Gauss-Bessel Beam Shaper [Vortex]

Digital Twin Specification

Twin Code: CF-BESV01
Twin Name: Gauss-Bessel Beam Shaper [Vortex]
Category: Component
Type: Function-Based
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Package: Light Shaping
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Description

This twin implements a quantized radial phase mask designed to transform a **collimated Gaussian input beam** into a Bessel-like beam with a ring-shaped intensity distribution, extended depth of focus, and optional orbital angular momentum (vortex phase). In a $2f$ configuration, the mask placed at the front focal plane produces the exact Fourier transform of the shaped field. The ideal Bessel beam (without vortex) requires a linear radial phase $\phi_{\text{ideal}}(\rho) = k_\rho \rho$; here the phase is quantized into Q levels. Lower Q (e.g., binary) yields multiple diffraction orders, while higher Q approaches a clean Bessel ring with a J_0^2 (or J_l^2 for vortex) profile.

An additional vortex phase $\exp(il\phi)$ (where l is the topological charge) can be imposed, imparting orbital angular momentum and creating a dark core even for the continuous phase mask. The output maintains the polarization state of the input.

Model Parameters

Design parameters:

- **Ring separation ratio (M)**: Controls the spacing between rings relative to the diffraction-limited focal spot size (default: 2).
  - $M = 1$: First ring just touches central spot (minimum resolution)
  - $M = 2$: Clear separation between rings (recommended default)
  - $M = 3$: Multiple rings well resolved
  - Warning issued if $M < 1$ (rings not resolved)
- **Topological charge (l)**: Integer vortex charge (default: 1). Sets the azimuthal phase variation $\exp(il\phi)$. Non-zero values produce a phase singularity and a dark core in the focal plane.
- **Quantization levels (Q)**: Number of discrete phase levels (2, 4, 8, 16, 0 for continuous phase). Default is 0 (continuous phase). The beam structure depends strongly on Q:
  - $Q = 0$ (continuous phase): Ideal Bessel phase – a single ring with J_0^2 profile (or J_l^2 for $l \neq 0$).
  - $Q = 8$: Close approximation to the ideal; most energy in the first ring, faint higher orders.
  - $Q = 4$: Moderate approximation; the first ring is dominant but may have slightly broadened shape.

- $Q = 2$: Binary phase mask – produces multiple rings (odd orders) and each ring may exhibit a double-peak structure due to the abrupt phase jumps and the nature of the Hankel transform. No central spot appears because the average transmission is zero.

- **Sampling accuracy (S)**: Multiplicative factor that increases the number of samples per 2π phase period beyond the base value (default: 1.0). The base number of samples per 2π is automatically set to $\max(16, Q)$ to ensure at least one sample per quantization level. The actual number of samples per period is then $S \times \max(16, Q)$. Increase S if you observe numerical artifacts or if the ring structure appears distorted.

- **Export Designed Phase**: When enabled, system simulation pauses at the shaper plane and an export dialogue opens and allows for the export of the designed phase.

Simulation Model

The shaper applies a phase mask consisting of two parts: a quantized radial Bessel phase and an optional vortex phase. Within the paraxial Fourier optics framework, the field in the focal plane is the exact Fourier transform of the shaped field.

Ideal Radial Phase

The ideal continuous phase for a Bessel beam is linear in the radial coordinate:

$$\phi_{\text{ideal}}(\rho) = k_\rho \rho, \tag{1}$$

where $\rho = \sqrt{x^2 + y^2}$ is the radial coordinate in the shaper plane. In practice, the phase is taken modulo 2π because only the value modulo 2π affects the field.

From Fourier optics, the transverse wavenumber k_ρ determines the first ring radius in the focal plane:

$$r_0 = \frac{\lambda f k_\rho}{2\pi} \tag{2}$$

The focal spot size for a collimated Gaussian beam with radius w_0 is:

$$\Delta r_{\text{focus}} = \frac{\lambda f}{\pi w_0} \tag{3}$$

The ring separation ratio $M = r_0/\Delta r_{\text{focus}}$ yields the simple design equation:

$$\boxed{k_\rho = \frac{2M}{w_0}} \tag{4}$$

Quantized Radial Phase

For a given quantization level $Q > 0$, the ideal phase modulo 2π is mapped to the nearest of Q equally spaced levels. The quantized phase is:

$$\Phi_{\text{Bessel}}(\rho) = \frac{2\pi}{Q} \cdot \text{round}\left(\frac{(k_\rho \rho \bmod 2\pi)}{2\pi/Q}\right), \tag{5}$$

where round rounds to the nearest integer. For the continuous case ($Q = 0$), we simply take $\Phi_{\text{Bessel}}(\rho) = k_\rho \rho \bmod 2\pi$.

Vortex Phase

The vortex phase adds an azimuthal dependence:

$$\Phi_{\text{vortex}}(\phi) = l\phi, \tag{6}$$

where l is the topological charge and $\phi = \text{atan2}(y, x)$. The total phase applied by the mask is the sum:

$$\Phi_{\text{total}}(\rho, \phi) = \Phi_{\text{Bessel}}(\rho) + \Phi_{\text{vortex}}(\phi). \tag{7}$$

The complex transmission of the mask is then $t(\rho, \phi) = \exp(i\Phi_{\text{total}})$.

