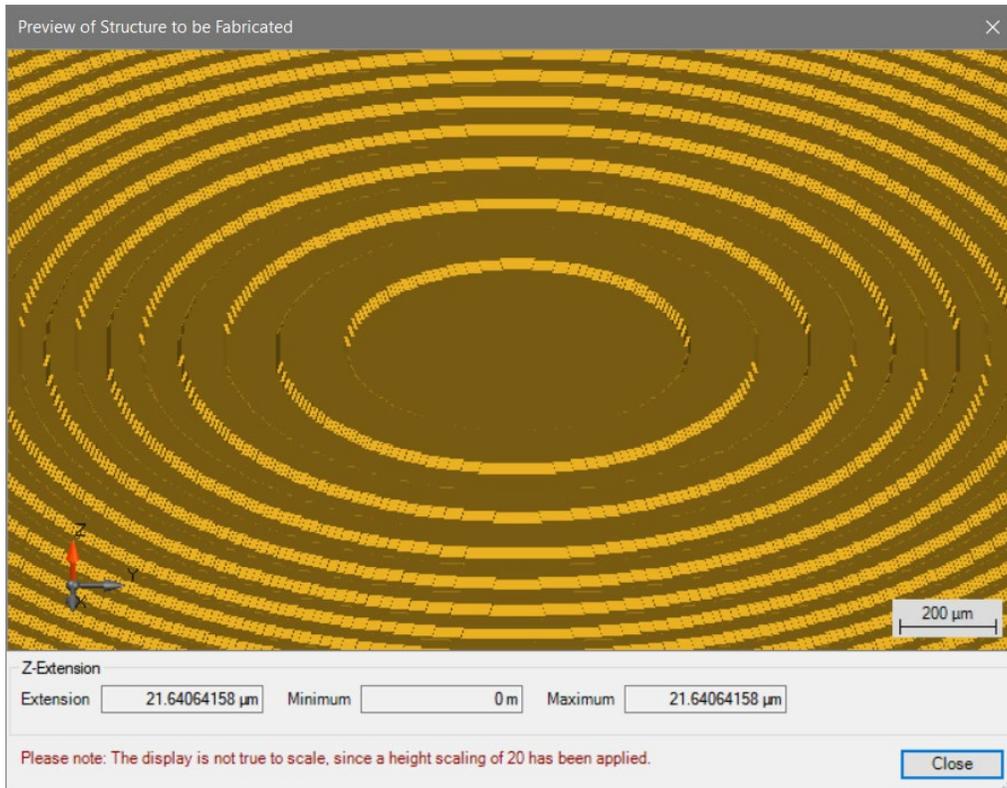


Structure of Workshop

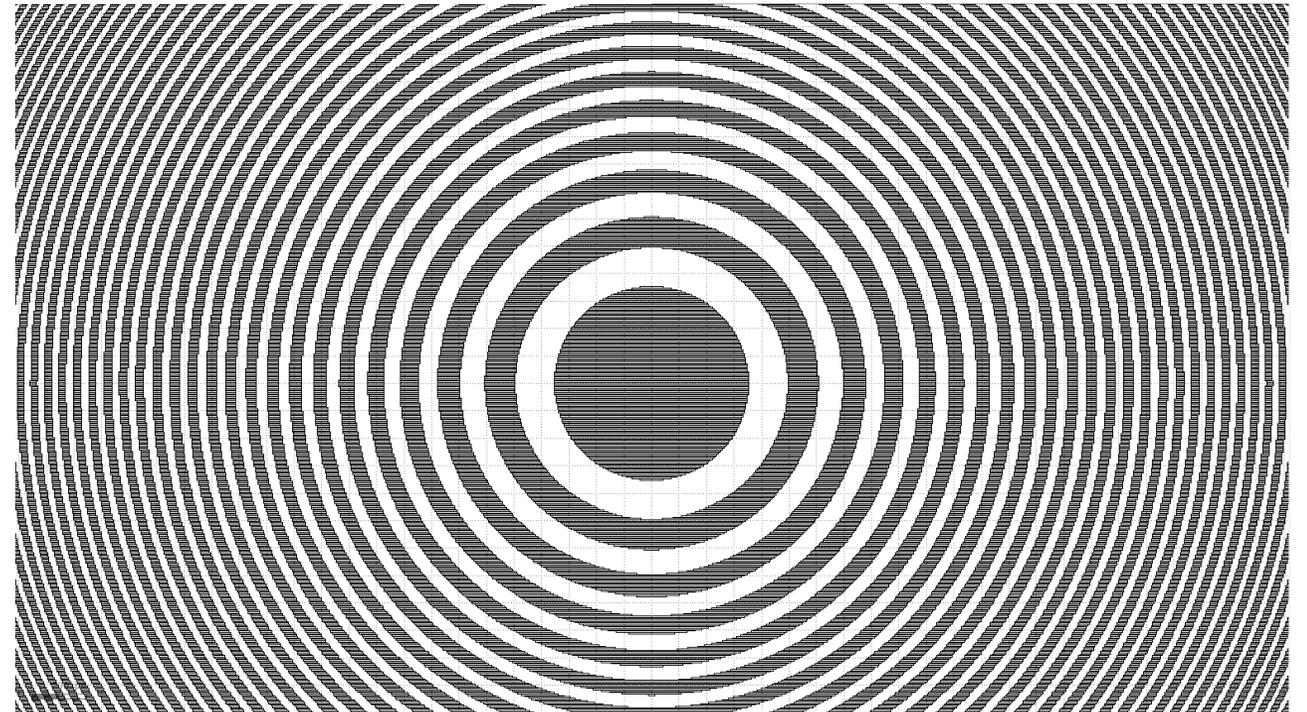
- Introduction of theory
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- Design of binary surfaces in OpticStudio
 - Akil Bhagat
- Physical-optics analysis of imported systems from OpticStudio: Diffractive lenses
 - Roberto Knoth
- Metalenses theory and modeling
 - Site Zhang
- **Fabrication export**
 - **Roberto Knoth**

Fabrication Export: Intraocular Lens (Binary \rightarrow 1 Mask)

Sampled Data Export



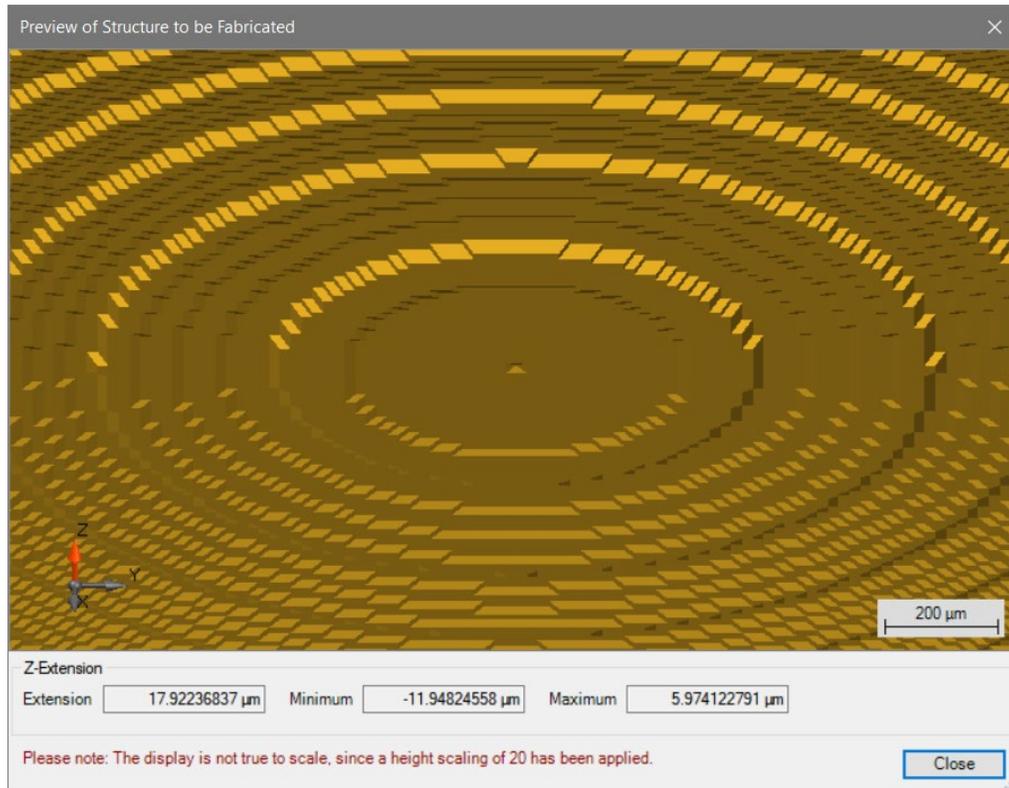
Structure Preview in VirtualLab Fusion



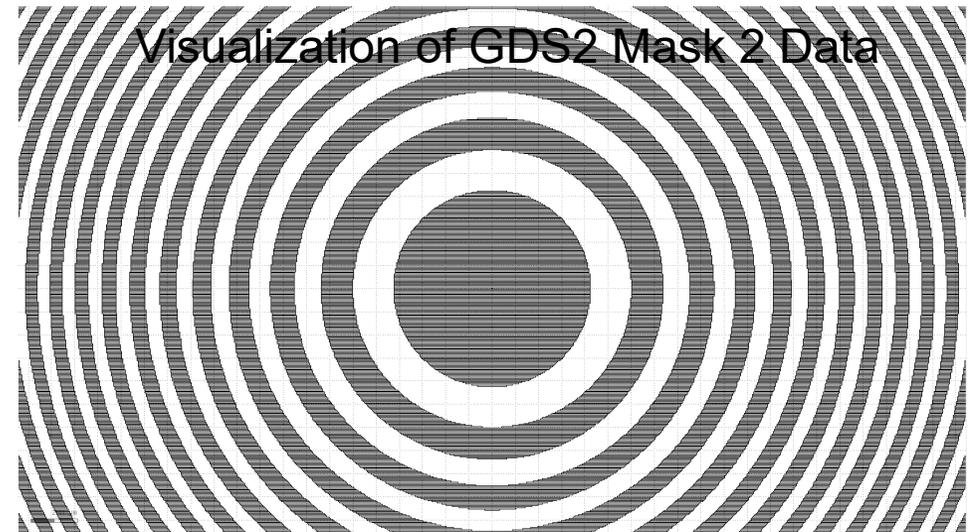
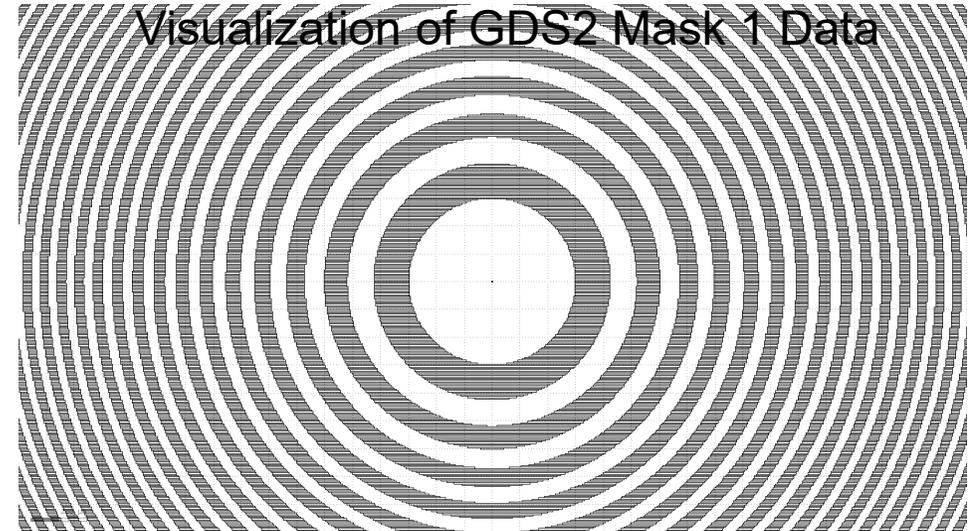
Visualization of GDS2 Mask Data

Fabrication Export: Eyepiece (4 Level → 2 Masks)