Beam Characteristics for Different Q

The Fourier transform (Hankel transform) of the quantized phase mask yields a pattern that depends on Q and l:

- For $Q = 0$ (continuous), the output is a single ring with intensity proportional to $J_l^2(k_\rho r)$ (where k_ρ relates to the radial frequency). For $l = 0$, this is the ordinary Bessel beam; for $l \neq 0$, it is a Bessel vortex beam with a dark core.
- For $Q = 2$ (binary), the mask acts as a radial square wave. Its Hankel transform produces multiple odd orders (1st, 3rd, 5th, ...) and, due to the sharp phase discontinuities and the finite Gaussian aperture, each order may split into a double-peak structure. This is a physical effect, not a numerical artifact.
- For $Q = 4$, the phase steps are $\pi/2$. The approximation is better; the first order becomes dominant and the double peaks typically disappear, though some asymmetry may remain.
- For $Q \geq 8$, the output closely approximates the ideal Bessel ring, with negligible higher orders and no double peaks.

Key Physical Principles

- **Fourier optics foundation:** With collimated input and shaper at distance f before the lens, the focal plane field is the exact Fourier transform with no quadratic phase distortion—essential for extended depth of focus.
- **Parameter Q:** Determines how faithfully the output approximates an ideal Bessel beam. Higher Q reduces artifacts (multiple orders, double peaks) and concentrates energy into the first ring.
- **Vortex charge l:** Imparts orbital angular momentum, creating a phase singularity and a dark core in the focal plane.

Typical Application Scenarios

1. **Laser processing with ring-shaped beams**: Use the ring-shaped intensity distribution for applications such as drilling or cutting where a ring-shaped focal spot is advantageous, e.g., for trepanning or creating annular heat affected zones. The vortex version can be used for drilling with a dark center.
2. **Optical trapping with ring traps and vortex beams**: The ring-shaped beam can trap particles in the dark core (for $l \neq 0$) or along the ring. Vortex beams are particularly useful for transferring orbital angular momentum to trapped particles.

3. **Microscopy with structured illumination**: Employ the ring beam for structured illumination microscopy techniques, where the extended depth of focus and dark core can enhance resolution or provide background suppression.

4. **Material processing with extended depth of focus**: Leverage the non-diffracting nature of the beam for processing materials with varying thickness or for drilling high aspect ratio holes. The vortex version can create annular heat affected zones.

5. **Beam shaping research**: Investigate the effects of phase quantization on Bessel beam generation, including the emergence of double peaks and multiple orders, and compare with theoretical predictions.

6. **Educational demonstrations**: Demonstrate the concept of Bessel beam generation, orbital angular momentum, and the impact of quantization using a simple phase mask.

Software Usage

This twin is available in the Digital Twin Hub. To achieve the optimal configuration for extended depth of focus, follow these steps:

System Setup

1. **Generate a Gaussian beam:** Place a Gaussian Beam Mode twin (`SF-GAUS01`) in your system.

2. **Collimate the beam (if needed):** If your source is not already collimated, add a collimation element:

   - Use an Ideal Lens [Collimation] twin (`CF-ILCO01`) for perfect aberration-free collimation, OR
   - Use a Spherical Lens twin (`CS-SLEN01`) with appropriate curvature to achieve collimation

   The beam at the shaper plane must have infinite wavefront curvature ($R_{\text{in}} \to \infty$).

3. **Add the beam shaper:** Place the Gauss-Bessel Beam Shaper [Vortex] twin (`CF-BESV01`) at a distance exactly equal to the focal length f before the Fourier lens.

4. **Configure the shaper:** Set the design parameters (M, l, Q, Sampling accuracy S).

5. **Add the Fourier lens:** Place a lens with focal length f in a 2-f configuration:

   - Use an Ideal Lens [2f-Setup] twin (`CF-ILSU02`) for perfect aberration-free Fourier transformation, OR
   - Use a Thin Lens twin (`CF-THLE01`) with focal length f placed at a distance f from the element plane.

6. **Observe the result:** Place field monitors (`DF-FMON01`) in the focal plane and beyond to observe the extended depth of focus of the ring-shaped Bessel-like beam.

Note:

- Input beam must be collimated at the shaper plane.
- Shaper must be positioned at distance f before the Fourier lens.
- For proper operation, the input field must be a Gaussian beam; other input fields produce unspecified outputs.

Exporting the designed phase for fabrication:

- Check the **Export Designed Phase** option in the shaper's dialogue.
- When enabled, system simulation pauses at the shaper plane and an export dialogue opens.
- The dialogue displays the current pixel size $\Delta x = \pi w_0/(MN)$ where $N = S \cdot \max(16, Q)$. The user can specify the desired number of sampling points (i.e., the grid size) for the exported mask. The minimum allowed number of points corresponds to the current N (i.e., the sampling that would be used internally if the simulation continued). Larger grid sizes increase resolution but also file size.
- Users can adjust the number of points based on fabrication constraints; the software automatically resamples the phase data accordingly. However, reducing the number of points below the minimum would degrade the mask and is not permitted.
- After closing the export dialogue, the simulation continues normally.

Note: The Export Designed Phase option must be disabled during parameter runs or parametric optimizations, as it pauses the simulation and opens an export dialogue, which interrupts automated sweeps.

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Related Twins: SF-GBES01, CF-BESP01, CF-PHMV01, SF-DONA01, SF-DONR01