Sampled Data Export

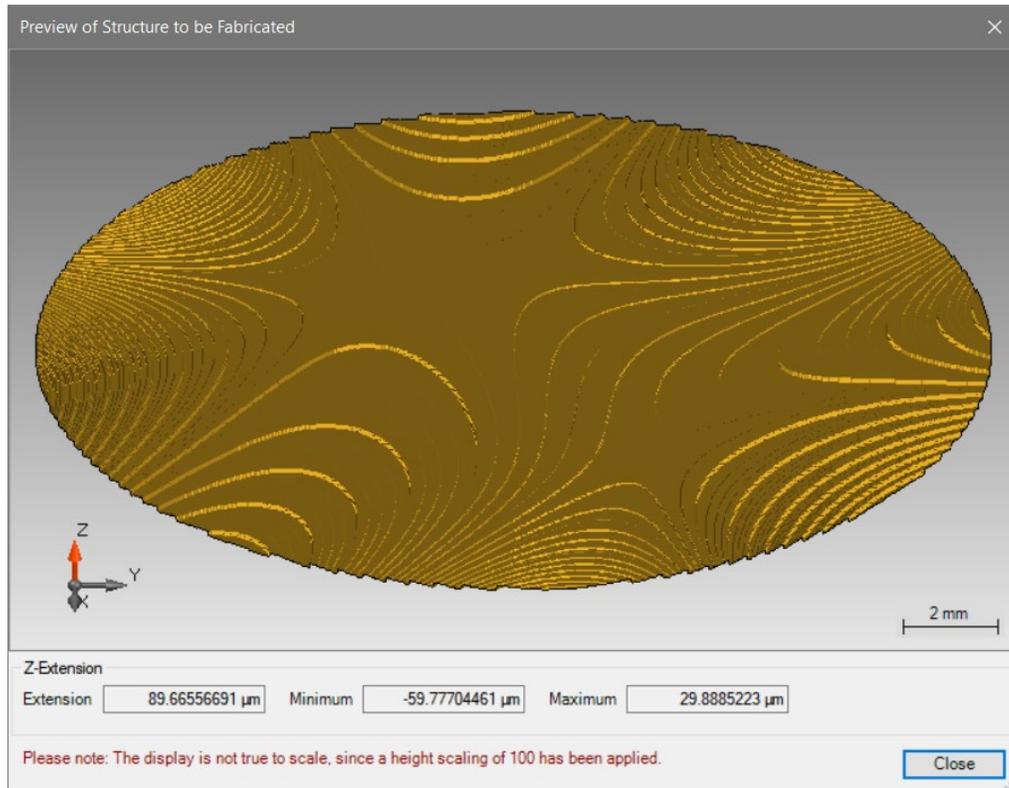


Structure Preview in VirtualLab Fusion

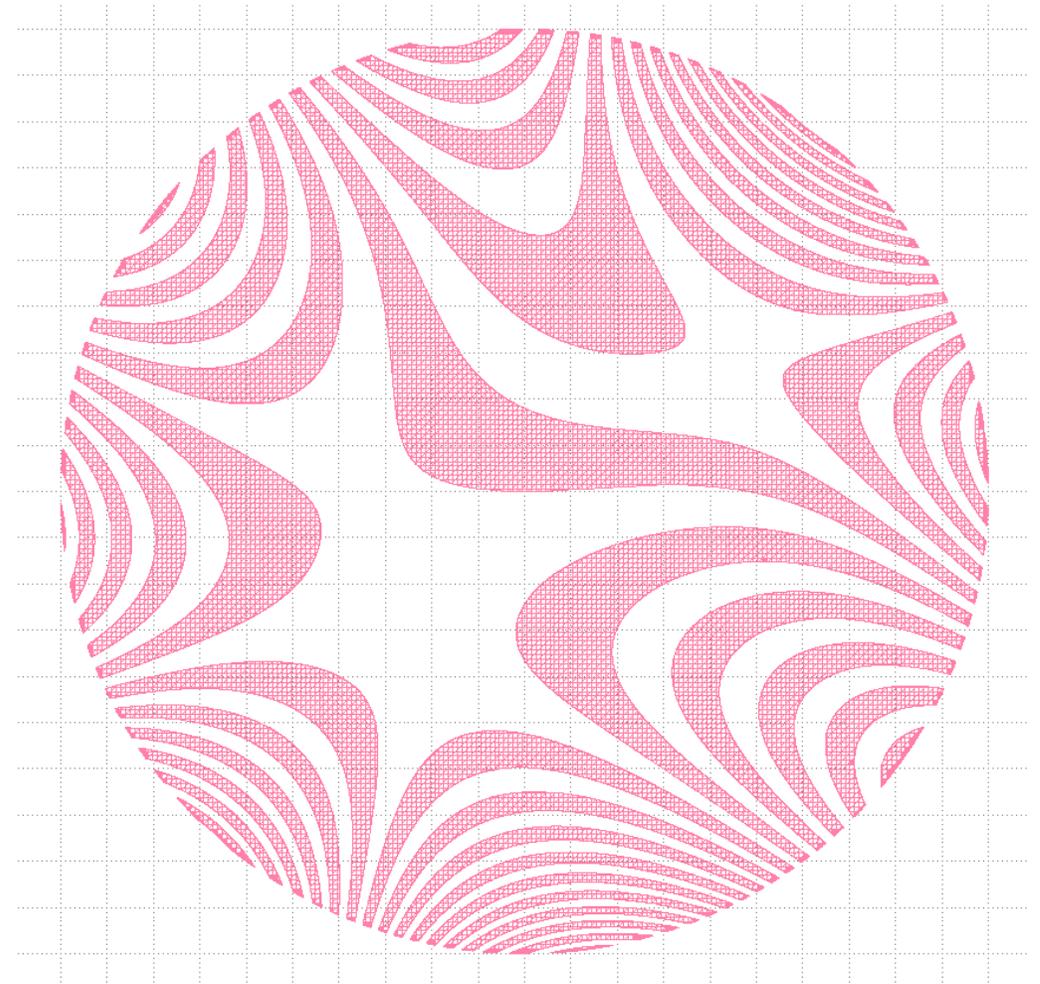


Fabrication Export: HOE (Binary \rightarrow 1 Mask)

Polygon Data Export



Structure Preview in VirtualLab Fusion



Visualization of GDS2 Mask Data